A NOTE ON ELASTIC IMPACT TIME DETERMINATION

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ABSTRACT

This note is intended to present an approximate method of determining direct impact time between two elastic bodies. The steps involved in impact time determination are given and the accuracy of the method based on comparision with results of the rigorous theory or experiment is illustrated. The scope of the method is also discussed.

Keywords. Impact, Impact time, Elastic impact, Direct Impact, Longitudinal wave propagation through solids.

INTRODUCTION

In many practical problems, *e.g.*, pile driving, high speed shearing, high energy rate forming, manufacturing with drop hammers, etc., it is necessary to determine the impact time which form a part of the cycle time of operation. The principles of stress wave propagation through solids provide a rigorous analysis of impact phenomena wherefrom impact time can be determined. Generally the bodies that undergo impact in the aforementioned problems are complicated in shape and are difficult to be rigorously analysed. An alternative method to determine impact time to a first degree of approximation is proposed here. The method can be easily applied even to cases involving complicated shape and/or loading where application of the stress wave theory is not so simple.

As an illustration four cases are discussed in this note. The first three are of a standard nature; results are compared with those of the rigorous analysis. The last case is a special application for which there is no rigorous analysis available; comparison with experimental results are made.

Nomenclature

A = area of cross-section ;
a = velocity of longitudinal stress waves ;
E = Young's modulus of a material ;
l = length ;
m = mass ;
K, k = spring stiffness ;
r = mass ratio $= m_1/m_2$;
ρ = mass density ;
t = time ;
W = weight ;
ω = circular frequency of vibrations ;
v = velocity ;
x = cartesian co-ordinate x.

Impact Time Determination

Impact time over which two impact elements remain in contact has generally two distinct phases for all elastic impacts. The distance between the two centres of gravity gradually reduces over a phase known as 'approach' period and then increases over a later phase known as 'recovery' period.

In the proposed method the impact process is visualised to take place with masses of the bodies concentrated at their centres of gravity and their relative motion being restrained by a massless 'bar spring' whose stiffness is fully determined by their configuration and material properties between their centres of gravity. The 'bar spring' gets compressed over the 'approach' period and this compression is relieved during the 'recovery' period. The termination of contact is characterised by no compression in the 'bar spring' and a separating relative velocity between the two bodies.

To determine stiffness K_{eq} (Fig. 1) of this 'bar spring' the portion between the plane of contact and the centre of gravity of each of the body is divided into number of segments each of which is represented by a spring. The nature of coupling among these springs are taken into account to obtain K_{eq}. Referred to Fig. 1 (*a*) and (*c*)

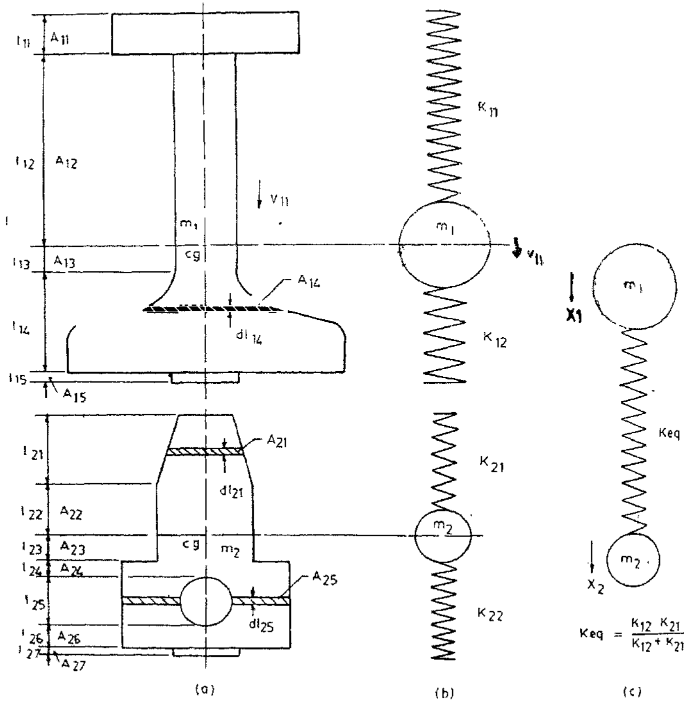

FIG. 1. A general scheme of impact.

