

ON THE TRANSFORM METHOD OF SOLUTION OF THE PROBLEM OF A GRIFFITH CRACK AT THE INTERFACE OF AN ELASTIC HALF-PLANE AND A RIGID FOUNDATION

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ABSTRACT

The stress and displacement fields are determined in a semi-infinite elastic media bonded to a rigid foundation, containing a crack at the interface. The elastic medium is assumed to be under shear. The problem has been solved in closed form within the linear theory of elasticity, assuming plain strain conditions to hold good. The well-known Fourier Transform method has been applied to reduce the mixed boundary value problem to a simultaneous set of dual integral equations involving trigonometric kernels. The set of dual equations have been solved by the usual technique of solving such equations and the displacement and stress field have been calculated from the present solution of these dual equations. It is observed, as usual, that the solution yields an oscillatory phenomenon near the ends of the crack and thus the present method of solution of the simultaneous set of dual equations gives a right answer to the question of validity of the Transform method of solving such crack problems, first asked by Erdogan [1]. The technique of solving the set of simultaneous dual equations is general and can be applied even to the set that arises while solving the same crack problem when the crack is opened by an equal and opposite pressure.

Key words: Transform Method, Griffith Crack, Simultaneous set of dual integral equations; Abel integral Equations; Riemann Hilbert Problem.

1. INTRODUCTION

In 1968 Erdogan [1] tackled the problem of an even number of cracks at the interface of two bonded dissimilar half-planes by the method of Fourier transforms. The physical problem at hand was reduced by Erdogan to a set of simultaneous dual integral equations involving trigonometric kernels. The equations of Erdogan are of the form:

$$\begin{aligned} \frac{2}{\pi} \int_0^{\infty} P(\zeta) \cos(y\zeta) d\zeta &= p_0(y), \quad (y \in L') \\ \frac{2}{\pi} \int_0^{\infty} Q(\zeta) \sin(y\zeta) d\zeta &= q_0(y), \quad (y \in L') \\ \frac{1}{\pi} \int_0^{\infty} [a_{11}P(\zeta) - a_{12}Q(\zeta)] \sin(y\zeta) d\zeta &= g_1(y), \quad (y \in L) \\ \frac{1}{\pi} \int_0^{\infty} [a_{21}P(\zeta) - a_{22}Q(\zeta)] \cos(y\zeta) d\zeta &= g_2(y), \quad (y \in L) \end{aligned} \quad (1)$$

where $L' = L_1' + L_2' + \dots + L_k'$, L' s being the cracks occupying portions of the half-line $0 \leq y < \infty$ and L is the complement of L' on $0 \leq y < \infty$. The infinite integrals in (1) and (2) are understood as:

$$\lim_{x \rightarrow 0^+} \int_0^{\infty} A(\zeta) e^{-x\zeta} \frac{\cos(\zeta y)}{\sin(\zeta y)} d\zeta = \int_0^{\infty} A(\zeta) \frac{\cos(\zeta y)}{\sin(\zeta y)} d\zeta$$

and the constants a_{11} , a_{12} , a_{21} , a_{22} are the bielastic constants depending on the materials of the two half-planes.

Erdogan next reduced the simultaneous equations (1) and (2) for the functions $P(\zeta)$ and $Q(\zeta)$ to a simultaneous singular integral equations with Cauchy type kernel for two other functions $p(t)$ and $q(t)$ defined by:

$$\begin{aligned} P(\zeta) &= \int_0^{\infty} p(t) \cos(\zeta t) dt, \\ Q(\zeta) &= \int_0^{\infty} q(t) \sin(\zeta t) dt. \end{aligned} \quad (3)$$

The system of singular integral equations was finally solved by the well-known technique of Mushkhelishvili [2] and the solution of these equations gave directly the behaviour of the functions $p(y)$ and $q(y)$, which, in Erdogan's notation represent the components of the stress on the interface of the two media. It was shown ultimately that the stresses at the tips of the cracks behave as:

$$\sigma_{ij} \sim A_{ij}(a) s^{-1/2} \frac{\sin}{\cos} \left(\omega \log \frac{s}{a} \right), \quad (4)$$

where σ_{ij} ($i, j = 1, 2$) are the stress components, s , a are polar coordinate referred to a tip as origin and a is a characteristic length. As the method

of solution outlined by Erdogan does not need calculating the inverse Fourier transforms under consideration to compute the quantities of physical interest, he had rightly raised a question about the legitimacy of the use of integral transforms in such problems.

