LARGE DEFLECTION OF A HEATED ELLIPTIC PLATE ON ELASTIC FOUNDATION

S. DATTA

(*Head of the Department, Mechanical Engineering, Jalpaiguri Government Engineering College, Jalpaiguri, West-Bengal, India*)

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ABSTRACT

Following Berger's method the large deflection of a heated elliptic plate with clamped edges and placed on elastic foundation has been investigated under stationary temperature distribution. The deflection is obtained in terms of Mathieu function of the first kind and of zero order.

Keywords: Berger's method; Mathieu function; Elliptic plate; Elastic foundation.

INTRODUCTION

In recent years there has been a rapid development of thermoelasticity stimulated by various engineering sciences. In the field of machine structures, mainly with aircraft, steam and gasturbines and in chemical and nuclear engineering, thermal stresses play an important and frequently even a primary role. Determination of thermal deflections of plates, especially of thin plates, is of vital importance in the design of machine structures, because excessive deflections may cause heavy undesirable thermal stresses.

The classical large deflection of thin plate problems usually lead to non-linear differential equations which cannot be exactly solved. H. M. Berger [1] has shown that if, in deriving the differential equations from the expressions for strain energy, the strain energy due to second invariant in the middle plane of the plate is neglected, a simple fourth order differential equation coupled with a non-linear second order equation is obtained. Although no complete explanation of the method is set forth, the stresses and deflections obtained by Berger himself for rectangular and circular plates agree well with those found from more precise analysis. This approximate method has been extended to orthotropic plates by Iwinski and Nowinski [2] and further boundary value problems associated with rectangular and circular plates have been solved by Nowinski [3]. Thein Wah and Robert Schmidt [4] and Nash and Modeer [5] obtained satisfactory results following this method Basuli [6] has extended this approximate method of Berger to problems under uniform load and heating under stationary temperature distribution.

Berger's technique of neglecting the second invariant of the middle surface strains has been applied by Sinha [7] to circular and rectangular plates placed on elastic foundation and under uniform transverse load.

In this paper the author has applied the method of Berger to investigate the large deflection of an elliptic plate placed on elastic foundation and heated under stationary temperature distribution. The foundation is assumed to be such that its reaction is proportional to the deflection. The deflection is obtained in terms of Mathieu function of the first kind and of zero order.

NOTATIONS

The following notations have been used in the paper:

D = Flexural rigidity of the plate $= \dfrac{Eh^3}{12(1-\nu^2)}$

E, ν, α = Young's modulus, Poisson's ratio and Coefficient of thermal expansion respectively.

h = Thickness of plate.

u, v = Displacement along the x and y axis respectively.

w = Lateral displacement

e_1 = First strain invariant;

$$= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2$$

e_2 = Second strain invariant.

K = Foundation reaction per unit area per unit deflection.

∇ = Laplacian operator.

FORMULATION OF PROBLEM

The strain energy due to bending and stretching of the middle surface of the plate is given by:

$$V_1 = \frac{D}{2} \int\int \left[(\nabla^2 W)^2 + \frac{12}{h^2} e_1^2 - 2(1-\nu) \left\{ \frac{12}{h^2} e_2 \right.\right.$$

$$+ \frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \Big\} \Big] \, dxdy \quad (1)$$

Combining the potential energy of the foundation reaction and also the potential energy due to heating with Eq. 1 and neglecting e_2, the modified energy expression for the total energy becomes:

$$V = \frac{D}{2} \int\int \Big[(\nabla^2 w)^2 + \frac{12}{h^2} e_1^2 - 2(1-\nu) \Big\{ \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \Big\} + \frac{K}{D} w^2 \Big] \, dxdy - \int\int \int_{-h/2}^{h/2} \frac{E\alpha T'}{1-\nu} \times (e_1 - z \nabla^2 w) \, dxdydz \quad (2)$$

in which T' is the temperature distribution at any point given by (Basuli [6])

$$T'(x, y, z) = T_0(x, y) + g(z) \, T(x, y) \quad (3)$$

and

$$\int_{-h/2}^{h/2} z g(z) \, dz = f(h); \qquad \int_{-h/2}^{h/2} g(z) \, dz = 0 \quad (4)$$

