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Second order fluid flow between two rotating porous discs of moderate rotation

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Abstract

The flow of second order fluid between two rotating porous discs is studied. The equations of motions are solved by a regular perturbation method for small Reynolds number. The effects of the viscoelasticity parameter, cross-viscosity parameter of the fluid, suction/injection parameter and rotation parameter on the velocity components, pressure distribution and skin friction have been discussed numerically and compared with Newtonian fluid case.

Key words: Porous, moderate rotation, viscoelasticity, cross-viscosity.

1. Introduction

The viscous laminar flow between porous discs has recently been studied by several authors. Elkouh[1-3] obtained solutions of laminar flow between non-rotating and rotating porous discs with equal suction and injection. Narayana and Rudraiah[4] studied the steady axisymmetric flow of a viscous incompressible fluid between two co-axial discs, one rotating and the other stationary, with uniform suction at the stationary disc. Wang[5] studied the symmetric viscous flow between two rotating porous discs with moderate rotation. The results were compared with those from numerical integration. In this paper we extend the problem of Wang[5], to the flow of second order fluid between two rotating porous discs.

The model of the second order fluid as suggested by Coleman and Noll[6] is used in the present analysis. The constitutive equations of an incompressible second order fluid are

$$\tau_{ij} = -pg_{ij} + \phi_1 A_{ij} + \phi_2 B_{ij} + \phi_3 A_i^k A_{kj}, \tag{1}$$

$$A_{ij} = v_{i,j} + v_{j,i}, \tag{2}$$

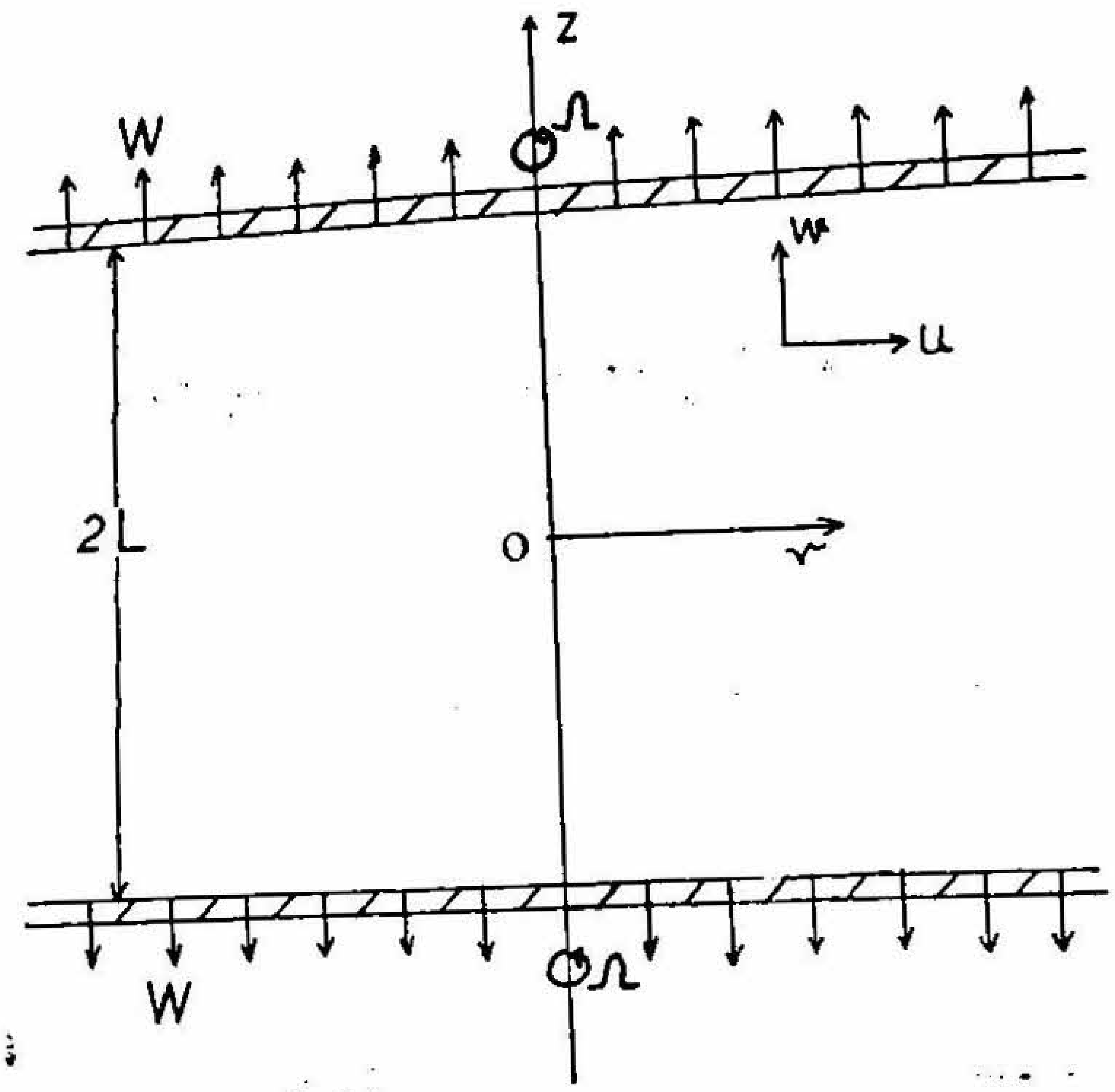

FIG. 1. The physical model.

and

$$B_{ij} = a_{i,j} + a_{j,i} + v_{m,i}\, v_{,j}^{m} + v_{m,j}\, v_{,i}^{m} \tag{3}$$

where τ_{ij} is the stress tensor, g_{ij} is the metric tensor, a_i and v_i are the acceleration and velocity vectors, ϕ_1, ϕ_2, ϕ_3 are the fluid parameters, p is the pressure and comma denotes covariant differentiation. The solution of 6·8 per cent polyisobutylene in cetane at 30° C behaves as a second order fluid and the values of the constants ϕ_1, ϕ_2 and ϕ_3 have been determined experimentally by Markovitz and Brown[7] and Markovitz[8].

2. Equations of motion

Consider two coaxial porous discs situated at $Z = \pm L$ and rotating with the same angular velocity as shown in fig. 1. Fluid is withdrawn from both discs with velocity W. Assuming that the gap with $2L$ is small compared to the diameter of the discs, the end effects are neglected. The flow field is symmetric about the $Z = 0$ plane and the Z-axis.

The incompressible axisymmetric equations of motion and continuity equation in cylindrical polar coordinates are

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right) = -\frac{\partial p}{\partial r} + \frac{\partial \tau rr}{\partial r} + \frac{\partial \tau rz}{\partial z} + \frac{\tau rr - \tau\theta\theta}{r},$$

$$\rho\left(u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r}\right) = \frac{\partial \tau r\theta}{\partial r} + \frac{\partial \tau\theta z}{\partial z} + 2\frac{\tau r\theta}{r}, \tag{5}$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{\partial \tau rz}{\partial r} + \frac{\partial \tau zz}{\partial z} + \frac{\tau rz}{r} \tag{6}$$

and

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{7}$$

where τ_{ij} are the stress components, ρ is the density and u, v, w are the velocity components in the directions r, θ, z respectively. The boundary conditions are

$$Z = \pm L,\ u = 0,\ v = r\Omega,\ w = \pm W. \tag{8}$$

3. Solution of the problem

Utilizing the symmetry of the problem, we define

$$u = rf'(\eta)\,W/L,\ v = rg(\eta)\,W/L,\ W = -2f(\eta)\,W$$

$$p = -\rho r^2 AW^2/2L^2 + \rho r^2 Q(\eta) + \rho P(\eta) \tag{9}$$

where $\eta = Z/L$, p is the modified pressure and A is a constant to be determined. Equations (4)–(7), using (9), give

$$f''' - R(f'^2 - 2ff'' - g^2) - 2KR(f''^2 + ff^{iv} + 2g'^2)$$
$$- SR(f''^2 + 2f'f''' + 3g'^2) + AR = 0 \tag{10}$$

or, after differentiating once we have

$$f^{iv} + 2R(ff''' + gg') - KR(4f''f''' + 2f'f^{iv}$$
$$+ 2ff^{v} + 8g'g'') - RS(4f''f''' + 2f'f^{iv} + 6g'g'') = 0, \tag{11}$$

$$g'' - 2R(f'g - fg') + 2KR(f''g' - fg''') + 2SR(f''g' - f'g'') = 0, \tag{12}$$

$$Q(\eta) = (2\nu_2 + \nu_3)\frac{W^2}{L^4}(f''^2 + g'^2), \tag{13}$$

$$P(\eta) = -2W^2f^2 - 2\nu_1 W + f'/L + \nu_2 W^2(4ff'' + 16f'^2)/L^2$$
$$+ 14\nu_3 f'^2/L^2 + B \tag{14}$$

where $\nu_1 = \phi_1/\rho$, $\nu_2 = \phi_2/\rho$, $\nu_3 = \phi_3/\rho$,

$R = \rho WL/\phi_1$ is the cross-flow Reynolds number (for suction case R is positive), $K = \phi_2/\rho L^2$ is the dimensionless viscoelasticity parameter, and $S = \phi_3/\rho L^2$ is the dimensionless cross-viscosity parameter.

