A DIGITAL KALMAN FILTER FOR RADAR TRACKING WITH FILTER GAINS INDEPENDENT OF SAMPLING INTERVAL

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ABSTRACT

In radar tracking using digital Kalman filter, it often becomes necessary to change the sampling rate. As the filter gains to be used in computation are dependent on the sampling rate, it becomes necessary to employ a large memory in the filter to store the time varying filter gains corresponding to each possible value of the sampling interval. In this paper, a method of reducing the total memory requirements and consequently, the number of computations, is proposed through a transformation which makes the filter gains independent of the sampling rate. A block diagram of the system for realizing the proposed digital filter using simple digital circuits such as adders, multipliers, shift registers, etc., which are well suited for integrated circuit (including both MSI and LSI) implementation, is also presented.

Key words: Digital filters, Digital signal processing, Radar tracking, Kalman fiilter, Reduced memory filter.

1. INTRODUCTION

The problem of radar tracking a large number of targets simultaneously is usually associated with assigning a suitable sampling rate for each target depending on its speed and strength of the return signal [1]. Even when tracking a single target, the sampling rate can be varied depending on its distance to the tracking radar. When a digital Kalman filter [2, 3] is employed for trajectory estimation a change in the sampling rate requires a corresponding change in the filter gains and these time varying filter gains have to be precomputed and stored in the memory of the filter, for each possible value of the sampling interval. However, this process can be simplified if the filter is modified in such a way that the time varying gains of the filter are made independent of the sampling interval. Then it is possible to choose any desired sampling interval depending on the targets to be tracked and use the same filter to process the radar signal at all sampling intervals. Therefore, in this paper a technique which makes the filter gains to be used in the radar signal processing independent of the sampling interval is described and the results of the computer simulation of the proposed digital filter are presented.

2. DIGITAL KALMAN FILTER FOR RADAR TRACKING

Suppose

$x_k = x(t = kT)$ is an n-dimensional state vector of the target, where T is the sampling interval

and

y_k is a q-dimensional measurement vector,

with

u_k as a p-dimensional process noise vector,

and

v_k as a q-dimensional measurement noise vector, then the process can be described by the following two equations (2, 3)

$$x_{k+1} = F_k x_k + G_k u_k \tag{1}$$

$$y_{k+1} = H_{k+1} x_{k+1} + v_{k+1} \tag{2}$$

where F_k, G_k and H_k are matrices of appropriate dimensions.

Then the optimum estimate $\hat{x}_{k+1}$, of x_{k+1} is given by

$$\hat{x}_{k+1} = F_k \hat{x}_k + K_{k+1}(y_{k+1} - H_{k+1} F_k \hat{x}_k) \tag{3}$$

A block diagram of the system for the implementation of the above equation is shown in Fig. 1.

