SYSTEM IDENTIFICATION FROM STEP RESPONSE

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ABSTRACT

Identification of systems is very essential in order to be able to provide necessary corrective measures to improve the overall system performance. A simple technique of approximating a given system from its step response is furnished in this paper. The method employed in this technique is based on some of the distinguishing features which could be noted in the step responses.

Key words: Identification, Step Response.

INTRODUCTION

Identification of a system is necessary in order to plan corrective measures for making the overall system perform in a desirable manner. In general, identification of a system involves the determination of the parameters of the approximating model of the system. This may be done by exciting the system with a suitable input and studying the output of the system.

Some of the methods of identification [1–4] for linear systems are: (i) classical methods, (ii) gradient techniques, (iii) stochastic approximation, etc. The transient response method, the sinusoidal response method, correlation techniques and learning models fall under the category of classical methods.

In the transient response technique, the system is excited by a step or impulse input and the output of the system is used [5] to estimate the parameters of the transfer function governing the system. In the frequency response method, a constant amplitude variable frequency input signal is used [6] in order to get the gain and phase characteristics of the system using which the system transfer function is obtained by asymptotic approximation. The correlation technique employs white noise test signal of amplitude small enough not to disturb the system, but applied for time long enough to give sufficient data to identify the system parameters. The learning model technique involves proper adjustment of the parameters of a simulated model depending on the difference in the outputs of this model and the actual system, when both are excited by the same input.

Since a step function can be easily generated and applied to the system, it would be convenient to approximate the system from its step response. A simple technique, based on some of the basic distinguishing features in the step responses, is discussed in this paper.

Step Response

The step response of any system may be broadly described by the following features:

(i) Starting value 'k'

(ii) Starting slope 'm'

(iii) Time to first peak 'T_1'

(iv) Magnitude at first peak $(1 + C_1)$

(v) Times to and Magnitudes at other stationary points.

Many of the possible responses, based on different values of the above, are sketched in Fig. 1 (*a*)–(*d*). A method is suggested in this paper by which it may be possible to build an initial model by observing the step response of the given system. This may be sufficient in most cases. Refinements may however be done on this model wherever necessary.

Approximating Transfer Functions

The transfer functions chosen for approximating the system, knowing its step response, are:

(i) Second order all-pole transfer function $G_2(s)$

(ii) Second order transfer function with a pair of complex poles and a real zero $G_{21}(s)$

(iii) Third order all-pole transfer function $G_3(s)$

(iv) Second order all-pass transfer function $G_{2A}(s)$

(v) Biquadratic transfer function $G_{2B}(s)$

where

$$G_2(s) = \frac{w_n^2}{s^2 + 2\delta w_n s + w_n^2}$$

$$G_{21}(s) = \frac{(s + a)\, w_n^2}{a\,(s^2 + 2\delta w_n s + w_n^2)}$$

$$G_3(s) = \frac{a w_n^2}{(s + a)\,(s^2 + 2\delta w_n s + w_n^2)}$$

$$G_{2A}(s) = \frac{s^2 - 2\delta w_n s + w_n^2}{s^2 + 2\delta w_n s + w_n^2}$$

and

$$G_{2B}(s) = \frac{s^2 + 2\delta_1 w_{n1} s + w_{n1}^2}{s^2 + 2\delta_2 w_{n2} s + w_{n2}^2}$$

Among the types of step responses sketched in Fig. 1 the step response of the type in Fig. 1 (*a*) is very common in the case of many systems. The