The constant B is determined from the pressure at the disc. The boundary conditions (8) are reduced to

$$f'(1) = f(0) = 0 = f''(0),\ f(1) = -1/2, \tag{15}$$

and

$$g'(0) = 0,\ g(1) = \Omega L/W = \beta. \tag{16}$$

The equations (11) and (12) are two simultaneous non-linear differential equations with boundary conditions (15) and (16). We assume that the suction/injection parameter R is very small. Then $f(\eta)$ and $g(\eta)$ can be expanded in terms of the small parameter R :

$$f(\eta) = f_0 + Rf_1 + R^2 f_2 + \ldots, \tag{17}$$

and

$$g(\eta) = g + Rg_1 + R^2 g_2 + \ldots. \tag{18}$$

Equations (11) and (12) using (17) and (18), we have

$$f_0^{iv} = 0, \tag{19}$$

$$f_1^{iv} + 2(f_0 f_0''' + g_0 g_0') - K(4f_0'' f_0''' + 2f_0' f_0^{iv} + 2f_0 f_0^{v} + 8g_0' g_0'') - S(4f_0'' f_0''' + 2f_0' f_0^{iv} + 6g_0' g_0'') = 0, \tag{20}$$

$$f_2^{iv} + 2(f_1 f_0''' + f_0 f_1''' + g_0 g_1' + g_1 g_0') - K(4f_0'' f_1''' + 4f_1'' f_0''' + 2f_0' f_1^{iv} + 2f_1' f_0^{iv} + 2f_0 f_1^{v} + 2f_1 f_0^{v}{}' + 8g_0' g_1'' + 8g_1' g_0'') - S(4f_0'' f_1''' + 4f_1'' f_0'' + 2f_0' f_1^{iv} + 2f_1' f_0^{iv} + 6g_0' g_1'' + 6g_1' g_0'') = 0, \tag{21}$$

$$g_0'' = 0, \tag{22}$$

$$g_1'' - 2(f_0' g_0 - f_0 g_0') + 2K(f_0'' g_0' - f_0 g_0''') + 2S(f_0'' g_0' - f_0' g_0'') = 0, \tag{23}$$

and

$$g_2'' - 2(f_0' g_1 + f_1' g_0 - f_0 g_0' - f_0 g_1) + 2K(f_0'' g_1' + f_1'' g_0' - f_0 g_1''' - f_1 g_0''') + 2S(f_0'' g_1' + f_1'' g_0' - f_0' g_1'' - f_1' g_0'') = 0. \tag{24}$$

The corresponding boundary conditions are

$$\begin{aligned} &f_0(0) = f_0''(0) = 0 = f_0'(1),\ f_0(1) = -1/2\ ;\\ &f_1(0) = f_1''(0) = 0 = f_1'(1) = f_1(1)\ ;\\ &f_2(0) = f_2''(0) = 0 = f_2'(1) = f_2(1)\ ;\\ &g_0'(0) = 0,\ g_0(1) = \beta\ ;\\ &g_1'(0) = 0 = g_1(1)\ ; \end{aligned} \tag{25}$$

and

$$g_2'(0) = 0 = g_2(1). \tag{26}$$

The solutions of the equations (19)-(24) with (25) and (26) are

$$f_0 = 1/4\,\eta^3 - 3/4\eta. \tag{27}$$

$$f_1 = 1/1120\,\eta^7 + 3/160\,\eta^5 - 39/1120\,\eta^3 + 19/1120\,\eta + (K+S)(3/40\,\eta^5 - 3/20\,\eta^3 + 3/40\,\eta), \tag{28}$$

$$
\begin{aligned}
f_2 = {} & 3/246400\,\eta^{11} - 1/3360\,\eta^9 + 531/235200\,\eta^7 - 51/16800\,\eta^5 \\
& + 443/1034880\,\eta^3 + 137/215600\,\eta + \beta^2(-1/840\,\eta^7 + 1/40\,\eta^5 \\
& - 13/280\,\eta^3 + 19/840\,\eta) + K^2(54/840\,\eta^7 - 189/600\,\eta^5 \\
& + 1836/4200\,\eta^3 - 261/1400\,\eta) + KS(207/1680\,\eta^7 - 621/1200\,\eta^5 \\
& - 5589/8400\,\eta^3 - 759/2800\,\eta) + S^2(99/1680\,\eta^7 - 243/1200\,\eta^5 \\
& + 1917/8400\,\eta^3 - 237/2800\,\eta) + K(-517/282240\,\eta^9 \\
& + 819/29400\,\eta^7 - 1011/11200\,\eta^5 + 18397/176400\,\eta^3 \\
& - 18839/470400\,\eta) + S(-11/6720\,\eta^9 + 141/5600\,\eta^7 \\
& - 87/1400\,\eta^5 + 929/16800\,\eta^3 - 187/11200\,\eta), \qquad (29)
\end{aligned}
$$

$$g_0 = \beta, \qquad (30)$$

$$g_1 = \beta(1/8\,\eta^4 - 3/4\,\eta^2 + 5/8), \qquad (31)$$

$$
\begin{aligned}
g_2 = {} & \beta[-3/2240\,\eta^8 + 1/80\,\eta^6 - 111/3360\,\eta^4 - 253/560\,\eta^2 \\
& + 3183/6720 + K(1/40\,\eta^6 - 3/40\,\eta^4 + 3/40\,\eta^2 - 1/40) \\
& + S(1/20\,\eta^6 - 3/40\,\eta^4 + 6/5\,\eta^2 - 47/40)]. \qquad (32)
\end{aligned}
$$

After $f(\eta)$ is known, the constant A is determined by

$$
\begin{aligned}
A = {} & f''_{(1)} - \beta^2 - f'''_{(1)}/R + K(2f''^2_{(1)} - f^{iv}_{(1)} + 4g'^2_{(1)})/2 \\
& + S(f''^2_{(1)} + 3g'^2_{(1)}). \qquad (33)
\end{aligned}
$$

When $K = 0 = S$, the solutions for $f(\eta)$ and $g(\eta)$ reduce to those obtained by Wang[5] who studied the symmetric viscous flow between two rotating porous discs with moderate rotation. When $k = 0 = S$, $\beta = 0$, the solution for $f(\eta)$ reduces to those obtained by Elkouh[1], who studied the small Reynolds number flow between two non-rotating porous discs.

4. Pressure distribution

The pressure on either disc is

$$p(r, 1) - p(r_{0,1}) = \frac{\rho w^2}{L^2}\left[-\frac{A}{2} + (2K + S)(f''^2_{(1)} + g'^2_{(1)})\right](r^2 - r_0^2). \qquad (34)$$

where r_0 is a certain distance in radial direction.
The dimensionless pressure coefficient is

$$
\begin{aligned}
p^* & = \frac{p(r,1) - p(r_0,1)}{\frac{1}{2}\rho w^2}\left(\frac{L}{r_0}\right)^2 \\
& = \left[\frac{27}{35} - \beta^2 - \frac{3}{2R} - \frac{207}{20}K - \frac{117}{20}K + R\left(-\frac{151}{5390} - \frac{34}{35}\beta^2 - \frac{1719}{175}K^2\right.\right. \\
& \quad \left. - \frac{2313}{175}KS - \frac{594}{175}S^2 - \frac{13288}{7350}K - \frac{298}{250}S\right)
\end{aligned}
$$

$$+ R^2 \left\{ \frac{447}{26950} + \beta^2 \left(\frac{6}{35} - \frac{106}{35} K + \frac{17}{35} S \right) - \frac{1809}{175} K^3 - \frac{5490}{175} K^2 S \right.$$
$$- \frac{3573}{175} KS^2 + \frac{108}{175} S^3 - \frac{30711}{3675} K^2 - \frac{14054}{1225} KS - \frac{156}{175} S^2$$
$$\left. \left. - \frac{99181}{80850} K - \frac{4036}{26950} S \right\} \right] 1 - r^2/r_0^2. \tag{35}$$

5. Skin friction

The dimensionless skin friction coefficient is

$$\tau^* = \frac{\tau}{\frac{1}{2}\rho w^2} \left(\frac{L}{r_0} \right). \tag{36}$$

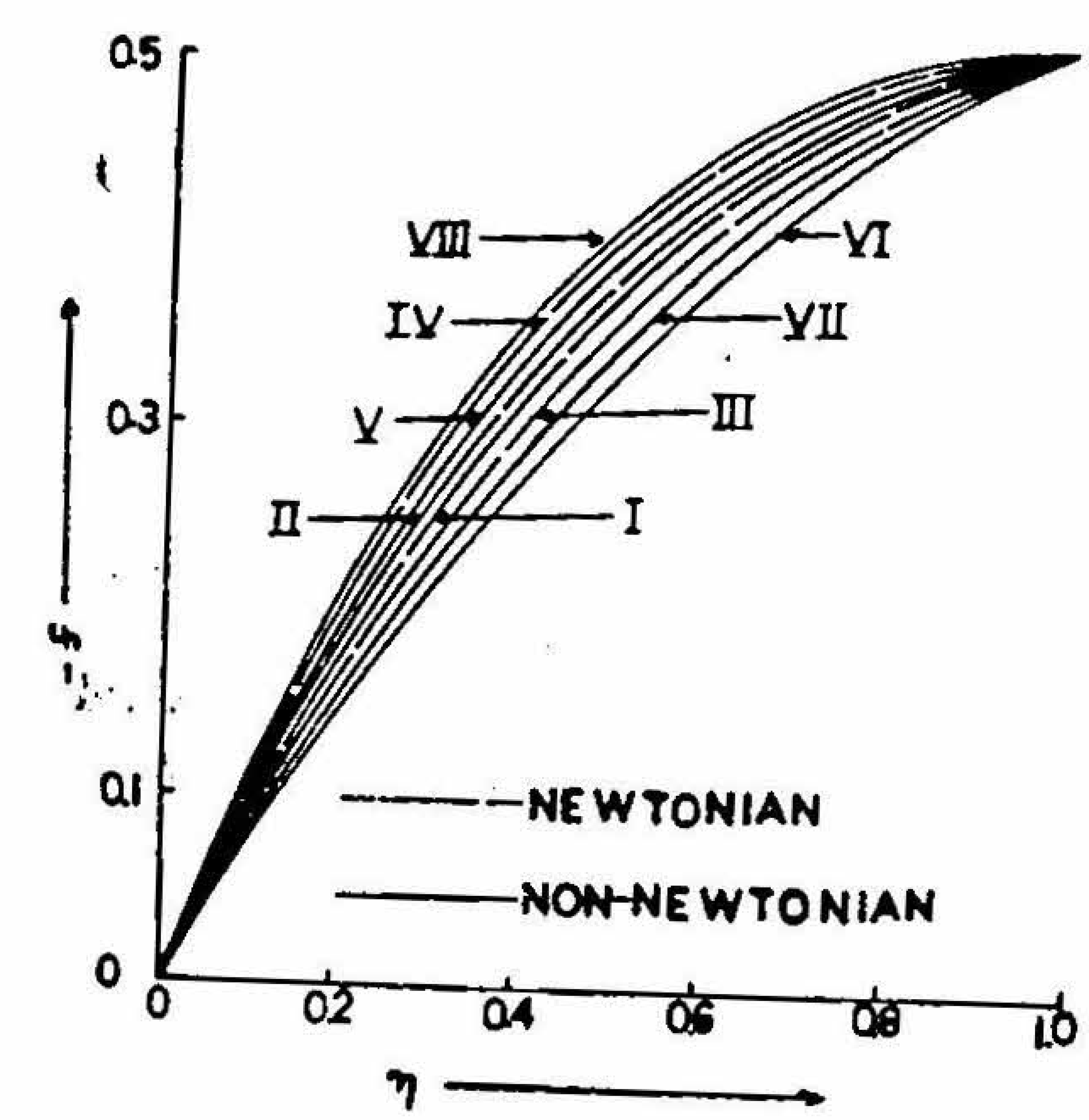

FIG. 2. Normal velocity distribution.

