EFFECTS OF ENVIRONMENT ON THE SURFACE WAVE CHARACTERISTICS OF A DIELECTRIC-COATED CONDUCTOR*—PART I

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1. ABSTRACT

The surface wave characteristics, such as guide wavelength, radial propagation constant (u_2), *etc., of a surface wave* (ϵ_0) *excited dielectric* (ϵ_1) *coated conductor as function of the dielectric constant* (ϵ_2) *of the environment in which the structure is immersed are studied. Results show* (*i*) *that surface wave solution exists as long as* $\epsilon_2 < \epsilon_1$; (*ii*) *existence of maximum of* u_2 *vs.* ϵ_2 *curves for a particular value of* ϵ_1 *and shift of the maximum as* ϵ_1 *is varied; and* (*iii*) *longitudinal power flow in the environmental medium decreases with increasing* ϵ_2 *and the rate of decrease depends upon the value of* ϵ_1. *It is concluded that the dielectric constant of the environmental medium has significant influence on the characteristics of a surface wave structure.*

Key words: Surface wave characteristics. Dielectric coated aerials.

2. INTRODUCTION

Several authors [1-19] have studied surface wave characteristics of electromagnetic surface wave structures immersed in air. But there seems to be no information in available published literature on the surface wave characteristics of electromagnetic structures surrounded by a medium other than air. In view of the importance of subsurface communication it has been considered worthwhile to study the effects of dielectric constant, loss tangent and other properties of the environmental medium on the surface wave properties of *e.m.* structures, beginning with the dielectric-coated conductor. The present report is concerned with the study of variation of the surface wave characteristics of a dielectric-coated conductor immersed in an infinitely extended lossless medium, as its dielectric constant is varied. The effects of a lossy medium will be reported later. Further work when ϵ_2 is a tensor is also under progress. It is intended to correlate the results

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of these investigations with the propagation characteristics of surface wave structures immersed in natural environments such as jungle, snow or inside the earth.

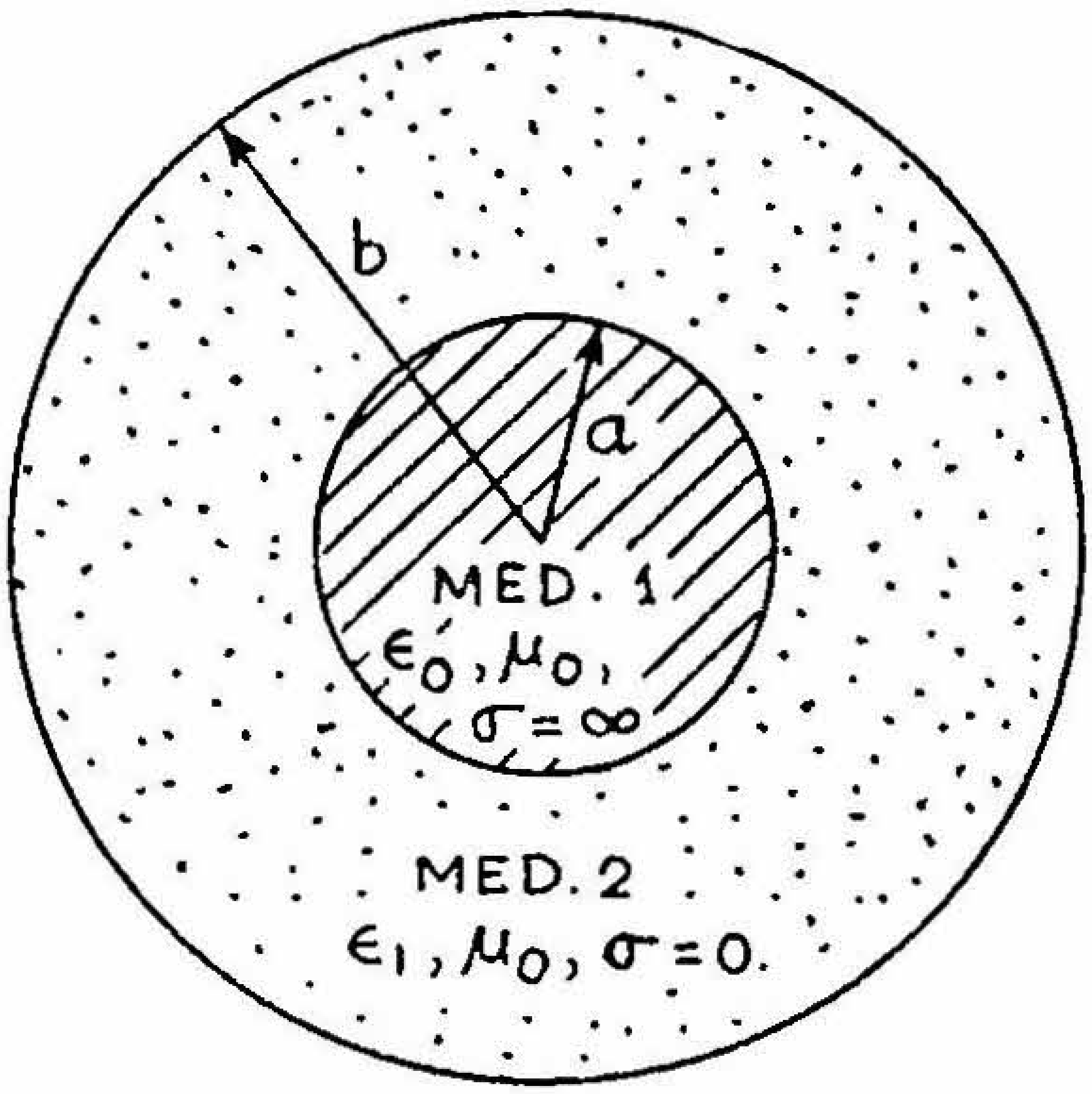


FIG. 1. Dielectric coated conductor surrounded by a medium of dielectric constant ϵ_2

3. Field Components

The field components for a conductor ($\sigma = \infty$) coated with a lossless dielectric (ϵ_1, μ_0, $\sigma = 0$) immersed in a medium (ϵ_2, μ_0, $\sigma = 0$) and excited in E_4 mode are (Fig. 1) in different media:

Medium 1: $0 \leq \rho \leq a$

$$Ez_1 = AJ_0(u_0\rho)\, e^{j\omega t - \gamma z}$$

$$E\rho_1 = \frac{\gamma}{u_0} AJ_1(u_0\rho)\, e^{j\omega t - \gamma z}$$

$$H\phi_1 = j\,\frac{-k_0^2}{\omega\mu_0 u_0} AJ_1(u_0\rho)\, e^{j\omega t - \gamma z} \tag{1}$$

Medium 2: $a \leq \rho \leq b$

$$Ez_2 = [BJ_0(u_1\rho) + CY_0(u_1\rho)]\, e^{j\omega t - \gamma x}$$

$$E\rho_2 = \frac{\gamma}{u_1}[BJ_1(u_1\rho) + CY_1(u_1\rho)]\, e^{j\omega t - \gamma z}$$

$$H\phi_2 = \frac{jk_1^2}{\omega\mu_0 u_1} [BJ_1(u_1\rho) + CY_1(\nu_1\rho)] e^{j\omega t-\gamma z} \tag{2}$$

