MODULAR TRAVELLING SALESMAN SCANS FOR PICTURE PROCESSING AND COMPUTER DISPLAYS

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ABSTRACT

Some scanning patterns are described, which map a two-dimensional sampling array into a one-dimensional sequence while retaining as many neighbourhood relations as possible. Their applications to picture-processing schemes and to computer output displays of multidimensional data are also discussed.

Key words : Scanning Patterns ; Picture Processing ; Computer Displays.

The object of the note is to consider some modular scanning patterns which have applications in picture processing and computer output displays. The scans will be reviewed first in order to appreciate their necessity and use.

Classification of Patterns

The problem of classification of a given two-dimensional array of discrete random variables, (such as a sampled version of a picture), into membership of one of the given set of patterns is a very important problem. For such a classification problem, certain reasonable assumptions must be made regarding the distributions of random variables, so that the criterion of classification which minimizes the probability of misclassification can be arrived at. It is most convenient to assume that the variables are all statistically independent or that their joint distribution is multivariate normal. But in many cases, these assumptions are unrealistic because there is strong local correlation. In the case of a one-dimensional data, one could consider the problem by assuming that the data arise as the result of a Markov process. In a case of a two-dimensional array either of the two approaches must be used. (*a*) The concept of a Markov process must be extended to a two-dimensional case. While this, in principle, is possible [1], it increases the dimensionality of the problem. (*b*) The alternative approach is to scan the two-dimensional array to get a one-dimensional sequence and to use the assumption of a Markovian chain for this sequence. Since the finite Markovian assumption implies that the distribution of a particular sample depends only on the values of the r samples on either side on the sample in a sequence, the scan must retain the two-dimensional neighbourhood relations as far as possible [1].

Bandwidth Reduction

In theory, bandwidth reduction of a signal does not necessarily have to imply redundancy reduction or data compression. Given a band-limited signal, it can be converted into another signal which requires lesser bandwidth, such that the former signal can be recovered from the latter without error. This mapping of a band-limited signal into another of lesser bandwidth does not presume any *a priori* knowledge of the signal. The bandwidth compression scheme called 'sampling mapping scheme' [2–3] works as follows. The input signal samples are partitioned into sets of N samples. Let the sample values be so normalized that they lie in the range 0–1. Then, as every set contains N numbers in the range 0–1, they can be considered as representing the co-ordinates of a point in an N-dimensional unit cube. The points in an N-dimensional unit cube can then be mapped on to the points on a unit line. This mapping then enables us to look upon a set of N sample values by a single number between 0 and 1. Each of these is considered to be a sample of a new band-limited signal which reduces the bandwidth requirement by a factor of N. Noise immunity of this band-limited signal is obviously poorer. The mapping from an N-dimensional unit cube onto the unit line must be such that the reverse mapping from the line to the cube must be continuous everywhere so that minute amount of channel noise does not result into large errors in the reconstructed signal. A mapping from an N-cube to a line may be thought of as a continuous curve which passes through each point of the cube. When the sample values are quantized and for the case of $N = 2$, these curves can be replaced by the scans that are a topic of this note.

Predictive Quantization :

One way of data compression is to employ predictive quantization schemes. For picture signals there are some planar prediction schemes [4]–[5] wherein the value of the sample is predicted on the basis of the samples in the two-dimensional neighbourhood of it that have been transmitted earlier. One of the drawbacks of these schemes is that the errors can propagate over long distances horizontally or vertically. Alternative to two-dimensional prediction schemes is to scan the picture to generate a one-dimensional sequence and apply one-dimensional prediction schemes to this sequence. The efficiency of the scheme would obviously depend on the scan used. For example, if the conventional television raster is used, the correlation across the scanning direction is not made use of at all. Thus, the scanned one-dimensional version of a two-dimensional picture signal must be obtained in a way which makes the most effective use of the two-dimensional correlation [6]. The advantage of this approach over planar prediction schemes is that the errors do not propagate over long distances horizontally or vertically. Some features of planar prediction schemes, such as choice from a set of prediction strategies based on some comparisons over the neighbourhood of the point to be predicted can be retained.