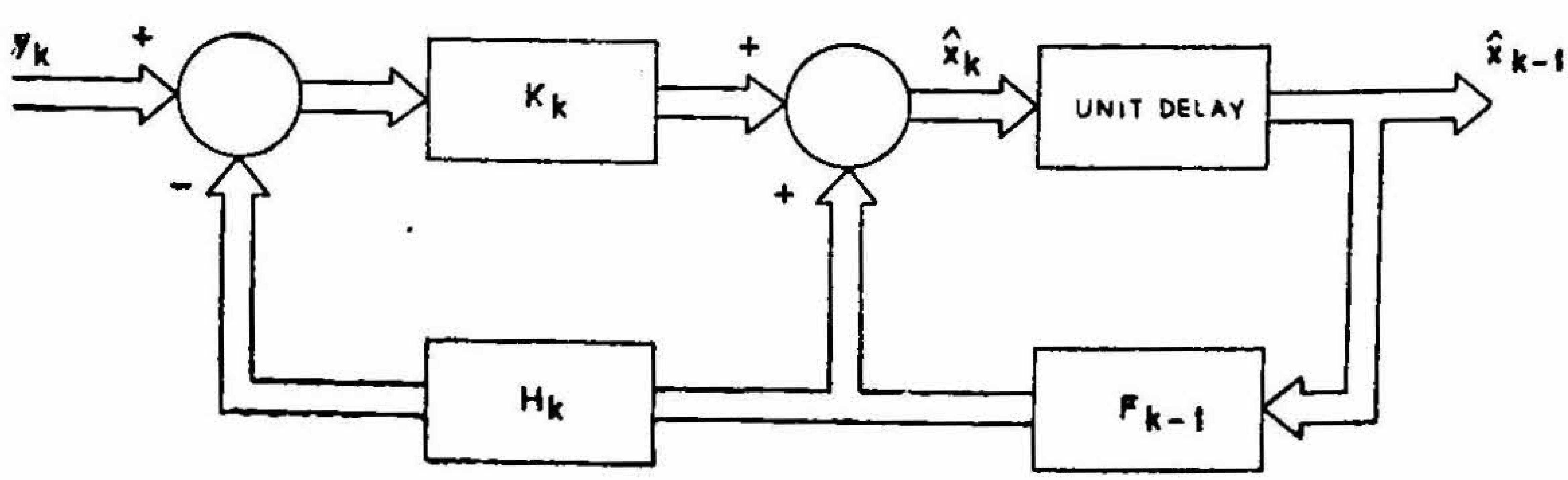

FIG. 1. Matrix block diagram for the digital Kalman filter.

The time-varying filter gains K_{k+1} are given by

$$K_{k+1} = M_{k+1}H_{k+1}\,[H_{k+1}M_{k+1}H'_{k+1} + R_{k+1}]^{-1} \tag{4}$$

with

$$M_{k+1} = F_kP_kF'_k + G_kQ_kG'_k \tag{5}$$

and

$$P_k = [I - K_kH_k)] \cdot M_k \tag{6}$$

where I is an identity matrix of order n and, Q_k and R_k are the covariance matrices of the process noise u_k and the measurement noise v_k respectively which are assumed to be two independent normal random variables with zero mean. It can be seen from the block diagram that the output $F_k\hat{x}_k$ of the unit F_k is the 'extrapolated' state of the target before the observation y_{k+1} is made. Similarly, the output $H_{k+1}F_k\hat{x}_k$ of the unit H_k is the 'extrapolated' observation.

In deriving a model for the process, it can be observed that a second degree curve fitted to the target trajectory would give sufficiently accurate estimates of the parameters of the trajectory [1]. With this in view, the mathematical model for the trajectory of a target can be described by equations (1) and (2) in which

$$F_k = F = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$G_k = G = g = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{8}$$

and

$$H_k = H = h = [\,1 \quad 0 \quad 0\,]. \tag{9}$$

It should be noted that the equations (1), (2), (7), (8) and (9) imply that the three dimensional state vector x_k consists of position, velocity and acceleration of the target with respect to the radar at $t = kT$. u_k is the scalar process noise and v_k is the scalar measurement noise. y_k is the scalar measurement of the position of the target.

Equation (7) shows that the matrix F depends on the sampling interval T. Therefore, whenever the sampling interval T is changed, the time varying filter gains will have to be recomputed. Consequently, if the digital filter is to be used for several different sampling intervals, the filter gains have to be precomputed for all possible values of the sampling interval and stored in the memory of the computer. Naturally such a situation calls for increased memory requirements for the digital filter and correspondingly the number of mathematical operations to be performed increases. However this situation can be overcome by normalizing the filter gains with respect to the sampling interval T, as described below.

3. NORMALIZATION OF THE FILTER GAINS WITH RESPECT TO SAMPLING INTERVAL

Suppose a new variable z_k is defined as

$$z_k = D\, x_k, \tag{10}$$

where

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T^2/2 \end{bmatrix} \tag{11}$$

with the transformation of state variables defined in (10), equations (1) and (2) with (7), (8) and (9) can now be expressed in terms of z_k as follows.

$$\left.\begin{aligned} z_{k+1} &= DFD^{-1} z_k + Dgu_k \\ &= \hat{F} z_k + D_g u_k \end{aligned}\right\} \tag{12}$$

and

$$\left.\begin{aligned} y_{k+1} &= hDz'_{k+1} + v_k \\ &= \hat{h} z_{k+1} + v_k \end{aligned}\right\} \tag{13}$$

