LARGE DEFLECTION OF A CIRCULAR PLATE ON ELASTIC FOUNDATION AND SUPPORTED AT SEVERAL POINTS

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ABSTRACT

Large deflections of a uniformly loaded circular plate placed on elastic foundation and supported at several points along the boundary have been analysed following Berger's method. A particular case, where the number of supports is two, has been treated fully. Numerical results have been presented in the form of graphs.

Key words : Large deflections, elastic foundation

INTRODUCTION

Small deflections of thin plates placed on elastic foundations have been examined by S. Timoshenko and S. Woinowsky Krieger [1] and several other authors on the assumption that strain due to stretching of the middle surface of the plate is negligible. When the deflections are moderately large, that is, on the order of thickness of the plate, then the forces in the middle surface of the plate must be taken into account. In the case of such large deflections of plates placed on elastic foundations, three differential equations for displacement and deflection may be written, but it is usually difficult to obtain the solutions of these equations because of their nonlinear character.

On the other hand, various problems of large deflections of plates not resting on elastic foundations have been examined by S. Way [2], S. Levy [3] and many other authors. But the methods used by them involve and require considerable computation. A simple and approximate, yet fairly accurate, method of analysing large deflections of plates was suggested by H. M. Berger [4]. The method uses the technique of neglecting the strain energy due to the second strain invariant of the middle surface strains in analysing large deflection of plates having axisymmetric deformation Berger's method reduces computation and although no complete explanation of this method is offered in, the stresses and deflections obtained for both rectangular and circular plates are in good agreement with those found in practical analysis. Berger's method has been applied successfully by Nowinsk [5] to his plate problems and Nash and Modeer [6 investigated the problems having no axial symmetry.

The technique of neglecting the second strain invariant in the expression corresponding to the total potential energy of the system has been successfully applied by Sinha [7] to determine large deflection of circular and rectangular plates placed on elastic foundations and under uniform lateral loads.

In this paper large deflection of a circular plate placed on elastic foundation and supported at several points along the boundary has been solved. The load is assumed to be uniformly distributed and the foundation is of the Winkler type. A complete analysis of a particular case, where the number of supports is two is given.

FORMULATION OF PROBLEM

For moderately large deflections, the strain displacement relationships and the strain energy of the middle plane of the plate are

$$\epsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{1}$$

$$\epsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \tag{2}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \tag{3}$$

$$V_1 = \frac{D}{2}\int\int\Big[(\nabla^2 w)^2 + \frac{12}{h^2}e_1^2 - 2(1-v) \times \left\{\frac{12}{h^2}e_2 + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right\}\Big]\,dx\,dy \tag{4}$$

in which e_1 and e_2 are the first and second middle surface strain invariants, respectively.

Neglecting e_2 and by adding the potential energy of the transverse load and of the foundation reaction, K, the modified energy equation becomes

$$V = \frac{D}{2}\int\int \Big[(\nabla^2 w)^2 + \frac{12}{h^2}e_1^2 - 2(1-v)$$

$$\times \left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right\} + \frac{K}{D}w^2 - \frac{2qw}{D}\Big]\,dxdy. \qquad (5)$$

Applying Euler's variational method to eq. 5 the following differential equations in polar co-ordinates are obtained [7]

$$\nabla^4 w - \alpha^2 \nabla^2 w + \frac{K}{D} w = \frac{q}{D} \qquad (6)$$

where α is a constant given by

$$\frac{\alpha^2 h^2}{12} = \frac{\partial u}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 + \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{1}{2r^2}\left(\frac{\partial w}{\partial \theta}\right)^2$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}. \qquad (7)$$