Medium 3: $\rho \geqslant b$

$$Ez_3 = DH_0^{(1)}(ju_2\rho) e^{j\omega t-\gamma z}$$

$$E\rho_3 = \frac{-j\gamma}{u_2} DH_1^{(1)}(ju_2\rho) e^{j\omega t-\gamma z}$$

$$H\phi_3 = \frac{k_2^2}{\omega\mu_A u_2} \cdot DH_1^{(1)}(ju_2\rho) e^{j\omega t-\gamma z} \tag{3}$$

where, the radial propagation constants (u) in media 1, 2 and 3 respectively are related to the axial propagation constant γ as follows:

$$u_0^2 = \gamma^2 + k_0^2$$
$$u_1^3 = \gamma^2 + k_1^2$$
$$u_1^2 = -(\gamma^2 + k_2^2) \tag{4}$$

where

$$k_0^2 = j\omega\mu_0\sigma$$
$$k_1^2 = \omega^2 \mu_0 \epsilon_1$$
$$k_2^2 = \omega^2 \mu_0 \epsilon_2$$

and the axial propagation constant is

$$\gamma = a + j\beta \simeq j\beta.$$

4. EXCITATION CONSTANTS

Using appropriate boundary conditions and field components in different media (2 and 3) the following relations between the excitation constants are obtained:

$$C = B\frac{J_0(u_1a)}{Y_0(u_1a)} \tag{5}$$

$$B = D\frac{u_1}{u_2} \cdot \frac{k_2^2}{jk_1^2} \cdot \frac{H_1^{(1)}(ju_2b)\, Y_0(u_1a)}{[J_1(u_1b)\, Y_0(u_1a) - J_0(u_1a)\, Y_1(u_1b)]} \cdot \tag{6}$$

Hence,

$$BB^* = DD^* \left[\frac{2u_1\epsilon_2}{\pi u_2\epsilon_1} \cdot \frac{Y_0(u_1a)\, K_1(u_2b)}{\{J_1(u_1b)\, Y_0(u_1a) - J_0(u_1a)\, Y_1(u_1b)\}}\right]^2 \cdot \tag{6 a}$$

5. Characteristic Equation

The transverse impedances at $\rho = b$ being equal, *i.e.*,

$$\left.\frac{Ez_2}{H\phi_2}\right|_b = \left.\frac{Ez_3}{H\phi_3}\right|_b$$

the characteristic equation is

$$\frac{\epsilon_1 u_2}{\epsilon_2 u_1} \cdot \frac{K_0(u_2 b)}{K_1(u_2 b)} + \left[\frac{J_0(u_1 b)\, Y_0(u_1 a) - J_0(u_1 a)\, Y_0(u_1 b)}{J_1(u_1 b)\, Y_0(u_1 a) - J_0(u_1 a)\, Y_1(u_1 b)}\right] = 0 \qquad (7)$$

which is obtained by writing $H_0^{(1)}(ju_2 b)$ and $H_1^{(1)}(ju_2 b)$ in terms of $K_v(u_2 b)$ and $K_1(u_2 b)$ respectively.

By using the following relation

$$u_1^2 = \omega^2 \mu_0 (\epsilon_1 - \epsilon_2) - u_2^2 \qquad (8)$$

which is obtained from eq. (4), and solving the characteristic equation (7) the radial propagation constants u_1 and u_2 are determined.

The guide wavelength $\lambda_g = 2\pi/\beta$ and the phase velocity $v_p = \omega/\beta$ are determined from the following relation between the axial and radial propagation constants when the media are assumed to be lossless.

$$\gamma = j\beta = j(u_2^2 + k_2^2)^{1/2}. \qquad (9)$$

6. Power Flow in Media 2 and 3

The longitudinal power flow in media 2 (Pz_2) and 3 (Pz_3) is calculated by using the relations

$$Pz_2 = \tfrac{1}{2} Re \int_{\phi=0}^{2\pi} \int_{\rho=a}^{b} E\rho_2 H\phi_2^* \, \rho d\rho d\phi \qquad (10)$$

$$Pz_3 = \tfrac{1}{2} Re \int_{\phi=0}^{2\pi} \int_{\rho=b}^{\infty} E\rho_3 H\phi_3^* \, \rho d\rho d\phi \qquad (11)$$

and using the functional relations (equations 5, 6) for the excitation constants.

$$Pz_2 = \frac{\pi\beta\omega\epsilon_1}{2u_1^2} \cdot \frac{BB^*}{Y_0^2(u_1 a)} [G(b) - G(a)] \qquad (10\,a)$$

where

$$G(r) = [J_1(u_1 r)\, Y_0(u_1 a) - J_0(u_1 a)\, Y_1(u_1 r)]^2 + [J_0(u_1 r)\, Y_0(u_1 a) - J_0(u_1 a)\, Y_0(u_1 r)]^2$$

$$- \frac{2}{u_1 r} [J_1(u_1 r)\, Y_0(u_1 a) - J_0(u_1 a)\, Y_1(u_1 r)]$$

$$\times [J_0(u_1 r)\, Y_1(u_1 a) - J_0(u_1 a)\, Y_0(u_1 r)] \tag{10 b}$$

where, $r = (a, b)$

$$Pz_3 = \frac{2\beta\omega\epsilon_2}{\pi u_2^2} DD^* b^2 \left[K_0^2(u_2 b) - K_1^2(u_2 b) + \frac{2}{u_2 b} K_0(u_2 b) K_1(u_2 b) \right]. \tag{11 a}$$

In deriving the expressions for Pz_2 and Pz_3, the following relations have been used appropriately.

$$J_0'(u_1\rho) = J_0(u_1\rho) - \frac{1}{u_1\rho} J_1(u_1\rho)$$

$$Y_1'(u_1\rho) = Y_0(u_1\rho) - \frac{1}{u_1\rho} Y_1(u_1\rho)$$

$$H_1^{(1)\prime}(ju_2\rho) = H_0^{(1)}(ju_2\rho) - \frac{1}{ju_2\rho} H_1^{(1)}(ju_2\rho)$$

$$H_0^{(1)}(ju_2 b) = -\frac{2}{\pi} jK_0(u_2 b)$$

$$H_1^{(1)}(ju_2 b) = -\frac{2}{\pi} K_1(u_2 b).$$

The total power flow in the longitudinal direction (z) is

$$P_T = Pz_2 + Pz_3.$$

Hence, the percentage of powerflow in media 2 and 3 are respectively.

$$P_2\% = \frac{Pz_2}{P_T} \times 100$$

$$P_3\% = \frac{Pz_3}{P_T} \times 100. \tag{12}$$

7. Constant Percentage Power Contour

The radius ρ_p of the constant percentage power (p) contour is determined from the relation

$$p = \left[1 - \frac{F(\rho_p)}{F(\rho_b)} \right] \tag{13}$$

where

$$F(\rho_p) = \rho_p^2 \left[K_0^2 (u_2\rho_p) - K_1^2 (u_2\rho_p) + \frac{2}{u_1\rho_p} K_0 (u_2\rho_p) K_1 (u_2\rho_p) \right]$$

$$F(\rho_b) = \rho_b^2 \left[K_0^2 (u_2 b) - K_1^2 (u_2 b) + \frac{2}{u_1 b} K_0 (u_2 b) K_1 (u_2 b) \right]. \quad (13\ a)$$