Recently Lowengrub [3] has solved the problem, same as the one that has been considered in this paper by the application of Fourier transforms and has claimed that transform methods do work well in such problems and the question raised by Erdogan [1] may be ignored. But, it may be worth pointing out here that, the method of solving the set of simultaneous dual integral equations of the problem as formulated by Lowengrub is once again the same method as the one which was outlined by Erdogan [1] and thus Erdogan's question has not been answered by Lowengrub [3]. Moreover, it may again be worth noting that there are several major and minor errors in Lowengrub's [3] method of solution of the problem which has been retackled in the preprint paper cf. formulac (3.15) [3], (3.16) [3], (3.19) [3], (3.21) [3] and so on. The Hilbert problem (3.16) in 3 is not the actual one for the physical problem under consideration and this has to be discarded. In this paper, we have handled the same problem of Lowengrub [3] in a different manner and tried to answer Erdogan's question in the right direction.

Here we consider the same problem of Lowengrub, *viz.*, the problem of a crack at the interface of an elastic half-plane bonded to a rigid foundation, when the elastic half plane is under constant shear at ∞ , and reduce the problem to the same set of dual integral equations *via* Fourier transforms as in [3]. This set of dual integral equations has been reduced to simultaneous Abel integral equations (cf. Gakhov [4]) by a technique that is usually used for solving dual integral equations (cf. [5], [6], [7]). Finally, the simultaneous Abel equations have been reduced to simultaneous Riemann Hilbert problems for sectionally analytic functions and have been solved by the technique of Mushkhelishvili [2].

It is observed that the displacement and stress near the ends of the crack behave in the same oscillatory fashion as was shown by Erdogan [1] for a more general problem and by Lowengrub [3] for the present problem by the method of integral transforms. It is emphasized that the present method of solution of the simultaneous dual integral equations in question is not the same as that of Erdogan [1] or Lowengrub [3], even though the calculation of the quantities of physical interest does not need obtaining the inversions of various integral transforms under consideration directly.

The method is general in character and a similar method is applicable for similar kind of three-dimensional axi-symmetric problems also. The corresponding axi-symmetric problems will be investigated shortly. Thus, it is hoped that the question of Erdogan [1] has been answered here in the right direction as he felt and hinted at the end of the conclusion of his paper [1]. The answer to Erdogan's question can now be given in the following words:

"Yes, the transform methods of solving the problems of cracks at the interface of two dissimilar elastic media are fully legitimate and the methods do give the same physically inadmissible solutions if the problems are treated under the theory of linear elasticity as has been pointed out by England [8]."

2. STATEMENT OF THE PROBLEM AND FORMULATION OF THE SYSTEM OF SIMULTANEOUS DUAL INTEGRAL EQUATIONS

The problem under consideration is that of determining the stress field when the elastic half-plane is under constant shear P at infinity.

The boundary conditions on the boundary $y = 0$ of the elastic half-plane $y \geq 0$, with the crack $-1 \leq x \leq 1$, can be expressed as: (cf. Lowengrub [3]) [see figure 1 for coordinate system].

$$\begin{aligned}\sigma_{yy}(x, 0) &= 0, \quad |x| \leq 1 \\ \sigma_{xy}(x, 0) &= -P, \quad |x| \leq 1 \\ u_x(x, 0) = u_y(x, 0) &= 0, \quad |x| > 1\end{aligned}\quad (2.1)$$

with the components of stress all vanishing at infinity.