Solution of Problem

Let the circular plate (Fig. 1) be of radius a, supported at several points along the boundary and placed on the elastic foundation. Let the centre of the plate be the origin and a diameter as the initial line, $\theta = 0$. The general solution of eq. 6 is

$$w = w_0 + w_1 \qquad (8)$$

in which w_0 is the large deflection of a plate placed on elastic foundation and simply supported along the entire boundary and w_1 satisfies the equation

$$\nabla^4 w_1 - \alpha^2 \nabla^2 w_1 + \frac{K}{D} w_1 = 0 \qquad (9)$$

Eq. (9) can be written in the form

$$(\nabla^2 - P_1^2)(\nabla^2 - P_2^2)\, w_1 = 0 \qquad (10)$$

where

$$P_1^2 + P_2^2 = \alpha^2 \qquad (11)$$

$$P_1^2 P_2^2 = \frac{K}{D}. \qquad (12)$$

Considering the number of points of support is i, and denoting the concentrated reactions at these points $N_1, N_2 \ldots N_i$, the expression for each reaction N_i is (1, P. 293)

$$\frac{N_i}{\pi a}\left[\tfrac{1}{2} + \sum_{m=1}^{\infty} \cos m\theta_i\right] \tag{13}$$

where $\theta_i = \theta - \psi_i$, ψ_i is the angle defining the position of the support i.

The intensity of the reactive forces at any point of the boundary is then given by the expression.

$$\sum_{i=1}^{i} \frac{N_i}{\pi a}\left[\tfrac{1}{2} + \sum_{m=1}^{\infty} \cos m\theta_i\right] \tag{13 a}$$

in which the summation is extended over all the concentrated reactions. Assuming that the plate is solid and considering that deflections and moments

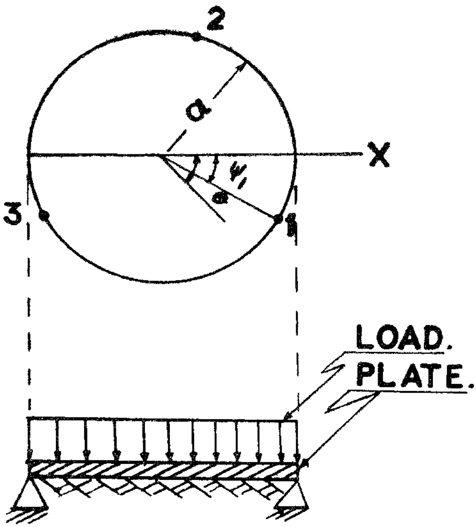

FIG. 1. Circular plate on foundation.

at the centre must be finite, the appropriate solution of eq. 9 can be taken in the form

$$w_1 = A_0 I_0 (P_1 r) + B_0 I_0 (P_2 r) + \sum_{m=1}^{\infty} [A_m I_m (P_1 r) + B_m I_m (P_2 r)]$$

$$\times \cos m\theta + \sum_{m=1}^{\infty} [A'_m I_m (P_1 r) + B'_m I_m (P_2 r)] \sin m\theta \qquad (14)$$

in which I_0 is the modified Bessel function of the first kind and zero order, and I_m is of the first kind and mth order. For determining the constants we have the following conditions at the boundary:

$$w \Big|_{\substack{r = a \\ \theta = 0,\, \pi}} = 0 \qquad (15)$$

$$\left[\frac{\partial^2 w_1}{\partial r^2} + \frac{\nu}{r}\frac{\partial w_1}{\partial r} + \frac{\nu}{r^2}\frac{\partial^2 w_1}{\partial \theta^2}\right]_{r=a} = 0 \qquad (16)$$

$$\left[Q_r - \frac{1}{r}\frac{\partial}{\partial \theta} M_{r\theta}\right]_{r=a} = -\sum_{i=1}^{i} \frac{Ni}{\pi a}\left[\tfrac{1}{2} + \sum_{m=1}^{\infty} \cos m\theta_i\right] \qquad (17)$$

where

$$Q_r = D \frac{\partial}{\partial r} [(\nabla^2 - \alpha^2) w_1] \qquad (17\,a)$$

$$Mr_\theta = (1 - v) D \left[\frac{1}{r}\frac{\partial^2 w_1}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial w_1}{\partial \theta}\right]. \qquad (17\,b)$$