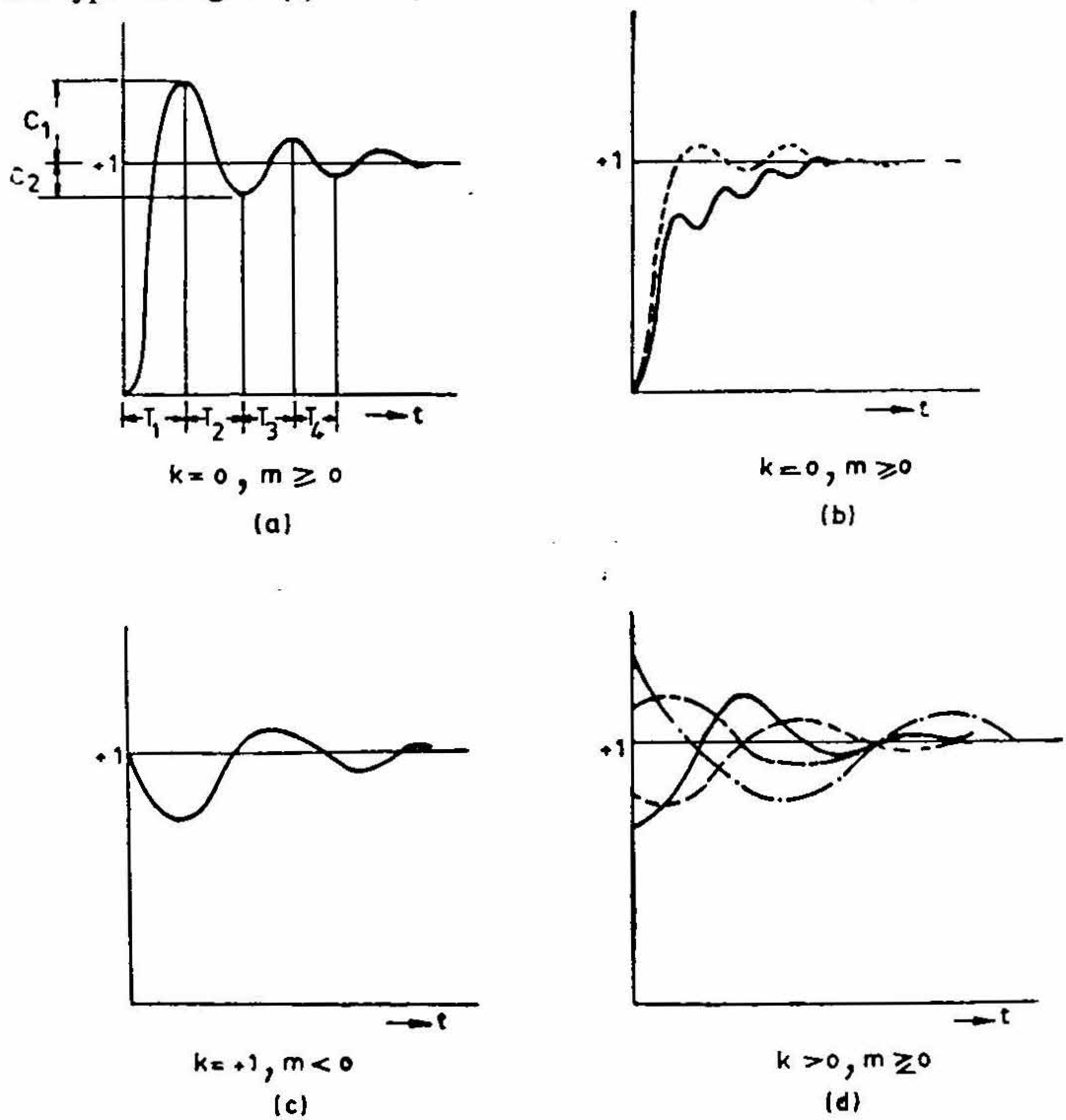


FIG. 1

transfer functions useful for approximation in this case and the methods of evaluating the parameters of these transfer functions are discussed in detail. For other types of responses, the suitable transfer functions are suggested along with a broad outline of getting the parameter values by trial and error in some cases.

The transfer functions $G_2(s)$, $G_{21}(s)$ and $G_3(s)$ may be considered for approximating the type of response sketched in Fig. 1 (*a*). It may be worthwhile noting some of the basic features distinguishing the step responses of these transfer functions as given in table below.

Transfer function	Features
$G_2(s)$	$T_1 = T_2 = T_3 = \cdots = \pi/w_n\sqrt{1-\delta^2}$
$G_{21}(s)$	$T_1 < T_2$
	$T_2 = T_3 = \cdots = \pi/w_n\sqrt{1-\delta^2}$
$G_3(s)$	$T_1 > T_2$
	$T_2 < T_3$
	$T_4 < T_5$

Hence depending on the relationship between T_1 and T_2 in the actual response of the system, one of the above transfer functions may first be chosen for approximation. The parameter values may then be evaluated as under:

IDENTIFICATION

$G_2(s)$.—Measuring C_1 and T_1 from the response, the following expressions may be used to obtain δ and w_n.

$$C_1 = \exp(-\delta/\sqrt{1-\delta^2}),$$

and

$$T_1 = \pi/w_n\sqrt{1-\delta^2}.$$

$G_{21}(s)$.—Three measurements are needed in this case to evaluate the three parameters δ, w_n and α. However, since α would affect the time scale only, this can be evaluated at the end. From the values of C_1 and C_2, it is possible to evaluate δ and $z\,(=w_n/a)$ after many trial and error calculations. These may however be easily evaluated by referring to C_1 and C_2 charts (provided in Fig. 2.1 and Fig. 2.2 respectively). These two plots (on transparent sheets) are superimposed one over the other such that the horizontal lines in the respective plots representing the measured values of C_1 and C_2 coincide. The value of δ may be evaluated as pertaining to those contours in Figs. 2.1 and 2.2 having the same value of δ and which cross on this horizontal line. The abscissa at this cross over point gives the value of z.