R	K	S	β	
0·1	0	0	0·5	I
−0·1	0	0	0·5	II
0·5	0	0	0·5	III
−0·5	0	0	0·5	IV
−0·1	−0·1	0·2	0·5	V
−0·1	−0·1	0·2	0·5	VI
−0·5	−0·1	0·2	0·5	VII
−0·5	−0·1	0·2	0·5	VIII

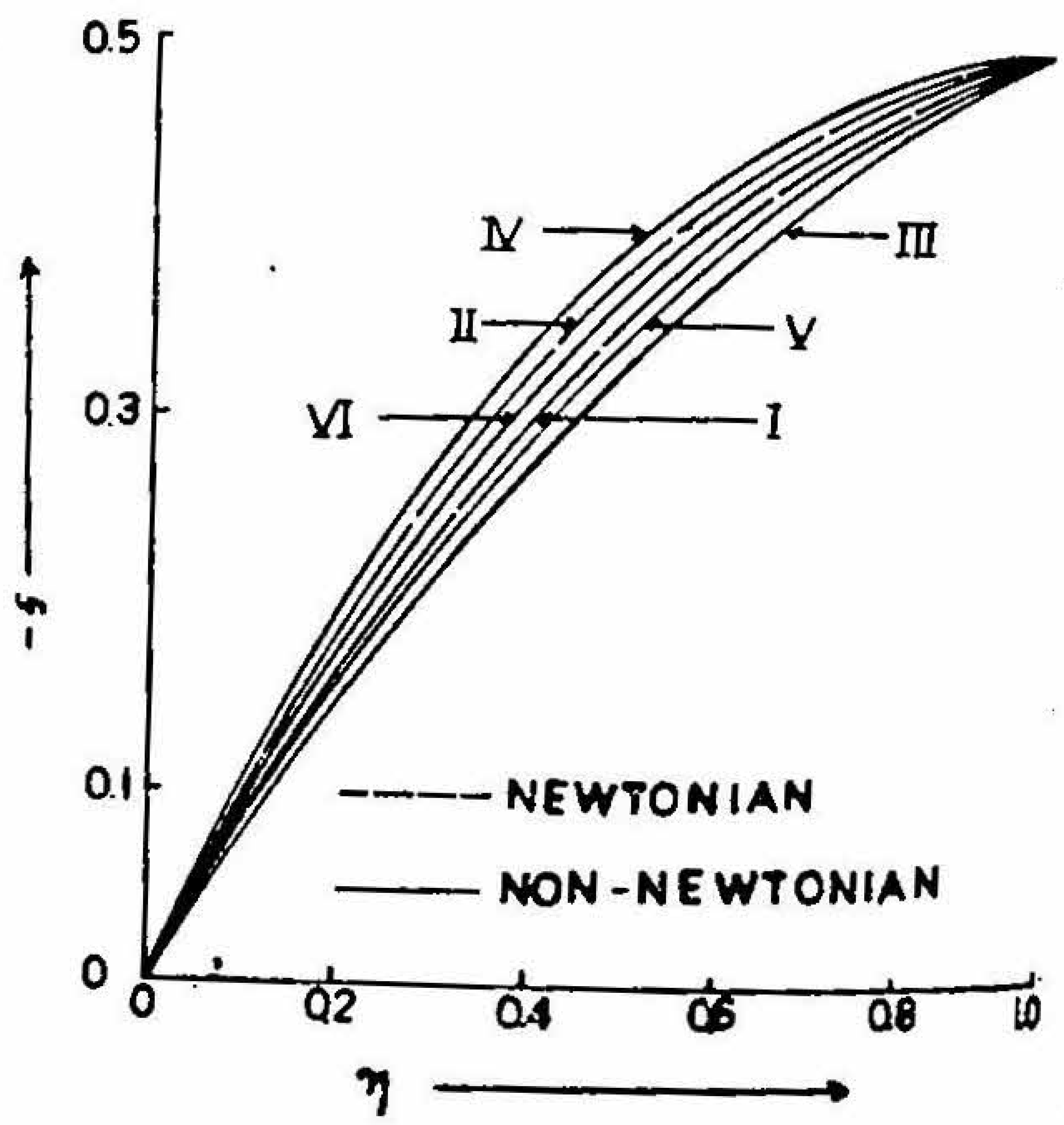

FIG. 3. Normal velocity distribution.

R	K	S	β	
0·1	0	0	0·5	I
−0·1	0	0	0·5	II
0·1	−0·1	0·2	0·5	III
−0·1	−0·1	0·2	0·5	IV
−0·1	−0·1	0·2	0·5	V
−0·1	−0·2	0·2	0·5	VI

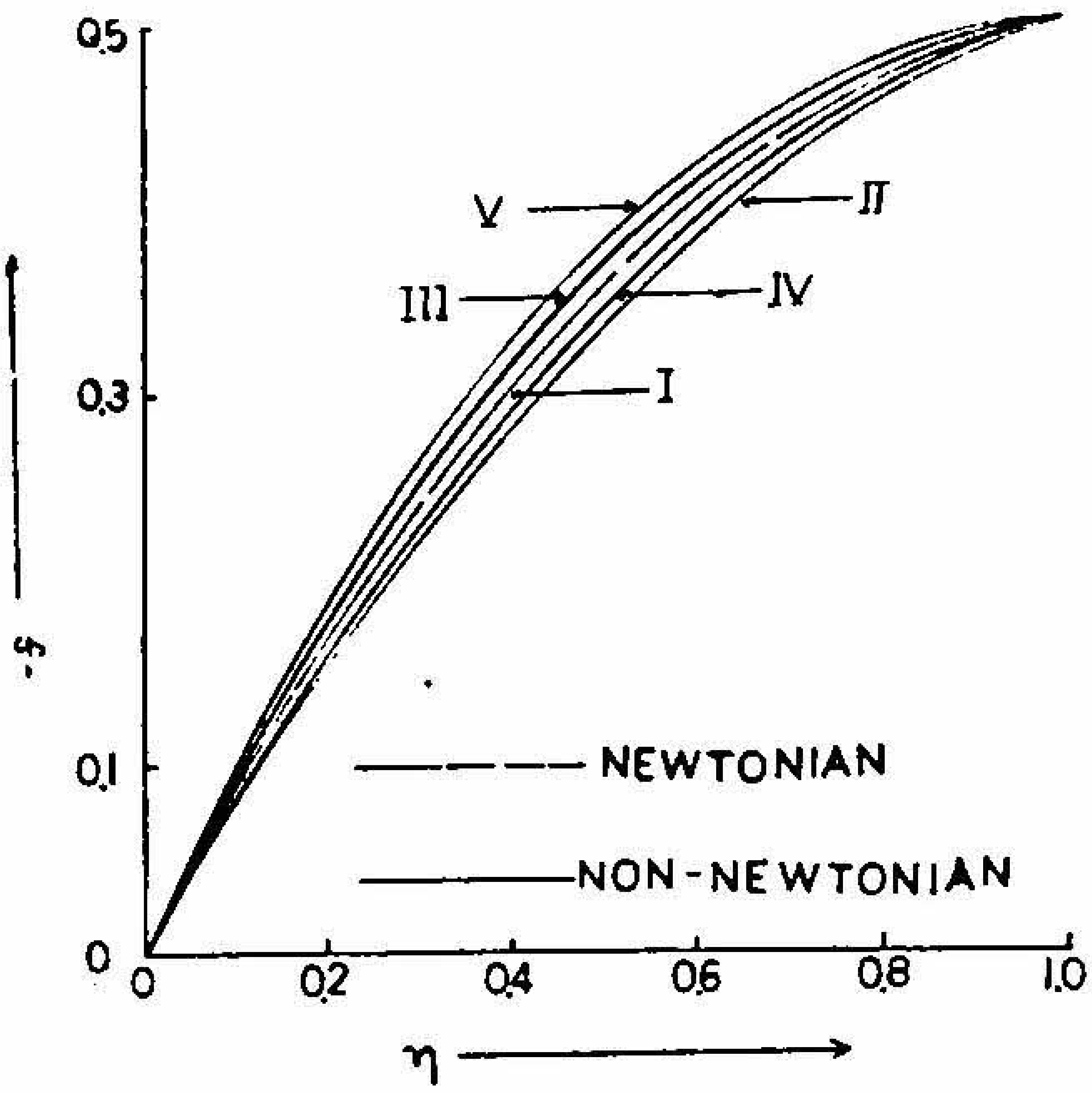

FIG. 4. Normal velocity distribution.