Consider a particular case when the plate is supported at two points which are the two end points of the diameter taken as the initial line from which θ is measured. Then

$$\psi_1 = 0, \quad \psi_2 = \pi.$$

Considering the above boundary conditions one gets after solving for the constants

$$A_0 = \frac{P}{\pi D a} \beta \psi_0 (a) \qquad (18)$$

$$B_0 = -\frac{P}{\pi D a} \beta \phi_0 (a) \qquad (19)$$

$$A_m = -\frac{P}{\pi D a}\frac{\mu_m(a)}{\{\beta_m(a)\mu_m(a) - \lambda_m(a)\eta_m(a)\}} \tag{20}$$

$$B_m = \frac{P}{\pi D a}\frac{\lambda_m(a)}{\{\beta_m(a)\mu_m(a) - \lambda_m(a)\eta_m(a)\}} \tag{21}$$

$$A'_m = 0 = B'_m \tag{22}$$

where $P = \pi a^2 q =$ total load on the plate

$$\beta = \frac{[\lambda_m(a) I_m(P_2 a) - \mu_m(a) I_m(P_1 a)]}{[\beta_m(a)\mu_m - ((a)\lambda_m(a)\eta_m a)] \times [I_0(P_2 a)\phi_0(a) - I_0(P_1 a)\psi_0(a} \tag{23}$$

$$\psi_0(a) = P_2^2 I_0''(P_2 a) + \frac{\nu}{a} P_2 I_1(P_2 a) \tag{24}$$

$$\phi_0(a) = P_1^2 I_0''(P_1 a) + \frac{\nu}{a} P_1 I_1(P_1 a) \tag{25}$$

$$\mu_m(a) = P_2^2 I''_m(P_2 a) + \frac{\nu}{a} P_2 I'_m(P_2 a) - \frac{\nu m^2}{a^2} I_m(P_2 a) \tag{26}$$

$$\beta_m(a) = P_2^2 P_1 I'_m(P_1 a) - (1-\nu)\left\{\frac{m^2}{a^3} I_m(P_1 a) - \frac{P_1 m^2}{a^2} I'_m(P_1 a)\right\} \tag{27}$$

$$\lambda_m(a) = P_1^2 I''_m(P_1 a) + \frac{\nu}{a} P_1 I'_m(P_1 a) - \frac{\nu m^2}{a^2} I_m(P_1 a) \tag{28}$$

$$\eta_m(a) = P_1^2 P_2 I'_m(P_2 a) - (1-\nu)\left\{\frac{m^2}{a^3} I_m(P_2 a) - \frac{P_2 m^2}{a^2} I'_m(P_2 a)\right\} \tag{29}$$

Thus the complete solution of eq. 6 is obtained in the following form

$$w = w_0 + A_0 I_0(P_1 r) + B_0 I_0(P_2 r) + \sum_{m=2,4,6,\ldots}^{\infty} [A_m I_m(P_1 r) + B_m I_m(P_2 r)] \cos m\theta \tag{30}$$

where

$$w_0 = \frac{q}{K} + A_0' I_0(P_1 r) + B_0' I_0(P_2 r) \tag{31}$$

$$A_0 = -\frac{q}{K}\left[\frac{P_2^2 I_0''(P_2 a) + P_2 \frac{v}{a} I_1(P_2 a)}{\phi(Pa)}\right] \tag{31 a}$$

$$B_0' = \frac{q}{K}\left[\frac{P_1^2 I_0''(P_1a) + P_1 \frac{v}{a} I_1(P_1a)}{\phi(Pa)}\right] \tag{31 b}$$

$$\phi(Pa) = \{I_0(P_1a)\, P_2^2 I_0''(P_2a) - I_0(P_2a)\, P_1^2 I_0''(P_1a)\}$$
$$+ \frac{v}{a}\{P_2 I_1(P_2a)\, I_0(P_1a) - P_1 I_1(P_1a)\, I_0(P_2a)\} \tag{31 c}$$