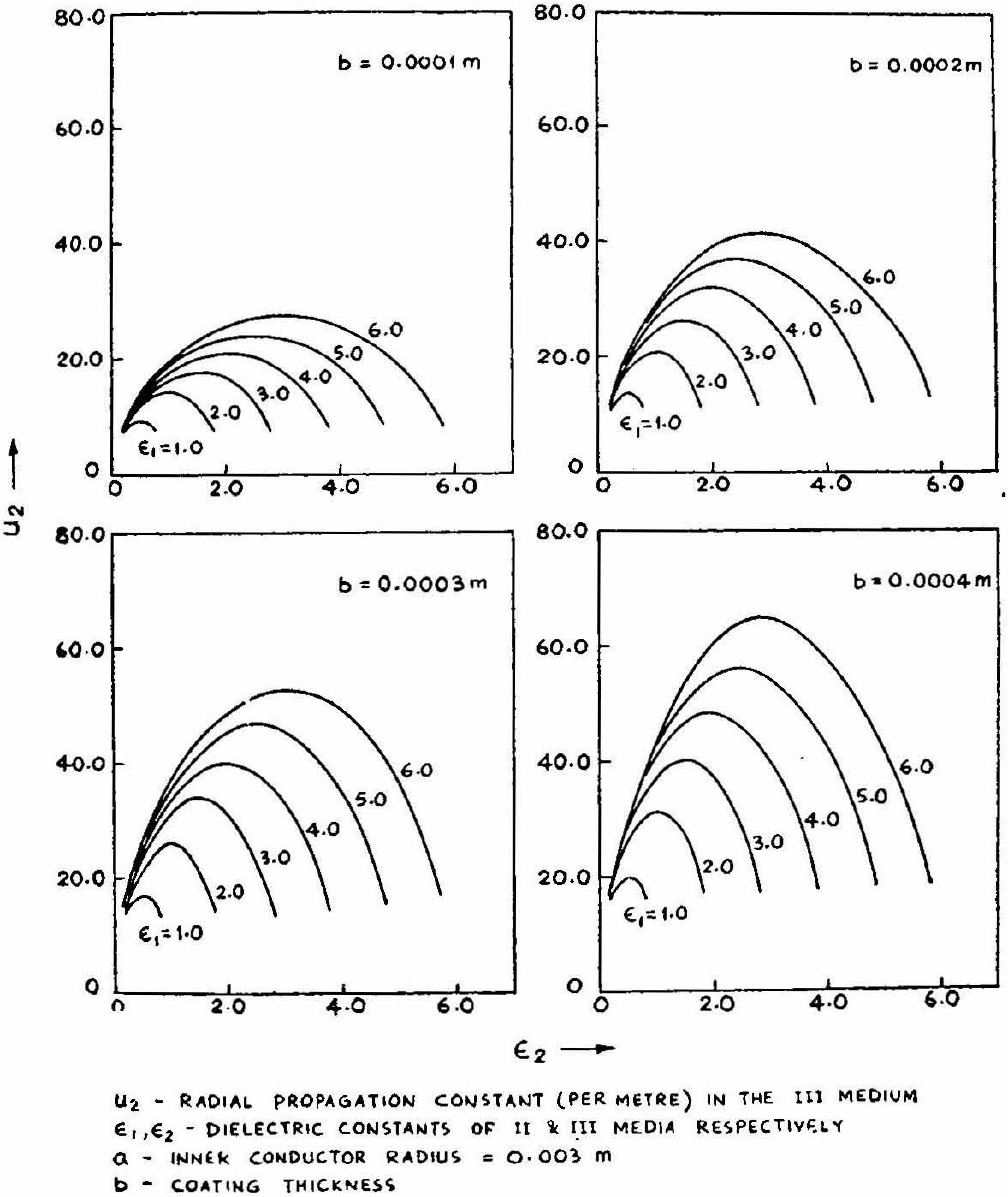


FIG. 2. Variation of U_2 with ϵ_2

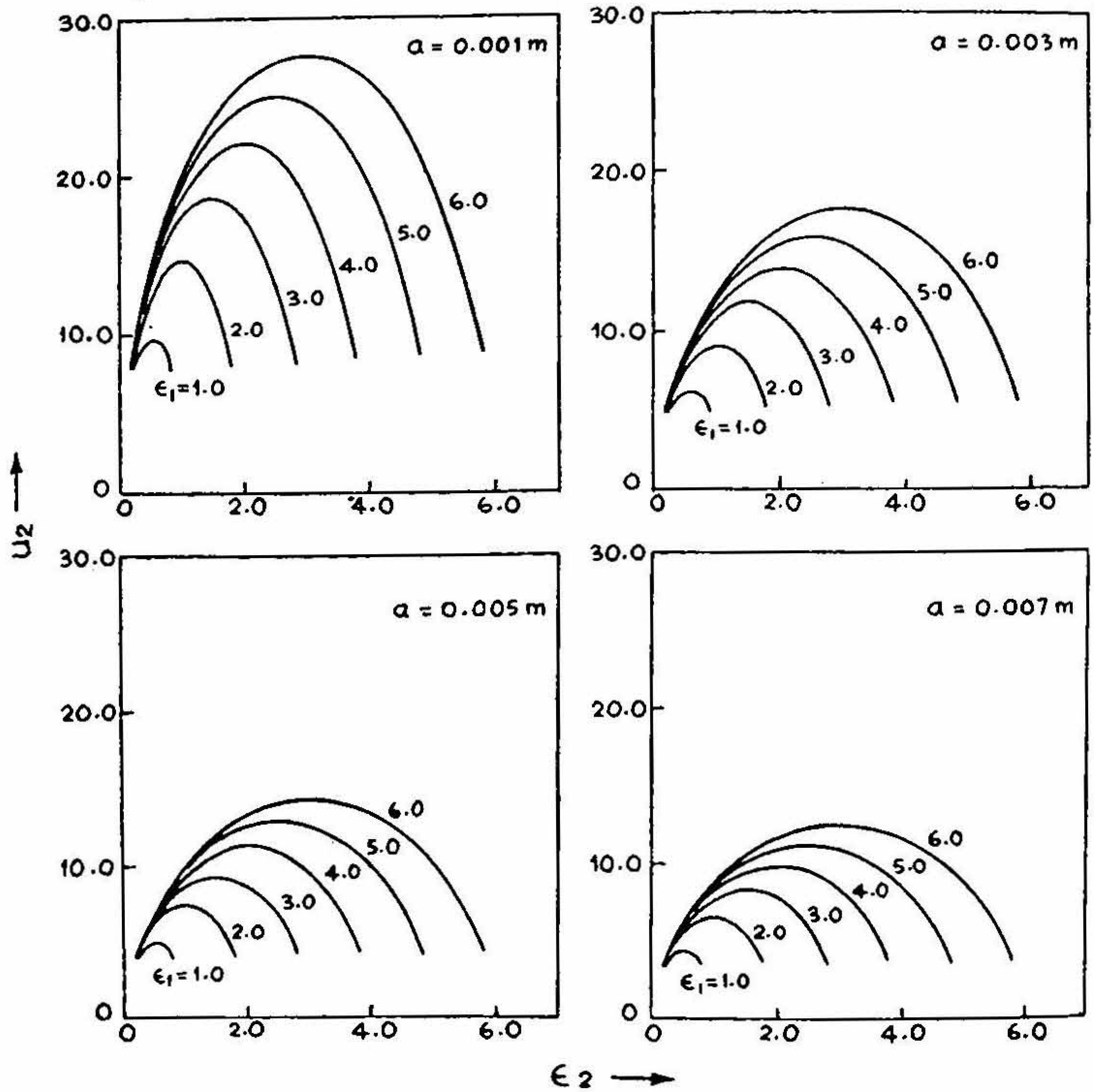
8. NUMERICAL EVALUATION

8.1. *Effect of* ϵ_2 *on the Radial Propagation Constant* u_2

The radial propagation constant u_2 is determined from the solution of the characteristic equation and its variation with *a*, *b*, ϵ_1 and ϵ_2 is shown graphically in Figs. 2 and 2 *a*.