Combining Equations 2, 3 and 4 one gets

$$V = \frac{D}{2} \int\int \Big[(\nabla^2 w)^2 + \frac{12}{h^2} e_1^2 - 2(1-\nu) \Big\{ \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \Big\} + \frac{K}{D} w^2 \Big] \, dxdy - \int\int \frac{E\alpha}{1-\nu} \left(T_0 e_1 h - f(h) \, T \nabla^2 w\right) \, dxdy. \quad (5)$$

According to the principle of minimum potential energy, the displacements that satisfy the equilibrium conditions make the potential energy, V, minimum. In order for the integral of Eq. 5 to be an extremum, the integrand, F, must satisfy the following Euler's equations of the calculus of variation:

$$\frac{\partial F}{\partial u} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial u_y}\right) = 0 \quad (6\,a)$$

$$\frac{\partial F}{\partial v} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial V_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial V_y}\right) = 0 \quad (6\,b)$$

$$\frac{\partial F}{\partial w}-\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial w_x}\right)-\frac{\partial}{\partial y}\left(\frac{\partial F}{\partial w_y}\right)+\frac{\partial^2}{\partial x^2}\left(\frac{\partial F}{\partial w_{xx}}\right)$$
$$+\frac{\partial^2}{\partial y^2}\left(\frac{\partial F}{\partial w_{yy}}\right)+\frac{\partial^2}{\partial x\partial y}\left(\frac{\partial F}{\partial w_{xy}}\right)=0. \qquad (6\,c)$$

Application on the Eqs. 6 *a*, 6 *b*, and 6 *c* to Eq. 5 yields:

$$\frac{\partial}{\partial x}\{e_1-(1+\nu)\,\alpha T_0\}=0 \qquad (7\,a)$$

$$\frac{\partial}{\partial y}\{e_1-(1+\nu)\,\alpha T_0\}=0. \qquad (7\,b)$$

$$\nabla^4 w-\frac{12}{h^2}\{e_1-(1+\nu)\,\alpha\,T_0\}\;\nabla^2 w+\frac{K}{D}\,w$$
$$+\frac{E\alpha f(h)}{D\,(1-\nu)}\;\nabla^2 T=0. \qquad (7\,c)$$

Eqs. 7 *a*, and 7 *b* prove that:

$$\{e_1-(1+\nu)\,\alpha T_0\}$$

is independent of x and y and therefore

$$e_1-(1+\nu)\,\alpha T_0=\text{constant}=\frac{\beta^2\,h^2}{12} \qquad (8\,a)$$

in which β is a normalised constant of integration, and

$$e_1=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \qquad (8\,b)$$

Considering Eq. 8 *a*, Eq. 7*c* reduces to

$$\nabla^2(\nabla^2-\beta^2)\,w+\frac{K}{D}\,w=-\,\frac{E\alpha\,f(h)}{D\,(1-\nu)}\;\nabla^2\,T \qquad (9)$$

SOLUTION OF PROBLEM

Let us take an elliptic plate of thickness, h. The centre of the plate in the middle surface is taken as the origin and the Z-axis downwards.

If there is no source of heat inside the plate the following differential equations must be satisfied for stationary temperature distribution (Nowacki [8])

$$\nabla^2 T_0 - \epsilon T_0 = -\frac{\epsilon_0}{2}(\theta_1 + \theta_2)$$

$$\nabla^2 T - \frac{12}{h^2}(1+\epsilon)\, T = -\frac{12\epsilon}{h^3}(\theta_1 - \theta_2) \tag{11}$$

in which θ_1 and θ_2 denote temperatures at the upper and lower media of the plate respectively.

If $\theta_1 = \theta_2$, Eq. (11) becomes

$$\nabla^2 T - \beta_1{}^2 T = 0 \tag{12}$$

In which

$$\beta_1{}^2 = (1+\epsilon)\frac{12}{h^2} \tag{13}$$

Transferring to elliptic co-ordinates (ξ, η) defined by $x + iy = d \cosh (\xi + i\eta)$, where $2d$ is the interfocal distance of the ellipse, Eq. 12 reduces to

$$\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial \eta^2} - \frac{\beta_1{}^2 d^2}{2}(\cosh\ 2\xi - \cos 2\eta)\, T = 0. \tag{14}$$