Walsh-Domain Picture Processing

Let $A(K)$, $K = 0, 1, \ldots, 2^n - 1$ be the signal samples and n an integer. Let $X(J)$, $J = 0, 1, \ldots, 2^n - 1$ be the Walsh coefficients. Then, it is shown that [7, 8] if

$$A(2K) \simeq A(2K+1); \quad K = 0, 1, \ldots, 2^{n-1} - 1 \tag{1}$$

then

$$X(2J+1) \simeq 0\ ; \quad J = 0, 1, \ldots, 2^{n-1} - 1 \tag{2}$$

Further, if

$$A(4K) \simeq A(4K+1) \simeq A(4K+2) \simeq A(4K+3), \quad K = 0, 1, \ldots, 2^{n-2} - 1 \tag{3}$$

then

$$\left.\begin{aligned} X(2J+1) &\simeq 0\ ; \quad J = 0, 1, \ldots, 2^{n-1} - 1 \\ X(4J+2) &\simeq 0\ ; \quad J = 0, 1, \ldots, 2^{n-2} - 1 \end{aligned}\right\} \tag{4}$$

and so on. Eqs. (1) and (3) represent correlated signals. Thus if the signal to be subjected to Walsh transformation is strongly correlated, many Walsh coefficients will be close to zero by Eqs. (2) and (4) and their generalization. As these coefficients represent only a small fraction of total signal-energy by virtue of the Parseval's theorem, they need not be transmitted. A one-dimensional signal which satisfies a generalization of Eqs. (1) and (3) can be constructed from a two-dimensional picture signal if the scanning pattern used maps two-dimensional neighbourhoods into one-dimensional neighbourhood as far as possible.

