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The effect of linear inertia on the squeeze film action between a curved circular plate and a flat plate

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Abstract

The effect of linear inertia on the action of the curved squeeze film between two circular plates when the upper plate moves normal to itself and approaches the lower plate with uniform velocity, is studied. Expressions for pressure, load capacity and response time are obtained by using the method of averaged inertia and the regular perturbation method. Pressure, load capacity and response time increase markedly with increasing curvature parameter but only slightly with increasing inertia parameter. Numerical values of the bearing characteristics obtained by the two methods are found to be nearly equal.

Key words: Linear inertia, squeeze film, curved circular plate, load capacity, response time, lubrication.

1. Introduction

It is usually assumed in hydrodynamic lubrication that the effect of inertia is negligible in comparison with viscous forces. However, the effect of inertia becomes more important when the lubricants used have high density and low viscosity or when the speed and temperature of the machine are extremely high. Agrawal[1-3] studied the effects of linear inertia in squeeze films between surfaces neglecting the curvature effect, in full and half journal bearings and in spherical bearing. He showed that the load capacity and time of approach increased owing to the linear inertia of the lubricant. Tichy and Winer[4] analysed the effect of linear inertia on the squeeze film action between two flat

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circular plates assuming unsteady conditions and using regular perturbation method. Bhat and Patel[5] deduced the rotational inertia effects on the squeeze film action between parallel porous circular plates and showed that pressure, load capacity and response time decreased owing to the rotational inertia. In fact, owing to elastic, thermal and uneven wear effects the bearing surfaces are far from flat. So, it is necessary to consider the curvature effects also.

Murti[6] introduced an exponential function to describe the curved film between two circular plates. He observed that the concavity of the upper plate increased the load capacity.

In this paper the analysis[6] is extended by considering the linear inertia effects. The problem is solved by using the method of averaged inertia and the regular perturbation method assuming quasi-steady conditions.

2. Analysis

The film thickness, as in fig. 1, is

$$h = h_0 e^{-Br^2}, \quad 0 \leqslant r \leqslant a, \tag{1}$$

where h_0 is the central film thickness, a is the radius of each plate and B is the curvature parameter. The upper plate moves normal to itself with uniform velocity $dh_0/dt = \dot{h}_0$ towards the lower plate which is fixed and flat. Following the usual assumptions of hydrodynamic lubrication applicable to thin film and retaining the inertia terms, equations governing the problem are