8.2. *Effect of* ϵ_2 *on the Guide Wavelength* λ_g

The variation of λ_g with ϵ_2 is determined from equation (9) and the relation λ_g between and β and is shown in Fig. 3.



u_2 - RADIAL PROPAGATION CONSTANT (PER METRE) IN THE III MEDIUM
ϵ_1, ϵ_2 - DIELECTRIC CONSTANTS OF II & III MEDIA RESPECTIVELY
a - INNER CONDUCTOR RADIUS
b - COATING THICKNESS = 0.00005 m

FIG. 2 *a*. Variation of u_2 with ϵ_2

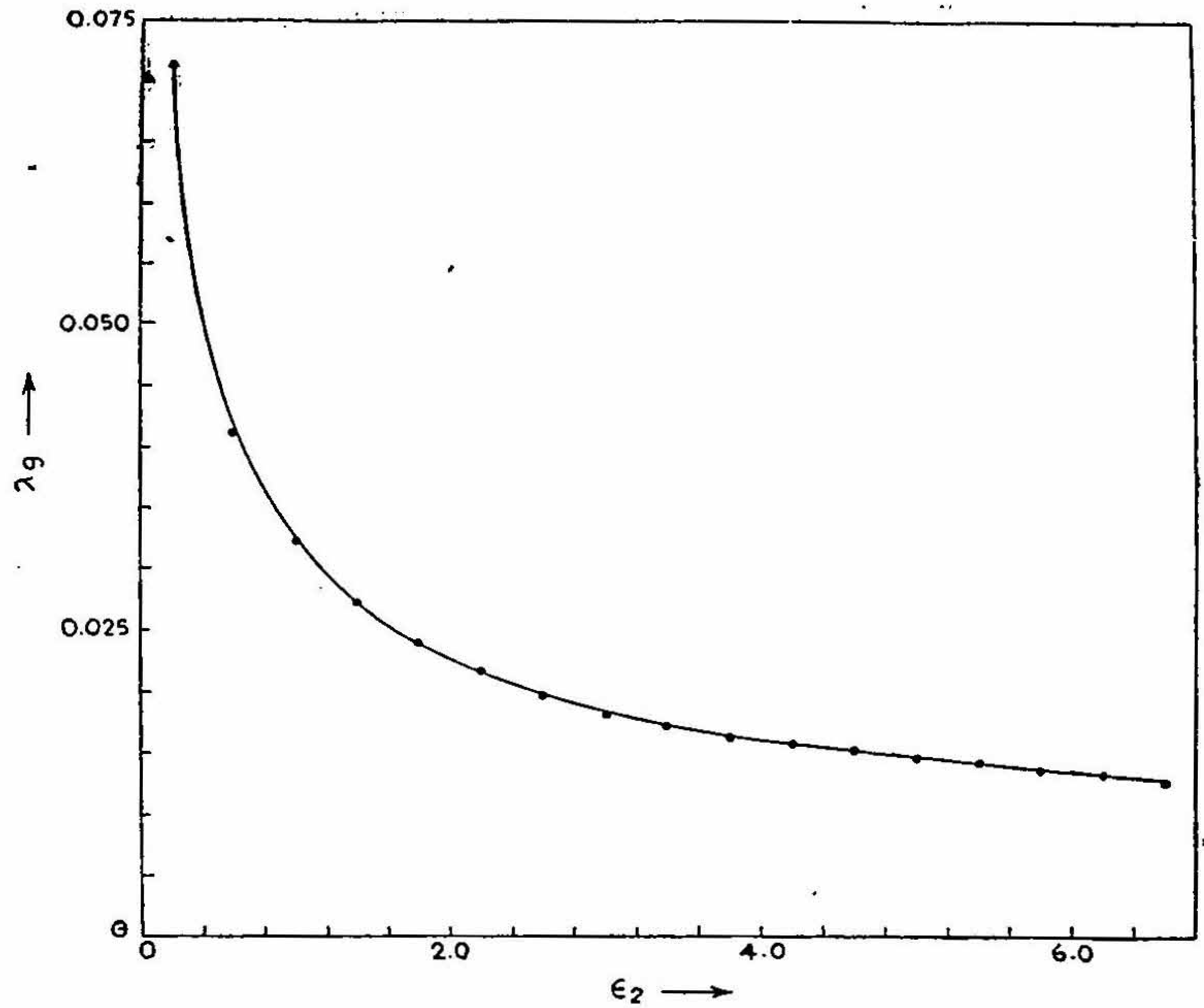


FIG. 3. γ_0 VS ϵ_2

8.3. *Effect of ϵ_2 on Constant Percentage Power Contour*

The variation of the constant percentage power contour with ϵ_2 is determined from eq. (13) and is shown in Fig. 4.

8.4. *Effect of ϵ_2 on $P_3\%$*

The percentage of powerflow in medium 3 ($P_3\%$) with respect to the total powerflow P_T is calculated as *a* function of ϵ_2 for different values of ϵ_1 and *a* by using eq. (12) and is shown in Fig. 5.

8.5. *Radial Field Spread*

The values of the u_2 for different *a*, ϵ_1 and ϵ_2 obtained from the solution of characteristic equation enable the determination of radial field spread of components E_z and E_ρ. Figure 6 shows the field decay in the radial direction for $\epsilon_1 = 6 \cdot 0$ and $\epsilon_2 = 3 \cdot 0$ for different values of *a*.