Simple calculations show that

$$\hat{F} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

$$\hat{h} = h = [\,1 \quad 0 \quad 0\,] \tag{15}$$

$$\hat{g} = \begin{bmatrix} 0 \\ 0 \\ T^2/2 \end{bmatrix}. \tag{16}$$

It is to be noted that with the above transformation of state variables, $\hat{F}$ has become independent of T, while retaining the independency of $\hat{h}$ with respect to T. However, the matrix $\hat{g}$ is still a function of T. In order to seek the advantage of having the filter gains independent of T, it is necessary that $\hat{g}$ also does not depend upon T. Therefore it is suggested that $\hat{g}$ is arbitrarily replaced by

$$g = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Under the above assumption $\hat{Q} = Q$, and the filter gains are made independent of the sampling interval T; This procedure implies that when $T^2/2 < 1$, the process noise is arbitrarily assumed to be larger than what it really is and that when $T^2/2 > 1$, the process noise is arbitrarily assumed to be smaller than what it really is. Whether or not the above assumption is justifiable can be judged based on the performance of the modified (normalized) filter equations as compared with that of the conventional one. Therefore, the performance of the two filters were studied (by simulating them on a digital computer) under different noise conditions and for several different values of the sampling interval T. The results of such a study are discussed in the next section.

4. Results of Computer Simulation

For the purpose of obtaining radar positional measurements, an aircraft flying on a straight line trajectory, with a constant offset of 1,000 ft from the ground and with a constant velocity of 880 ft/sec was considered as a first example. The radar was assumed to be stationed on the ground in the vertical plane along the straight line trajectory of the aircraft.

Measurement noise having mean zero and a variance of $(15\text{ ft})^2$ and $(25\text{ ft})^2$ were injected into the measurements. A Process noise having mean zero and a standard deviation ranging from 1 ft/sec/sec upto 5 ft/sec/sec were considered during the simulation of the trajectory in order to get positional range measurements.

The recursive equations of the conventional digital Kalman filter and also of the modified digital Kalman filter were programmed on a digital computer (IBM 360/44). The experiment was repeated for cases when the positional measurement was range, X-coordinate and elevation angle; the injected noises being appropriately changed. In each of the above cases

TABLE I

Performance of digital Kalman filter—Conventional

Assumed Initial Covariance Matrix is given below:

100·000	0·0	0·0
0·0	50·000	0·0
0·0	0·0	50·000

Assumed Initial State Variables are:

88015·000	0·0	0·0

Variance of Input Noise = 1·0000; Variance of Measurement Noise = 225·0000; Sampling Interval in Seconds = 0·500; Number of Samples = 25.

These are Filter Gains

K	GK1K	GK2K	GK3K
2	0·3349	0·0831	0·0185
3	0·3363	0·1865	0·0671
4	0·3989	0·3008	0·1182
5	0·4699	0·3779	0·1440
6	0·5122	0·3998	0·1430
7	0·5243	0·3862	0·1291
8	0·5187	0·3594	0·1129
9	0·5055	0·3309	0·0984
10	0·4901	0·3050	0·0865
11	0·4749	0·2828	0·0772
12	0·4609	0·2643	0·0699
13	0·4484	0·2492	0·0644
14	0·4377	0·2369	0·0602
15	0·4286	0·2273	0·0572
16	0·4211	0·2199	0·0550
17	0·4151	0·2144	0·0536
18	0·4105	0·2105	0·0527
19	0·4071	0·2078	0·0522
20	0·4047	0·2061	0·0519
21	0·4030	0·2051	0·0518
22	0·4020	0·2045	0·0517
23	0·4014	0·2043	0·0518
24	0·4010	0·2041	0·0518
25	0·4009	0·2040	0·0518

TABLE I—(*Contd.*)