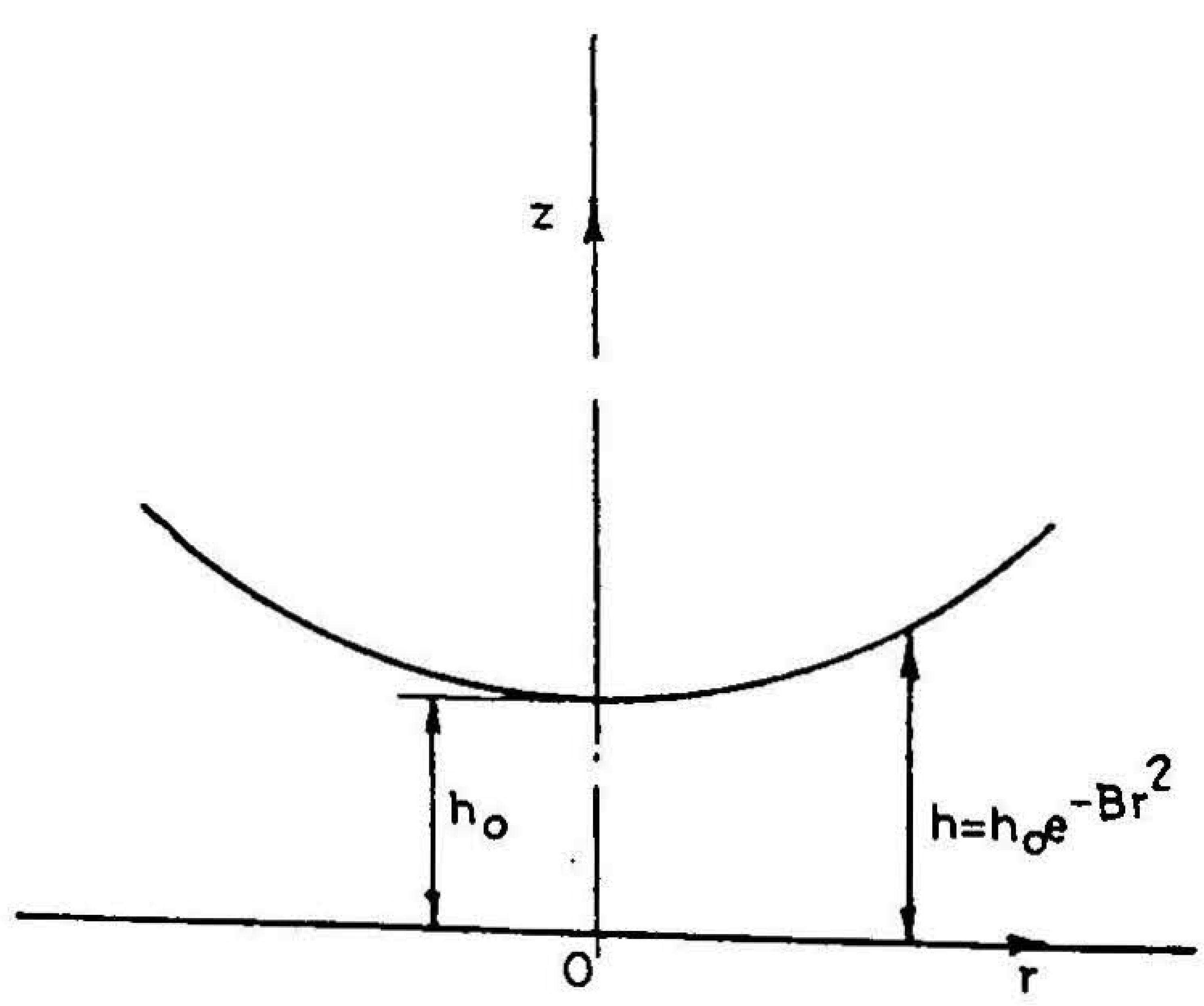

FIG. 1. Geometry and co-ordinate system.

$$\rho\left(u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z}\right)=-\frac{dp}{dr}+\mu\frac{\partial^2 u}{\delta z^2} \tag{2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial v}{\partial z}=0, \tag{3}$$

where u, v are the velocity components of the lubricant in the radial and axial directions r and z, and ρ, p, μ are respectively the density, pressure and viscosity of the lubricant.

Introducing the following dimensionless quantities

$$\left.\begin{aligned}&U=-\frac{u}{\dot{h}_0},\ V=-\frac{v}{\dot{h}_0},\ R=\frac{r}{h_0},\ Z=\frac{z}{h_0},\ E=-\frac{h_0\dot{h}_0\rho}{\mu}\\&P=-\frac{h_0 p}{\mu\dot{h}_0},\ H=\frac{h}{h_0}=e^{-\bar{B}R^{-2}},\ \bar{B}=Bh_0^2,\end{aligned}\right\} \tag{4}$$

equations (2) and (3) transform to

$$E\left(U\frac{\partial U}{\partial R}+V\frac{\partial U}{\partial Z}\right)=-\frac{dP}{dR}+\frac{\partial^2 U}{\partial Z^2} \tag{5}$$

and

$$\frac{1}{R}\frac{\partial}{\partial R}(RU)+\frac{\partial V}{\partial Z}=0. \tag{6}$$

The associated initial or boundary conditions are

$$U(R,0)=0,\ U(R,H)=0 \tag{7a}$$

$$V(R,0)=0,\ V(R,H)=-1 \tag{7b}$$

$$\frac{dP}{dR}(0)=0,\ P\left(\frac{a}{h_0}\right)=0. \tag{7c}$$

Equation (5) is non-linear. We solve it using (i) the method of averaged inertia and (ii) the regular perturbation method.