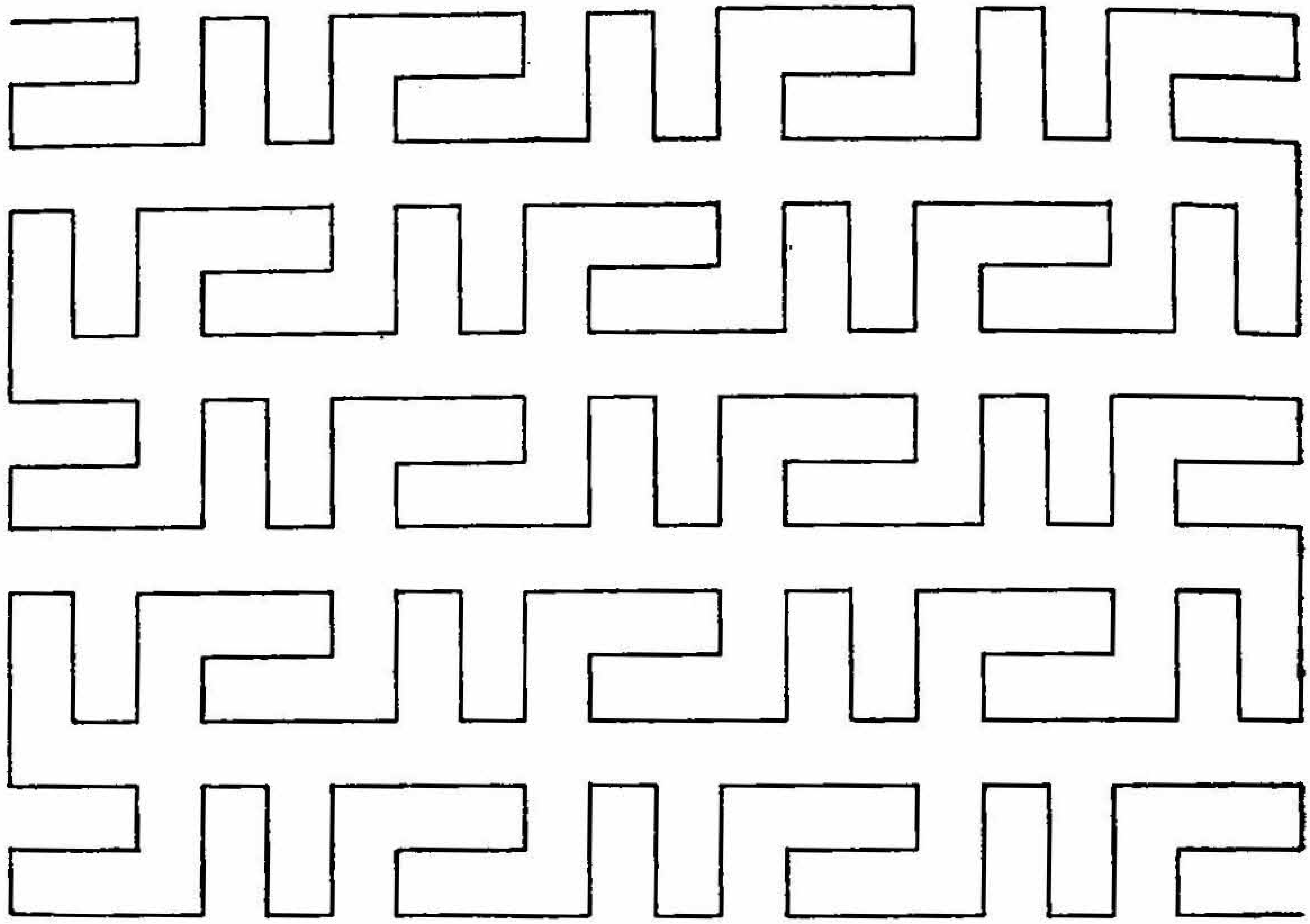

FIG. 1 (*a*)