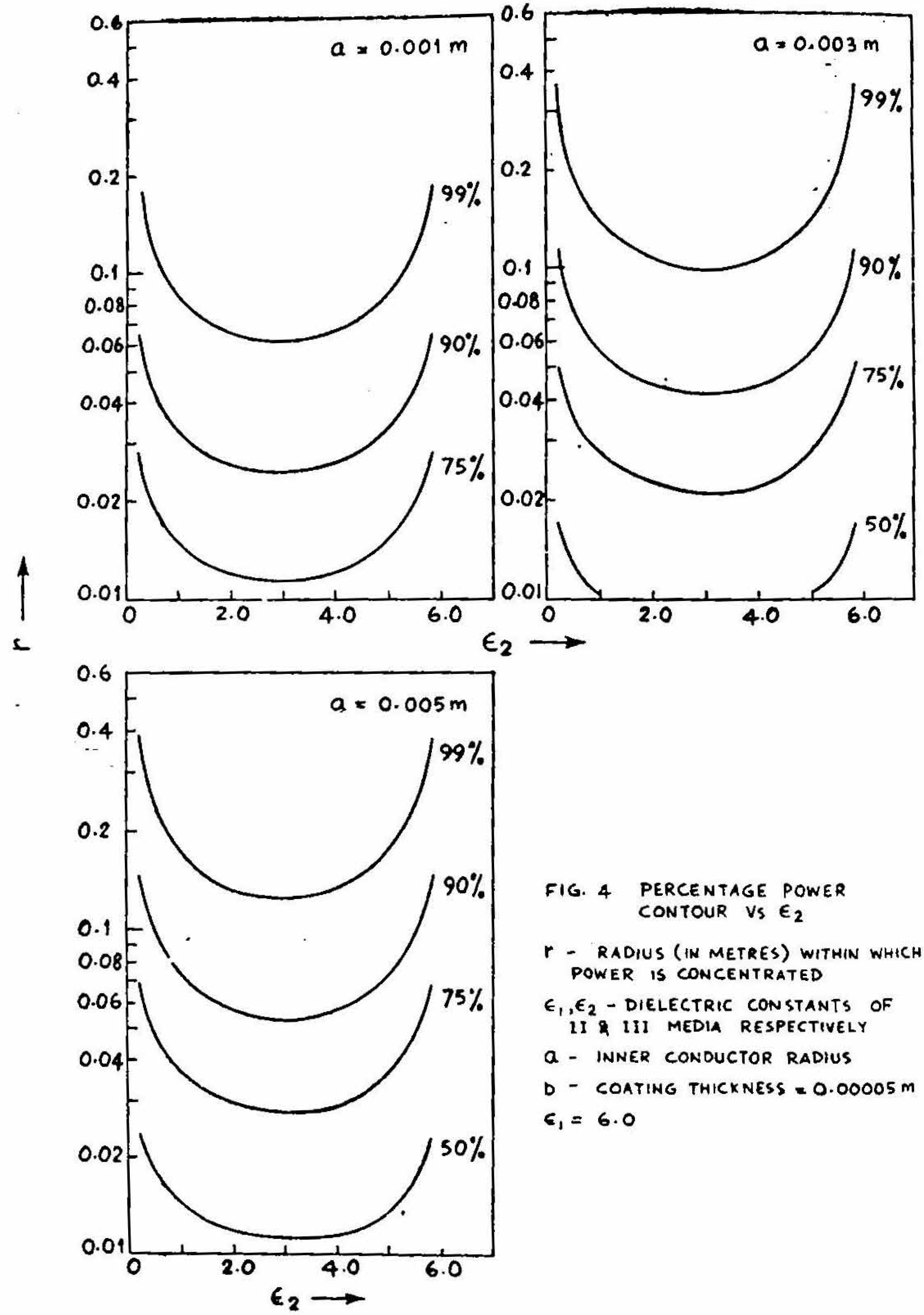

a = 0.001 m
a = 0.003 m
a = 0.005 m
99%
90%
75%
50%
0.6
0.4
0.2
0.1
0.08
0.06
0.04
0.02
0.01
0
2.0
4.0
6.0
r
ϵ_2 →
FIG. 4 PERCENTAGE POWER CONTOUR Vs ϵ_2
r - RADIUS (IN METRES) WITHIN WHICH POWER IS CONCENTRATED
ϵ_1, ϵ_2 - DIELECTRIC CONSTANTS OF II & III MEDIA RESPECTIVELY
a - INNER CONDUCTOR RADIUS
b - COATING THICKNESS = 0.00005 m
ϵ_1 = 6.0

FIG. 4

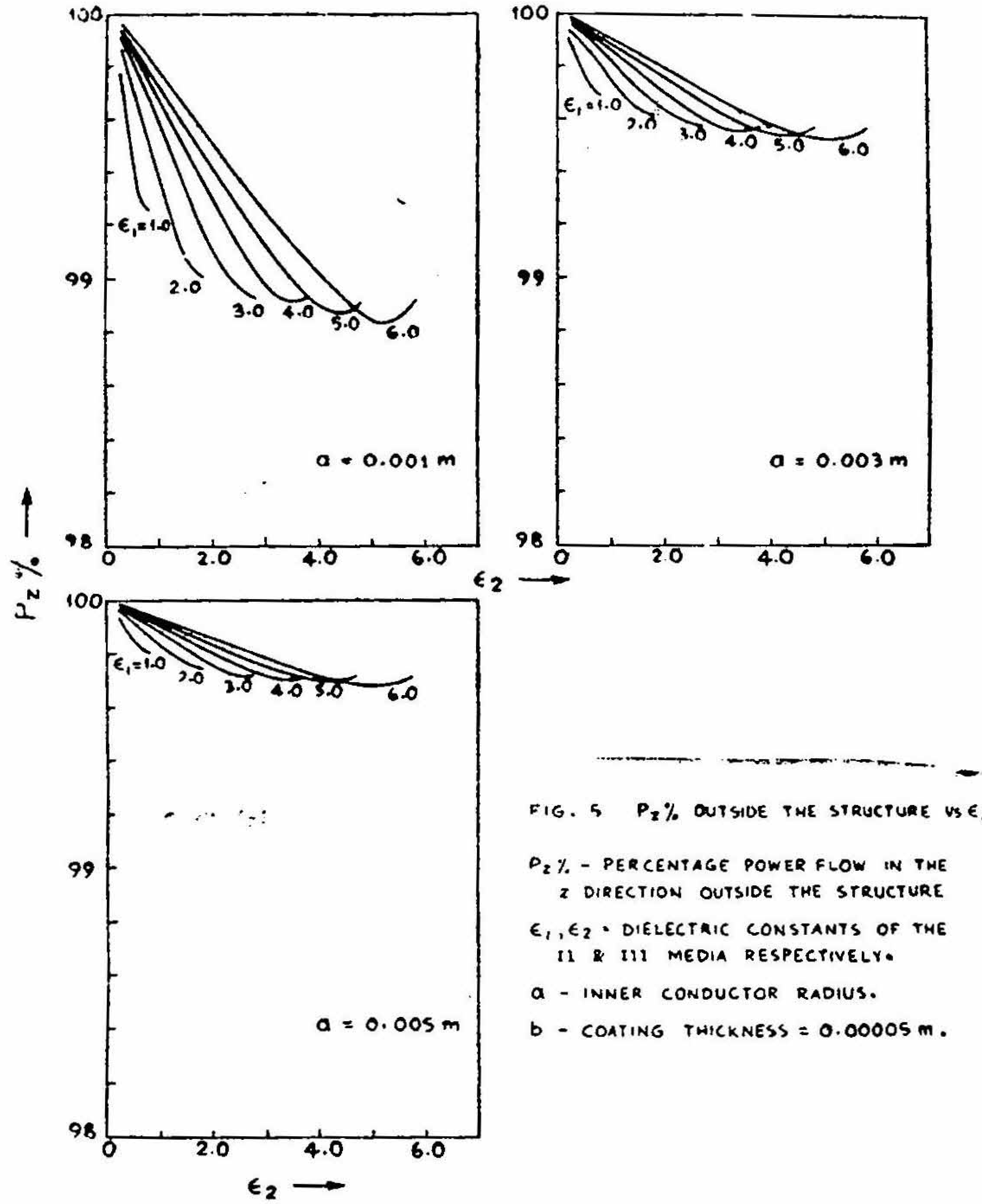


FIG. 5

9. Conclusions

The analysis leads to the following conclusions regarding the effect of the dielectric constant of the environmental medium on the surface wave characteristics of a dielectric-coated conductor excited in E_0-mode.

(i) Surface wave solutions exist as long as $\epsilon_2 < \epsilon_1$.