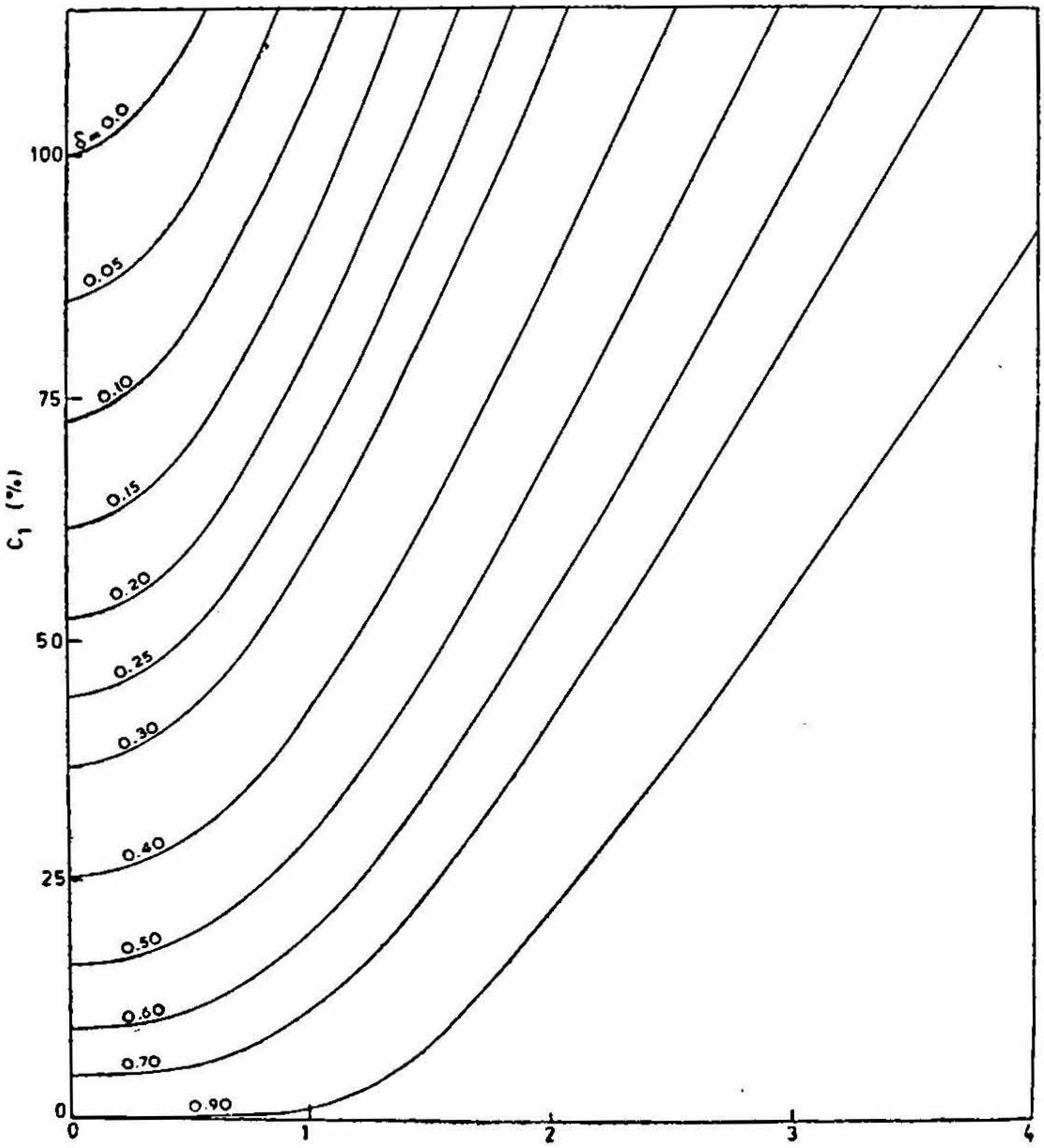


FIG. 2.1

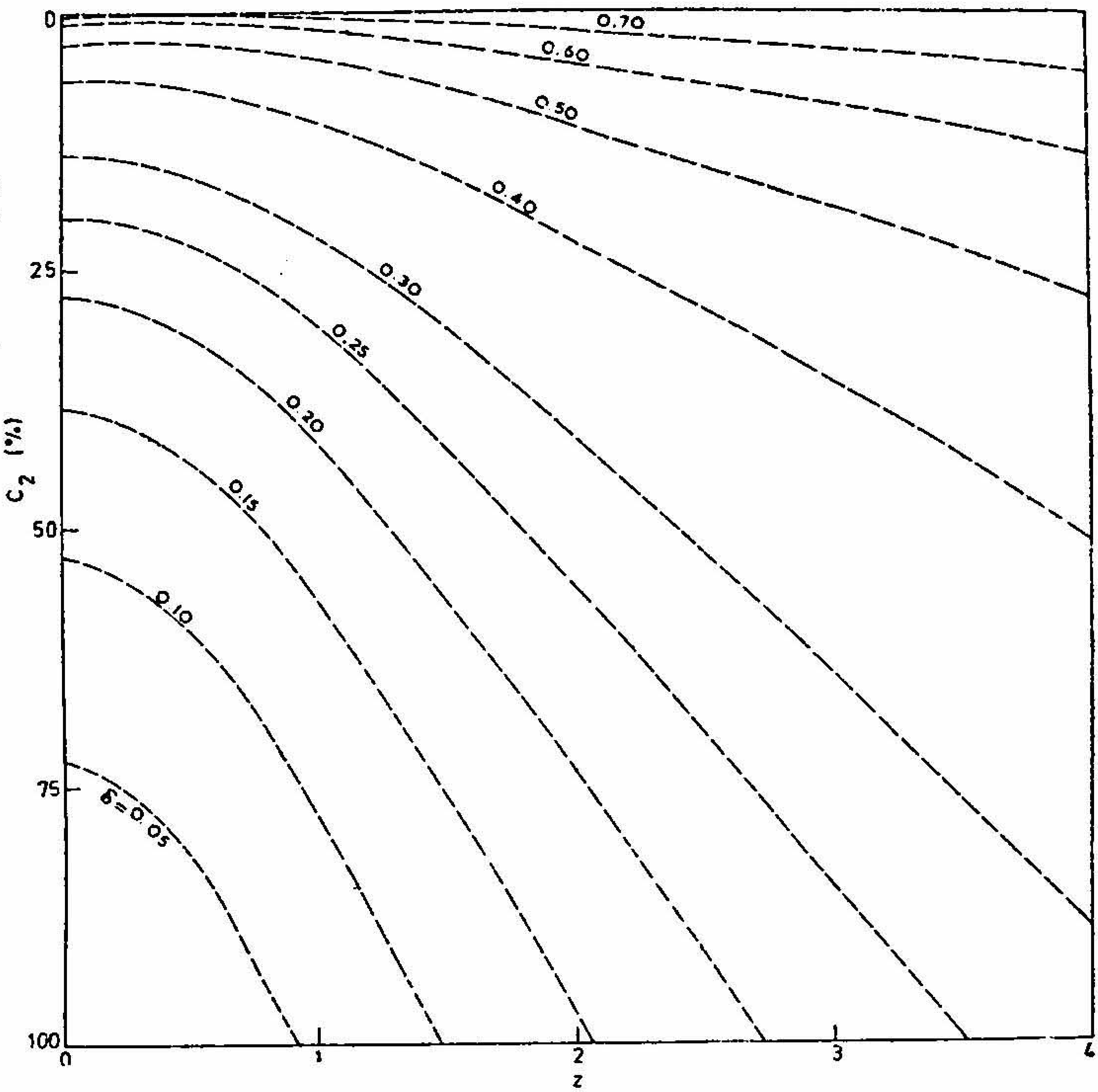


FIG. 2.2

Knowing δ and z and measuring T_2 from the response, a may be obtained using the following expression

$$a = \frac{\pi}{zT_2\sqrt{1-\delta^2}}.$$

$G_3(s)$.—As in the earlier case, three measurements, say C_1, C_2 and T_1, would be required to evaluate δ, w_n and a. Using the measured values of C_1 and C_2, the charts in Figs. 3.1 and 3.2 may be used as in the earlier case to obtain δ and $y\,(=w_n^2/a^2)$. Since there is no simple expression for T_1 in the case of $G_3(s)$, the time aT_1 plot provided in Fig. 3.3 may be used. Knowing δ and y from the earlier step, aT_1 may be directly read from Fig. 3.3. Measuring T_1 from the response, a may be easily evaluated.