R	K	S	β	
0·1	0	0	0·5	I
0·1	−0·1	0·2	0·5	II
−0·1	−0·1	0·2	0·5	III
0·1	−0·1	0·3	0·5	IV
−0·1	−0·1	0·3	0·5	V

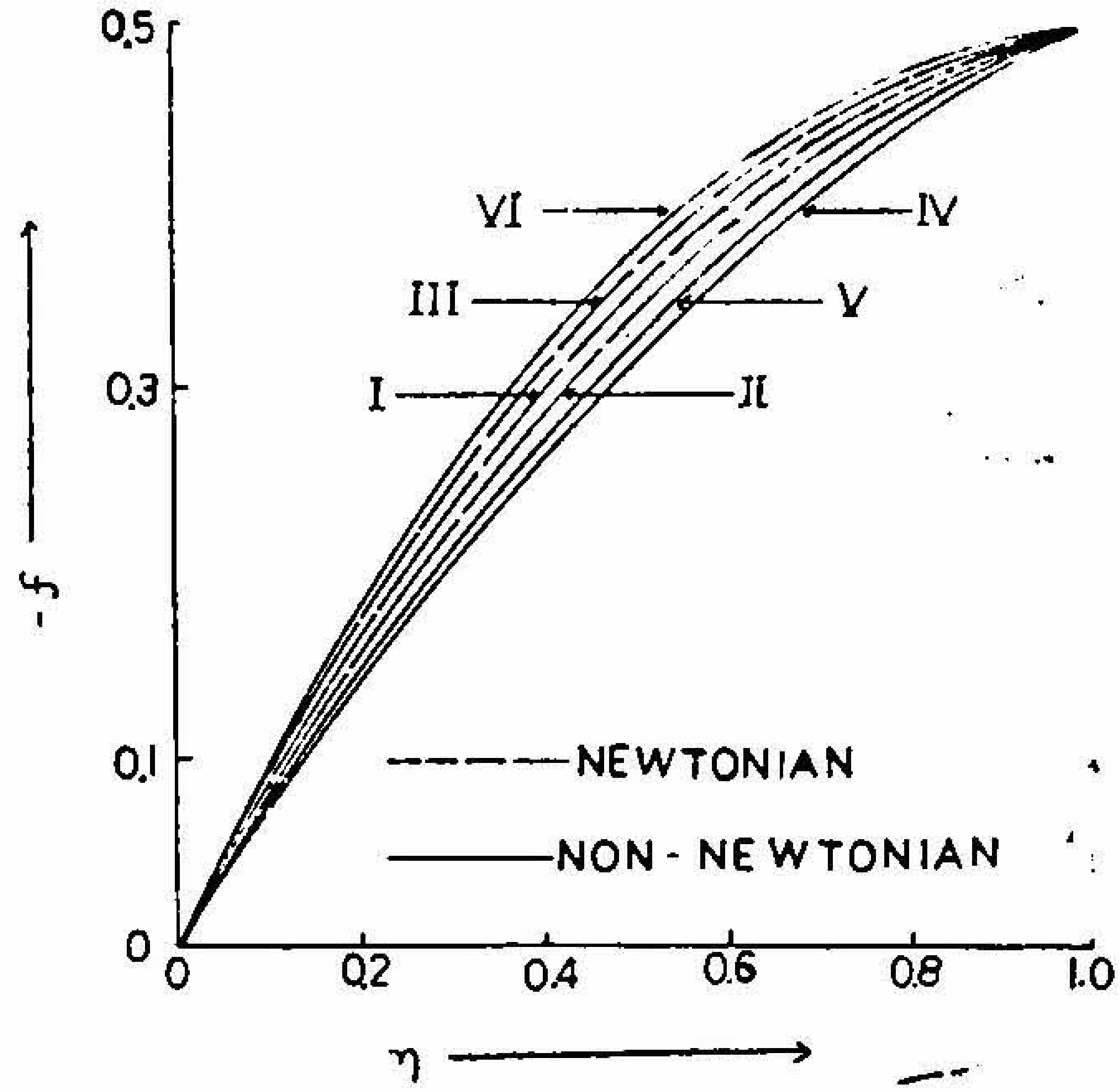

FIG. 5. Normal velocity distribution.

R	K	S	β	
0·1	0	0	0·5	I
0·1	0	0	1·0	II
−0·1	0	0	1·0	III
0·1	−0·1	0·2	0·5	IV
0·1	−0·1	0·2	1·0	V
−0·1	−0·2	0·2	1·0	VI

Hence, skin friction coefficient at $\eta = 1$ is

$$\tau^* = -\left[2f''/R + 4K(f'f'' - ff''') - (4\,Sf'f'')\right]\left(\frac{r}{r_0}\right)\eta = +1 \tag{37}$$

$$\begin{aligned}
&= -\Bigg[3/R + \frac{9}{35} + \frac{21}{5}K + \frac{6}{5}S + R\left(\frac{447}{13475} + \frac{12}{35}\beta^2\right. \\
&\quad \left. + \frac{2754}{525}K^2 + \frac{846}{175}KS - \frac{72}{175}S^2 + \frac{2927}{2940}K + \frac{2}{35}S\right) \\
&\quad + KR^2\left(\frac{844}{2695} + \frac{68}{35}\beta^2 - \frac{2916}{525}K^2 - \frac{414}{175}KS + \frac{558}{175}S^2\right. \\
&\quad \left. + \frac{1003}{3675}K + \frac{373}{175}S\right)\Bigg]\left(\frac{r}{r_0}\right).
\end{aligned} \tag{38}$$

where τ is the sheer stress at the disc.

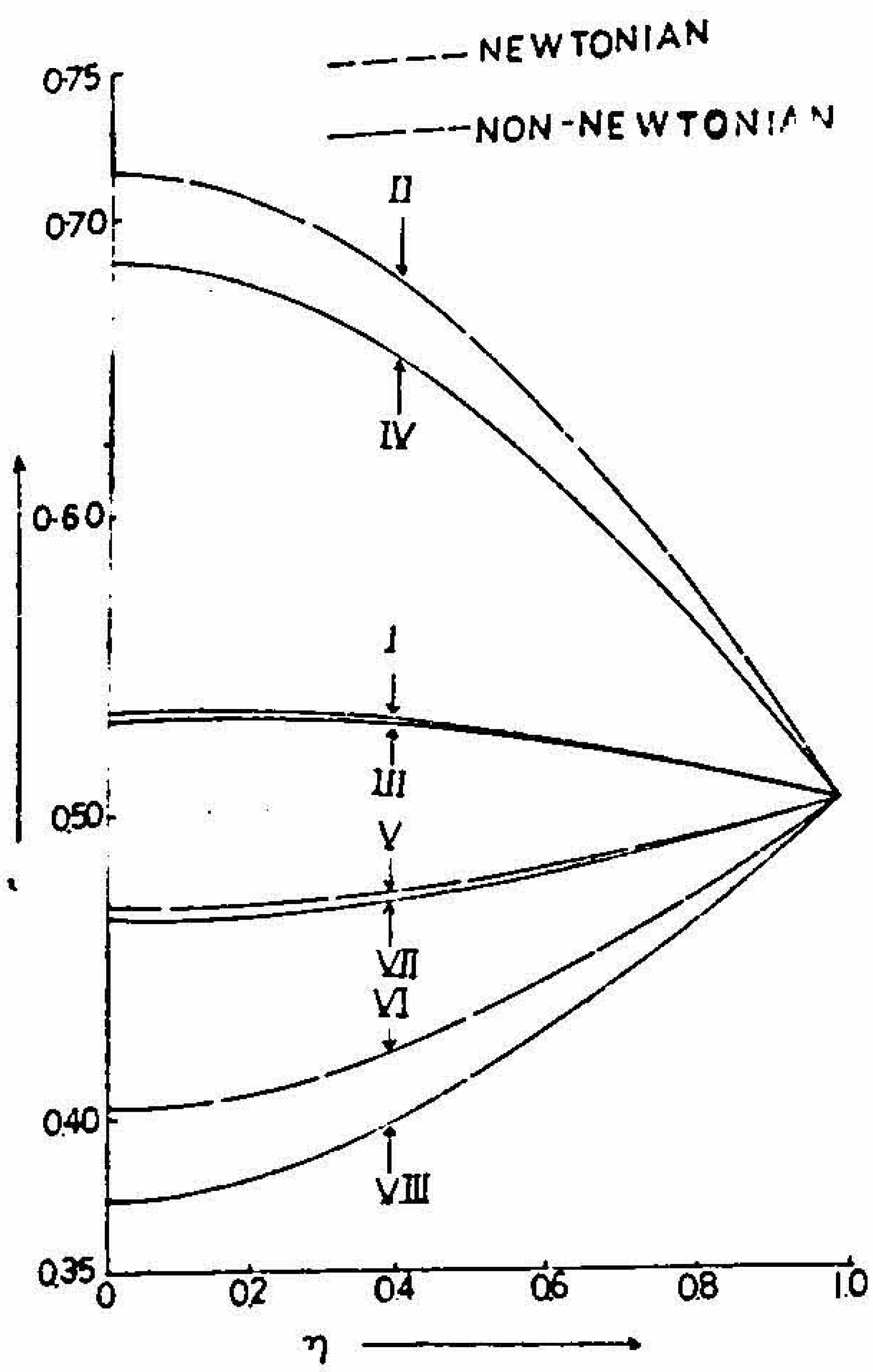

FIG. 6. Azimuthal velocity distribution.

R	K	S	β	
0·1	0	0	0·5	I
0·5	0	0	0·5	II
0·1	−0·1	0·2	0·5	III
0·5	−0·1	0·2	0·5	IV
−0·1	0	0	0·5	V
−0·5	0	0	0·5	VI
−0·1	−0·1	0·2	0·5	VII
−0·5	−0·1	0·2	0·5	VIII