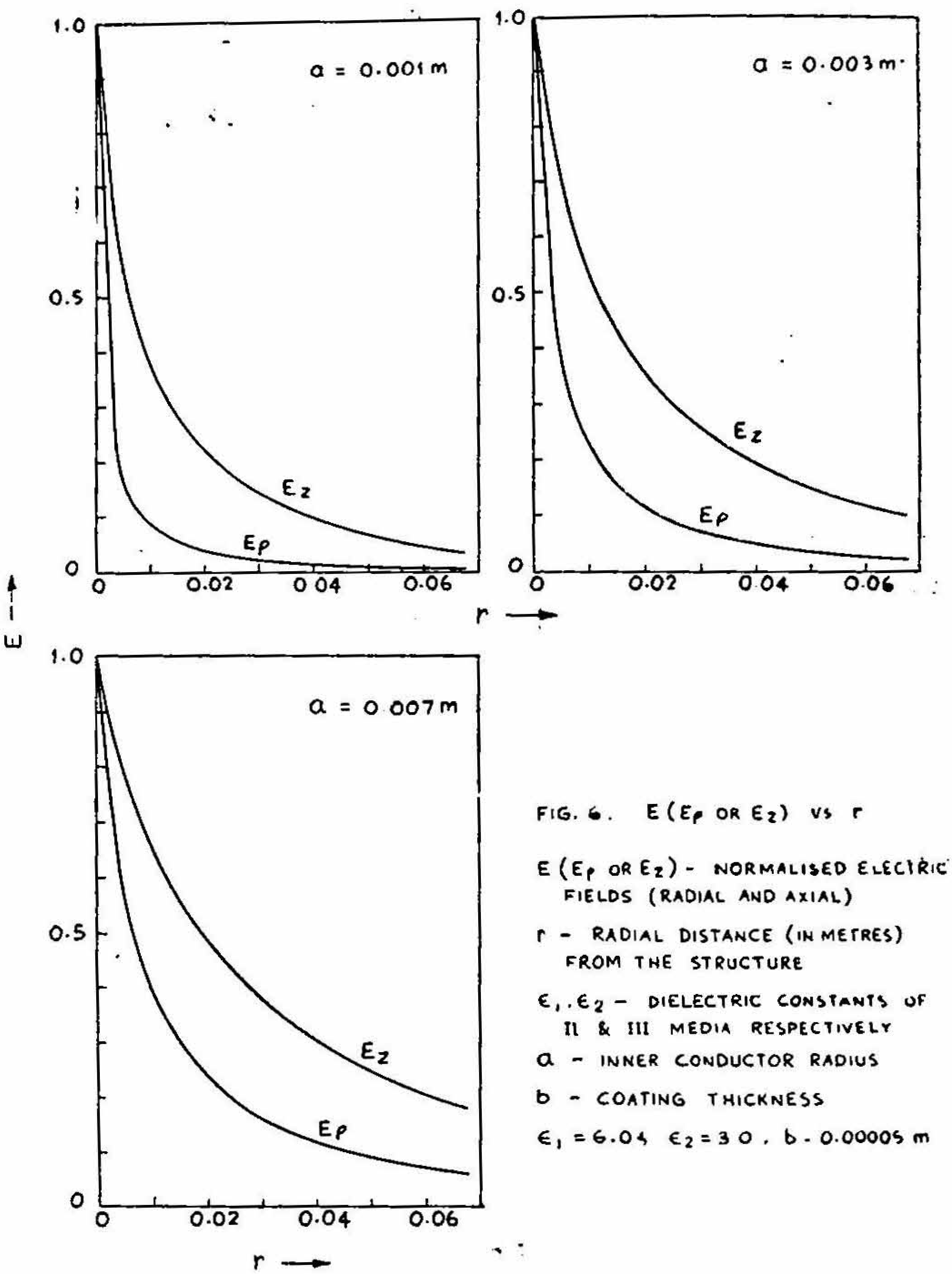
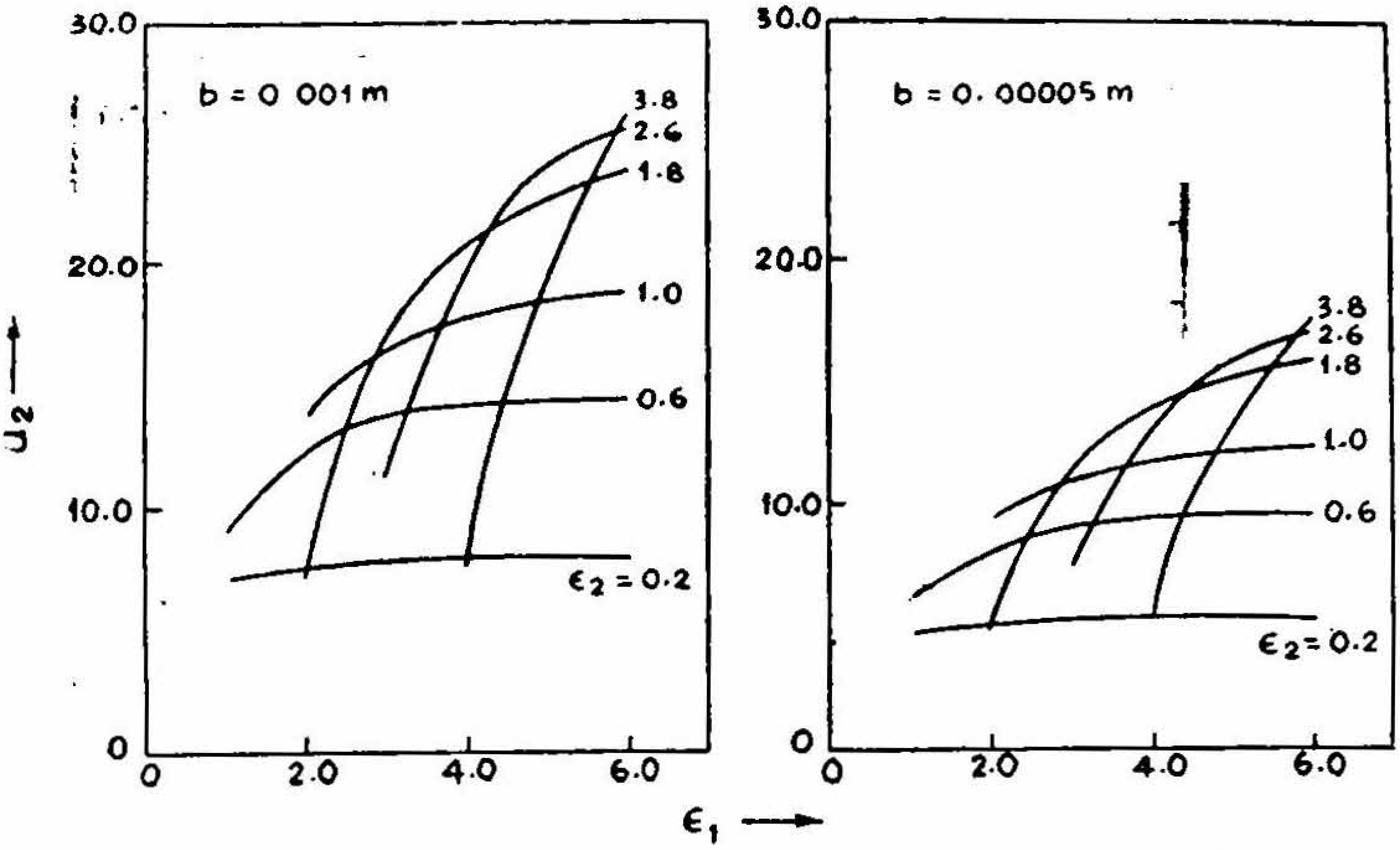
