STABILIZATION OF AN ELECTROHYDRAULIC SERVO USING DYNAMIC PRESSURE FEEDBACK

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ABSTRACT

This paper discusses the advantages and disadvantages of different methods of stabilization of poorly damped electrohydraulic servos. Using linear analysis, it is shown that the technique of dynamic pressure feedback can be employed to stabilize the servo without deteriorating its performance in other respects. An example is worked out to illustrate the design procedure.

Key words : Electrohydraulic Servo, Stabilization, Dynamic Pressure Feedback,

1. INTRODUCTION

In high power applications, it is a common practice to use electrohydraulic control systems to conveniently exploit the advantages of low power flexibility of electronics to achieve signalling and logic functions and high power capability of hydraulics to provide large forces. Such systems are said to possess electrical nerve and hydraulic muscle. Examples of such systems are found in ship steering systems, earthmoving equipment, position control of aircraft control surfaces, radar antenna, anti-aircraft guns, etc.

Inadequate damping is an inherent characteristic of hydraulic systems. Failure to take this fact into account has led some designers to accuse[1] them of not complying with theoretical predictions and having surprises in store. It is therefore necessary to recognize hydraulic systems as those having insufficient damping and evolve suitable design methods so that the overall system possesses adequate damping and performs satisfactorily.

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2. STATEMENT OF THE PROBLEM

When an electrohydraulic actuator shown in Fig. 1 is used with position feedback for position control of the load, the system is found to exhibit an objectionably large resonance peak towards the end of or in the frequency range of interest. This resonance peak appears as a result of low damping together with a low natural frequency which is equal to the square-root of the ratio between the equivalent compliance (associated with those of the actuator mounting, piston rod and linkages and the hydraulic fluid) and the mass of the load. Therefore, the problem is to evolve a design method which increases damping sufficiently or improves relative stability but does not appreciably deteriorate the performance of the servo in other respects.

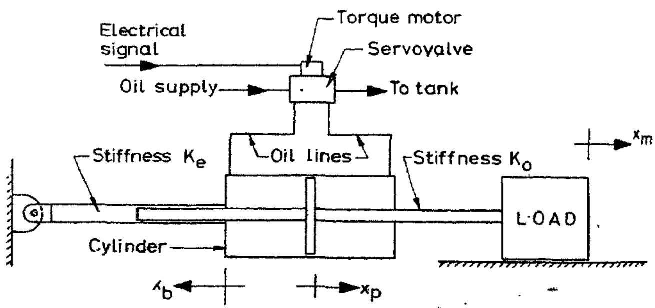

FIG.

3. METHODS OF IMPROVING DAMPING

The actuator compliance and the inertia of the load form a spring-mass system with small damping. This damping is essentially derived from small leakages across the valve and actuator piston, seal and bearing friction and a small structural damping. In well-designed actuators, the leakages and friction are deliberately kept low for better system performance.

One obvious method of increasing damping is to introduce additional leakage across the control valve and the actuator piston. But these techniques are unacceptable because of the additional power loss they cause (especially in an emergency situation in aircraft when stored energy is utilized) and the reduction in steady state accuracy and output stiffness they result in.

The use of internal dampers in the valve or increasing damping by feeding the load pressure (in an appropriate sense) to the ends of the spool suffer from large valve forces and flutter problems in aircraft applications. External dashpots fitted at the load can be used but at the expense of weight, space and other mounting problems.

Electrical compensating networks have been suggested[2] to control the undesirable nature of the second order dynamics, but these networks need to be frequently adjusted as the characteristics of the actuator drift with temperature and other variations. It is reported[2] that variation in natural frequency of the system due to changes in temperature and aeration of the oil can be as much as 40% of the theoretical value.

The above methods while introducing additional damping deteriorate the system performance in other respects. It appears that if additional damping could be introduced only around the resonance frequency, then the system might be expected to perform satisfactorily. Dynamic leakage technique[1] allows a certain amount of fluid to be leaked from one actuator chamber to the other in the frequency range of interest thereby introducing additional damping only around the resonance frequency. Dynamic pressure feedback (DPF) technique senses the differential pressure across the actuator and feeds this information through a suitable high pass filter to the torque motor of the electrohydraulic valve (EHV). The high pass filter is selected in such a way that additional damping is introduced around the resonance frequency. These two methods have been found helpful in stabilizing a hydraulic servo having an objectionable resonance peak (as a result of a lightly-damped complex pole-pair) in the frequency range of interest.

In the following, a linear model for an electrohydraulic servo (considering small perturbations about the mid-stroke position of the actuator) is developed and analyzed thereby establishing the need for DPF. An example is worked out to show how DPF can improve the system performance.