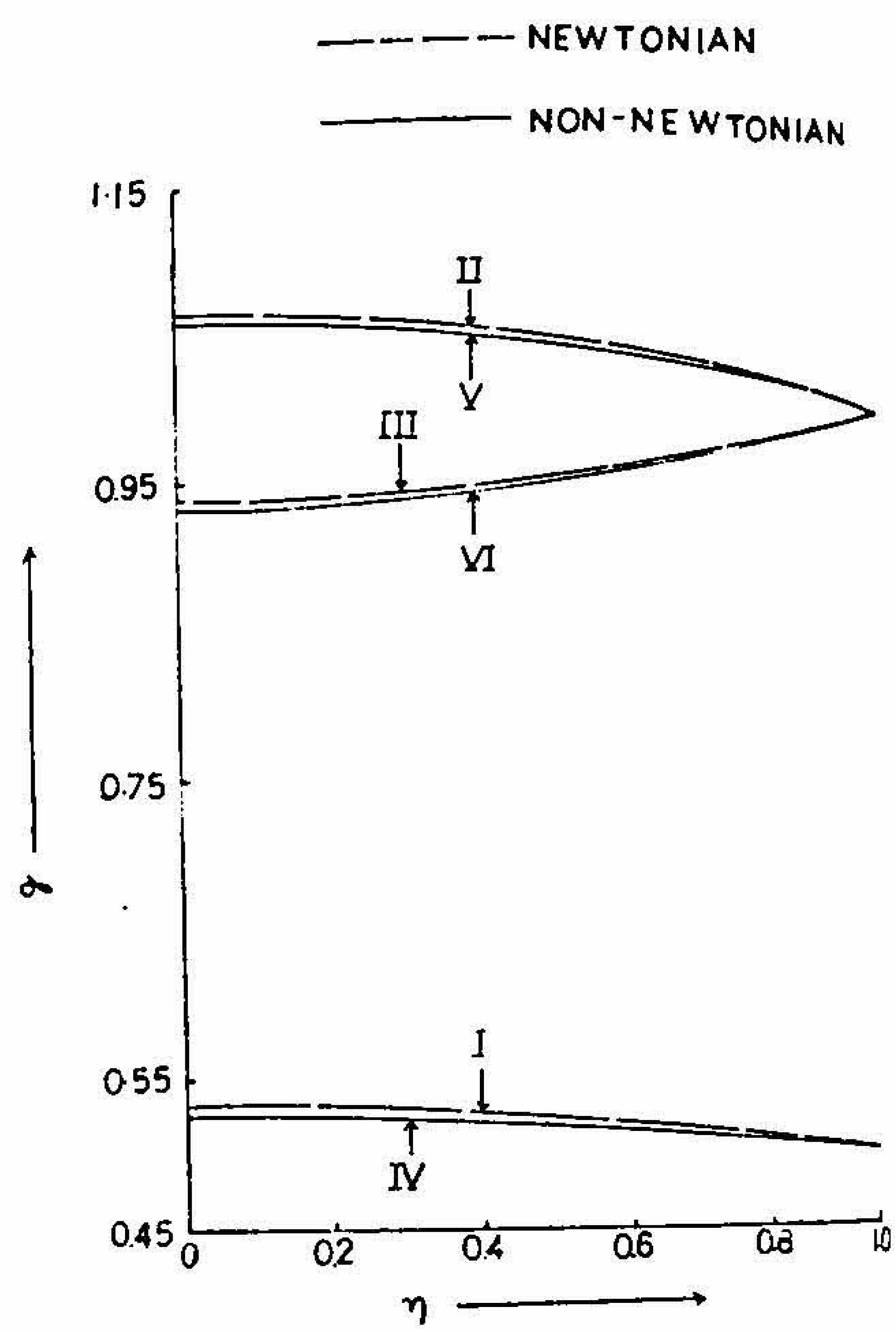

FIG. 7. Azimuthal velocity distribution.

R	K	S	β	
0·1	0	0	0·5	I
0·1	0	0	1·0	II
−0·1	0	0	1·0	III
0·1	−0·1	0·2	0·5	IV
0·1	−0·1	0·2	1·0	V
−0·1	−0·1	0·2	1·0	VI

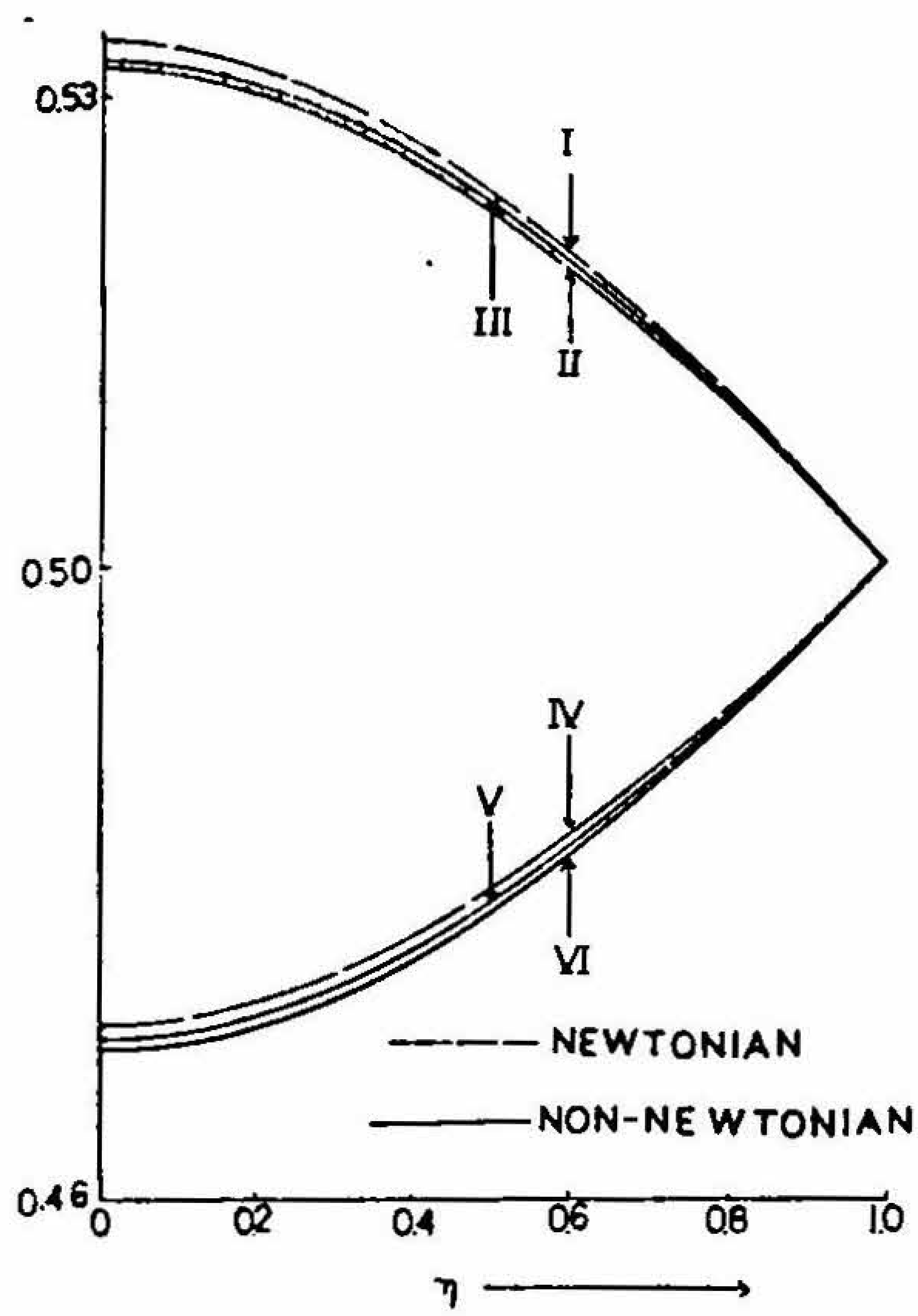

FIG. 8. Azimuthal velocity distribution.

R	K	S	β	
0·1	0	0	0·5	I
0·1	−0·1	0·2	0·5	II
0·1	−0·2	0·2	0·5	III
−0·1	0	0	0·5	IV
−0·1	−0·1	0·2	0·5	V
−0·1	−0·2	0·2	0·5	VI

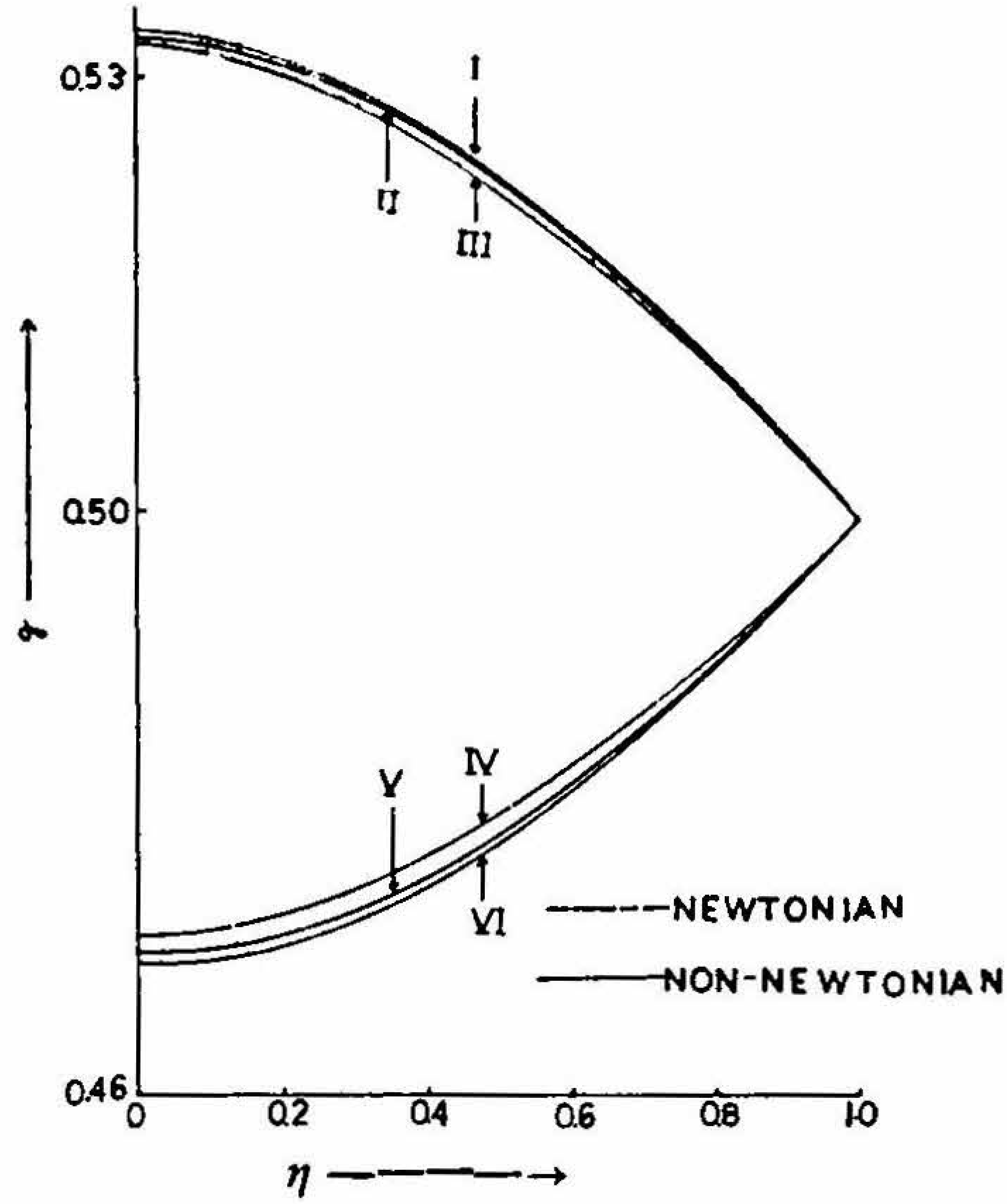

FIG. 9. Azimuthal velocity distribution.