K	Actual, Range Feet X (K)	Estimated Range Feet XH1K	Estimated Velocity Feet/Second XH2K	Estimated Acceleration Feet/Second/ Second XH3K	One Step Prediction Range Error Feet
2	87565·625	87861·500	− 38·099	− 8·467	−458·250
3	87125·563	87594·313	− 179·351	− 57·785	−734·625
4	86685·438	87168·000	− 456·616	−155·384	−825·750
5	86245·563	86592·000	− 798·296	−255·984	−698·500
6	85805·688	85953·875	−1087·781	−313·738	−403·938
7	85365·750	85360·813	−1251·891	−316·158	− 18·750
8	84925·875	84810·250	−1330·251	−291·123	221·813
9	84485·438	84282·000	−1362·339	−257·387	342·938
10	84045·813	83795·688	−1349·682	−217·286	463·438
11	83606·000	83325·625	−1320·128	−179·574	488·625
12	83165·813	82875·313	−1276·694	−144·332	504·000
13	82725·875	82440·063	−1225·964	−112·578	493·250
14	82285·938	82022·688	−1168·704	− 83·719	479·250
15	81845·938	81605·375	−1116·383	− 60·025	414·375
16	81405·938	81194·375	−1065·579	− 39·795	367·563
17	80966·125	80775·188	−1024·207	− 24·478	285·813
18	80526·000	80361·500	− 984·398	− 11·449	247·313
19	80086·188	79945·000	− 950·705	− 1·557	189·688
20	79646·188	79531·313	− 919·924	6·387	153·125
21	79206·250	79124·625	− 889·964	13·143	130·500
22	78766·313	78722·000	− 862·657	18·388	101·375
23	78326·438	78302·813	− 848·382	19·676	24·875
24	77886·375	77887·688	− 835·150	20·536	61·625
25	77446·250	77462·563	− 829·995	19·239	25·063

different noise conditions and several different sampling intervals were considered. The results of computer simulation showed that the performance of the modified digital filter was never inferior to that of the conventional digital filter; the basis for comparison was the accuracy of estimating the positional, velocity and acceleration measures of the target and also the accuracy of predicting the positional measurement one sampling interval ahead. Typical computer results obtained for both the conventional digital

TABLE II

Performance of digital Kalman filter—Normalised with respect to sampling interval

Assumed Initial Covariance Matrix is given below:

100·000	0·0	0·0
0·0	50·000	0·0
0·0	0·0	50·000

Assumed Initial State Variables are:

88015·000	0·0	0·0

Variance of Input Noise = 1·000; Variance of Measurement Noise = 225·0000; Sampling Interval in Seconds = 0·500; Number of Samples = 25. These are Filter Gains.

K	GK1K	GK2K	GK3K
2	0·4194	0·1935	0·0645
3	0·6010	0·4184	0·1496
4	0·7105	0·4657	0·1438
5	0·7157	0·4169	0·1120
6	0·6884	0·3615	0·0869
7	0·6570	0·3171	0·0701
8	0·6289	0·2839	0·0590
9	0·6055	0·2598	0·0520
10	0·5872	0·2433	0·0479
11	0·5740	0·2328	0·0457
12	0·5653	0·2270	0·0447
13	0·5602	0·2242	0·0444
14	0·5602	0·2242	0·0444
15	0·5568	0·2230	0·0445
16	0·5565	0·2230	0·0446
17	0·5563	0·2229	0·0446
18	0·5562	0·2227	0·0445
19	0·5560	0·2226	0·0445
20	0·5559	0·2224	0·0445
21	0·5557	0·2223	0·0444
22	0·5556	0·2223	0·0444
23	0·5556	0·2223	0·0444
24	0·5556	0·2222	0·0444
25	0·5556	0·2222	0·0444

TABLE II—(*Contd.*)