Substitution of the values of the constants A_0', B_0', A_0, B_0, A_m and B_m into eq. 30 yields

$$\frac{w}{h} = \left(\frac{qa^4}{Dh}\right)\left[\frac{1}{K_F}\left\{1 + \frac{\left[P_1^2 I_0''(P_1a) + P_1 \frac{v}{a} I_1(P_1a)\right] I_0(P_2r)}{\phi(Pa)}\right.\right.$$
$$\left. - \frac{\left[P_2^2 I_0''(P_2a) + P_2 \frac{v}{a} I_1(P_2a)\right] I_1(P_1r)}{\phi(Pa)}\right\}$$
$$+ \frac{1}{a^3}\left\{\beta\psi_0(a)\, I_0(P_1r) - \beta\phi_0(a)\, I_0(P_2r)\right.$$
$$\left.\left. - \sum_{m=2,4,6,\ldots}^{\infty} \left[\frac{\mu_m(a)\, I_m(P_1r) - \lambda_m(a)\, I_m(P_2r)}{\beta_m(a)\,\mu_m(a) - \lambda_m(a)\,\eta_m(a)}\right] \cos m\theta\right\}\right]. \tag{32}$$

As $P_1 \rightarrow 0$ and $P_2 \rightarrow 0$, eq. 32 reduces to

$$w = w_0 + \frac{Pa^2}{2\pi D}\frac{1}{(3+\nu)}\left\{2\log 2 - 1 + \frac{1+\nu}{1-\nu}\left(2\log 2 - \frac{\pi^2}{12}\right)\right.$$
$$- \sum_{m=2,4,6,\ldots}^{\infty}\left[\frac{1}{m(m-1)} + \frac{2(1+\nu)}{m^2(m-1)(1-\nu)}\right.$$
$$\left.\left. - \frac{(r/a)^2}{m(m+1)}\right]\left(\frac{r}{a}\right)^m \cos m\theta\right\} \tag{33}$$

as obtained by Timoshenko [1] in the corresponding small deflection problem for a plate supported at two points on the boundary.

The normalised constant a can be determined from Eqs. 7 and 30. Since we are interested only in the lateral displacement w, the radial and cross-radial displacements u and v have been eliminated by choosing suitable expressions for u and v, compatible with their boundary conditions and integrating over the whole area of the plate. The radial and cross-radial displacements have been assumed in the forms

$$u = \Sigma\ U(r) \cos m\theta \tag{34}$$

$$v = \Sigma\ V(r) \sin m\theta \tag{35}$$

subject to the boundary conditions $U(a) = V(a) = 0$. Multiplying both sides of the equation 7 by $rdrd\theta$ and integrating between the limits 0 to a and 0 to 2π, one gets

$$\int_0^a \int_0^{2\pi} rU'(r) \cos m\theta\, drd\theta + \int_0^a \int_0^{2\pi} U(r) \cos m\theta\, drd\theta$$

$$+ \int_0^a \int_0^{2\pi} mV(r) \cos m\theta\, drd\theta + \frac{1}{2} \int_0^a \int_0^{2\pi} r \left(\frac{\partial w}{\partial r}\right)^2 drd\theta$$

$$+ \frac{1}{2} \int_0^a \int_0^{2\pi} \frac{1}{r} \left(\frac{\partial w}{\partial \theta}\right)^2 drd\theta = \int_0^a \int_0^{2\pi} \frac{\alpha^2 h^2}{12} r\, drd\theta.$$

After evaluating the integrals the following equation leading to α is obtained.

$$\frac{\alpha^2 h^2 a^2}{12} = -\frac{1}{2} A_0'^2 P_1^2 a^2 \left\{\frac{1}{4} [I_0(P_1 a) + I_2(P_1 a)]^2\right.$$

$$\left. - \left[1 + \frac{1}{P_1^2 a^2}\right] I_1^2(P_1 a)\right\} - \frac{1}{2} B_0'^2 P_2^2 a^2$$

$$\times \left\{\frac{1}{4} [I_0(P_2 a) + I_2(P_2 a)]^2 - \left[1 + \frac{1}{P_2^2 a^2}\right] I_1^2(P_2 a)\right\}$$