3. Solution by the method of averaged inertia

In this method, the inertia terms of equation (5) are replaced by their average value across the film. Thus equation (5) is expressed as

$$\frac{\partial^2 U}{\partial Z^2}=X(R) \tag{8}$$

where

$$X(R)=\frac{dP}{dR}+\frac{E}{H}\int_0^H\left(U\frac{\partial U}{\partial R}+V\frac{\partial U}{\delta Z}\right)dZ. \tag{9}$$

Solving equation (8) using conditions (7a) yields

$$U = \frac{1}{2} X(R)\,(Z^2 - HZ). \tag{10}$$

Substituting from equation (10) in equation (6) and then integrating it on the interval (0, H) using conditions (7b) we obtain

$$X(R) = -\frac{6R}{H^3}, \tag{11}$$

since $X(0)$ is finite.

Making use of equations (10) and (11) in (6) and then integrating it on the interval (0, Z), we obtain

$$V = \frac{2Z^3}{H^3}(1 + 3\bar{B}R^2) - \frac{3Z^2}{H^2}(1 + 2\bar{B}\,R^2) \tag{12}$$

Using equations (4), (10) - (12) in (9) and then solving it under the conditions (7c) the dimensionless pressure is obtained as

$$P^* = \frac{h_0^3 P}{a^2} = -\frac{1}{B^*}(e^{3B^*} - e^{3B^* r^{*2}}) + \frac{3E}{20B^*}[(1 + B^*)\,e^{2B^*} - (1 + B^* r^{*2})\,e^{2B^* r^{*2}}], \tag{13}$$

where

$$B^* = Ba^2 \text{ and } r^* = r/a. \tag{14}$$

The load capacity of w of the bearing is given by

$$w = 2\pi \int_0^a r\,p\,dr$$

and in dimensionless form it is expressed as

$$W^* = -\frac{h_0^3\,w}{2\pi\,\mu\,a^4\,\dot{h}_0} = \frac{1}{6B^{*2}}[(3B^* - 1)\,e^{3B^*} + 1] + \frac{3E}{160B^{*2}}[1 + (4B^{*2} + 2B^* - 1)\,e^{2B^*}]. \tag{15}$$

Equation (15) is written in terms of W_0 and W_a, where W_0 is the inertia free load capacity and W_a is the coefficient of E, and using the dimensionless quantities

$$\bar{h} = \frac{h_0}{h_1},\ \bar{t} = \frac{wth_1^2}{2\pi\,\mu\,a^4},\ \bar{e} = \frac{w\,\rho\,h_1^4}{2\pi\,\mu^2\,a^4},$$

the time-height relation for a given load w is obtained from it as

$$\bar{t} = -\frac{1}{2}\int_1^{\bar{h}_0} \frac{1}{\bar{h}^{-3}}[W_0 + (W_0^2 + 4W_a\,\bar{e}\,\bar{h}^4)^{1/2}]\,d\bar{h}$$

$$= \frac{1}{4}\left[\frac{W_0 + (W_0^2 + 4W_a \bar{e}\bar{h}_2^4)^{1/2}}{\bar{h}_2^2} - W_0 - (W_0^2 + 4\,W_a \bar{e})^{1/2}\right.$$

$$\left. + 2\,(W_a\bar{e})^{1/2} \log\left\{\frac{(W_0^2 + 4\,W_a\bar{e}\,\bar{h}_2^4)^{1/2} - (4W_a\bar{e}\bar{h}_2^4)^{1/2}}{(W_0^2 + 4W_a\bar{e})^{1/2} - (4W_a\bar{e})^{1/2}}\right\}\right], \quad (16)$$

where h_1 and h_2 are the initial and the final values of the central film thickness h_0.

4. Solution by the regular perturbation method

In this method, it is assumed that U, V and P can be expanded in terms of the squeeze Reynold number E as follows :

$$U = U_0 + EU_1 + E^2U_2 + \ldots\ldots \quad (17a)$$

$$V = V_0 + EV_1 + E^2V_2 + \ldots\ldots \quad (17b)$$

$$P = P_0 + EP_1 + E^2P_2 + \ldots\ldots \quad (17c)$$

Substituting equations (17) into equations (5) - (7) and collecting terms of like powers of E, we have, after neglecting second and higher powers of E,