Solution of Eq. 14 can be taken in the following form

$$T = \sum_{m=0}^{\infty} C_{2m}\, Ce_{2m}(\xi, -q)\, ce_{2m}(\eta, -q) \tag{15}$$

in which $Ce_{2m}(\xi, -q)$ and $ce_{2m}(\eta, -q)$ are modified Mathieu function and ordinary Mathieu function of the first kind and of order $2m$ respectively, and

$$q = \frac{\beta_1{}^2 d^2}{4} \tag{16}$$

While solving a problem of bending of a plate with an elliptic hole, by taking a single Mathieu function of the second orcer instead of taking Mathieu functions of all orders, Naghdi [9] has shown that the results are satisfactory for larger elliptic holes. In this paper also similar approximation is made by taking Mathieu function of zero order and on this assumption Eq.15 reduces to

$$T = C_o\, Ce_o(\xi, -q)\, ce_o(\xi, -q) \tag{17}$$

The following boundary condition is imposed on T

$$T = \text{Constant} = K_1 \text{ on } \xi = \xi_0$$

with the above boundary condition Eq. 17 yields

$$K_1 = C_0 C_{eo}(\xi_0, -q)\, c_{eo}(\xi, -q) \tag{18}$$

Multiplying Eq. 18 by $c_{eo}(\eta, -q)$ and integrating with respect to η from 0 to 2π and using the orthogonality relation and normalisation (Mclachlan [10]) one gets

$$C_0 = \frac{2A_0^{(0)} K_1}{C_{eo}(\xi_0, -q)} \tag{19}$$

in which $A_0^{(0)}$ is the first Fourier Coefficient in the expansion of $c_{eo}(\eta, -q)$ Therefore

$$T = \frac{2A_0^{(0)} K_1}{C_{eo}(\xi_0, -q)} C_{eo}(\xi, -q)\, c_{eo}(\eta, -q) \tag{20}$$

is determined.

Changing Eq. 9 to elliptic Co-ordinates and substituting the expresion for $\nabla^2 T$ one gets

$$(\nabla^2 - P_1^2)(\nabla^2 - P_2^2) w = \lambda C_{eo}(\xi, -q)\, c_{eo}(\eta, -q) \tag{21}$$

in which

$$P_1^2 + P_2^2 = -\beta^2 \tag{22}$$

$$P_1^2 P_2^2 = \frac{K}{D} \tag{23}$$

$$\lambda = -\frac{Eaf(h)}{D(1-\nu)} \frac{2\beta_1^2 A_0^{(0)} K_1}{C_{eo}(\xi_0, -q)} \tag{24}$$

$$\nabla^2 = \frac{2}{d^2(\cosh 2\xi - \cos 2\eta)}\left[\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2}\right] \tag{25}$$

Complimentary function of Eq. 21 is given by

$$\begin{aligned} W = {} & B_0 C_{eo}(\xi, -q_1)\, c_{eo}(\eta, -q_1) \\ & + D_0 C_{eo}(\xi, -q_2)\, c_{eo}(\eta, -q_2) \end{aligned} \tag{26}$$

in which

$$q_1 = \frac{P_1^2 d^2}{4}\ ;\ q_2 = \frac{P_2^2 d^2}{4} \tag{27}$$

Clearly the particular integral of Eq. 21 is

$$\frac{\lambda}{(\beta_1^2 - P_2^2)(\beta_1^2 - P_1^2)} C_{eo}(\xi, -q)\, c_{eo}(\eta, -q) \tag{28}$$

Thus the complete solution of Eq. 21 is

$$W = B_0 C_{eo}(\xi, -q_1)\, c_{eo}(\eta, -q_1) + D_0 C_{eo}(\xi, -q_2)\, c_{eo}(\eta, +q_2)$$

$$+ \frac{\lambda}{(\beta_1^2 - P_2^2)(\beta_1^2 - P_1^2)} C_{eo}(\xi, -q) c_{eo}(\eta, -q) \tag{29}$$

If the outer boundary of the plate $\xi = \xi_0$ be clamped, the boundary conditions are

$$(w)_{\xi=\xi_0} = \left(\frac{\partial w}{\partial \xi}\right)_{\xi=\xi_0} = 0 \tag{30}$$