$$\frac{1}{K_{eq}} = \left(\frac{1}{E_1 A_{13}/l_{13}} + \frac{1}{E_1 / \int\limits_{l_{14}} \frac{dl_{14}}{A_{14}}} + \frac{1}{E_1 A_{15}/l_{15}} \right) + \left(\frac{1}{E_2 / \int\limits_{l_{21}} \frac{dl_{21}}{A_{21}}} + \frac{1}{E_2 A_{22}/l_{22}} \right) \tag{1}$$

$$= \left(\frac{1}{k_{13}} + \frac{1}{k_{14}} + \frac{1}{k_{15}} \right) + \left(\frac{1}{k_{21}} + \frac{1}{k_{22}} \right) \tag{1 a}$$

where k_{13}, k_{14} etc. are the stiffnesses corresponding to the segments l_{13}, l_{14} etc. respectively as shown in Fig. 1.

This scheme of analysis thus calls for representing each impact element by a rigid mass attached to two massless springs (Fig. 1 *b*). The magnitude of the rigid mass is equal to that of the impact element and is placed at its centre of gravity. The stiffness of the two springs are determined by the body geometry and material property between its centre of gravity and the respective ends. Consequently K_{11}, K_{12}, K_{21} and K_{22} are also determined from a relation of the type (1). That is

$$\frac{1}{K_{12}} = \frac{1}{k_{13}} + \frac{1}{k_{14}} + \frac{1}{k_{15}}, \qquad \frac{1}{K_{11}} = \frac{1}{k_{11}} + \frac{1}{k_{12}}$$

$$\frac{1}{K_{21}} = \frac{1}{k_{21}} + \frac{1}{k_{22}}, \qquad \frac{1}{K_{22}} = \frac{1}{k_{23}} + \frac{1}{k_{24}} + \cdots + \frac{1}{k_{27}} \tag{1 b}$$

It may be noted that

$$\frac{1}{K_{eq}} = \frac{1}{K_{12}} + \frac{1}{K_{21}} \tag{1 c}$$

For the orientation, as shown in Fig. 1, the springs of stiffness K_{11} and K_{22} need not be included in the equivalent system. So they are disregarded in Fig. 1 (*c*).

In general, impact time t_i is determined in four steps: (i) determine the 'bar spring' stiffness K_{eq}, (ii) solve equations of motion of the equivalent system, (iii) obtain an equation in time t by applying the condition that, at the instant of separation, there is no compression in the equivalent spring K_{eq}, and (iv) solve the equation, obtained in the previous step, for a root which also satisfies the separating relative velocity conditions. These steps are illustrated in what follows.

For the case shown in Fig. 1 the equation of motion of the two bodies are

$$m_1 \frac{d^2x_1}{dt^2} = - K_{eq} (x_1 - x_2) \tag{2 a}$$

$$m_2 \frac{d^2x_2}{dt^2} = + K_{eq} (x_1 - x_2) \tag{2 b}$$

where x_1 and x_2 are two local coordinates, The relative motion $x = x_1 - x_2$ is then given by

$$\frac{d^2 x}{dt^2} = - \frac{K_{eq} (m_1 + m_2)}{m_1 m_2}$$
$$= - (K_{eq} / m_{eq}) \, x \qquad (2\,c)$$

where

$$\frac{1}{m_{eq}} = \frac{1}{m_1} + \frac{1}{m_2} \qquad (2\,d)$$

The solution of this equation can be written as

$$x = x_{max} \sin \omega t$$

where

$$\omega^2 = K_{eq} / m_{eq} \qquad (2\,e)$$

x_{max} = maximum compression of the spring K_{eq}.

For $x = 0$, $t = 0$, π/ω, $2\pi/\omega$ .... It can be shown that $t = \pi/\omega$ satisfies the separating relative velocity condition. Because at $t = \pi/\omega$, $dx/dt = - \omega x_{max}$. Therefore, the impact time $t_i = \pi/\omega$.

For this example the approach occurs during the first half of the contact period followed by the recovery in the second half. The spring K_{eq} gets compressed during the former stage and this compression is gradually relieved during the latter. The gradual recovery finally leads to loss of contact between the two bodies and hence the end of impact.

From the known relative motion, motion of each body can then be determined from equations (2 *a*) and (2 *b*).