K	Actual Range Feet X (K)	Estimated Range Feet XH1K	Estimated Velocity Feet/Second XH2K	Estimated Acceleration Feet/Second/ Second XH3K	One Step Range Prediction Error Feet
2	87565·625	87822·813	− 177·387	−118·258	−458·250
3	87125·563	87351·063	− 749·106	−484·789	−612·500
4	86685·438	86742·313	−1218·999	−625·292	−244·250
5	86245·563	86174·188	−1392·282	−550·421	167·125
6	85805·688	85648·500	−1416·142	−429·552	347·688
7	85365·750	85192·313	−1335·818	−299·163	465·313
8	84925·875	84757·375	−1241·212	−197·595	430·125
9	84485·438	84317·625	−1163·543	−126·898	339·625
10	84045·813	83903·188	−1075·130	− 67·125	312·125
11	83606·000	83486·313	−1003·895	− 26·019	225·063
12	83165·813	83074·875	− 941·552	3·657	166·000
13	82725·875	82664·750	− 891·462	22·778	107·625
14	82285·938	82261·063	− 848·653	35·287	70·375
15	81845·938	81841·688	− 830·507	35·487	1·125
16	81405·938	81417·625	− 823·299	31·277	− 23·625
17	80966·125	80972·250	− 837·725	19·258	− 67·438
18	80526·000	80528·813	− 849·674	10·633	− 48·438
19	80086·188	80078·688	− 865·612	2·137	− 47·750
20	79646·188	79632·938	− 875·052	− 2·064	− 23·625
21	79206·250	79199·188	− 872·749	− 0·731	7·500
22	78766·313	78773·750	− 864·251	2·814	19·928
23	78326·438	78328·500	− 873·595	− 1·487	− 24·188
24	77886·375	77894·938	− 871·587	− 0·386	6·188
25	77446·250	77452·625	− 876·892	− 2·431	− 11·500

Kalman filter and for the modified digital Kalman filter are given in Tables I and II. Figures 2 and 3 show graphically the range estimation error and one step range prediction error of the two filters. It can be clearly seen that the modified filter is better than the conventional filter.

The effect of changing the sampling interval on the performance of both the conventional and modified digital filters was also studied. It was found that for sampling intervals less than 0·05 sec the performance of