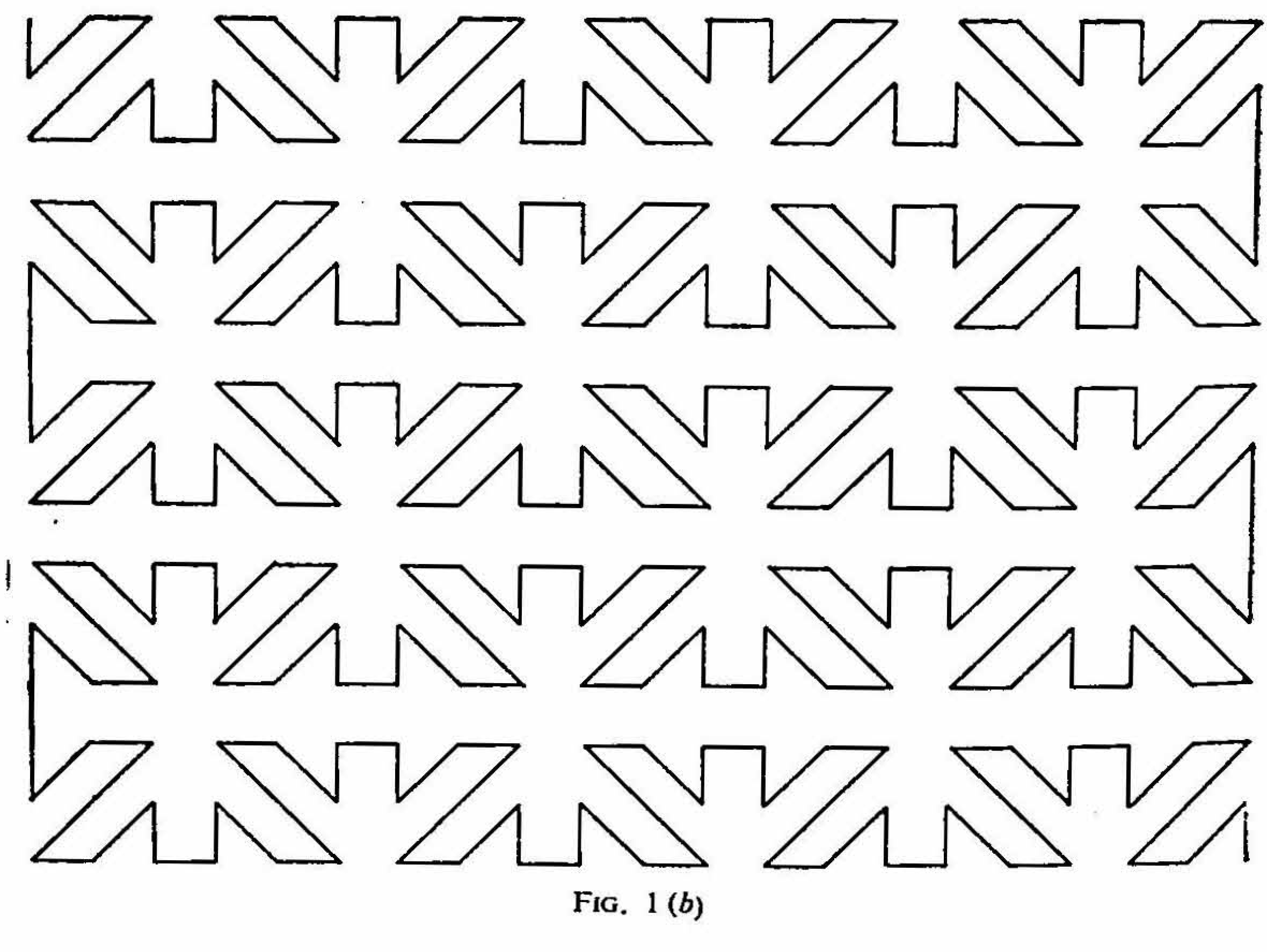

FIG. 1 (*b*)