EXAMINATION OF STANDARD CASES

For the impact of a bar of uniform cross-section A and length l against a rigid surface the 'bar spring' corresponds to a bar of cross-section A and length $l/2$. Therefore $K_{eq} = AE \, (l/2)^{-1}$. Since mass $m_{eq} = m = Al\rho$ impact time $t_i = 1 \cdot 11 \; (2l/a)$.

Likewise for the impact of two identical bars of uniform cross-section A and length l the 'bar spring' corresponds to a bar of cross-section A and length l. Therefore, $K_{eq} = AE/l$. Since $m_{eq} = m/2 = Al\rho/2$, in this case, impact time $t_i = 1 \cdot 11 \; (2l/a)$. In each of these two cases there is a difference of about 11% with the result of stress wave theory.

It is possible to determine impact time when motion of the impact elements are restrained, and/or, when constant or time dependent forces act on them during impact. Under such circumstances the springs K_{11} and K_{22} (Fig. 1 *b*) may have to be included in the equivalent system. However, impact time determination steps remain unaltered.

An example which belongs to the category of retrained motion is the well known case of impact of a rigid mass against a built-in bar. Referred to Figs. 1 and 2, $K_{11} = K_{12} = 0$ and $K_{eq} = K_{21} = 2\ AE/l = K_{22}$. On analysis

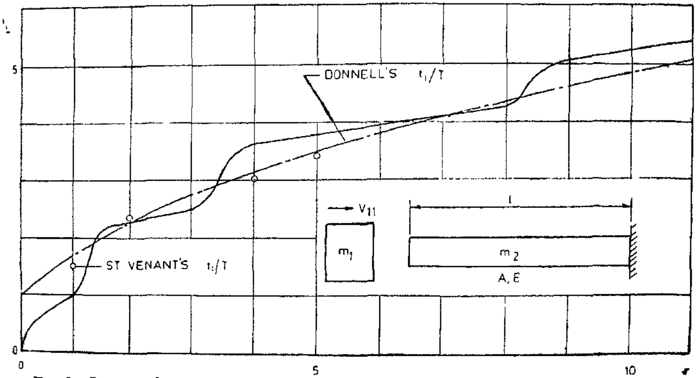

FIG. 2. Impact of a rigid mass against a built-in bar—comparison of impact time.

along the similar lines as given in the previous case, it is found that impact time t_i is a root of the transcendental equation

$$\frac{\sin \omega\,(2r + 1 - \sqrt{1 + 4r^2})^{1/2}\, t}{\sin \omega\,(2r + 1 + \sqrt{1 + 4r^2})^{1/2}\, t}$$

$$= \left(\frac{2r + 1 - \sqrt{1 + 4r^2}}{2r + 1 + \sqrt{1 + 4r^2}}\right)^{1/2} \left(\frac{1 + \sqrt{1 + 4r^2}}{1 - \sqrt{1 + 4r^2}}\right)$$

and what satisfies the separating relative velocity condition, $dx_1/dt < dx_2/dt$. Note that, in the above equation, $\omega^2 = AE/(m_1 l)$ and $r = m_1/m_2$.

Impact time t_i variation with mass ratio r is compared with the results of rigorous theory [1–4] in Fig. 2. Note that $T = 2l/a$, *i.e.*, the time taken by longitudinal stress waves to travel back and forth once through the bar. Except for mass ratios less than 1·5 the correlation is good.

EXAMINATION OF A SPECIAL CASE

Analysis was done for a case of the type shown in Fig. 1 when there is a constant force P, acting on the impacting body in the direction of its motion. As there is no rigorous analysis available for such a case theoretical impact time predictions are experimentally verified.