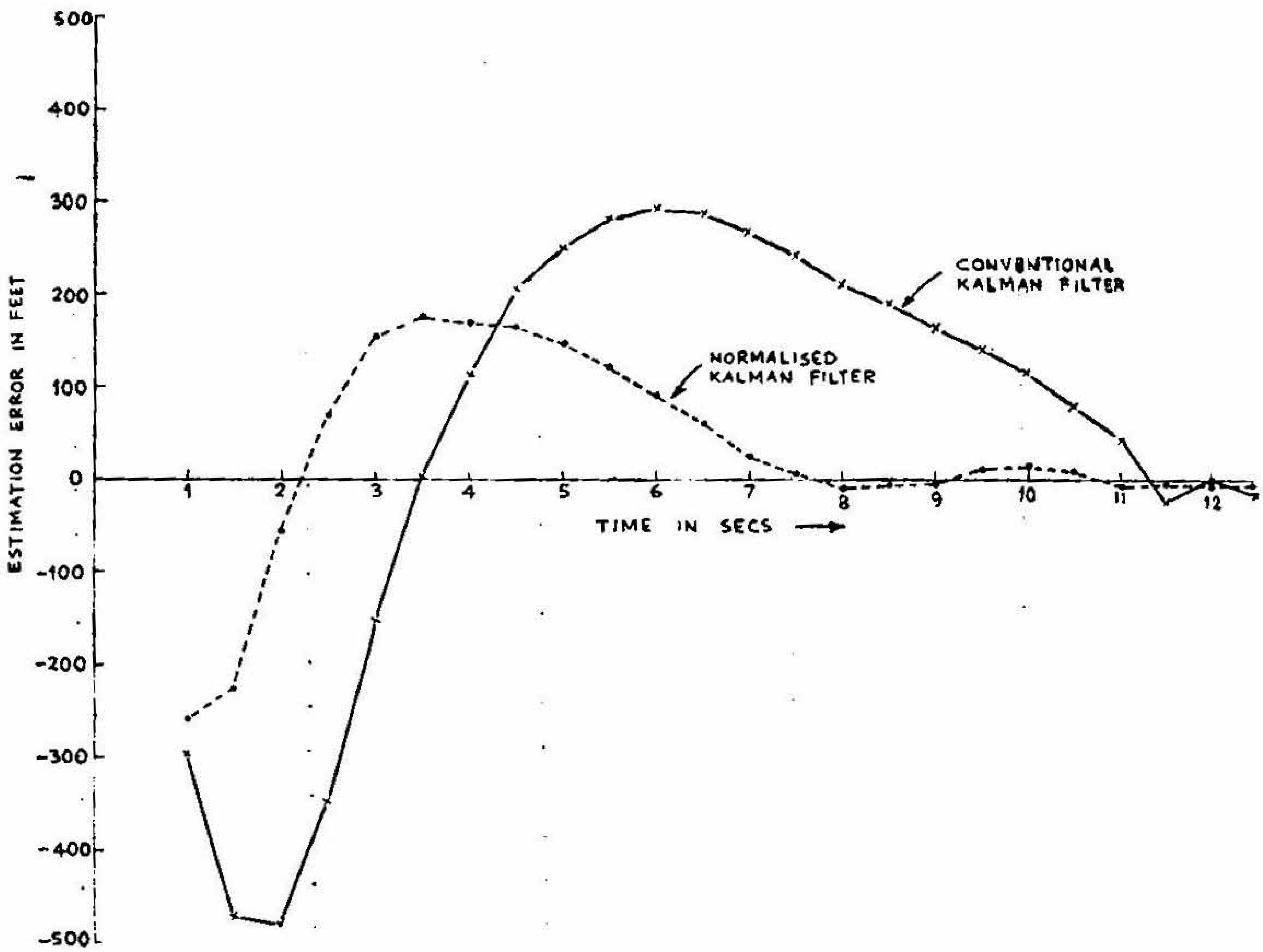

FIG. 2. Comparison of the performances of the conventional and normalised digital Kalman filters—PLOTS OF RANGE ESTIMATION ERROR.

both the filters were very poor. This poor performance can probably be attributed to the small value of signal to noise ratio that would result at low values of sampling intervals. A theoretical justification for such a behaviour is under study.

In the above experiments a straight line target with a constant velocity was considered only as a first example. However, the filter equations and the transformation suggested do not require that the acceleration be zero. As a second example, therefore, the target was assumed to take a constant 2-*g* turn after moving a certain distance along the straight line course. It is to be noted that in this case the velocity is no longer constant. The performances of the two filters were once again studied (by simulating them on a digital computer) under different noise conditions and for several different values of the sampling interval *T*. The results revealed that the modified digital filter behaved in the same manner as in the previous case.