R	K	S	β	
0·1	0	0	0·5	I
0·1	−0·1	0·2	0·5	II
0·1	−0·1	0·3	0·5	III
−0·1	0	0	0·5	IV
−0·1	−0·1	0·2	0·5	V
−0·1	−0·1	0·3	0·5	VI

6. Discussion

In this paper the problem of steady laminar flow of an incompressible second order fluid between two rotating porous discs has been studied. The basic equations have been solved by the perturbation method in which the suction/injection parameter is taken as the small perturbation parameter. From (17) and (18) it is found that the maximum value of $R \leqslant 2 \cdot 30$ for Newtonian case and that for non-Newtonian case $R \leqslant 2 \cdot 45$. Also, in the present analysis two discs rotating with the same angular velocity in the same sense have been considered.

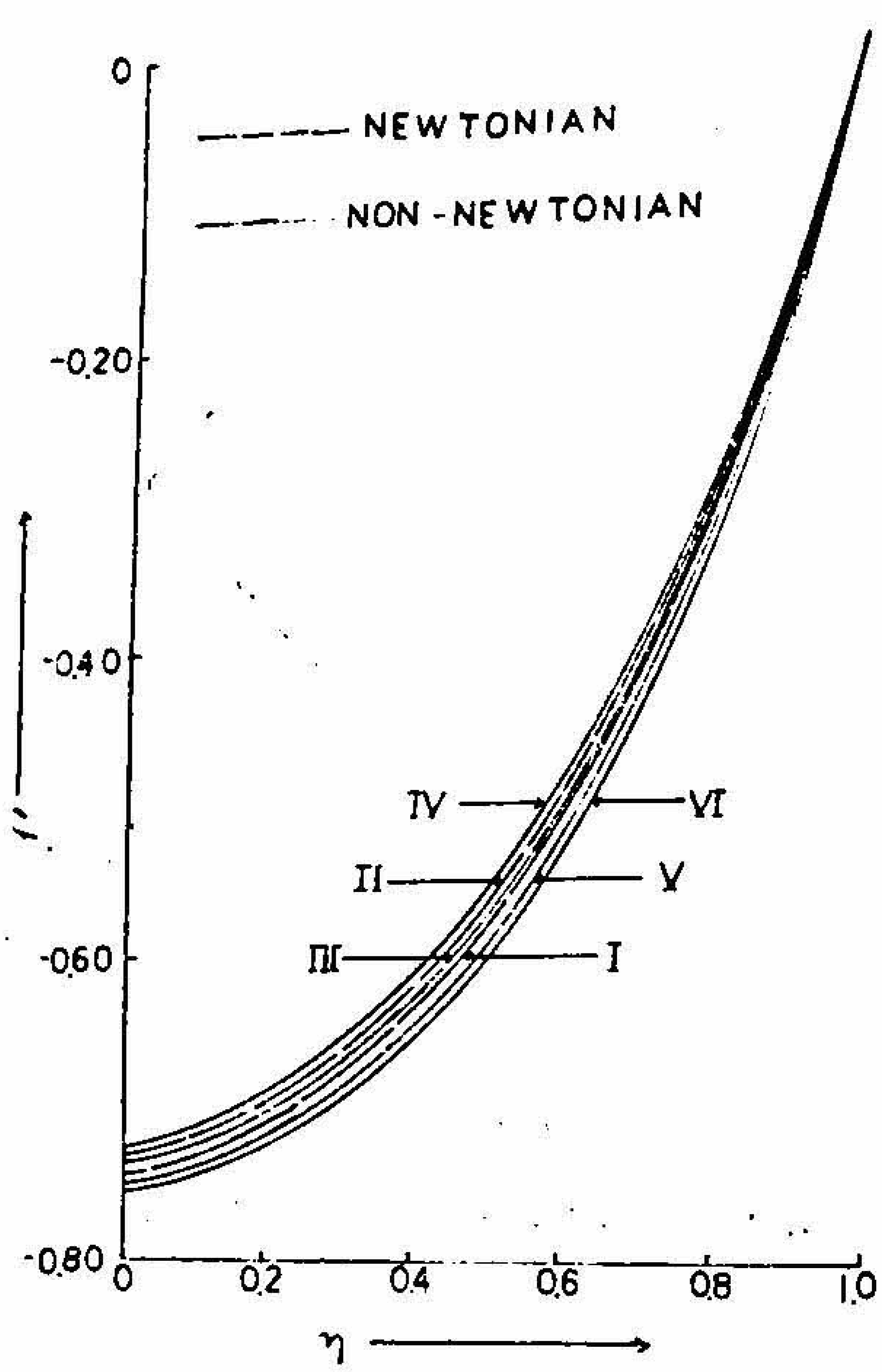

FIG. 10. Radial velocity distribution.

R	K	S	β	
0·1	0	0	0·5	I
0·5	0	0	0·5	II
0·1	−0·1	0·2	0·5	III
0·5	−0·1	0·2	0·5	IV
−0·1	−0·1	0·2	0·5	V
−0·5	−0·1	0·2	0·5	VI

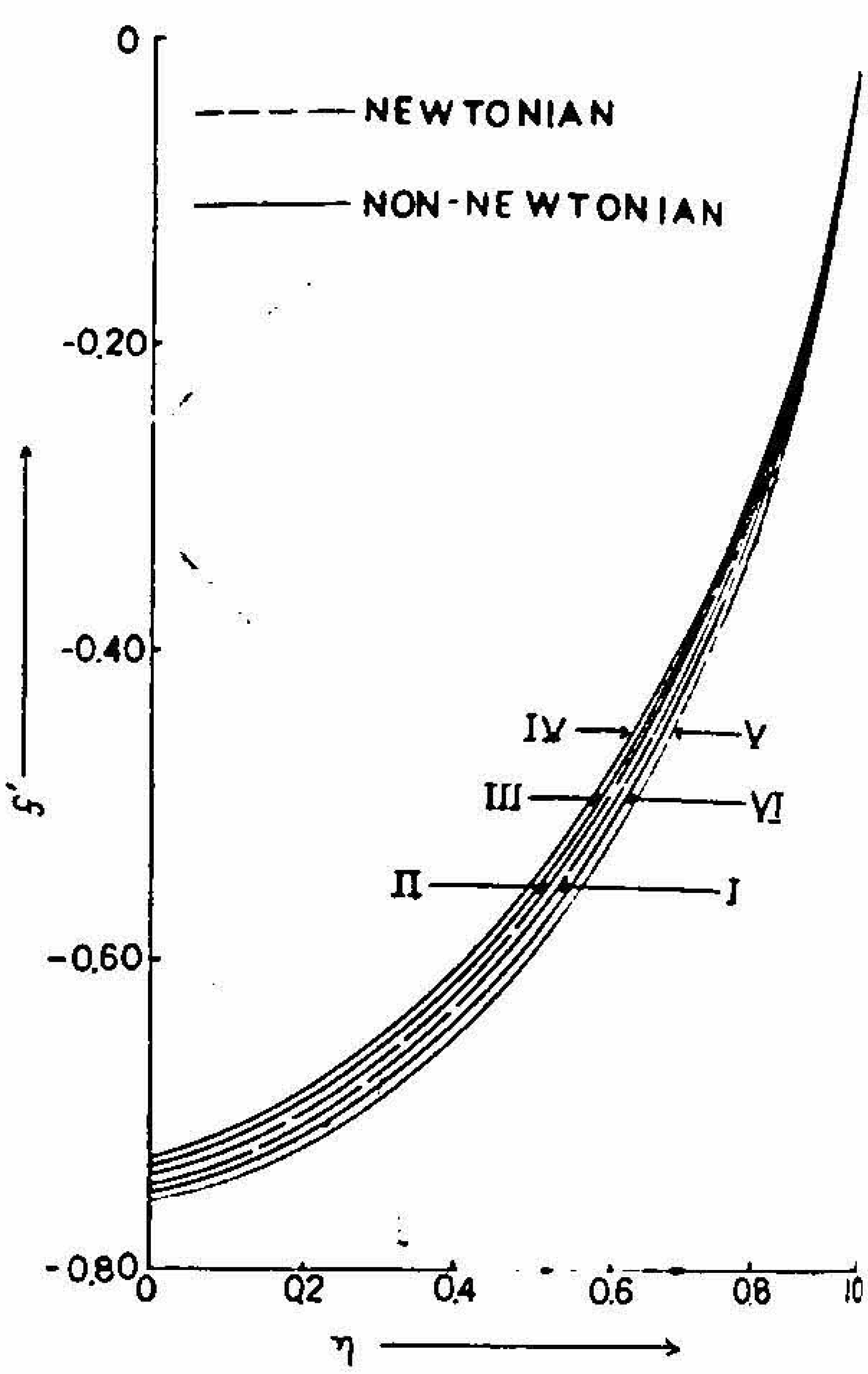

FIG. 11. Radial velocity distribution.

R	K	S	β	
0·1	0	0	0·5	I
0·1	0	0	1·0	II
0·1	−0·1	0·2	0·5	III
0·1	−0·1	0·2	1·0	IV
−0·1	−0·1	0·2	0·5	V
−0·1	−0·1	0·2	1·0	VI

Figures 2 to 13 show the velocity distribution, *i.e.*, $f(\eta)$ (Normal velocity), $g(\eta)$ (Azimuthal velocity) and $f'(\eta)$ (Radial velocity) for various values of cross-flow Reynold's number R (= $-0\cdot5, -0\cdot1, 0\cdot1, 0\cdot5$), viscoelasticity parameter ($K = -0\cdot2, -0\cdot1, 0$), cross-viscosity parameter S (= $0, 0\cdot2, 0\cdot3$) and rotation parameter β (= $0\cdot5, 1\cdot0$).