Using Eq. 30 in Eq. 29 one gets the following two conditional equations

$$B_0 C_{eo}(\xi_0, -q_1)\, c_{eo}(\eta, -q_1) + D_0 C_{eo}(\xi_0, -q_2)\, c_{eo}(\eta, -q_2)$$

$$+ \frac{\lambda}{(\beta_1^2 - P_2^2)(\beta_1^2 - P_1^2)} C_{eo}(\xi_0, -q)\, c_{eo}(\eta, -q) = 0 \tag{31 a}$$

$$B_0 C'_{eo}(\xi_0, -q_1)\, c_{eo}(\eta, -q_1) + D_0 C'_{eo}(\xi_0, -q_2)\, c_{eo}(\eta, -q_2)$$

$$+ \frac{\lambda}{(\beta_1^2 - P_2^2)(\beta_1^2 - P_1^2)} C'_{eo}(\xi_0, -q)\, c_{eo}(\eta, -q) = 0 \tag{31 b}$$

Multiplying Eqs. 31 *a* and 31 *b* by $c_{eo}(\eta, -q_1)$ and integrating with respect to η from 0 to 2π and using the orthogonality relation and normalisation one gets

$$B_0 = -\frac{\lambda\phi_2}{\pi\psi\phi_3}\{ C_{eo}(\xi_0, -q_2)\, C'_{eo}(\xi_0, -q) - C_{eo}(\xi_0, -q)$$

$$C'_{eo}(\xi_0, -q_2)\} \tag{32}$$

$$D_0 = \frac{\lambda\phi_2}{\psi\phi_3\phi_1}\{ C_{eo}(\xi_0, -q_1)\, C'_{eo}(\xi_0, -q)$$

$$- C_{eo}(\xi_0, -q)\, C'_{eo}(\xi_0, -q_1)\} \tag{33}$$

in which

$$\psi = (\beta_1^2 - P_2^2)(\beta_1^2 - P_1^2)$$

$$\phi_1 = 2\bar{A}_0^{(0)} \bar{\bar{A}}_0^{(0)} + \sum_{r=1}^{\infty} \bar{A}_{2r}^{(0)} \bar{\bar{A}}_{2r}^{(0)}$$

$$\phi_2 = 2A_0^{(0)} \bar{A}_0^{(0)} + \sum_{r=1}^{\infty} \bar{A}_{2r}^{(0)} A_{2r}^{(0)}$$

$$\phi_3 = C_{eo}(\xi_0, -q_2)\, C'_{eo}(\xi_0, -q_1) - C_{eo}(\xi_0, -q_1)\, C'_{eo}(\xi_0, -q_2)$$

$\bar{A}_{2r}^{(0)}$, $\bar{\bar{A}}_{2r}^{(0)}$ and $A_{2r}^{(0)}$ are the Fourier Coefficients in the expansion of $ce_0(\eta, q_1)$, $ce_0(\eta, -q_2)$ and $ce_0(\eta, -q)$ respectively.

To determine the constant β^2, Eq. 8 is transformed into elliptic Co-ordinates in the form

$$h_1 h_2 \left\{\frac{\partial}{\partial \xi}\left(\frac{u_\xi}{h_2}\right) + \frac{\partial}{\partial \eta}\left(\frac{u_\eta}{h_1}\right)\right\} + \frac{1}{2} h_1 h_2 \left\{\left(\frac{\partial w}{\partial \xi}\right)^2 + \left(\frac{\partial w}{\partial \eta}\right)^2\right\}$$

$$= \frac{\beta^2 h^2}{12} + (1 + \nu)\, \alpha\, T_0 \qquad (34)$$

in which

$$h_1 = h_2 = \frac{1}{d\sqrt{\sinh^2 \xi + \sin^2 \eta}}$$

The boundary conditions for u_ξ and u_η are

$$u_\xi = 0 = u_\eta \text{ at } \xi = \xi_0 \qquad (35)$$

Let

$$u_\xi = \sum_{n=0}^{\infty} P(\xi) \cos 2n\eta \qquad (36)$$

$$u_\eta = \sum_{n=1}^{\infty} G(\xi) \sin 2n\eta \qquad (37)$$

subject to the conditions

$$P(\xi_0) = G(\xi_0) = 0$$

Substituting Eqs. 29, 36, and 37 in Eq. 34 and integrating over the surface of the plate one gets

$$\int_0^{2\pi}\int_0^{\xi_0} \left\{\left(\frac{\partial w}{\partial \xi}\right)^2 + \left(\frac{\partial w}{\partial \eta}\right)^2\right\} d\xi d\eta$$

$$= d^2 \left\{\frac{\beta^2 h^2}{6} + 2(1 + \nu)\alpha T_0\right\} \int_0^{2\pi}\int_0^{\xi_0} (\sinh^2 \xi + \text{Sin}^2 \eta)\, d\xi d\eta \qquad (38)$$