Then following Lowengrub [3] and Sneddon [9], the displacements and stresses on $y = 0$ are expressed, in terms of Fourier Transforms, as:

$$u_x(x, 0) = (1 + \eta)(3 - 4\eta) \frac{P}{E} F_c [A(\xi); x], \quad (2.2)$$

$$u_y(x, 0) = (1 + \eta)(3 - 4\eta) \frac{P}{E} F_s [B(\xi); x], \quad (2.3)$$

$$\sigma_{yy}(x, 0) = -P \frac{d}{dx} F_c [(1 - 2\eta)A(\xi) + 2(1 - \eta)B(\xi); x] \quad (2.4)$$

$$\sigma_{xy}(x, 0) = P \frac{d}{dx} [x - F_s \{2(1 - \eta)A(\xi) + (1 - 2\eta)B(\xi); x\}]. \quad (2.5)$$

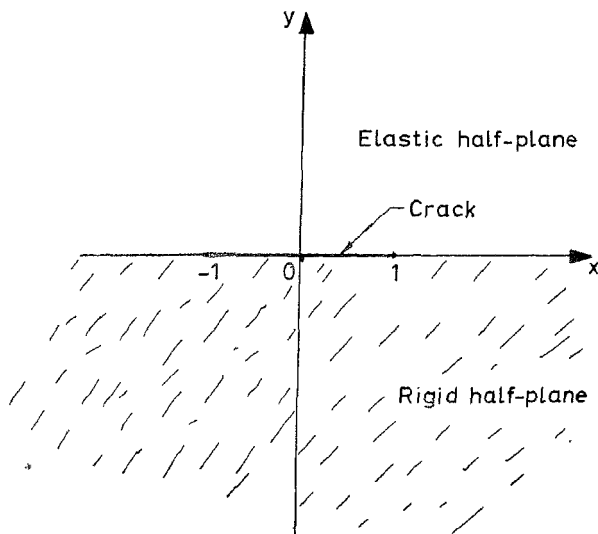


FIG. 1.

Here η is the Poisson's ratio and E is the Young's modulus of the elastic half-plane.

The definition of the Fourier transforms have been taken as:

$$F_c [A(\xi); x] = \int_0^{\infty} A(\xi) \cos(\xi x) d\xi$$

and

$$F_s [A(\xi); x] = \int_0^{\infty} A(\xi) \sin(\xi x) d\xi. \quad (2.6)$$

It is easily verified that the expressions (2.2)–(2.5) will satisfy the boundary conditions (2.1) if $A(\xi)$ and $B(\xi)$ are solutions of the following simultaneous system of dual integral equations:

$$F_s [a_1 A(\xi) + a_2 B(\xi); x] = x, \quad 0 < x < 1 \quad (2.7)$$

$$F_c [a_2 A(\xi) + a_1 B(\xi); x] = 0, \quad 0 < x < 1 \quad (2.8)$$

$$F_c [A(\xi); x] = 0, \quad x > 1 \quad (2.9)$$

$$F_s [B(\xi); x] = 0, \quad x > 1 \quad (2.10)$$

where $a_1 = 2(1 - \eta)$ and $a_2 = 1 - 2\eta$.

A similar set of dual integral equations arise in the case when $\sigma_{xy}(x, 0) = -q(x)$, in which case the right hand side of (2.7) is replaced by $Q(x)$ where $Q'(x) = q(x)$.

One may see the formulation of similar set of dual integral equations in the case when the crack is opened by equal and opposite pressure, in [3].

In the next section, we shall deal with the solution of the set of equations (2.7)–(2.10) in detail whilst a similar technique is applicable to the case when the crack is opened by pressure.

3. SOLUTION OF THE SET OF DUAL INTEGRAL EQUATIONS

To solve the set of equations (2.7)–(2.10), we assume, as is usually done (cf. [5], [6], [7]), that $A(\xi)$ and $B(\xi)$ are given in terms of two other unknown continuous functions $g_1(t)$ and $g_2(t)$, as:

$$A(\xi) = \int_0^1 g_1(t) J_0(\xi t) dt,$$

and

$$B(\xi) = \int_0^1 g_2(t) \left(\int_0^{\xi} J_0(ut) du \right) dt, \quad (3.1)$$

where $J_n(x)$ is Bessel's function of the first kind of order n .

The assumptions of $A(\xi)$ and $B(\xi)$ in these forms are the results of suggestions from a paper of Jones [10].