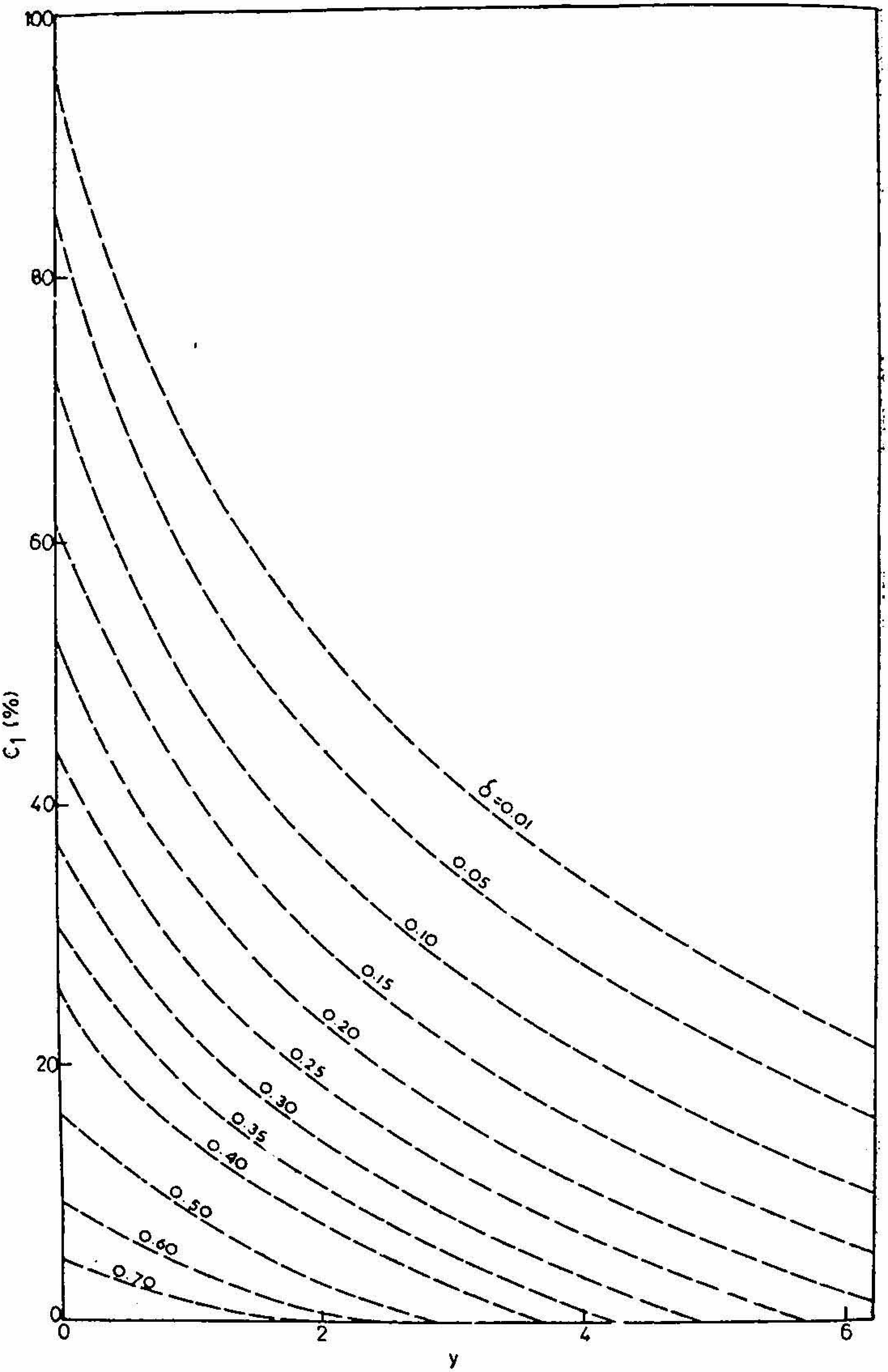

100
80
60
40
20
0
C_1 (%)
0
2
4
6
y
δ=0.01
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.50
0.60
0.70