After evaluating the integrals the following equation leading to β is obtained.

$$B_0^2 \left[(2\{\bar{A}_0^{(0)}\}^2 + \sum_{r=1}^{\infty} \{\bar{A}_{2r}^{(0)}\}^2)\{\sum_{r=1}^{\infty} 4r^2 \{A_{2r}^{(0)}\}^2 \psi_1\right.$$

$$+ \sum_{\substack{r=1 \\ r \neq s}}^{\infty} \sum_{s=1}^{\infty} 2rs(-1)^r(-1)^s A'^{(0)}_{2r} A'^{(0)}_{2s} \psi_2\}$$

$$+ (\sum_{r=1}^{\infty} 4r^2 \{\bar{A}^{(0)}_{2r}\}^2) \{(A'^{(0)}_0)^2 \xi_0 + A'^{(0)}_0 \sum_{r=1}^{\infty} A'^{(0)}_{2r} (-1)^r \psi_5$$

$$+ \sum_{r=1}^{\infty} A'^{(0)}_{2r} \psi_4 + \tfrac{1}{2} \sum_{\substack{r=1 \\ r \neq s}}^{\infty} \sum_{s=1}^{\infty} (-1)^r(-1)^s A'^{(0)}_{2r} A'^{(0)}_{2s} \psi_3\}]$$

$$+ D_0^2 [(2\{\bar{\bar{A}}^{(0)}_0\}^2 + \sum_{r=1}^{\infty} \{\bar{\bar{A}}^{(0)}_{2r}\}^2) \{\sum_{r=1}^{\infty} 4r^2 \{A''^{(0)}_{2r}\}^2 \psi_1$$

$$+ \sum_{\substack{r=1 \\ r \neq s}}^{\infty} \sum_{s=1}^{\infty} 2rs(-1)^r(-1)^s A''^{(0)}_{2r} A''^{(0)}_{2s} \psi_2\}$$

$$+ (\sum_{r=0}^{\infty} 4r^2 \{\bar{\bar{A}}^{(0)}_{2r}\}^2) \{(A''^{(0)}_0)^2 \xi_0 + A''^{(0)}_0 \sum_{r=1}^{\infty} A''^{(0)}_{2r} (-1)^r \psi_5$$

$$+ \sum_{r=1}^{\infty} A''^{(0)}_{2r} \psi_4 + \tfrac{1}{2} \sum_{\substack{r=1 \\ r \neq s}}^{\infty} \sum_{s=1}^{\infty} (-1)^r(-1)^s A''^{(0)}_{2r} A''^{(0)}_{2s} \psi_3\}]$$

$$+ 2B_0 D_0 [(2\bar{A}^{(0)}_0 \bar{\bar{A}}^{(0)}_0 + \sum_{r=1}^{\infty} \bar{A}^{(0)}_{2r} \bar{\bar{A}}^{(0)}_{2r}) \{\sum_{r=1}^{\infty} 4r^2 A'^{(0)}_{2r} A''^{(0)}_{2r} \psi_1$$

$$+ \sum_{\substack{r=1 \\ r \neq s}}^{\infty} \sum_{s=1}^{\infty} 2rs(-1)^r(-1)^s A'^{(0)}_{2s} A''^{(0)}_{2r} \psi_2\}$$

$$+ (\sum_{r=1}^{\infty} 4r^2 \bar{A}^{(0)}_{2r} \bar{\bar{A}}^{(0)}_{2r}) \{A''^{(0)}_0 A'^{(0)}_0 \xi_0 + A''^{(0)}_0 \sum_{r=1}^{\infty} (-1)^r A''^{(0)}_{2r} \psi_5$$

$$+ A'^{(0)}_0 \sum_{r=1}^{\infty} (-1)^r A''^{(0)}_{2r} \psi_5 + \sum_{r=1}^{\infty} A''^{(0)}_{2r} A'^{(0)}_{2r} \psi_4$$

$$+ \tfrac{1}{2} \sum_{\substack{r=1 \\ r \neq s}}^{\infty} \sum_{s=1}^{\infty} (-1)^r(-1)^s A''^{(0)}_{2r} A'^{(0)}_{2s} \psi_3\}]$$