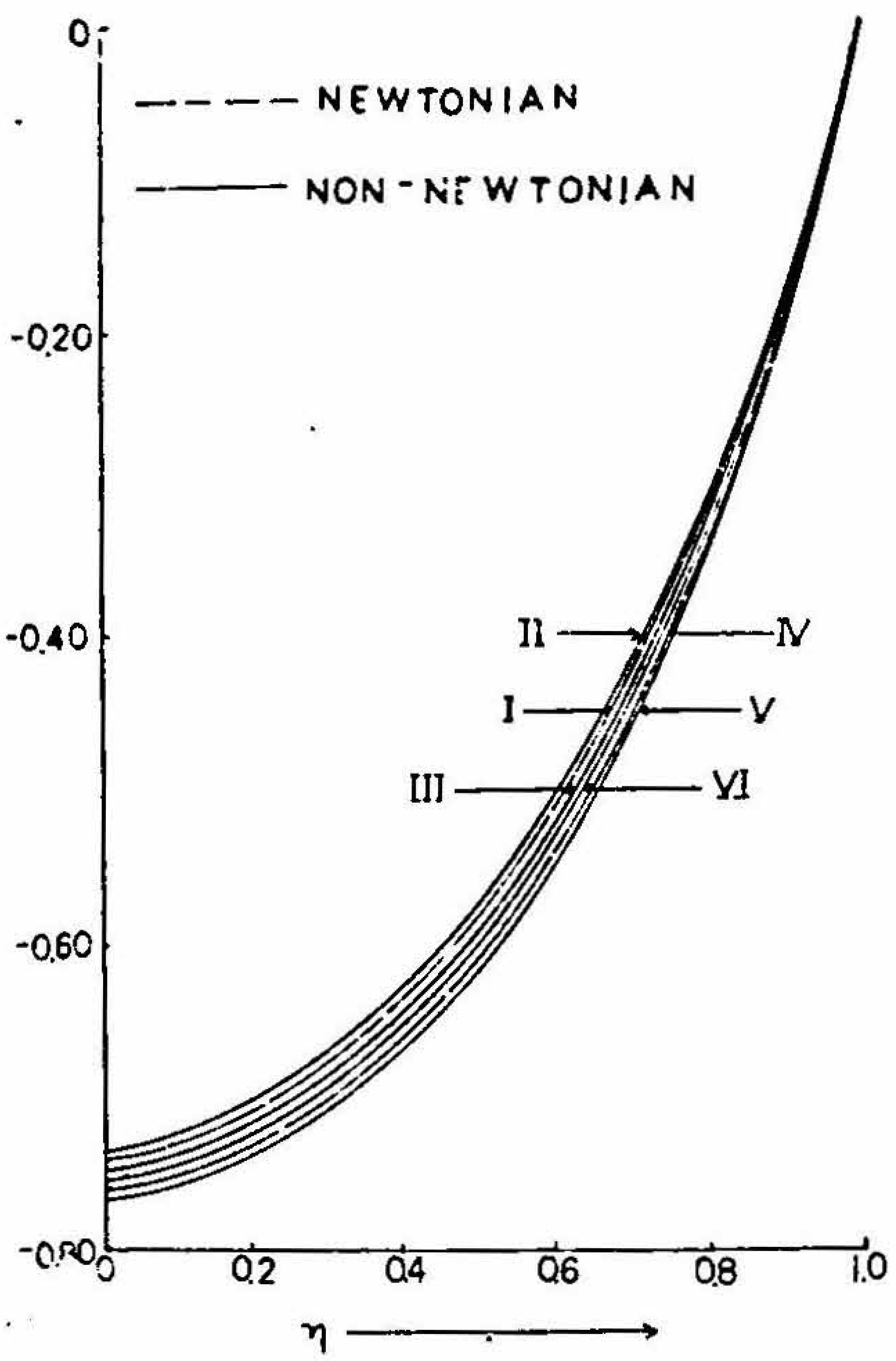

FIG. 12. Radial velocity distribution.

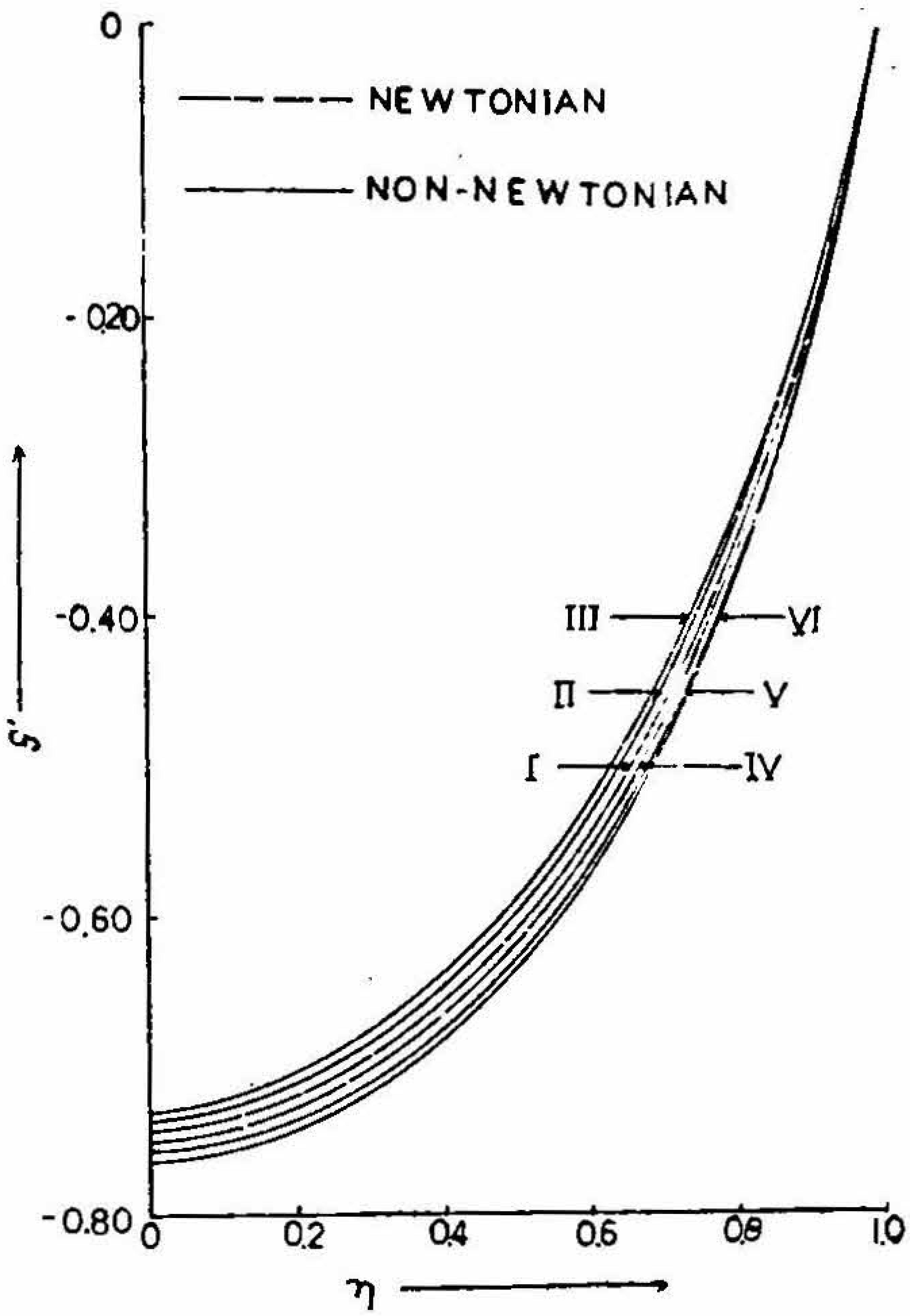

FIG. 13. Radial velocity distribution.

R	K	S	β	
0·1	0	0	0·5	I
0·1	−0·1	0·2	0·5	II
0·1	−0·2	0·2	0·5	III
0·1	0	0	0·5	IV
−0·1	−0·1	0·2	0·5	V
−0·1	−0·2	0·2	0·5	VI

R	K	S	β	
0·1	0	0	0·5	I
0·1	−0·1	0·2	0·5	II
0·1	−0·1	0·3	0·5	III
−0·1	0	0	0·5	IV
−0·1	−0·1	0·2	0·5	V
−0·1	−0·1	0·3	0·5	VI

The normal velocity distribution $f(\eta)$ is shown in figs. 2 to 5. The normal velocity increases towards the plate from $\eta = 0$ to $\eta = 1$. The magnitude of the normal velocity increases with the increasing value of suction parameter R and decreases with the increasing value of rotation parameter β when K and S are constants. The magnitude of the normal velocity increases with the decreasing value of K when suction parameters R, β and S are constants ; also, it increases with the increasing value of S when suction parameters R, β and K are constants. But the behaviour of the magnitude of the normal velocity is reverse in the case of injection parameter R. The magnitude of the normal velocity decreases more with the increasing value of β when injection parameters R, K and S are constants. The magnitude of the normal velocity in the case of non-Newtonian fluid is less in comparison with the Newtonian fluid case with various values of suction parameter R and rotation parameter β when K and S are fixed, but it is greater in comparison to the Newtonian fluid case with the decreasing value of K.

The azimuthal velocity distribution $g(\eta)$ is shown in figs. 6 to 9. The azimuthal velocity decreases towards the plate from $\eta = 0$ to $\eta = 1$ in both Newtonian and non-Newtonian fluids for suction parameter R. But the behaviour of the azimuthal velocity is opposite in the case of injection parameter R. The magnitude of the azimuthal velocity of non-Newtonian fluid is less in comparison with Newtonian fluid. The azimuthal velocity increases with the increasing value of β when R is constant in both the fluids. The magnitude of the azimuthal velocity of non-Newtonian fluid increases with the decreasing value of K when R, S and β are constants and the magnitude is less when compared to Newtonian case. But the magnitude of the azimuthal velocity of non-Newtonian fluid decreases with the increasing value of S when R, K and β are constants.