FIG. 3.1

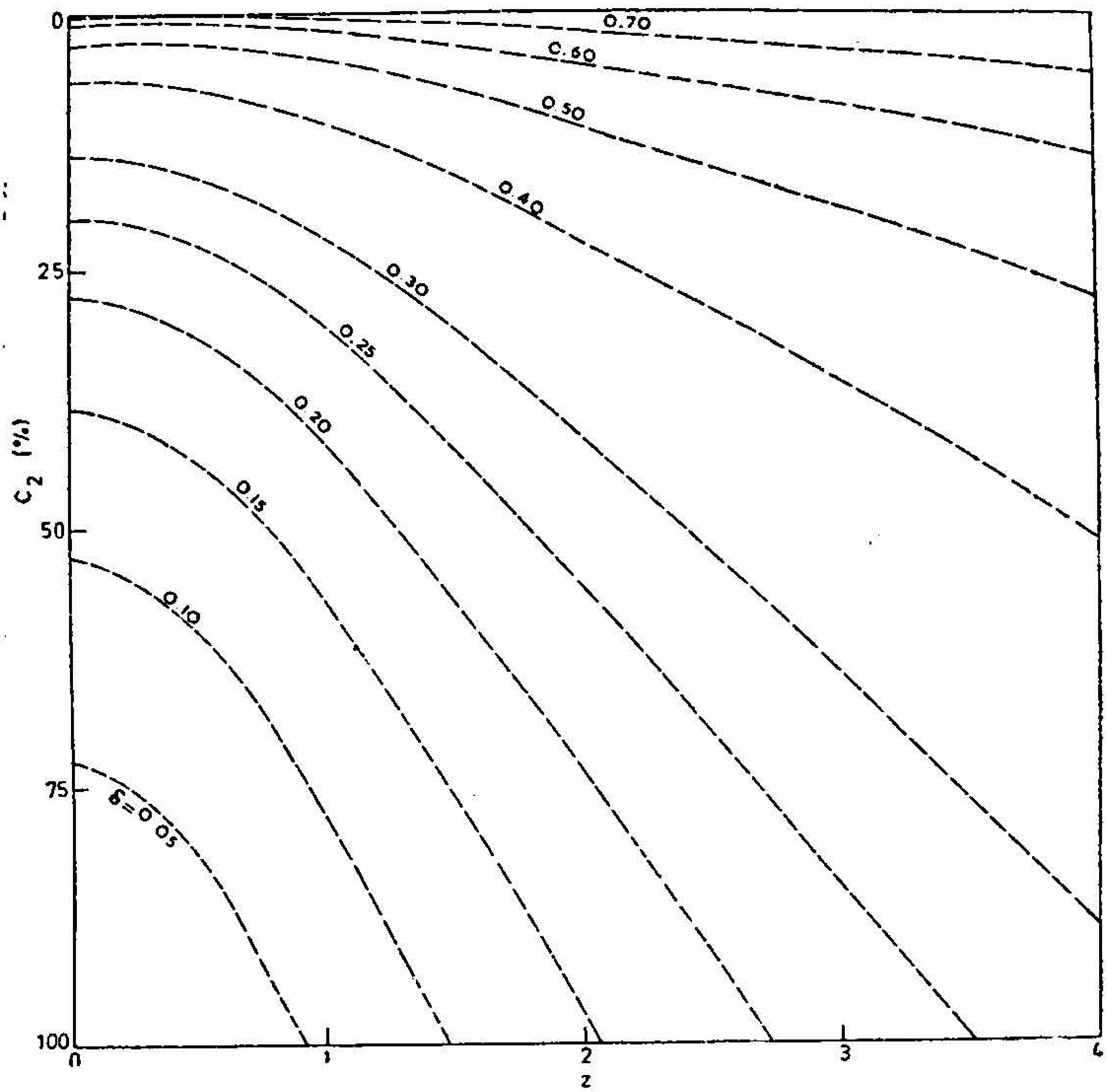
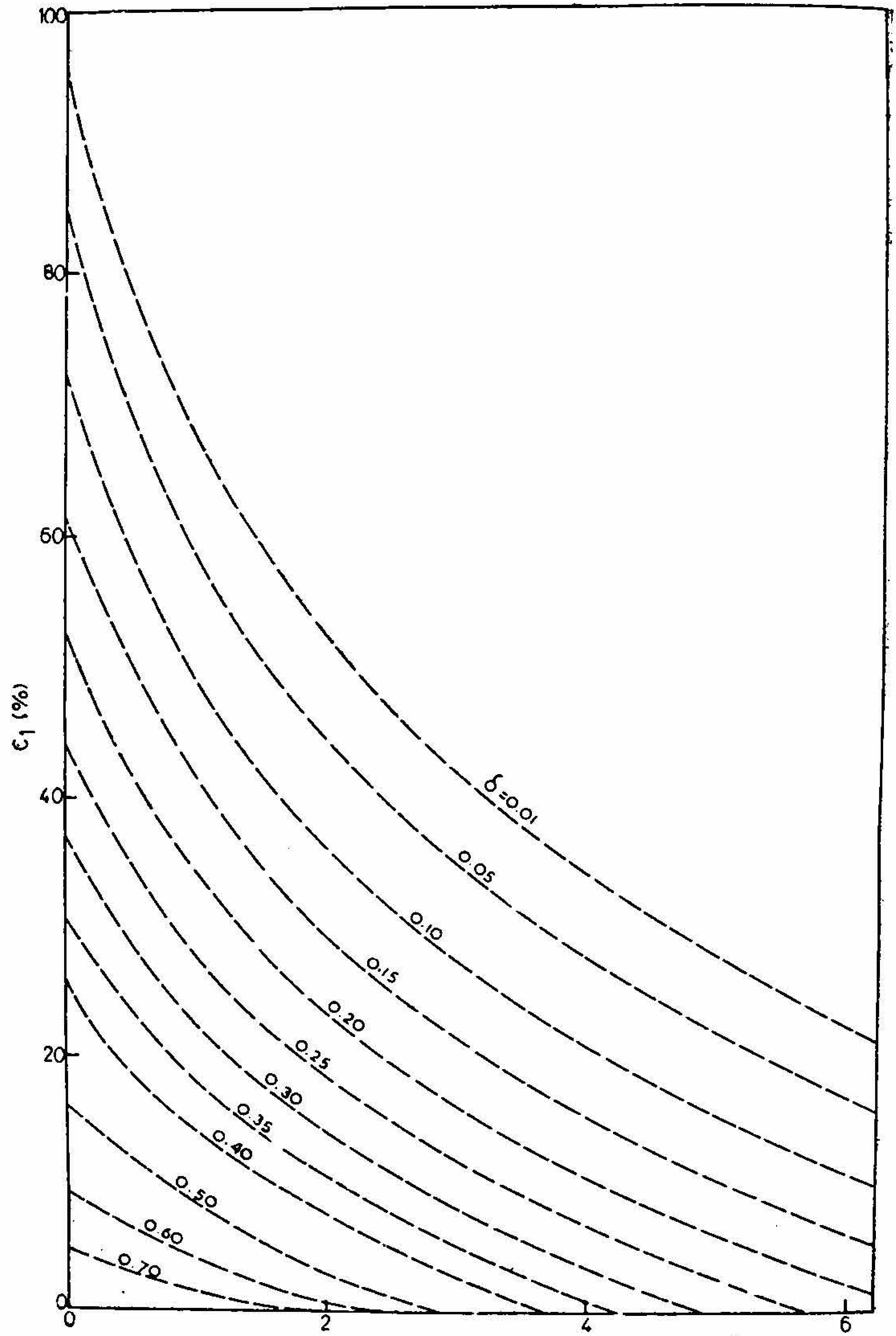