$$+ \frac{2B_0\lambda}{\psi} [(2A^{(0)}_0 \bar{A}^{(0)}_0 + \sum_{r=1}^{\infty} A^{(0)}_{2r} \bar{A}^{(0)}_{2r}) \{\sum_{r=1}^{\infty} 4r^2 A'^{(0)}_{2r} a^{(0)}_{2r} \psi_1$$

$$+ \sum_{\substack{r=1 \\ r \neq s}}^{\infty} \sum_{s=1}^{\infty} 2rs(-1)^r(-1)^s A'^{(0)}_{2s} a^{(0)}_{2r} \psi_2 + (\sum_{r=1} 4r^2 A^{(0)}_{2r} \bar{A}^{(0)}_{2r})$$

$$\times \{a^{(0)}_0 A'^{(0)}_0 \xi_0 + a^{(0)}_0 \sum_{r=1}^{\infty} (-1)^r A'^{(0)}_{2r} \psi_5 + A'^{(0)}_0 \sum_{r=1}^{\infty} (-1)^r a^{(0)}_{2r} \psi_5$$

$$+ \sum_{r=1}^{\infty} a^{(0)}_{2r} A'^{(0)}_{2r} \psi_4 + \tfrac{1}{2} \sum_{\substack{r=1 \\ r \neq s}}^{\infty} \sum_{s=1}^{\infty} (-1)^r(-1)^s a^{(0)}_{2r} A'^{(0)}_{2s} \psi_3\}]$$

$$+ \frac{2D_0\lambda}{\psi} [(2A_0^{(0)} \bar{\bar{A}}_0^{(0)} + \sum_{r=1}^{\infty} A_{2r}^{(0)} \bar{\bar{A}}_{2r}^{(0)}) \{ \sum_{r=1}^{\infty} 4r^2 A_{2r}^{\prime\prime(0)} a_{2r}^{(0)} \psi_1$$

$$+ \sum_{\substack{r=1 \\ r\neq s}}^{\infty} \sum_{s=1}^{\infty} 2rs (-1)^s (-1)^r A_{2p}^{\prime\prime(0)} a_{2r}^{(0)} \psi_2\} + (\sum_{r=1}^{\infty} 4r^2 A_{2r}^{(0)} \bar{\bar{A}}_{2r}^{(0)})$$

$$\times \{a_0^{(0)} A_0^{\prime\prime 0} \xi_0 + a_0^{(0)} \sum_{r=1}^{\infty} (-1)^r A_{2r}^{\prime\prime(0)} \psi_5 + A_0^{\prime\prime(0)} \sum_{r=1}^{\infty} (-1)^r a_{2r}^{(0)} \psi_5$$

$$+ \sum_{r=1}^{\infty} a_{2r}^{(0)} A_{2r}^{\prime\prime(0)} \psi_4 + \tfrac{1}{2} \sum_{\substack{r=1 \\ r\neq s}}^{\infty} \sum_{s=1}^{\infty} (-1)^r (-1)^s a_{2r}^{(0)} A_{2s}^{\prime\prime(0)} \psi_3\}]$$

$$+ \frac{\lambda^2}{\psi^2} [(2 \{A_0^{(0)}\}^2 + \sum_{r=1}^{\infty} \{A_{2r}^{(0)}\}^2) \{ \sum_{r=1}^{\infty} 4r^2 \{a_{2r}^{(0)}\}^2 \psi_1$$

$$+ \sum_{\substack{r=1 \\ r\neq s}}^{\infty} \sum_{s=1}^{\infty} 2rs (-1)^r (-1)^s a_{2r}^{(0)} a_{2s}^{(0)} \psi_2\} + (\sum_{r=1}^{\infty} 4r^2 \{A_{2r}^{(0)}\}^2)$$

$$\times \{(a_0^{(0)})^2 \xi_0 + a_0^{(0)} \sum_{r=1}^{\infty} a_{2r}^{(0)} (-1)^r \psi_5 + \sum_{r=1}^{\infty} a_{2r}^{(0)} \psi_4$$

$$+ \tfrac{1}{2} \sum_{\substack{r=1 \\ r\neq s}}^{\infty} \sum_{s=1}^{\infty} (-1)^r (-1)^s a_{2r}^{(0)} a_{2s}^{(0)} \psi_3\}]$$