4. Modelling of the System

For the purpose of this analysis, the EHV can be treated as a component which provides an oil flow rate that is proportional to the current flowing through its torque motor. Let this transfer function be k_b $(m^3/s/A)$ and let k_a (A/V) be the gain of the amplifier preceding the EHV.

In the analysis of actuators, such as those shown in Fig. 1, it is usual to assume[1, 2, 3] a double-rod-end cylinder having a piston area of A (m^2) on either side and consider small perturbations of the piston about the mid-stroke position at which the damping is shown[4] to be minimum. Then the dynamics governing the actuator motion can be represented[1, 2, 3] by

$$q_m = A\ (\dot{x}_p + \dot{x}_b) + c_1 p_m + (V/2\beta)\ \dot{p}_m \tag{1}$$

where q_m (m^3/s) and p_m (N/m^2) respectively represent the changes in flow rate into or out of the actuator and pressure differential across the actuator, x_p (m) and x_b (m) represent respectively the motions of the piston and the actuator body as shown in Fig. 1, c_1 $(m^3/s/N/m^2)$ denotes the leakage coefficient to account for leakage past the piston, V (m^3) represents one-half the cylinder volume, β (N/m^2) the bulk modulus of the oil and the dots over the letters represent differentiation with respect to time.

The equation describing the motion of the load can be written as

$$A\ p_m = M\ddot{x}_m + Bx_m + f \tag{2}$$

where x_m (m) represents the load displacement, M $(N/m/s^2)$ denotes load mass, B $(N/m/s)$ denotes viscous frictional coefficient and f (N) the external force which may be acting on the load. The following relations are also evident from Fig. 1.

$$A\ p_m = K_e\ x_b \tag{3}$$

$$= K_0\,(x_p - x_m) \tag{4}$$

where K_e (N/m) and K_0 (N/m) respectively represent the stiffnesses of the actuator attachment and the piston rod, linkages, etc. that connect the load to the actuator. Using eqns. (1)-(4) above, the block diagram of the electro-hydraulic actuator can be drawn as shown inside the dotted rectangle in Fig. 2.

Position feedback can be implemented by sensing the load position or the piston position. If a potentiometer mounted on a rigid frame and connected to the load is used, the signal fed back from this potentiometer will be proportional to the load displacement x_m. If, on the other hand, a potentiometer mounted on the actuator body and connected to the piston rod is used, the signal fed back will be proportional to (x_p+x_b) and not x_p alone as might be mistaken. The block diagram of Fig. 2 is so

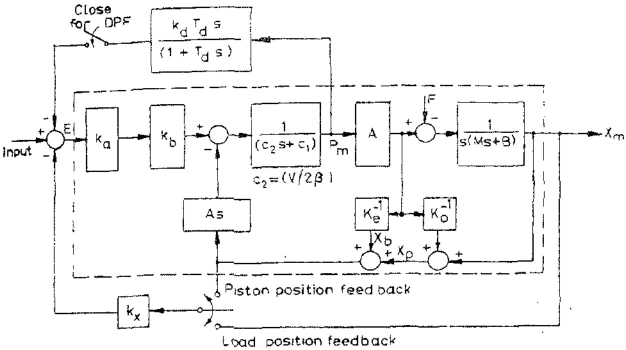

FIG. 2

drawn that it can be used either for load position feedback or piston position feedback with a feedback gain of k_x (V/m).

For implementing DPF, the differential pressure across the actuator can be sensed by a suitable transducer and fed back through a high pass filter having a transfer function of

$$H_d(s) = \frac{k_d T_d s}{(1 + T_d s)} \tag{5}$$

where k_d $(V/N/m^2)$ is the gain of the DPF loop and T_d is the time constant. The complete block diagram of the system incorporating both position and dynamic pressure feedback is shown in Fig. 2.

5. Analysis of the System

The forward transfer function of the block diagram of Fig. 2 or the transfer function of the electrohydraulic actuator, assuming the load force $f = K_f x_m$ where K_f (N/m) represents the equivalent stiffness of the external load, can be derived as

$$G(s) = \frac{X_m(s)}{E(s)} = \frac{k_a k_b / A}{sD(s) + c_1 K_f / A^2} \tag{6}$$

where

$$D(s) = (M/K_{eq})\, s^2 + [(B/K_{eq}) + (c_1 M/A^2)]s + [1 + (c_1 B/A^2) + (K_f/K_{eq})]$$

and

$$(1/K_{eq}) = (1/K_0) + (1/K_e) + (V/2\beta A^2).$$

Since the leakage coefficient c_1 and the viscous damping coefficient B are very small quantities, it can be seen that $G(s)$ will have a negative real pole very near the origin and a lightly-damped complex pole-pair.