$$+ 2A_0' B_0' P_1 P_2 \frac{a}{P_2^2 - P_1^2} [-\tfrac{1}{2} P_1 I_1(P_2 a) \{I_0(P_1 a)$$

$$+ I_2(P_1 a)\} + \tfrac{1}{2} P_2 I_1(P_1 a) \{I_0(P_2 a) + I_2(P_2 a)\}]$$

$$+ \sum_{m=2,4,6,\ldots}^{\infty} \left[A^2{}_m P_1^2 \left\{-\frac{1}{8} a^2 \left[\frac{1}{4} \{I_{m-2}(P_1 a) + I_m(P_1 a)\}^2\right.\right.\right.$$

$$\left. \left\{1 + \frac{(m-1)^2}{P_1^2 a^2}\right\} I_m^2(P_1 a)\right]$$

$$-\frac{1}{8} a^2 \left[\frac{1}{4} \{I_m(P_1 a) + I_{m+2}(P_1 a)\}^2 - \left\{1 + \frac{(m+1)^2}{P_1^2 a^2}\right\}\right.$$

$$\times I^2_{m+1}(P_1a)\Big] + \frac{1}{2}\sum_{\substack{n=0\\ n\neq t}}^{\infty}\sum_{t=0}^{\infty}\left(\frac{P_1}{2}\right)^{2m+2n+2t}\cdot\phi\Big\}\cdot\frac{1}{2}$$

$$+ B_m{}^2P_2{}^2\Big\{-\frac{1}{8}a^2\Big[\frac{1}{4}\{I_{m-2}(P_2a)+I_m(P_2a)\}^2$$

$$+\Big\{1+\frac{(m-1)^2}{P_2{}^2a^2}\Big\}I^2_{m-1}(P_2a)\Big] - \frac{1}{8}a^2\Big[\frac{1}{4}\{I_m(P_2a)$$

$$+ I_{m+2}(P_2a)\}^2 + \Big\{1+\frac{(m+1)^2}{P_2{}^2a^2}\Big\}I^2_{m+1}(P_2a)\Big]$$

$$+\frac{1}{2}\sum_{\substack{n=0\\ n\neq t}}^{\infty}\sum_{t=0}^{\infty}\left(\frac{P_2}{2}\right)^{2m+2n+2t}\cdot\phi\Big\}\frac{1}{2}+\frac{1}{2}A_mB_mP_1P_2$$

$$\times\Big\{\sum_{\substack{n=0\\ n\neq t}}^{\infty}\sum_{t=0}^{\infty}\Big[\frac{\left(\frac{P_1}{2}\right)^{m+2n-1}\left(\frac{P_2}{2}\right)^{m+2t-1}a^{2m+2n+2t}}{(2m+2n+2t)\,\underline{|n}\,\underline{|t}\,\Gamma(m+n)\,\Gamma(m+t)}$$

$$+\left(\frac{P_2}{2}\right)^{m+2n-1}\left(\frac{P_2}{2}\right)^{m+2t+1}\phi$$

$$+\frac{\left(\frac{P_1}{2}\right)^{m+2n+1}\left(\frac{P_2}{2}\right)^{m+2t-1}a^{2m+2n+2t+2}}{(2m+2n+2t+2)\,\underline{|n}\,\underline{|t}\,\Gamma(m+t)\,\Gamma(m+n+2)}$$

$$+\frac{\left(\frac{P_1}{2}\right)^{m+2n+1}\left(\frac{P_2}{2}\right)^{m+2t+1}a^{2m+2n+2t+4}}{(2m+2n+2t+4)\,\underline{|n}\,\underline{|t}\,\Gamma(m+n+2)\,\Gamma(m+t+2)}\Big]\Big\}\frac{1}{2}$$

$$+A_m{}^2\Big\{\sum_{n=0}^{\infty}\left(\frac{P_1}{2}\right)^{2m+4n}\phi_1+\sum_{\substack{n=0\\ n\neq t}}^{\infty}\sum_{t=0}^{\infty}\left(\frac{P_1}{2}\right)^{2m+2n+2t}\psi\Big\}\frac{m^2}{2}$$