$$= \frac{d^2}{2} \left\{ \frac{\beta^2 h^2}{6} + 2(1+\nu)\alpha\, T_0 \right\} \sinh 2\,\xi_0 \tag{39}$$

where

$$\psi_1 = \frac{\text{Sinh } 4r\xi_0}{8r} - \frac{\xi_0}{2}$$

$$\psi_2 = \frac{\text{Sinh } \overline{2r+2s}\,\xi_0}{2r+2s} - \frac{\text{Sinh } \overline{2r-2s}\,\xi_0}{2r-2s}$$

$$\psi_3 = \frac{\text{Sinh } \overline{2r+2s}\,\xi_0}{2r+2s} + \frac{\text{Sinh } \overline{2r-2r}\,\xi_0}{2r-2s}$$

$$\psi_4 = \frac{\xi_0}{2} + \frac{\text{Sinh } 4r\xi_0}{8r}$$

$$\psi_5 = \frac{\text{Sinh } 2r\,\xi_0}{2r}$$

and

$a_{2r}^{(0)}$, $A_{2r}^{\prime(0)}$, and $A_{2r}^{\prime\prime(0)}$ are the Fourier Coefficients in the expansions of $Ce_0(\xi, -q)$, $Ce_0(\xi, -q_1)$, and $Ce_0(\xi, -q_2)$ respectively.

Since β is determined, w is determined completely.

Numerical Calculation

To find the deflection at a given point, one has to start from Eq. 39 with an assumed value of β leading to the corresponding value of λ. With this value of λ and considering Eqs. 32 and 33 the deflection will be obtained from Eq. (29).

For numerical calculation the following values have been assumed:

$$\xi = 0,\ \eta = \frac{\pi}{2},\ \xi_0 = 3,\ d^2 = 2{\cdot}5,\ h = 1,\ f(h) = h,$$

$$K_F = \frac{K}{D}\ \xi_0^4 = 100,\quad \epsilon = 0{\cdot}03,\quad \nu = 0{\cdot}3,\quad \alpha T_0 = 2{\cdot}5 \times 10^{-3}.$$

The interfocal distance $2d$ being assumed and the values of β^2, P_1^2 and P_2^2 being known, the values of q, q_1 and q_2 are determined. q, q_1 and q_2 being

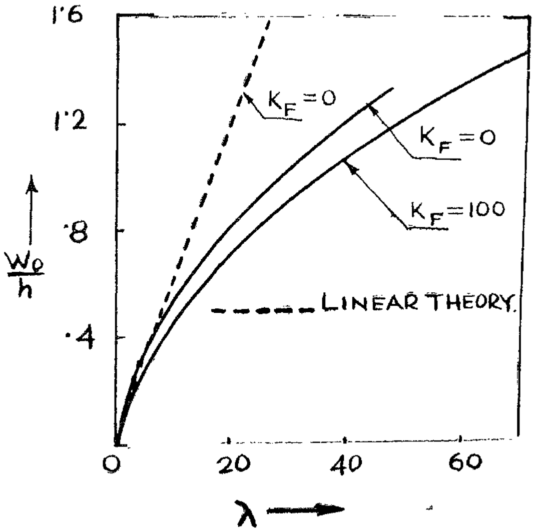

Fig. 1. Load-Deflection Curve

known the corresponding values of the Fourier Coefficients as well as those of Mathieu functions are determined. The maximum deflection W_0 is obtained at the centre of the plate. These deflections are graphically presented in Fig. 1 in which W_0/h for $K_F = 0$ and $K_F = 100$ are plotted against the non-dimensional load function λ. By setting $\beta \to 0$ the deflections according to the linear theory is obtained. For comparison Fig. 1 also includes a straightline which represents small deflections for $K_F = 0$. The results obtained in this study could not be compared in absence of any known results.

Conclusions

From Fig. 1 it is observed that the error according to the linear theory increases progressively with the increase in load function. Te solution proposed in this study is rapidly convergent and no computational difficulty other than computational effect is involved. The parameter q for the series $c_{eo}(\xi, q)$ may be real or imaginary and the corresponding coefficients can be computed with accuracy. The numerical results presented in this study are obtained by taking the first two terms of the series and sufficient for practical purposes. Since the deflection at any point is known the corresponding stresses can bew be easily estimated.

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