Considering the system of Fig. 2 with position feedback only, the open loop transfer function can be written as $k_x\, G(s)$ or $k_x\, G(s)\, H_x(s)$ depending on whether load position feedback or piston position feedback is employed where

$$H_x(s) = \frac{X_p + X_b}{X_m} = \frac{(Ms^2 + Bs + K_f)(K_0 + K_e) + K_0 K_e}{K_0 K_e} \qquad (7)$$

In the case of piston position feedback, the open loop transfer function $k_x\, G(s)\, H_x(s)$ will have, besides a negative real pole, a pair of lightly-damped complex poles and zeros which cancel each other when the leakage coefficient c_1 and the inverse of the liquid spring stiffness, *viz.*, $(V/2\,\beta A^2)$ are zero.

Considering the root locus of the system of Fig. 2 with load position feedback alone, it can be seen that the lightly-damped complex pole-pair of the open loop transfer function will soon move into the right half plane as the gain k_a is increased thus making the system unstable. Similarly considering the root locus (as k_a is varied) of the system with piston position feedback alone, it can be seen that the system may be unconditionally or conditionally stable depending on whether the two branches of the root locus connecting the pairs of lightly-damped complex poles and zeros remain in the left half plane or enter the right half plane. In either case, the closed loop transfer function $G_p(s)$ of Fig. 2 with position feedback alone will have a lightly-damped complex pole-pair in the left half plane besides a negative real pole for any value of the gain k_a within the stability limit. This lightly-damped pole-pair is the one that is responsible for producing an objectionably high resonance peak in the frequency response. Besides, such a system will have a small stability margin. Thus there is a need for system compensation. It is shown in the following that the necessary compensation can be effected by employing DPF.

In order to understand the effect of DPF, assume that the position feedback loop is initially closed and a suitable value for the gain k_a chosen (within its stability limit) before closing the DPF loop so that the transfer function $G_p(s)$ of system with position feedback alone becomes the forward transfer function for the system with DPF. Then the open loop transfer function of the system with DPF is given by $G_p(s)\ H_d(s)\ H_p(s)$ where

$$H_p(s) = \frac{P_m(s)}{X_m(s)} = \frac{Ms^2 + Bs + K_f}{A} \ . \tag{8}$$

The transfer function $H_p(s)$ above can have two negative real zeros or a complex pair of zeros depending on the actual values of the parameters M, B and K_f. The root locus of the system with DPF as the gain k_d is varied is sketched in Figs. 3 (*a*) and (*b*) for the two cases wherein $H_p(s)$ has two

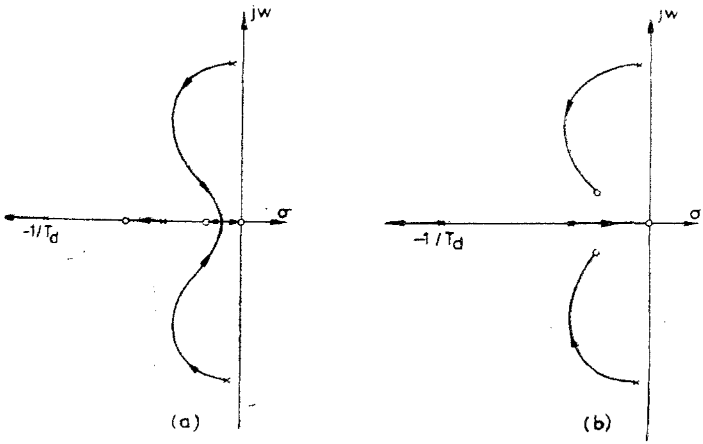

FIG. 3

negative real zeros and complex pair of zeros respectively. From this figure it can be seen that, by properly choosing the values of T_d and k_d, the damping ratio corresponding to the complex poles of the overall closed loop transfer function can be sufficiently increased thereby reducing or eliminating the resonance peak in the frequency response. A suggested value[5] for $(1/T_d)$ is approximately equal to one-third the natural frequency of the complex pole-pair of $G_p(s)$.

It is necessary that the overall system or the servo keep its commanded position under load disturbances. An idea about how well the servo maintains its position under load disturbances can be obtained by considering the mechanical impedance $[AP_m/(-X_m)]$ and the output stiffness $[F/(-X_m)]$ of the system. Higher values for these quantities indicate that the servo will be able to maintain better its commanded position under load disturbances. The mechanical impedance and the output stiffness of the system of Fig. 2, assuming load position feedback and zero command, can be written as

$$\frac{AP_m}{-X_m} = \frac{s + (k_a k_b k_x/A)}{(c_1/A^2) + (s/K_{eq}) + [k_a k_b k_d T_d s/A^2(1+T_d s)]}$$