Then, substituting (3.1) in (2.7)–(2.10), and interchanging the orders of integration, we see that the equations (2.9) and (2.10) are automatically satisfied and the equations (2.7) and (2.8) give rise to the following simultaneous system of Abel's integral equations:

$$a_1 x \int_0^x \frac{g_1(t) dt}{\sqrt{(x^2 - t^2)}} + a_2 \int_x^1 \frac{g_2(t) dt}{\sqrt{(t^2 - x^2)}} = x^2 \quad (3.2)$$

and

$$(0 < x < 1)$$

$$a_2 x \int_x^1 \frac{g_1(t) dt}{\sqrt{(t^2 - x^2)}} - a_1 \int_0^x \frac{g_2(t) dt}{\sqrt{(x^2 - t^2)}} = 0. \quad (3.3)$$

In the above manipulations, the following results have been used, which will be necessary in the sequel:

for $0 < x < 1$,

$$\begin{aligned}
 F_c [A(\xi); x] &= \int_x^1 \frac{g_1(t) dt}{\sqrt{(t^2 - x^2)}} \\
 F_s [A(\xi); x] &= \int_0^x \frac{g_1(t) dt}{\sqrt{(t^2 - x^2)}} \\
 F_c [B(\xi); x] &= -\frac{1}{x} \int_0^x \frac{g_2(t) dt}{\sqrt{(x^2 - t^2)}} \\
 F_s [B(\xi); x] &= \frac{1}{x} \int_x^1 \frac{g_2(t) dt}{\sqrt{(t^2 - x^2)}}
 \end{aligned} \tag{3.4}$$

And for $x > 1$,

$$\begin{aligned}
 F_s [A(\xi); x] &= \int_0^1 \frac{g_1(t) dt}{\sqrt{(x^2 - t^2)}} \\
 \text{and} \\
 F_c [B(\xi); x] &= -\frac{1}{x} \int_0^1 \frac{g_2(t) dt}{\sqrt{(x^2 - t^2)}}
 \end{aligned} \tag{3.5}$$

We shall now present the method of solving the system of equations (3.2) and (3.3) for the functions $g_1(t)$ and $g_2(t)$ ($0 < t < 1$). To this end, we introduce two sectionally analytical function $\phi(z)$ and $\psi(z)$ of the complex variable $z = x + iy$, analytic in the entire z -plane cut along the segment $-1 \leq x \leq 1$ of the real axis and vanishing as $|z| \rightarrow \infty$. These functions are defined by:

$$\phi(z) = \int_0^1 \frac{g_1(t) dt}{\sqrt{(z^2 - t^2)}} \quad \dots$$

and

$$\psi(z) = \int_0^1 \frac{g_2(t) dt}{\sqrt{(z^2 - t^2)}} \tag{3.6}$$

We observe that the limiting values $\phi \pm(x)$ and $\psi \pm(x)$ of the functions (3.6) on the line $y=0$, as $y \rightarrow \pm 0$, are given by (cf. Green and England [11])

For $0 < x < 1$:

$$\phi \pm(x) = \int_0^x \frac{g_1(t) dt}{\sqrt{(x^2 - t^2)}} \mp i \int_x^1 \frac{g_1(t) dt}{\sqrt{(t^2 - x^2)}} \quad (3.7)$$

$$\psi \pm(x) = \int_0^x \frac{g_2(t) dt}{\sqrt{(x^2 - t^2)}} \mp i \int_{-x}^1 \frac{g_2(t) dt}{\sqrt{(t^2 - x^2)}}$$

and for $-1 < x < 0$,

$$\begin{aligned} \phi \pm(x) &= - \int_0^{-x} \frac{g_1(t) dt}{\sqrt{(x^2 - t^2)}} \mp i \int_x^1 \frac{g_1(t) dt}{\sqrt{(t^2 - x^2)}} \\ \psi \pm(x) &= - \int_0^{-x} \frac{g_2(t) dt}{\sqrt{(x^2 - t^2)}} \mp i \int_{-x}^1 \frac{g_2(t) dt}{\sqrt{(t^2 - x^2)}} \end{aligned} \quad (3.8)$$

We again see that the following functions :

$$[\phi^+(x) + \phi^-(x)] = 2 \int_0^x \frac{g_1(t) dt}{\sqrt{(x^2 - t^2)}}$$

and

$$[\psi^+(x) + \psi^-(x)] = 2 \int_0^x \frac{g_2(t) dt}{\sqrt{(x^2 - t^2)}} \quad (3.9)$$

are odd functions of x , whilst the functions

$$[\phi^+(x) - \phi^-(x)] = -2i \int_x^1 \frac{g_1(t) dt}{\sqrt{(t^2 - x^2)}}$$

and

$$[\psi^+(x) - \psi^-(x)] = -2i \int_x^1 \frac{g_2(t) dt}{\sqrt{(t^2 - x^2)}} \quad (3.10)$$

are even functions of x for $x \in (-1, 1)$.