$$+B_m{}^2\Big\{\sum_{n=0}^{\infty}\left(\frac{P_2}{2}\right)^{2m+4n}\phi_1+\sum_{\substack{n=0\\ n\neq t}}^{\infty}\sum_{t=0}^{\infty}\left(\frac{P_2}{2}\right)^{2m+2n+2t}\psi\Big\}\frac{m^2}{2}$$

$$+2A_mB_m\Big\{\sum_{\substack{n=0\\ n\neq t}}^{\infty}\sum_{t=0}^{\infty}\left(\frac{P_1}{2}\right)^{m+2n}\left(\frac{P_2}{2}\right)^{m+2t}\psi\Big\}\frac{m^2}{2}\Big] \qquad (36)$$

where

$$\phi = \frac{a^{2m+2n+2t+2}}{(2m+2n+2t+2)\,\lfloor n\,\lfloor t\,\Gamma(m+n)\,\Gamma(m+t+2)}$$

$$\phi_1 = \frac{a^{2m+4n}}{(2m+4n)\,\{\lfloor n\,\Gamma(m+n+1)\}^2}$$

$$\psi = \frac{a^{2m+2n+2t}}{(2m+2n+2t)\,\lfloor n\,\lfloor t\,\Gamma(m+n+1)\,\Gamma(m+t+1)}\,.$$

Thus the deflection, w is completely determined. The expressions for the bending and twisting moment can now be determined.

$$\begin{aligned} M_r = -D\Big[& P_1^2(A_0' + A_0)\,I_0''(P_1 r) + P_2^2(B_0' + B_0)\,I_0''(P_2 r) \\ & + \sum_{m=2,4,6,\ldots}^{\infty} \{P_1^2 A_m I''_m(P_1 r) + P_2^2 B_m I''_m(P_2 r)\}\cos m\theta \\ & + \nu\Big\{\frac{P_1}{r}(A'_0 + A_0)\,I_1'(P_1 r) + \frac{P_2}{r}(B_0' + B_0)\,I_1'(P_2 r) \\ & + \frac{1}{r}\sum_{m=2,4,6,\ldots}^{\infty} [P_1 A_m I'_m(P_1 r) + P_2 B_m I'_m(P_2 r)]\cos m\theta \\ & - \frac{1}{r^2}\sum_{m=2,4,6,\ldots}^{\infty} m^2[A_m I_m(P_1 r) + B_m I_m(P_2 r)]\cos m\theta)\Big\}\Big] \end{aligned} \tag{37}$$

$$\begin{aligned} M_\theta = -D\Big[& \frac{P_1}{r}(A_0' + A_0)\,I_1'(P_1 r) + \frac{P_2}{r}(B_0' + B_0)\,I_1'(P_2 r) \\ & + \frac{1}{r}\sum_{m=2,4,6,\ldots}^{\infty} \{P_1 A_m I_m'(P_1 r) + P_2 B_m I_m'(P_2 r)\}\cos m\theta \\ & - \frac{1}{r^2}\sum_{m=2,4,6,\ldots}^{\infty} m^2\{A_m I_m(P_1 r) + B_m I_m(P_2 r)\}\cos m\theta \\ & + \nu\Big\{P_1^2(A_0' + A_0)\,I_0''(P_1 r) + P_2^2(B_0' + B_0)\,I_0''(P_2 r) \end{aligned}$$

$$+ \sum_{m=2,4,6,\ldots}^{\infty} [P_1^2 A_m I_m'' (P_1 r) + P_2^2 B_m I_m'' (P_2 r)] \cos m\theta \Big\} \Big] \tag{38}$$

$$M_{r\theta} = (1 - \nu) D \Big[-\frac{1}{r} \sum_{m=2,4,6,\ldots}^{\infty} m \{P_1 A_m I_m' (P_1 r) + P_2 B_m I_m' (P_2 r)\}$$

$$\times \sin m\theta + \frac{1}{r^2} \sum_{m=2,4,6,\ldots}^{\infty} m \{A_m I_m (P_1 r) + B_m I_m (P_2 r)\}$$

$$\times \sin m\theta \Big]. \tag{39}$$

The stresses can be calculated from the expressions

$$\sigma_r \frac{6M_r}{h^2}; \quad \sigma_\theta = \frac{6M_\theta}{h^2}; \quad \tau_{r\theta} = \frac{6M_{r\theta}}{h^2} \tag{40}$$