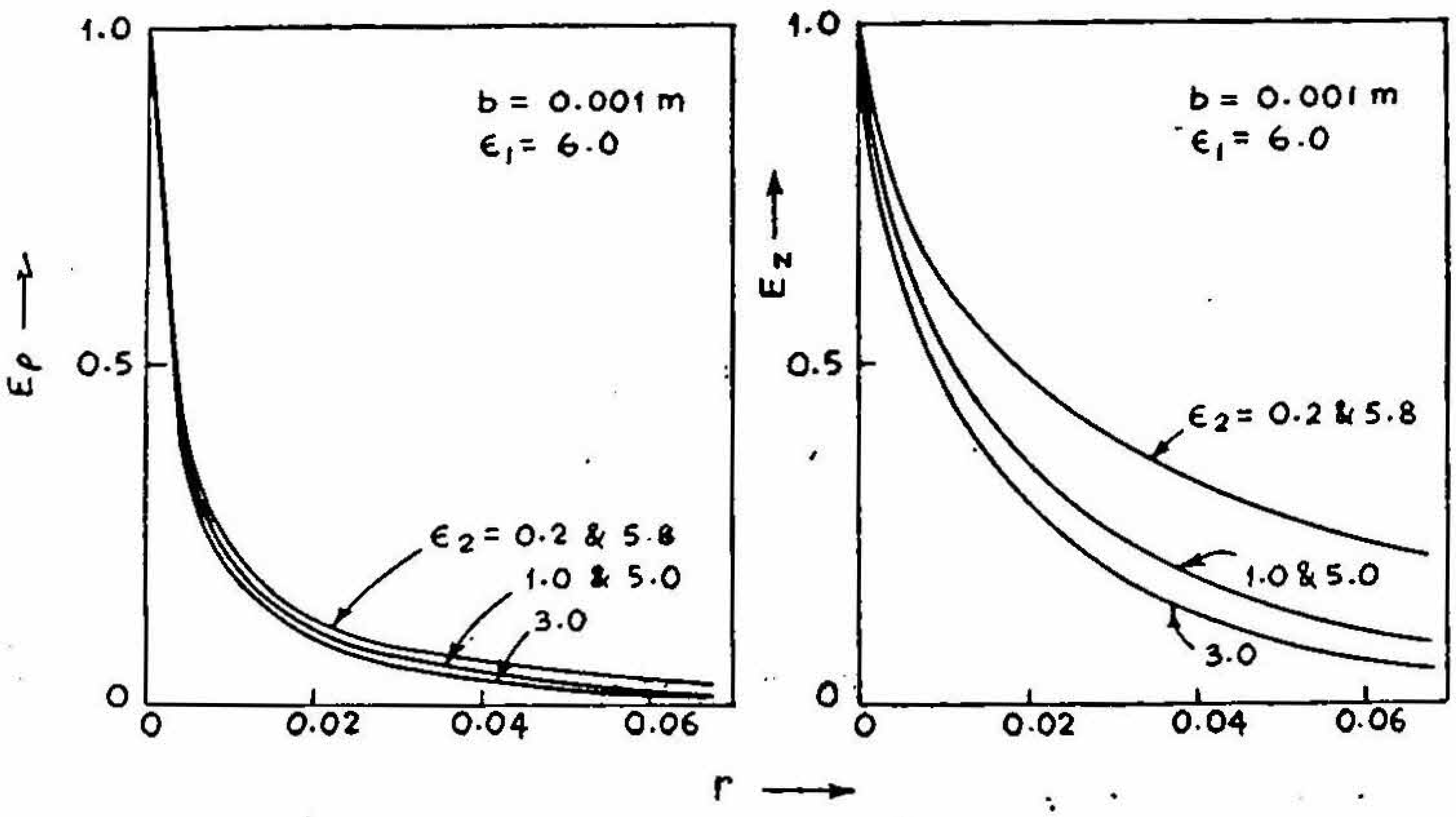