We may now express the system of equations (3.2) and (3.3) in the following forms of simultaneous Riemann-Hilbert problems for the sectionally analytic functions $\phi(z)$ and $\psi(z)$

$$a_1 [\phi^+(x) + \phi^-(x)] + \frac{ia_2}{x} [\psi^+(x) - \psi^-(x)] = 2x \quad (3.11)$$

and

$$a_2 [\phi^+(x) - \phi^-(x)] + \frac{ia_1}{x} [\psi^+(x) + \psi^-(x)] = 0. \quad (3.12)$$

Subtracting (3.12) from (3.11) and adding the two equations, respectively we reduce the problem to that of solving two independent Riemann-Hilbert problems for the functions:

$$\lambda(z) = \phi(z) - \frac{i}{z} \psi(z)$$

and

$$\mu(z) = \phi(z) + \frac{i}{z} \psi(z) \quad (3.13)$$

as given by:

$$\lambda^+(x) + (3 - 4\eta) \lambda^-(x) = 2x \quad (-1 < x < 1) \quad (3.14)$$

and

$$(3 - 4\eta) \mu^+(x) + \mu^-(x) = 2x \quad (-1 < x < 1). \quad (3.15)$$

The solutions of the two Hilbert problems (3.14) and (3.15) are obtained by the technique of Muskhelishvili [2] in the forms:

$$\lambda(z) = + \frac{1}{\pi i} \lambda_0(z) \int_{-1}^1 \frac{t dt}{\lambda_0^+(t)(t-z)}, \quad (3.16)$$

and

$$\mu(z) = + \frac{1}{\pi i} \frac{\mu_0(z)}{(3 - 4\eta)} \int_{-1}^1 \frac{t dt}{\mu_0^+(t)(t-z)}, \quad (3.17)$$

where $\lambda_0(z)$ and $\mu_0(z)$ are the solutions of the homogeneous problems (3.14) and (3.15) which are taken in the forms:

$$\lambda_0(z) = (z^2 - 1)^{1/2} \left(\frac{z+1}{z-1} \right)^{-in} \quad (3.18)$$

and

$$\mu_0(z) = (z^2 - 1)^{1/2} \left(\frac{z-1}{z+1} \right)^{-in}, \quad (3.19)$$

where

$$n = \frac{1}{2\pi} \ln(3 - 4\eta), \quad (3.20)$$

and those branches of λ_0 and μ_0 are taken which take such that

$$\lim_{|z| \rightarrow \infty} \frac{\lambda_0(z), \mu_0(z)}{z} = 1.$$

We observe that the limiting values of the functions λ_0 and μ_0 on the line $y = 0$ ($-1 < x < 1$) are given by:

$$\begin{aligned} \lambda_0^\pm(x) = & \pm ie^{\pm\pi n} \sqrt{1-x^2} \left\{ \cos \left[n \ln \left| \frac{x+1}{x-1} \right| \right] \right. \\ & \left. - i \sin \left[n \ln \left| \frac{x+1}{x-1} \right| \right] \right\} \end{aligned} \quad (3.21)$$

and

$$\begin{aligned} \mu_0^\pm(x) = & \pm ie^{\pm\pi n} \sqrt{1-x^2} \left\{ \cos \left[n \ln \left| \frac{x+1}{x-1} \right| \right] \right. \\ & \left. + i \sin \left[n \ln \left| \frac{x+1}{x-1} \right| \right] \right\} \end{aligned}$$

The knowledge of the functions $\lambda(z)$ and $\mu(z)$ are sufficient for our purpose, as is seen from (3.13) and (3.9), of the determination of the unknown functions $g_1(t)$ and $g_2(t)$, as the latter functions can then be determined by the usual formula for the inversion of Abel's integral equation. But, for the determination of the quantities of physical interest, the displacement and stress, we shall see in the next section that we do not need to obtain the function $g_1(t)$ and $g_2(t)$ in their explicit forms at all.

Thus, as far as the solution of the mathematical problem is concerned, complete solution of the problem at hand has been obtained here.