Numerical Calculation

To obtain deflection for a given value of plate radius 'a' and foundation modulus 'K_F' one has to start from the equation (36) with an assumed value of 'α' in order to obtain the corresponding value of the load function qa^4/Dh. Once this relationship is obtained the corresponding deflection w/h can be calculated by eq. 32. For $a = 50$ mm, $h = 0 \cdot 75$ mm, $v = 0 \cdot 3$ and $K_F = 80$ deflections have been presented in Fig 2.

Concluding Remarks

An examination of the eq. 32 will reveal that the deflection (w/h) depends on K_F, the plate radius 'a' and on the value of the angle, θ. For a given value of the load function eq. 32 can be written as

$$\left(\frac{w}{h}\right)_{\substack{r=0\\ \theta=0}} = K_1 \left(\frac{qa^4}{Dh}\right); \quad \left(\frac{w}{h}\right)_{\substack{r=a\\ \theta=\pi/2}} = K_2 \left(\frac{qa^4}{Dh}\right) \tag{41}$$

where K_1 and K_2 are two numerical constants, K_2 being greater than K_1. Because of the reactive forces at the two points of support, deflections on the diameter at $\theta = 0$ will be less than those on the diameter at $\theta = \pi/2$. Maximum deflection will occur at the boundary at $\theta = \pm \pi/2$. Deflections according to the linear theory have also been plotted in Fig. 2 and it is

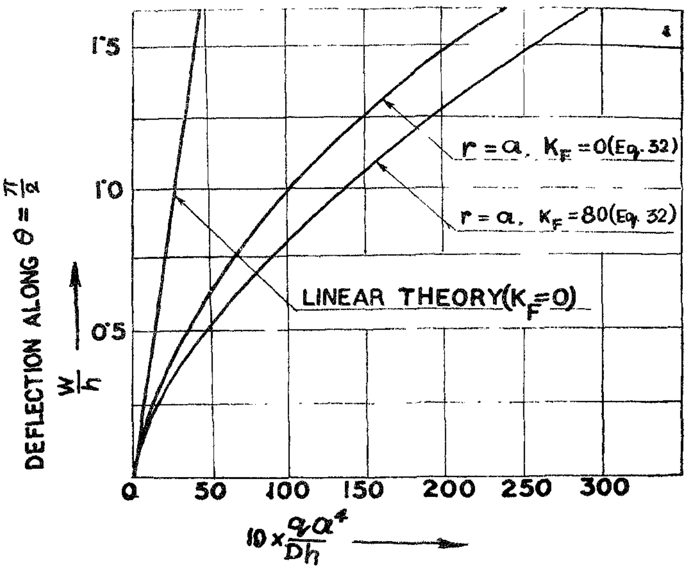

FIG. 2. Load deflection curve.

clear that the errors of the linear theory increases as the load increases. In order to study the variation of moments, eqs. 37, 38 and 39 are plotted in Fig. 3 for various values of (r/a) and for the angles at which they become maximum. It is observed that the maximum bending moments, their magnitudes being unequal, are developed at $r = 3a/4$, $\theta = \pm \pi/2$ and the twisting moment is maximum at $r = a$, $\theta = \pm \pi/4$, $\pm 3\pi/4$.

As the plate must be in equilibrium on the supports, the foregoing analysis for two simple supports represents the worst condition when the deflections and stresses are maximum for a given load function. With the increase in the number of supports, w_1 in eq. 8 decreases. For an infinitely large number of supports, w in eq. 8 will approach to w_0 in the limit and the point of maximum bending moments will shift to the centre of the plate. $(M_r)_{max}$ being equal to $(M_\theta)_{max}$ in that case.