The equations of motion of the two bodies are

$$m_1 \frac{d^2 x_1}{dt^2} = P - K_{eq} (x_1 - x_2) \tag{4 a}$$

$$m_2 \frac{d^2 x_2}{dt^2} = K_{eq} (x_1 - x_2) \tag{4 b}$$

The relative motion is then given by

$$\frac{d^2 x}{dt^2} + \omega^2 x = P/m_1 \tag{4 c}$$

where

$$\omega^2 = K_{eq} m_1 m_2/(m_1 + m_2).$$

The solution of equation (4 c) can be written as

$$x = \frac{P}{m_1 \omega^2} (\cos \omega t - 1) - \frac{v_{11}}{\omega} \sin \omega t. \tag{4 d}$$

Impact time is then determined using the condition that, at the instant of separation, $x = 0$, and $dx/dt < 0$. Impact time so obtained can be written in the following form

$$t_i = \pi + 2\theta \tag{4 e}$$

where

$$\tan \theta = P/(m_1 v_{11} \omega)$$

$$v_{11} = \text{impacting velocity}.$$

This problem was studied in connection with the dynamics of high speed shearing process. The impact elements are the platen of the machine and the moving tool. Experimental details are reported elsewhere [5].

Impact time was measured by a standard contact circuit [6] operating between the platen and the moving tool. This contact circuit is capable of producing two distinct voltage signals one corresponding to contact between the two bodies and the other corresponding to no contact between them. The duration of the voltage signals corresponding to contact between the two bodies therefore gives impact time directly. A record of impact time is shown in Fig. 3. Experimentally impact times were found to have good reproducibility.

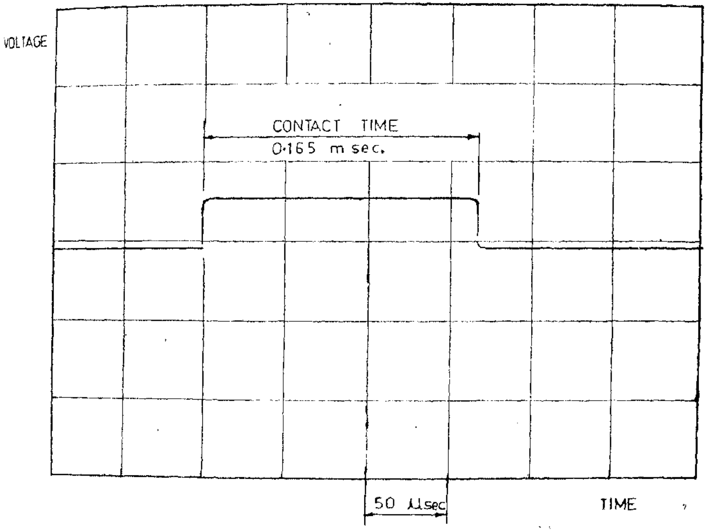

FIG. 3. A record of experimental impact time.

Theoretical impact time calculations are based on equation (4 *e*). The spring stiffnesses K_{12} and K_{21} (Fig. 1) were calculated for the existing geometry of the plate (m_1) and the moving tool (m_2) using equation (1 *b*). The moduli of elasticity used in such calculations are given in Table I. The axial load P and the corresponding impacting velocity v_{11} that are necessary for impact time calculation are also given in the same table.

Theoretical and experimental impact time are compared in Table I.

Theoretically, the effect of force P and the impacting velocity v_{11} on impact time t_i is negligible. Experimental observations substantiate the effect. The difference between experimental and theoretical values are of the order of 10–13%.

TABLE I.

Comparison of theoretical and experimental impact time

Weight of impacting body W_1 (Fig. 1) = $69 \cdot 8$ kg
Weight of impacted body $W_2 = 12 \cdot 06$ kg
Spring stiffness $K_{12} = 11 \cdot 9 \times 10^6$ kg/cm
Spring stiffness $K_{21} = 8 \cdot 5 \times 10^6$ kg/cm
Elastic Moduli $E_1 = E_2 = 2 \cdot 1 \times 10^6$ kg/cm^2

Force P(kg)	Velocity v_{11}(m/sec)	Impact time, t_i (msec)	
		Theoretical	Experimental
704	5·48	0·1441	0·168
740	5·78	0·1441	0·160
1400	6·56	0·14415	0·165
1570	7·04	0·14416	0·165
1609	7·35	0·14416	0·165
1828	7·70	0·14416	0·165

DISCUSSION

The proposed method provides a simplified picture of the mechanism of impact phenomenon whereby elastic impact time is easily determined. The method can be easily applied to practical situations which often involve complicated geometric shape and/or constant or time dependent forces.

For the three standard cases a difference of about 11% is observed with the results of rigorous theory. For the special case considered, a difference of 10–13% with experimental values is noted.

The method can be used for the study of temporal motion of the impact elements with the same order of accuracy.

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