FIG. 6

(ii) The radial propagation constant u_2 first increases, attains a maximum value and then decreases to a low value with increasing ϵ_2 for a particular value of ϵ_1. The magnitude of maximum u_2 increases and shifts with increase of ϵ_1.



(i) U_2 vs ϵ_1



(ii) E vs r

U_2 - RADIAL PROPAGATION CONSTANT (IN METRES) IN THE III MEDIUM
ϵ_1, ϵ_2 - DIELECTRIC CONSTANTS OF II & III MEDIA RESPECTIVELY
a - INNER CONDUCTOR RADIUS = 0.003 m
b - COATING THICKNESS
E - NORMALISED ELECTRIC FIELDS - E_ρ & E_z
r - RADIAL DISTANCE (IN METRES) FROM THE STRUCTURE
ϵ_2 = 1 CORRESPONDS TO HARMS-GOUBAU LINE

FIG. 7

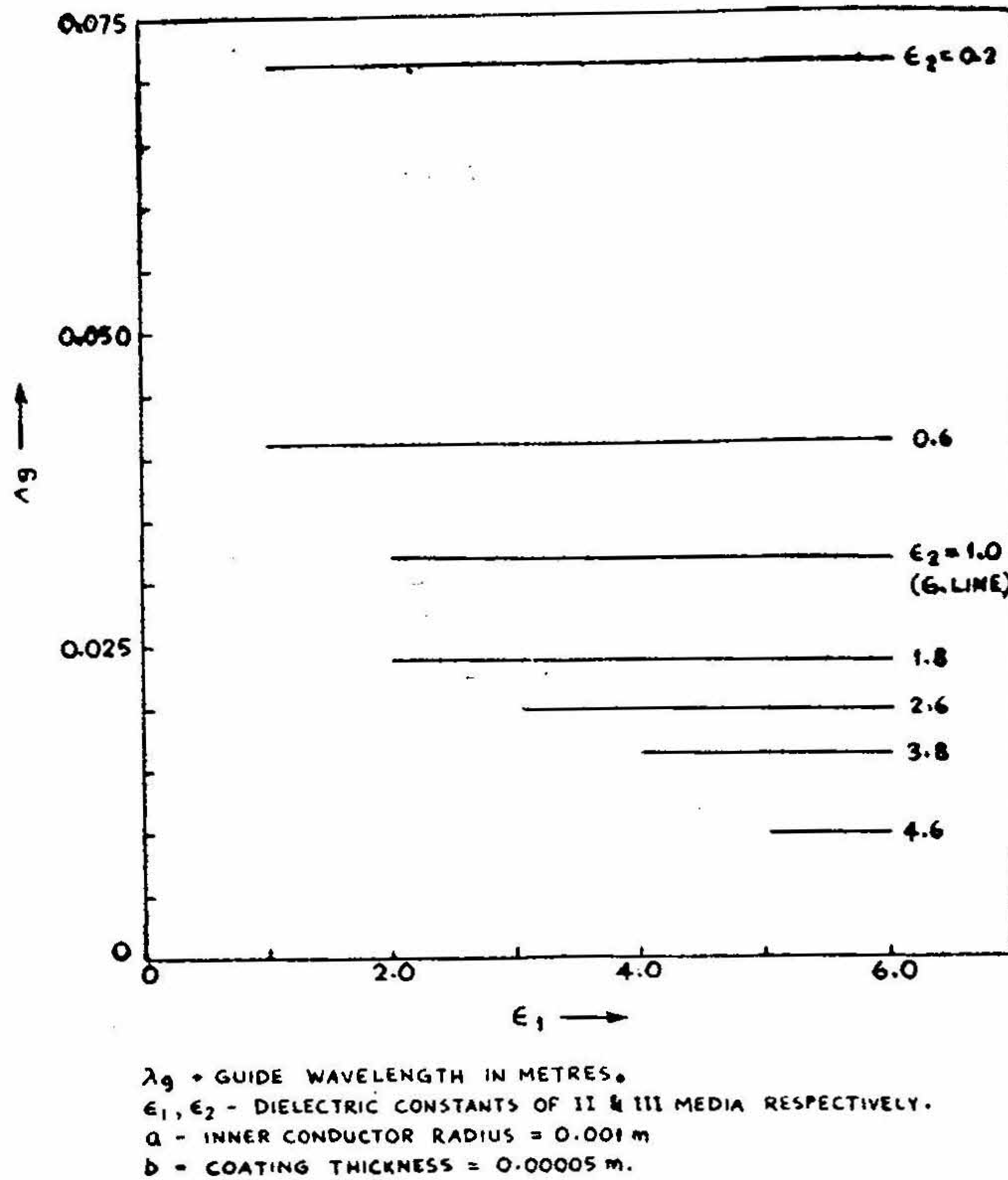


FIG. 8. λg vs ϵ_1

(iii) The magnitude of u_2 decreases with increasing values of a, but it increases with increasing values of b, for any particular value of ϵ_1 and ϵ_2 (Fig. 9).

(iv) The radial field decay differs significantly from Harms-Goubau line ($\epsilon_2 = 1$) when $\epsilon_2 > 1$.

(v) The surface wave field becomes more strongly bound than the Harms-Goubau line ($\epsilon_2 = 1$) when ϵ_2 is in the range $(1 < \epsilon_2 < \epsilon_1 - 1)$ and becomes more loosely bound when ϵ_2 is in the range $(1 > \epsilon_2 > \epsilon_1 - 1)$.

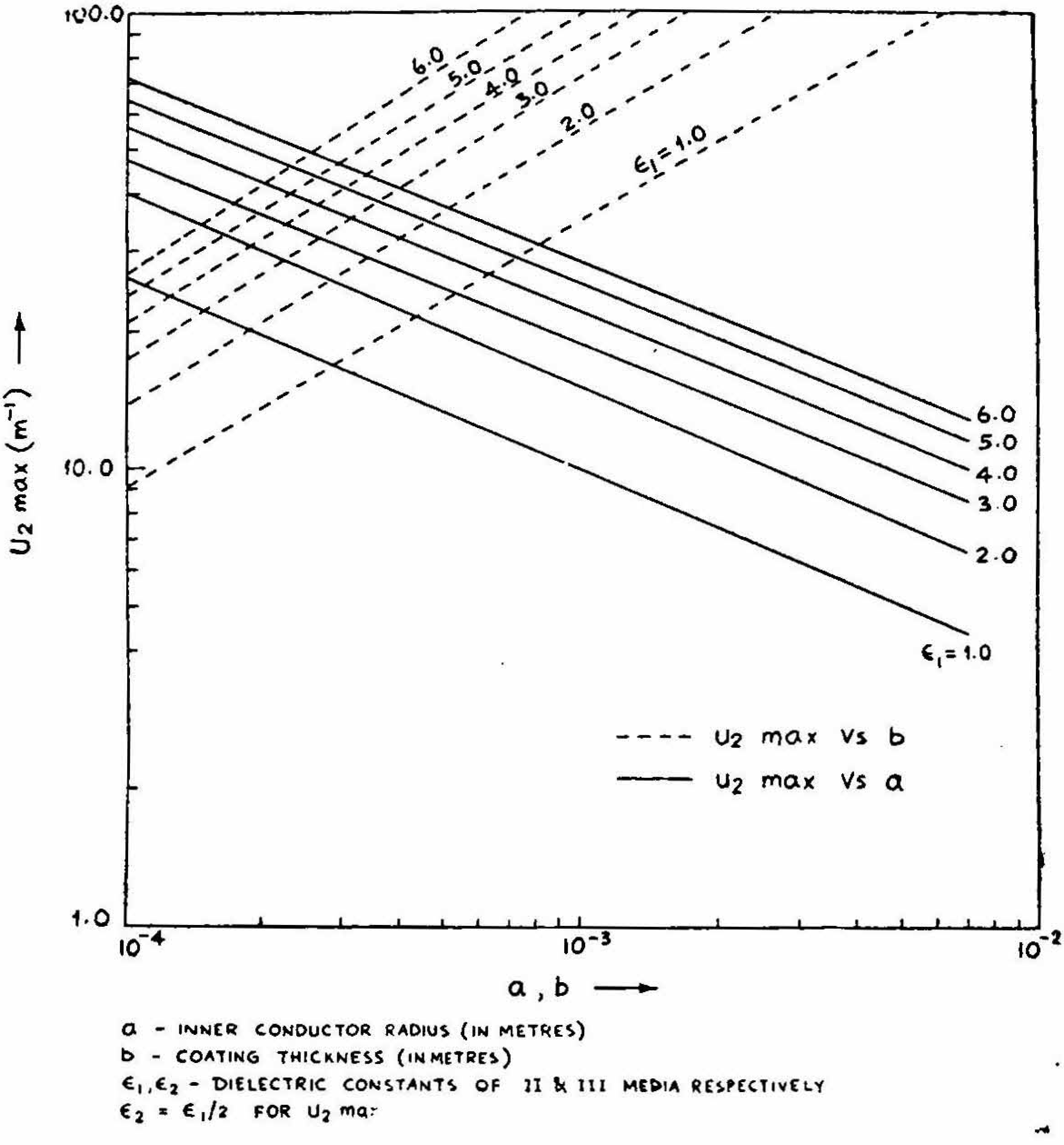


FIG. 9

This is evident from Figs. 2, 2*a*, 5 and 7. The attachment of the surface wave field to the structure is independent of ϵ_1 for all ϵ_2 within the range specified (Fig. 8).

(vi) Radii for different constant percentage power contour first decreases, then remains fairly constant and finally increases with increasing ϵ_2. This corresponds to the surface wave field being more and more strongly bound with increasing ϵ_2, then remaining practically independent of ϵ_2 within a certain range and finally becoming more and more loosely bound with further increase of ϵ_2. This is consistent with the observations made in (ii) and (iii).

(vii) Comparison of the characteristics of the surface wave line when $\epsilon_2 > 1$ but remains less than ϵ_1 with that of Harms-Goubau line ($\epsilon_2 = 1$) (Figs. 7 and 8) shows that the former can be said to guide more strongly bound surface wave than the latter.

It may therefore be said that by a proper selection of the combination of a, ϵ_1 and ϵ_2, the surface wave energy can be mostly concentrated in medium 2 and hence permitting long distance communication by surface wave, if the medium (2) is of very low loss.

Further work on the effect of lossy environment is under progress and the results will be reported later.

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