4. DETERMINATION OF THE STRESS AND DISPLACEMENT

From (2.2), (2.3), (3.4), (3.10) and (3.13) we observe that the displacement components on the crack surface are given by

[for $0 \leq x < 1$]:

$$u_x(x, 0) = -\frac{(1+\eta)(3-4\eta)P}{4Ei} [\{\lambda^+(x) - \lambda^-(x)\} + \{\mu^+(x) - \mu^-(x)\}] \quad (4.1)$$

and

$$u_y(x, 0) = -\frac{(1+\eta)(3-4\eta)P}{4E} [\{\lambda^+(x) - \lambda^-(x)\} - \{\mu^+(x) - \mu^-(x)\}]. \quad (4.2)$$

Now, by the method of Mushkhelishvili [12], p. 445-447, we find that from (3.16) and (3.17),

$$\lambda(z) = \frac{1}{2(1-\eta)} [z - \lambda_0(z)] \quad (4.3)$$

and

$$\mu(z) = +\frac{1}{2(1-\eta)} [z - \mu_0(z)]. \quad (4.4)$$

Finally, using (4.3), (4.4) and (3.21) we obtain from (4.1) and (4.2),

$$\begin{aligned} u_x(x, 0) &= +\frac{(1+\eta)(3-4\eta)P}{4E(1-\eta)} (e^{\pi n} + e^{-\pi n}) \sqrt{1-x^2} \\ &\times \cos \left\{ n \ln \left| \frac{x+1}{x-1} \right| \right\} \\ &= +\frac{1+\eta}{E\sqrt{3-4\eta}} P \sqrt{1-x^2} \cos \left\{ n \ln \left| \frac{x+1}{x-1} \right| \right\} \end{aligned} \quad (4.5)$$

and

$$u_y(x, 0) = +\frac{1+\eta}{E\sqrt{3-4\eta}} P \sqrt{1-x^2} \sin \left\{ n \ln \left| \frac{x+1}{x-1} \right| \right\} \quad (4.6)$$

The equations (4.7) and (4.8) give the correct forms of the displacements on the crack surface. (cf. Lowengrub [3], England [8]).

The stress, on the line $y=0$, for $|x| > 1$, can be calculated by means of the results (2.4), (2.5), (3.5), (3.13), (4.3) and (4.4). The components of the stress are given by:

$$\begin{aligned} \sigma_{yy}(x, 0) \Big|_{|x|>1} &= +P \frac{d}{dx} \left[\sqrt{x^2-1} \sin \left(\eta \ln \left| \frac{x+1}{x-1} \right| \right) \right] \\ &= -\frac{P}{\sqrt{x^2-1}} \left[2\eta \cos \left(\eta \ln \left| \frac{x+1}{x-1} \right| \right) - \sin \left(\eta \ln \left| \frac{x+1}{x-1} \right| \right) \right] \end{aligned} \quad (4.7)$$

and

$$\begin{aligned} \sigma_{xy}(x, 0) \Big|_{|x| > 1} &= P \frac{d}{dx} \left[\sqrt{x^2 - 1} \cos \left(\eta \ln \left| \frac{x+1}{x-1} \right| \right) \right] \\ &= \frac{P}{\sqrt{x^2 - 1}} \left[x \cos \left(\eta \ln \left| \frac{x+1}{x-1} \right| \right) + 2\eta \sin \left(\eta \ln \left| \frac{x+1}{x-1} \right| \right) \right] \quad (4.8) \end{aligned}$$

Equations (4.7) and (4.8) give the correct expressions of the components of stress, as compared with those obtained by Lowengrub [3]. Infact, the expressions for the components of displacement and stress as calculated in [3] are in error, as there are errors in the expressions $X(z)$ given by eqn. (3.19) [3] and hence $X(x)$ given by (3.21) [3] and these errors are propagated to the latter discussions in the paper [3].

CONCLUSION

The method of solving the set of dual integral equations presented here is straightforward and the technique is more useful. But the calculations here are a little involved as compared to those of Lowengrub [3] or Erdogan [1]. The main aim of the present paper has been to answer Erdogan's question about the legitimacy of the use of integral transforms.

The present technique of handling the set of simultaneous dual integral equations has left no doubt now about the use of integral transforms to such problems.

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