FIG. 2.2

Knowing δ and z and measuring T_2 from the response, a may be obtained using the following expression

$$a = \frac{\pi}{zT_2\sqrt{1-\delta^2}}.$$

$G_3(s)$.—As in the earlier case, three measurements, say C_1, C_2 and T_1, would be required to evaluate δ, w_n and a. Using the measured values of C_1 and C_2, the charts in Figs. 3.1 and 3.2 may be used as in the earlier case to obtain δ and $y\,(=w_n^2/a^2)$. Since there is no simple expression for T_1 in the case of $G_3(s)$, the time aT_1 plot provided in Fig. 3.3 may be used. Knowing δ and y from the earlier step, aT_1 may be directly read from Fig. 3.3. Measuring T_1 from the response, a may be easily evaluated.



100
80
60
40
20
0
ϵ_1 (%)
δ=0.01
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.50
0.60
0.70
0
2
4
6
y

FIG. 3.1

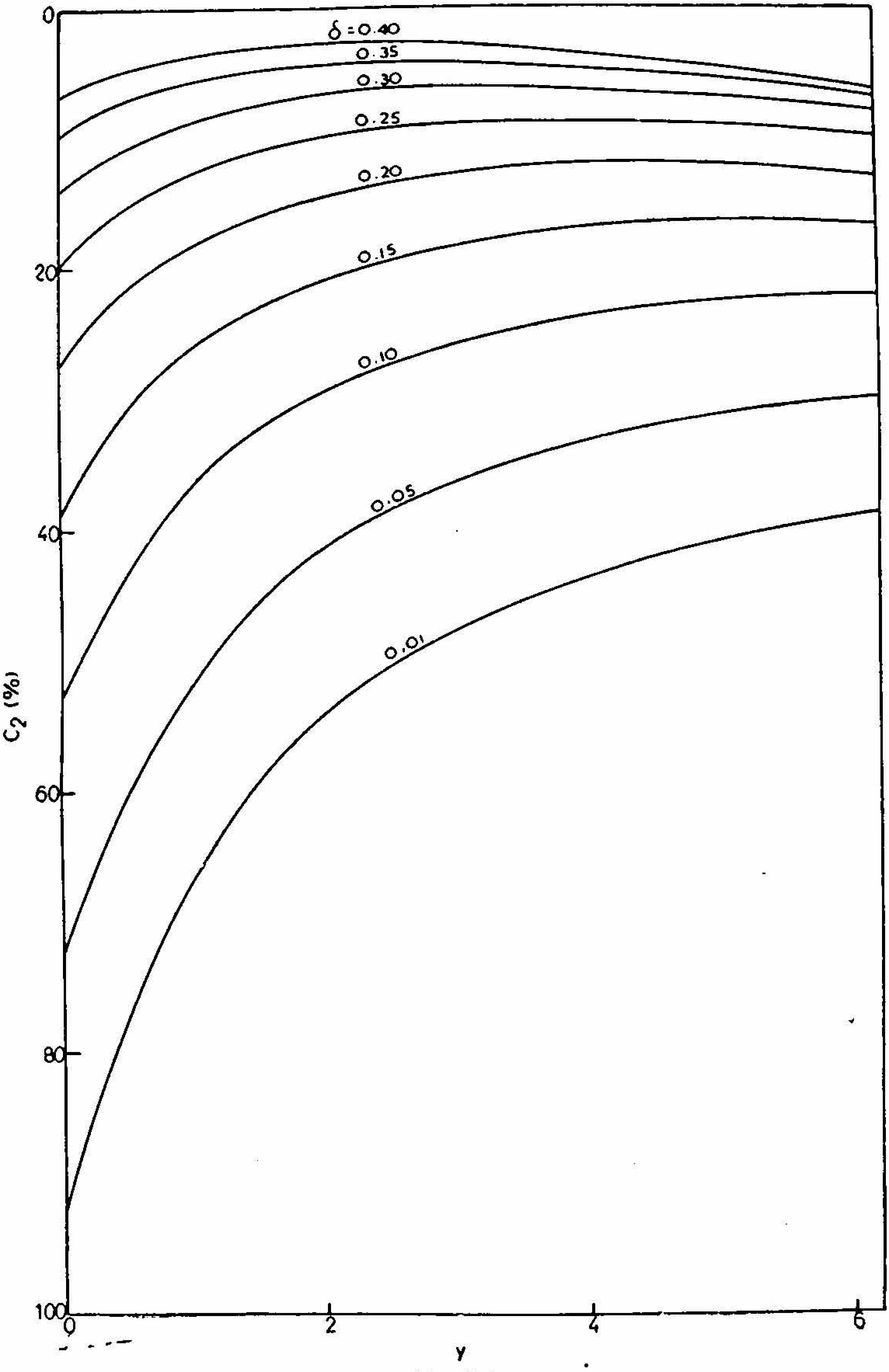


Fig. 3.2

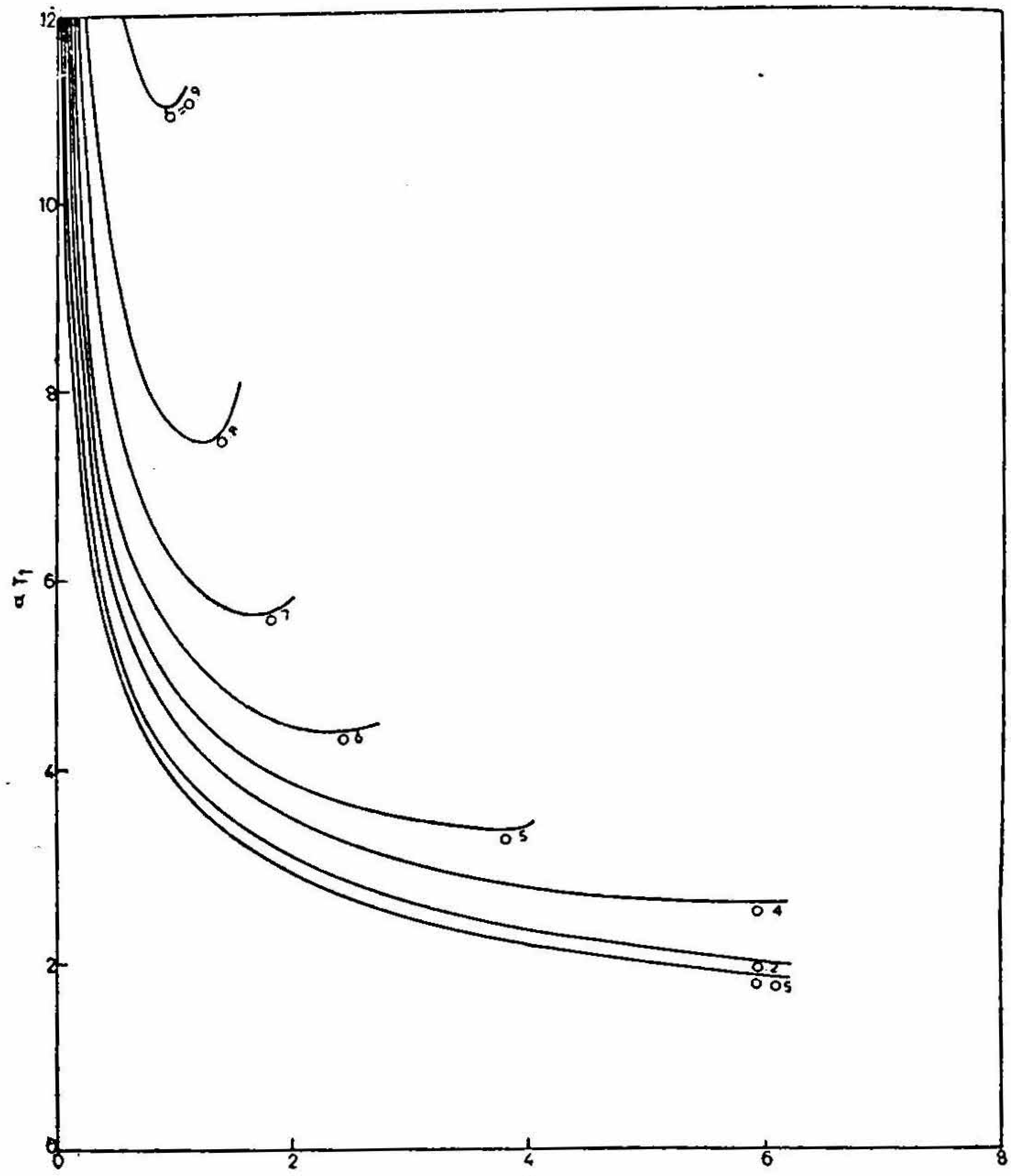


FIG. 3.3

Example.—Consider the step response shown (solid line) in Fig. 4.

The values of C_1, C_2 and T_1 in this response are $C_1 = 0\cdot165$, $C_2 = 0\cdot08$ and $T_1 = 5\cdot5$ seconds. Using Figs. 3.1 and 3.2, as explained earlier, the values of δ and y may be obtained as $\delta = 0\cdot3$, $y = 2\cdot2$. The value of αT_1 as interpolated from Fig. 3.3 for these values of δ and y is $3\cdot2$. Hence the value of α is