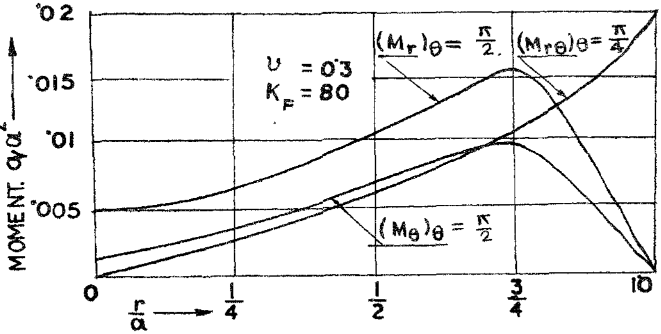

FIG. 3. Moment curve.

The present study can be extended to any number of supports, provided the supports are so chosen as not to disturb the equilibrium of the plate. For example, if three equidistant supports are chosen, $\psi_1 = 0$, $\psi_2 = 2\pi/3$, $\psi_3 = 4\pi/3$, the differential equations together with the boundary condition remaining unchanged. If the plate is clamped on the supports, the boundary conditions and the concentrated reactions at the supports will change totally demanding a separate investigation.

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REFERENCES

[1] Timoshenko, S and Woinowski-Krieger, S. *Theory of Plates and Shells,* McGraw-Hill Book Co., Inc., New York, N.Y., 2nd edition, 1959, pp. 259–278.

[2] Way .. Bending of circular plates with large deflectinos, *Transactions, ASME,* 1934, **56**, 627.

[3] Levy, S. .. *Bending of Rectangular Plates with Large Deflections,* Technical Note No. 846, N.A.C.A., Washington, D.C., May 1942.

[4] Berger, H. M. .. A new approach to the analysis of large deflection of plates, *Journal of Applied Mechanics, ASME*, 1955, **22**, 465–472

[5] Nowinski, J. .. Some mixed boundary-value problems for plates with large deflections, *MRC Technical Summary Report*, No 42, Mathematics Research Centre, U.S. Army, The University of Wisconnsin, 1958, p. 17

[6] Nash, W. A. and Modeer, J. R. Certain approximate analysis of the non-linear behaviour of plates and shallow shells, *Proceedings of the Symposium on the Theory of Thin Elastic Shells of the International Union of Theoretical and Applied Mechanics*, 1959, North Holland Pub. Co., Amsterdam, 1960, pp. 331–354.

[7] Sinha, S. N .. Large deflections of plates on elastic foundations, *Journal o the Engineering Mech. Division, Proceedings of the ASCE, EM*1, 1963, pp 1–24.

NOTATION

The following symbols have been used in this paper:

a = plate radius

$A_0', A_0, B_0', B_0, Am, Bm$ = Constants

D = flexural rigidity of the plate $= \dfrac{Eh^3}{12(1-\nu^2)}$

E = Young's modulus

e_1 = first invariant of middle surface strains

= $\epsilon_x + \epsilon_y$ in rectangular co-ordinates

= $\epsilon_r + \epsilon_\theta$ in cylindrical co-ordinates

e_2 = second invariant of middle surface strains

= $\epsilon_x\epsilon_y - \frac{1}{4}\gamma_{xy}^2$ in rectangular co-ordinates

= $\epsilon_r\epsilon_\theta$ in cylindrical co-ordinates in case of circular symmetry

h = plate thickness

I_0, I_m = Modified Bessel's function of the first kind and of the zero order and mth order respectively.

K = foundation reaction per unit area per unit deflection

K_F	= dimensionless foundation modulus $= \frac{K}{D} a^4$.
M	= moment
q	= uniform lateral load
r, θ	= polar co-ordinates
u, v	= radial and crossradial displacements
V_1	= strain energy
w	= deflection in z-direction
σ	= direct stress
τ	= shear stress
ϵ	= direct strain
γ	= shear strain
ν	= Poisson's ratio
Γ	= Gamma function.