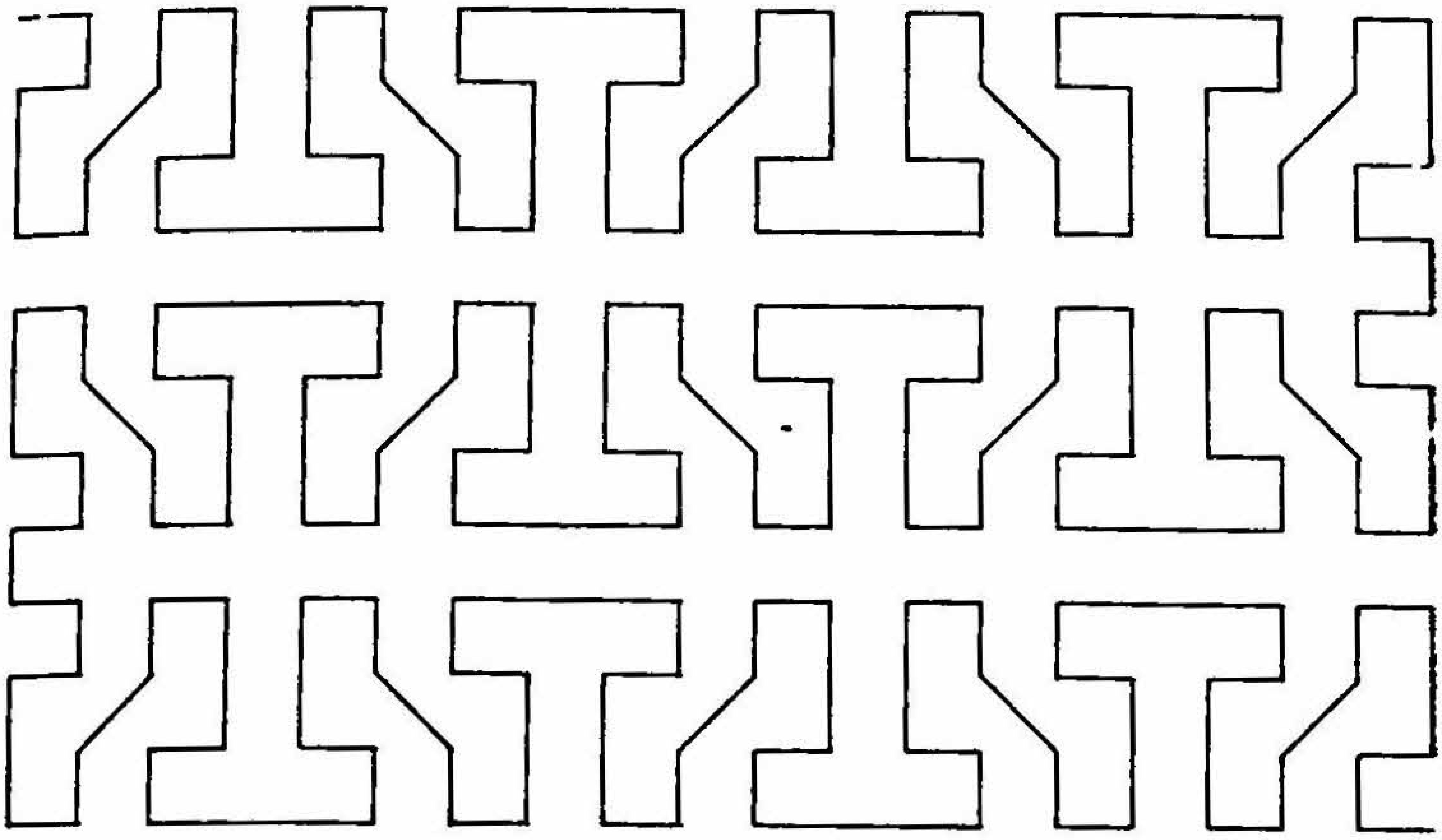

FIG. 1 (*c*)