$$\alpha = \frac{3\cdot2}{5\cdot5} = 0\cdot582.$$

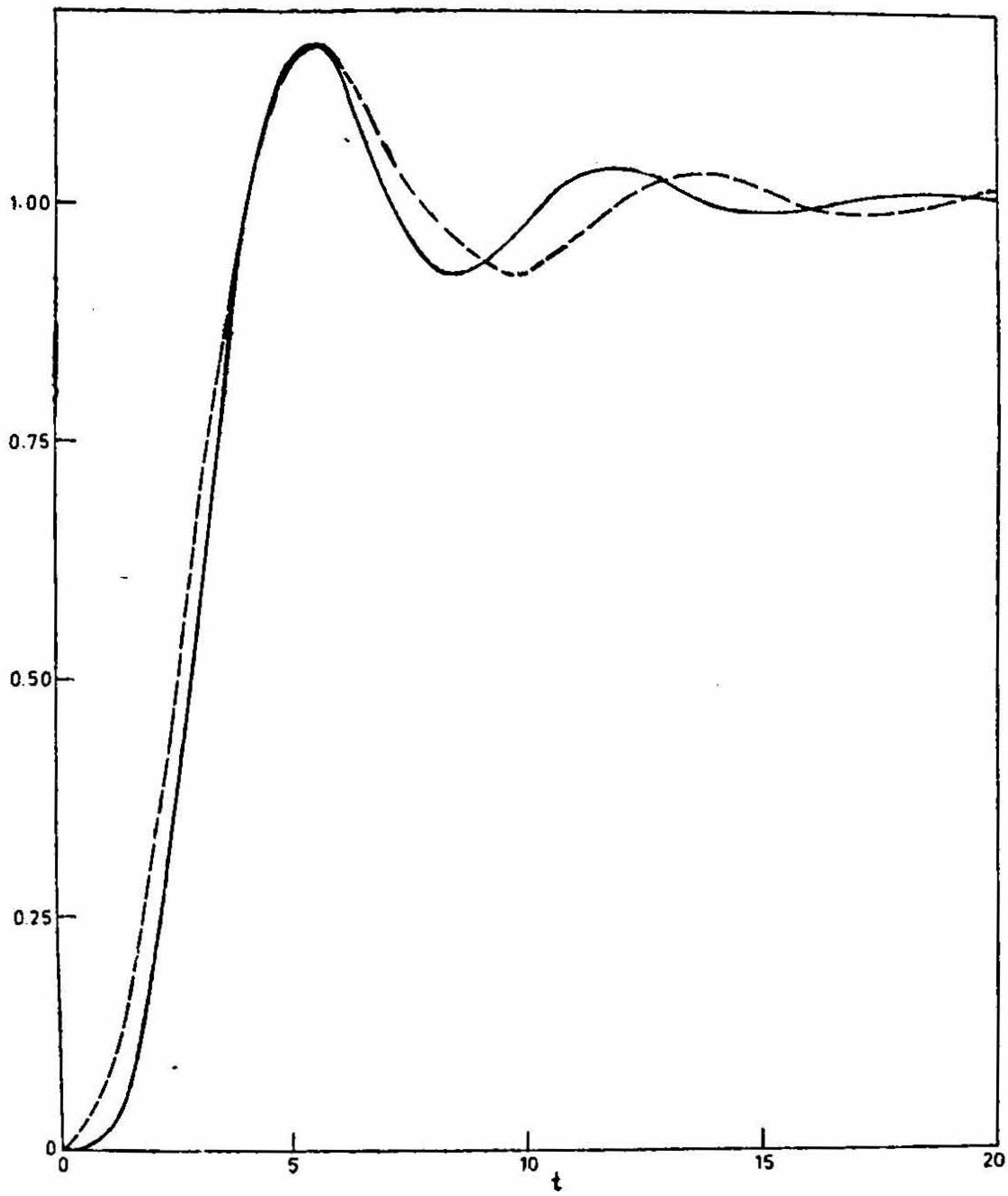


FIG. 4

The approximating transfer function may thus be obtained as

$$F_3(s) = \frac{0 \cdot 4336}{(s + 0 \cdot 582)(s^2 + 0 \cdot 5179s + 0 \cdot 7451)}.$$

The step response of this transfer function is shown (by dotted line) in Fig. 4 for comparison.

If the given response has a very slow rise initially, then this portion of the response can be approximated by a pure time delay; the rest of the response (after shifting the origin by an amount equal to the time delay) being approximated by a suitable transfer function as described earlier. This procedure can be expected to result in a better approximation. In the example considered above (Fig. 4), the approximation could be improved in a similar way if necessary.

OTHER TYPES OF RESPONSES AND IDENTIFICATION

Some systems give rise to step responses of the types sketched in Fig. 1 (*b*), (*c*) or (*d*). A brief procedure suggesting the method to be adopted in these cases is furnished below.

Type 1 (*b*).—In the step response of $G_3(s)$, for values of y beyond a particular y_{12} (for a given δ), the second peak becomes the absolute maximum of the response [7], [8]. Similarly the absolute maximum of the response shifts to the third, fourth peaks and so on as y lies in the range $y_{12} < y < y_{23}$, $y_{23} < y < y_{34}$ and so on. For $y = y_{12}$, y_{23} or y_{34} and so on, the response has two equal peaks (indicated by the suffix numbers). The $(\delta - y)$ plane and the s-plane decomposition based on the types of response along with the maximum overshoot contours [9] in the s-plane may be used to evaluate δ and y by trial and error. Referring to Fig. 3.3 for αT_1 charts and measuring T_1 from the response, α may be evaluated as before.

Type 1 (*c*).—Measure C_1 and T_1 from the response. The following expressions for C_1 and T_1 in the case of $G_{2A}(s)$ may be used

$$C_1 = 4\delta \exp\left[-(\delta \cos^{-1}\delta)/\sqrt{1-\delta^2}\right] \; : \; \delta \text{ is evaluated}$$

$$w_n T_1 = (1/\sqrt{1-\delta^2})(\cos^{-1}\delta) \; : \; w_n \text{ is obtained.}$$

Type 1 (*d*).—Since $G_{2B}(s)$ exhibits similar responses, this may be chosen as the approximating transfer function. From the values of k and m, as measured from the given response, the following may be evaluated

(i) $\beta^2 = (w_{n1}/w_{n2})^2 = 1/k$

(ii) $\delta_1 = (\delta_2 - m\sqrt{1-\delta_2^2})/\beta$.

However, to evaluate δ_2, the following expressions may be used.

$$C_1 = 1 - \frac{e^{-(\delta_2/\sqrt{1-\delta_2^2})\,\tau_m}}{\sqrt{1-\delta_2^2}} \cdot \left[\sin(\tau_m + \phi) - \frac{2}{\beta^2}(\delta_2 - m\sqrt{1-\delta_2^2}) \times \sin\tau_m + \frac{1}{\beta^2}\sin(\tau_m - \phi)\right]$$

where

$$\tau_m = \tan^{-1}\left[\frac{2m(1-\delta_2^2)}{(\beta^2 - 1 + 2\delta_2 m\sqrt{1-\delta_2^2})}\right],$$

and

$$\phi = \cos^{-1}(\delta_2).$$

Considerable effort may be required to solve the above by trial and error to evaluate δ_2. This process may be simplified if C_1 charts similar to that in the earlier case can be prepared. Knowing δ_2, m and β^2, $(w_{n2}T_1)$ may be evaluated. Comparing this with the time in the actual response, w_{n2} may be obtained.

Conclusion

The identification technique suggested above will be very useful for quickly obtaining an approximate model of the system based on which any necessary compensating system may be planned.

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