In figs. 10 to 13, the radial velocity distribution is shown. The radial velocity, in suction case, increases towards the plate from $\eta = 0$ to $\eta = 1$. The magnitude of the radial velocity of Newtonian and non-Newtonian fluid decreases with the increasing value of suction parameter R when K, S and β are constants. Also, the magnitude of the radial velocity of Newtonian and non-Newtonian fluid increases with the increasing value of rotation parameter β, when R, K and S are constants.

In the case of injection parameter R, the magnitude of the radial velocity of Newtonian fluid increases with the increasing value of β but increases with the increasing value of R (injection) when β is constant. In non-Newtonian fluid case, the magnitude of the radial velocity increases with the decreasing value of K when injection parameters R, S and β are constants, but decreases with the increasing value of S when injection parameters R, K and β are constants. Also, the magnitude of the radial velocity increases with the increasing value of β when injection parameter R, K and S are constants for both the fluids.

Pressure distribution on the either disc is shown in fig. 14 and compared with Newtonian fluid. It may be seen that in the case of injection, the magnitude of pressure in non-Newtonian case is less than that in Newtonian case while in the case of suction this phenomenon is reversed. The magnitude of pressure decreases with increase in injection parameter R when other parameters are fixed, but opposite behaviour in the case of suction.

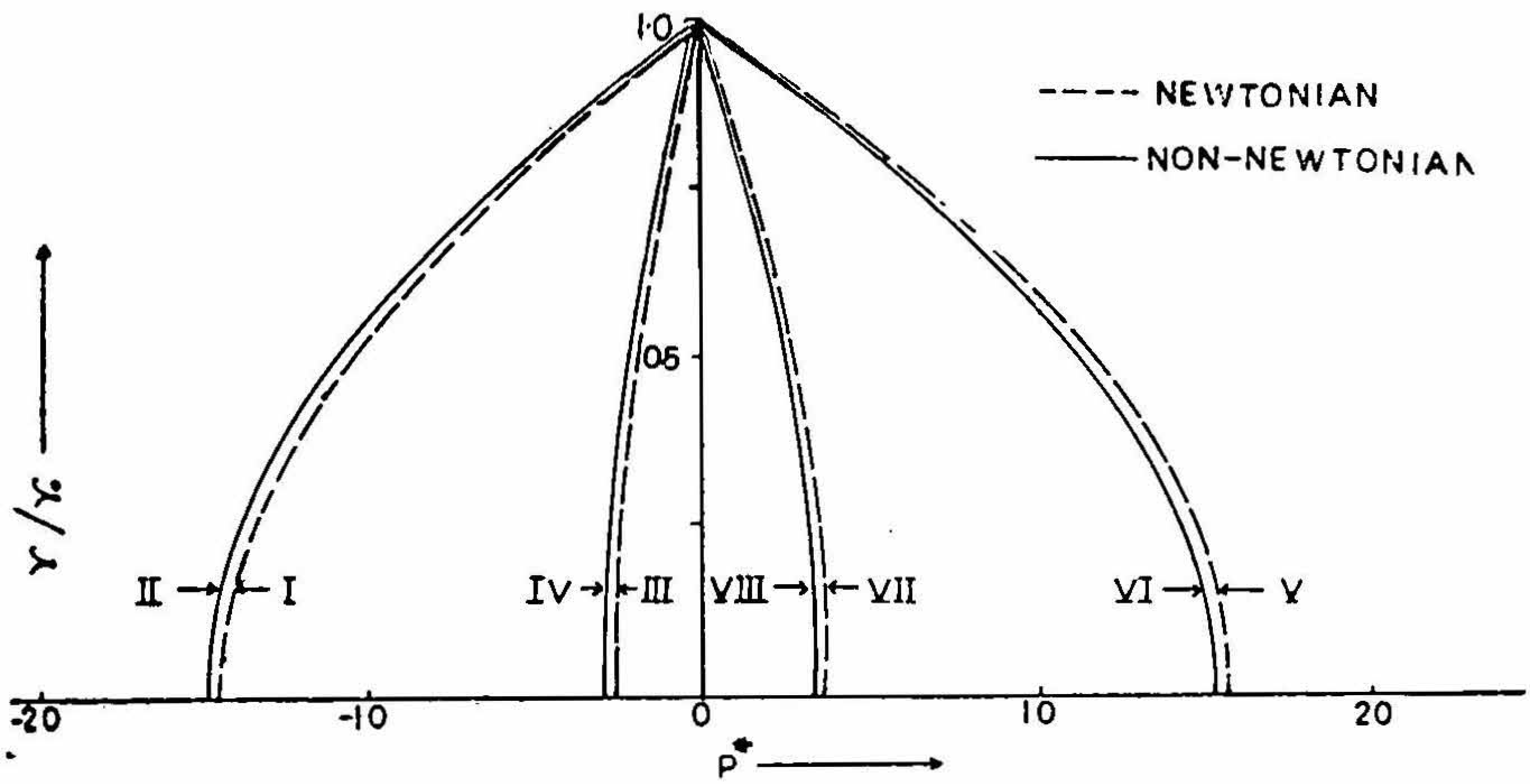

FIG. 14. Pressure distribution on either disc against r/r_0.

R	K	S	β	
0·1	0	0	0·5	I
0·1	−0·1	0·2	0·5	II
0·5	0	0	0·5	III
0·5	−0·1	0·2	0·5	IV
−0·1	0	0	0·5	V
−0·1	−0·1	0·2	0·5	VI
−0·5	0	0	0·5	VII
−0·5	−0·1	0·2	0·5	VIII

In fig. 15 coefficient of skin friction at the disc $\eta = 1$ is plotted against r/r_0. In the case of injection, the megnitude of this coefficient for non-Newtonian fluid is larger than that for Newtonian fluid and *vice versa* for suction. The magnitude of coefficient of skin friction also decreases with increase in injection parameter R when other parameters are fixed but behaviour is opposite in the case of suction.

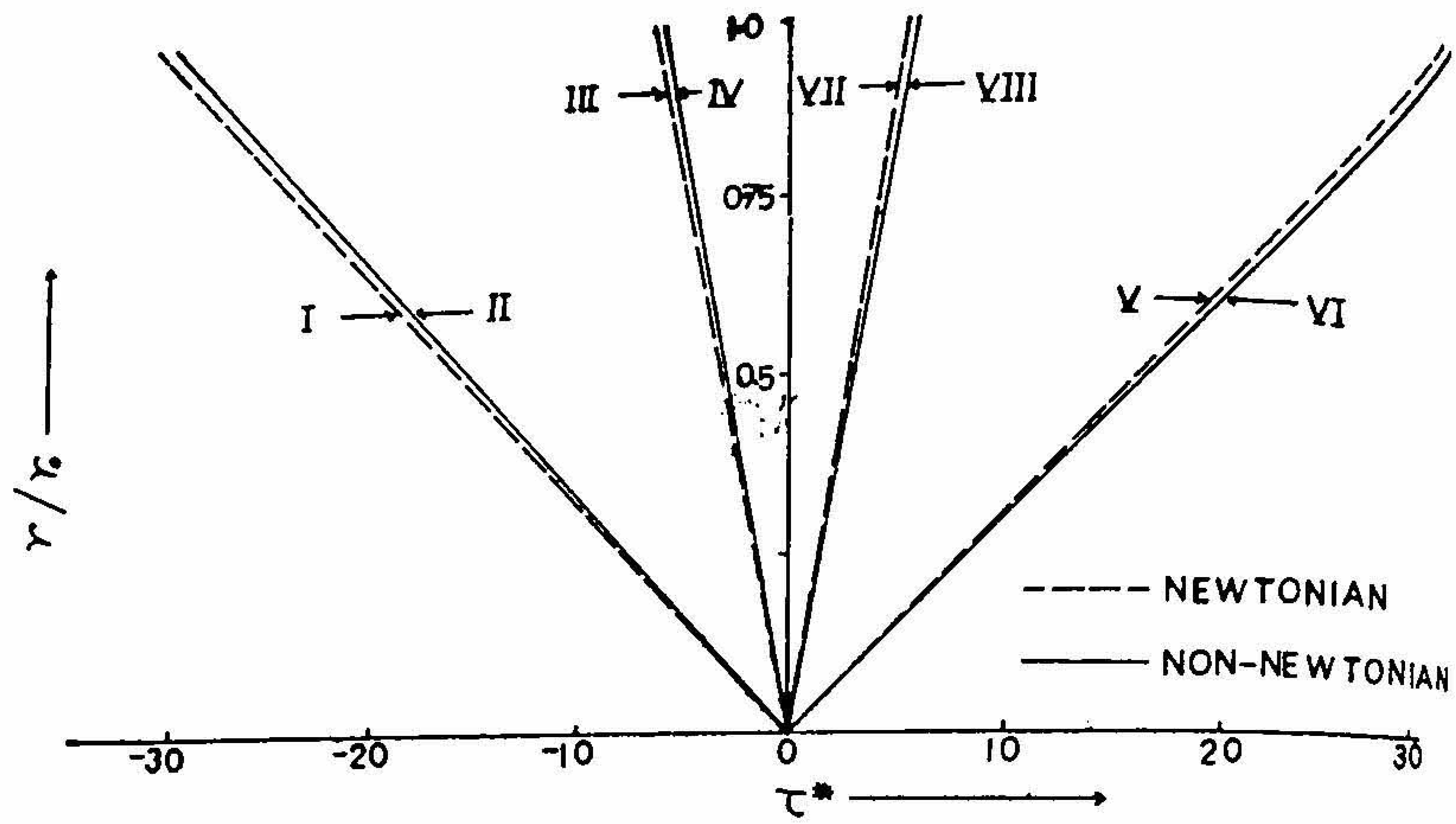

FIG. 15. Coefficient of skin friction at the disc $\eta = 1$, against r/r_0.

R	K	S	β	
0·1	0	0	0·5	I
0·1	−0·1	0·2	0·5	II
0·5	0	0	0·5	III
0·5	−0·1	0·2	0·5	IV
−0·1	0	0	0·5	V
−0·1	−0·1	0·2	0·5	VI
−0·5	0	0	0·5	VII
−0·5	−0·1	0·2	0·5	VIII

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