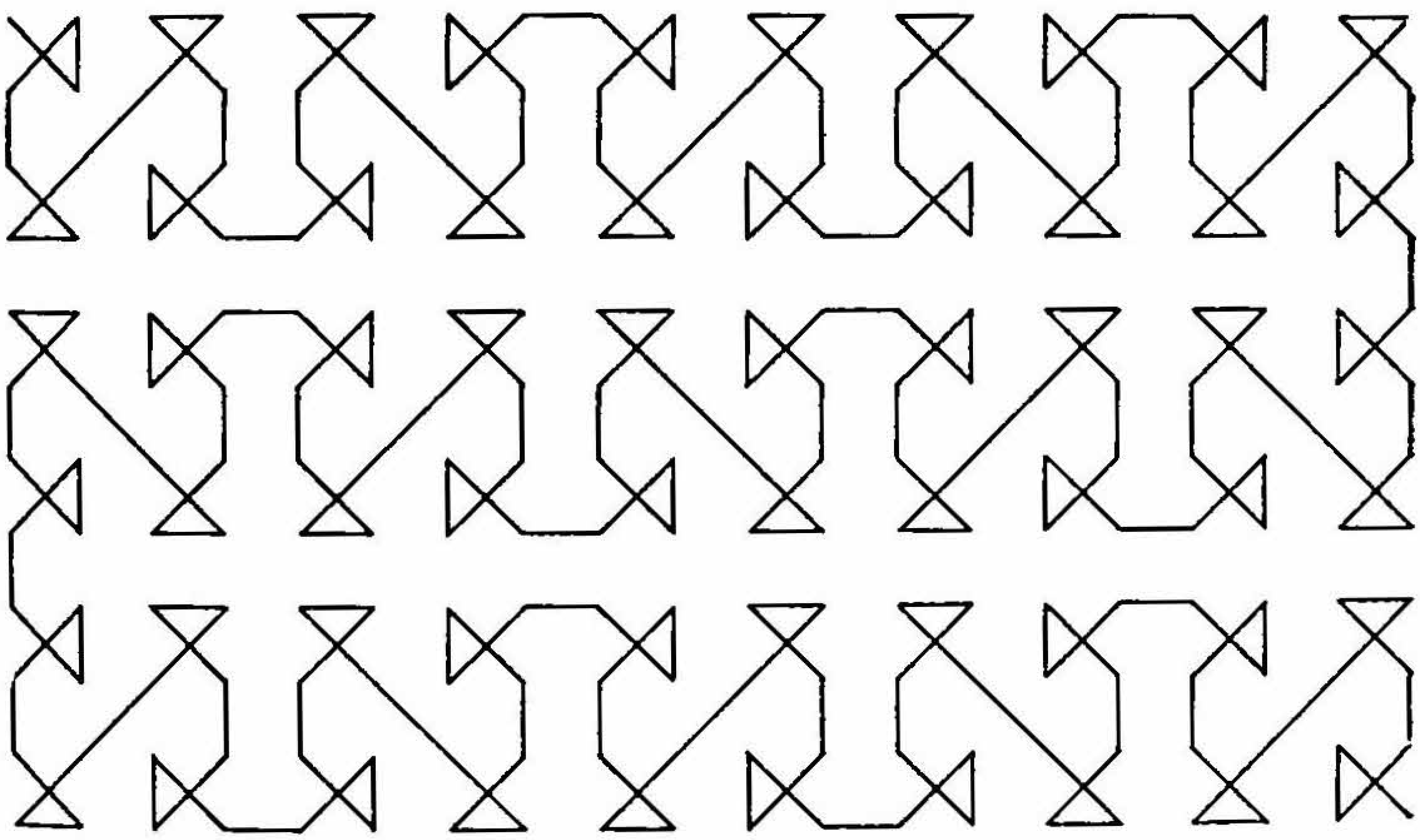

FIG. 1 (*d*)

FIG. 1. Four modular travelling salesman scans.

Computer Displays

With the increased use of a computer for data processing, the problem of displaying the processed data is acquiring increased importance. One of the problems in this area is that of displaying a real-valued function of two or more variables. It is suggested [9] that the problem can be tackled by mapping a multi-dimensional bounded domain on which a function is defined onto a bounded interval of the real line. It is desirable to have the mapping with the property that the neighbouring points in the bounded interval are mapped from the neighbouring points in the bounded multi-dimensional domain. It facilitates the visual judgment of the effects of parameter changes on the function being displayed.

Some Useful Scans

Many of such scanning patterns are available in the literature [1], [3], [6], [9]. Four more are shown in Figs. 1 (*a*) to 1 (*d*). Two of them are nomenclatured swastik scan and octal star scan but not explicitly identified as the nomenclature is descriptive enough. The property of these scans and some in the literature referred to as grid-filling curves [1], space-filling curves [3], meril scan [6], etc., is that the neighbouring points in the bounded interval are mapped from the neighbours in the two-dimensional bounded domain. In fact the grid is partitioned into disjoint neighbourhoods and the scanning pattern enters the adjacent neighbourhood only after exhausting all the sampling points in the previous neighbourhood. In addition, the partitioning in the disjoint neighbourhoods could be recursive. The scans are modular in the sense that if the partitioning is pursued till the last recursive stage, the scanning pattern is identical within every neighbourhood upto mirror images or orientation. They are ' travelling salesman ' scans in the sense that the scanning raster like a travelling salesman wants to visit every sampling point just once in every frame, from every sampling point wants to switch over to only the neighbouring point and wants to cover entire subarea before entering the adjacent subarea. Every grid-filling curve does not meet the requirements of the applications listed in this note. For example, dovetail scan [9] does not always move from one sampling point to its neighbour, though it completes a neighbourhood before entering the next. As another example, a scan that starts on the periphery and gradually spirals in is grid-filling but does not retain the two-dimensional neighbourhood relations very effectively in the scanned output.

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