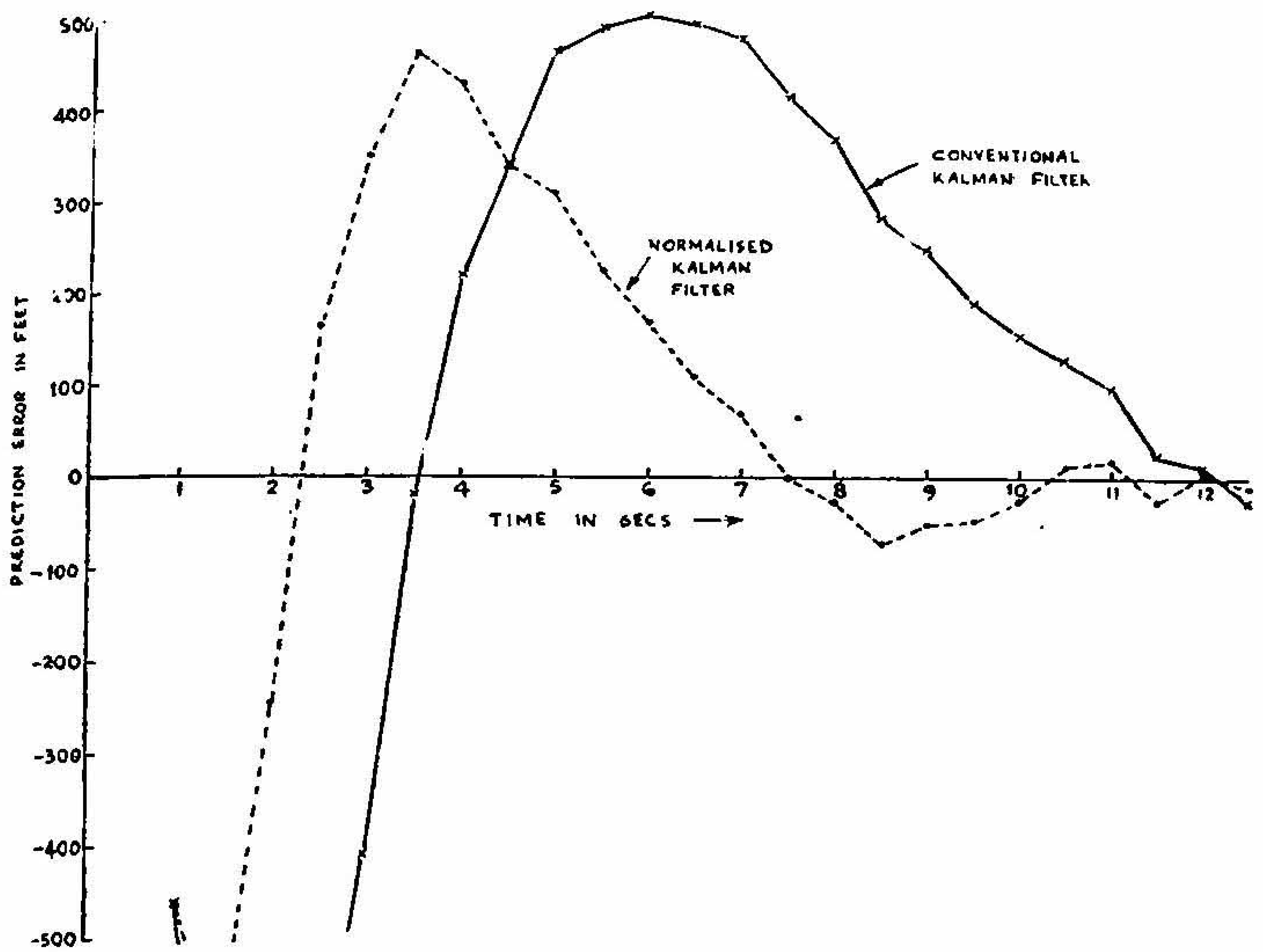

FIG. 3. Comparison of the performances of the conventional and normalised digital Kalman filters—PLOTS OF ONE-STEP RANGE PREDICTION ERROR.

Based on these studies, one can arrive at the conclusion that the modifications and assumptions suggested in the previous section would indeed help one to realize a more economical digital filter for radar tracking applications.

5. THE PROPOSED SYSTEM

The block diagram of the proposed digital Kalman filter, which can be easily constructed using ICs, is indicated in Fig. 4. y_k is the input to the system and represents the radar measurement obtained at the k-th sampling instant. $\hat{z}_{1k}$, $\hat{z}_{2k}$ and $\hat{z}_{3k}$ are the outputs of the system which are scaled down by a factor of 1, T and $T^2/2$ respectively, and correspond to the range ($\hat{x}_{1k} = \hat{z}_{1k}$), velocity ($\hat{x}_{2k} = \hat{z}_{2k}/T$), and acceleration ($\hat{x}_{3k} = 2\hat{z}_{3k}/T^2$) of the target at that instant. The time varying filter gains, K_{1k}, K_{2k} and K_{3k}, which are independent of the sampling interval, are precomputed and stored in the memory of the filter.