and

$$\frac{F}{-X_m} = Ms^2 + Bs + \frac{AP_m}{-X_m} .$$

It may be noted that at low frequencies, the above two quantities are approximately equal. The low and high frequency values of the mechanical impedance are respectively given by $(k_a k_b k_x A/c_1)$ and K_{eq}. The zero frequency value of the mechanical impedance depends on the leakage coefficient c_1 and some other parameters of the system while its infinite frequency value depends only on the equivalent stiffness. The zero frequency value of the output stiffness is very high because the leakage coefficient c_1 is very small. However, an increase in leakage at higher loads will decrease the value of the output stiffness at low frequencies. Now it is clear why it is important to minimize leakage while designing the actuator and the associated valve. It may be noted that direct pressure feedback as obtained by making $T_d \to \infty$ decreases the zero frequency value of the mechanical impedance while DPF maintains it as if only load position feedback existed. At intermediate frequencies, the mechanical impedance looks like a complex spring, its value being dependent on frequency.

6. Design Example

Let the given data be:

$k_b = 31 \cdot 13 \times 10^{-3}\ m^3/s/A$

$A = 7 \cdot 52 \times 10^{-4}\ m^2$

Stroke of the actuator $= 10^{-1}\ m$

$c_1 = 0$

$\beta = 52 \cdot 75 \times 10^7\ N/m^2$

$M = 1{,}000\ kg$

$B = 21{,}100\ N/m/s$

$K_e = 9 \times 10^7\ N/m$

$K_0 = 4{\cdot}5 \times 10^7\ N/m$

$K_f = 0$

$k_x = 160\ V/m$

It is desired that the magnitude response of the overall servo monotonically decrease with frequency.

Substituting the above values in eqn. (6), the forward transfer function can be obtained as

$$G(s) = \frac{4{\cdot}3 \times 10^5\, k_a}{s[(s + 10{\cdot}55)^2 + (101{\cdot}5)^2]}\ .$$

Considering load position feedback alone, the maximum value of the gain k_a for stability can be calculated as $3{\cdot}2 \times 10^{-3}\ A/V$. Choosing $k_a = 2{\cdot}37 \times 10^{-3}\ A/V$, the transfer function $G_p(s)$ of the system with load position feedback alone becomes

$$G_p(s) = \frac{G}{1 + k_x G} = \frac{1{,}020}{(s + 15{\cdot}8)[(s + 2{\cdot}65)^2 + (101{\cdot}6)^2]}$$

The lightly-damped complex pole-pair of $G_p(s)$ gives rise to a resonance peak of about 9 db as shown by curve A of Fig. 4 Now, by employing DPF

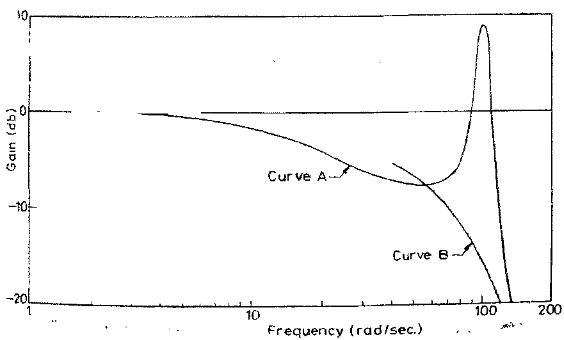

FIG. 4

and finally selecting $T_d = (1/30)$ and $k_d = 9 \cdot 2 \times 10^{-8}$ $V/N/m^2$ after considering various combinations, the overall closed loop transfer function $M(s)$ can be obtained as

$$k_x M(s) = \frac{1 \cdot 58 \times 10^5 (s + 30)}{(s + 16 \cdot 8)(s + 51 \cdot 5)[(s + 50 \cdot 5)^2 + (54)^2]}$$

The magnitude response plot of $k_x M(s)$ with frequency is as shown by curve B of Fig. 4 which monotonically decreases as desired.

7. CONCLUSION

This paper has considered different methods of stabilization of poorly-damped electrohydraulic servos and discussed their advantages and disadvantages. Using linear analysis, it was shown that dynamic pressure feedback can be advantageously used to improve relative stability of the servo without deteriorating the system performance in other respects. An example is worked out to illustrate the design procedure.

REFERENCES

1. Guillon, M. — *Hydraulic Servo Systems*, Butterworths, 1969.
2. Merritt, H. E. — *Hydraulic Control Systems*, John Wiley, 1967.
3. Blackburn, J. F., Reethof, G. and Shearer, J. L. (Editors) — *Fluid Power Control*, John Wiley, 1960.
4. Rao, S. N. — Damping in oil hydraulic control systems, *Fluid Power News* 1975, 1 (1).
5. Edwards, J. W. — Analysis of an electrohydraulic aircraft control surface servo and comparison with test results, NASA TN D-6928, 1972.