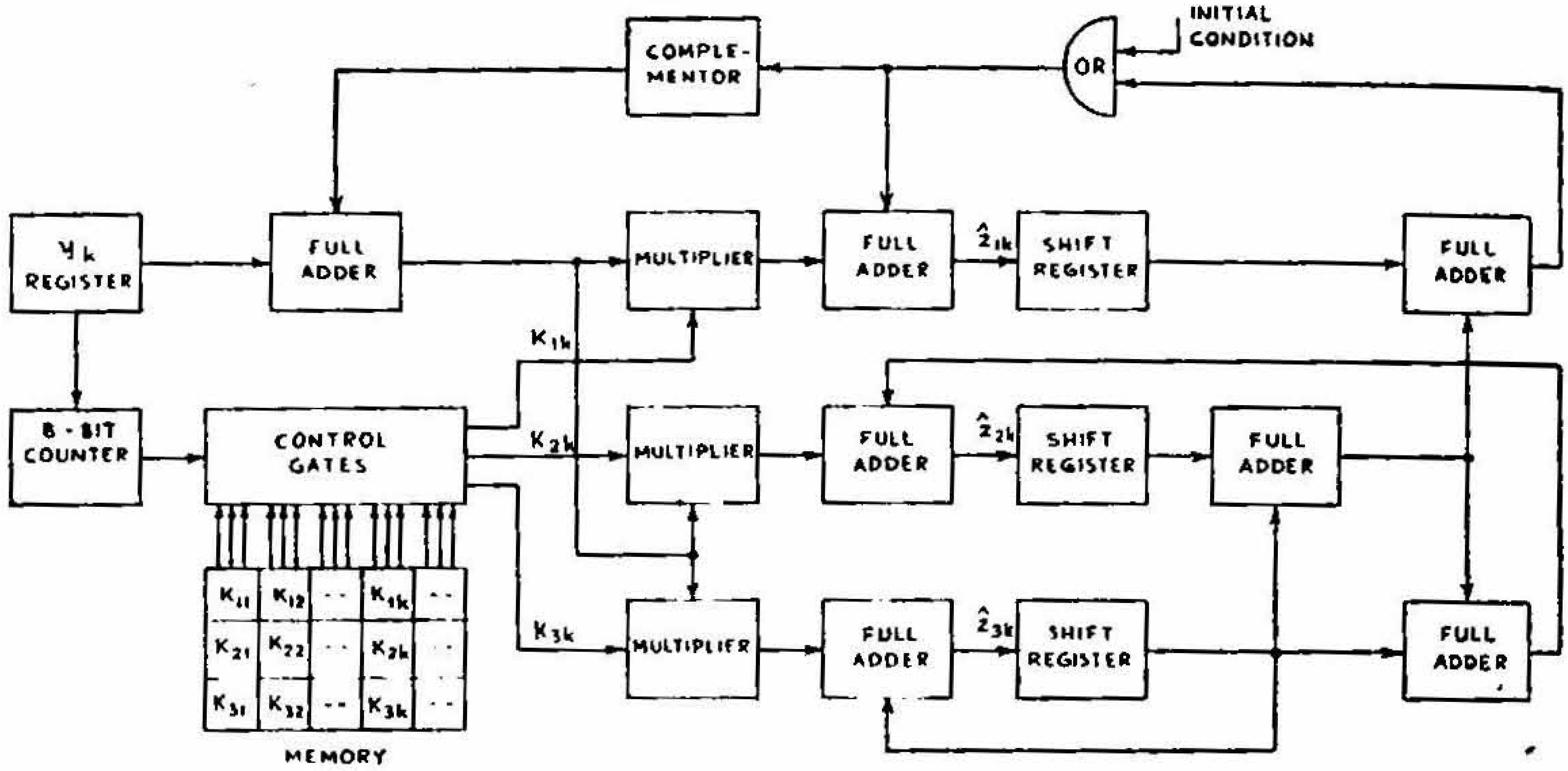

FIG. 4. Block diagram for realization of the modified digital Kalman filter.

The proposed system is designed for the serial arithmetic operation which reduces both complexity of the circuitry involved and the hardware requirements considerably. Keeping in view both the capacity of the available ICs (including both MSIs and LSIs) and the required accuracy in the computation, a word length of 16 bits (the most significant bit being reserved for the sign) has been chosen for the arithmetic operation. This gives an accuracy of about 16 feet in computation for a range of 100 miles. The subtraction is carried out by the two's complement addition which also automatically takes account of the sign of the numbers being added [4]. The counter and the control gates select the appropriate numbers from the memory depending on the sampling instant kT, and feed to the multiplier. The multiplication is performed using accumulators by the shifting and adding process [5]. The shift registers provide the required unit bit delays.

6. CONCLUSION

It has been shown in this paper that the proposed transformation simplifies the conventional digital Kalman filter through a smaller memory, and less number of computations. Thus, it is possible to realise a more economical digital filter for radar tracking with provision for changing the sampling intervals as desired.

7. ACKNOWLEDGEMENT

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