$$0 = -\frac{dP_0}{dR} + \frac{\partial^2 U_0}{\partial Z^2} \quad (18a)$$

$$U_0\frac{\partial U_0}{\partial R} + V_0\frac{\partial U_0}{\partial Z} = -\frac{dP_1}{dR} + \frac{\partial^2 U_1}{\partial Z^2} \quad (18b)$$

$$\frac{1}{R}\frac{\partial}{\partial R}(RU_0) + \frac{\partial V_0}{\partial Z} = 0 \quad (19a)$$

$$\frac{1}{R}\frac{\partial}{\partial R}(RU_1) + \frac{\partial V_1}{\partial Z} = 0 \quad (19b)$$

$$U_0(R, 0) = 0,\ U_0(R, H) = 0 \quad (20a)$$

$$U_1(R, 0 = 0,\ U_1(R, H) = 0 \quad (20b)$$

$$V_0(R, 0) = 0,\ V_0(R, H) = -1 \quad (20c)$$

$$V_1(R, 0) = 0,\ V_1(R, H) = 0 \quad (20d)$$

$$\frac{dP_0}{dR}(0) = 0,\ P_0\left(\frac{a}{h_0}\right) = 0 \quad (20e)$$

$$\frac{dP_1}{dR}(0) = 0,\ P_1\left(\frac{a}{h_0}\right) = 0. \quad (20f)$$

Solving equation (18a) with conditions (20a) yields U_0.

Substituting U_0 into equation (19a) and then solving it under conditions (20c) and (20e) we have

$$P_0 = \frac{1}{\bar{B}}\left(e^{3\bar{B}a^2/h_0^2} - e^{3\bar{B}R^2}\right) \quad (21)$$

Using equation (21), U_0 is obtained as

$$U_0 = \frac{3R}{H^3}(HZ - Z^2) \tag{22}$$

Substituting equation (22) into equation (19a) and then integrating it on the interval $(0, Z)$ with $V_0(R, 0) = 0$, we obtain

$$V_0 = \frac{2Z^3}{H^3}(1 + 3\bar{B}R^2) - \frac{3Z^2}{H^2}(1 + 2\bar{B}R^2) \tag{23}$$

Making use of equations (22) and (23) in equation (18b) and then integrating it with conditions (20b) yields

$$U_1 = \frac{1}{2}\frac{dP_1}{dR}(Z^2 - HZ) + \frac{R}{20}\Big[30\,\bar{B}R^2\left(\frac{Z}{H}\right)^4 + 6(1 - 6\,\bar{B}R^2)\left(\frac{Z}{H}\right)^5$$
$$+ 2(-1 + 6\,\bar{B}R^2)\left(\frac{Z}{H}\right)^6 - 2(2 + 3\,\bar{B}R^2)\frac{Z}{H}\Big] \tag{24}$$

Substituting for U_1 from equation (24) into equation (19b) and then integrating it with respect to Z on the interval $(0, H)$ using conditions (20d), we obtain an equation for P_1. Solving it using conditions (20f) yields

$$P_1 = \frac{27}{280\,\bar{B}}[(2\bar{B}\,a^2/h_0^2 + 1)\,e^{2\bar{B}a^2/h_0^2} - (2\bar{B}R^2 + 1)\,e^{2\bar{B}R^2}] \tag{25}$$

Using equations (21) and (25) in (17c) the dimensionless pressure up to the first order of E can be expressed as

$$P^* = \frac{1}{B^*}(e^{3B^*} - e^{3B^* r^{*2}}) + \frac{27E}{280B^*}[(2B^* + 1)$$
$$\times\, e^{2B^*} - (2B^*\, r^{*2} + 1)\, e^{2B^* r^{*2}}] \tag{26}$$

The dimensionless load capacity is

$$W^* = W_0 + EW_p, \tag{27}$$

where

$$W_p = \frac{27}{280}\, e^{2B^*}. \tag{28}$$

Proceeding as in the first method, we obtain an expression for $\bar{t}$ which is the same as in equation (16) with W_a replaced by W_p.

5. Results and discussion

Setting the inertia parameters E and $\bar{e}$ equal to zero, the results agree with those of Murti[6] who neglected the inertia effects.

Making the curvature parameter B^* tend to zero we obtain the results for parallel circular plates which agree with those of Tichy and Winer[4] when restricted to quasi-steady conditions.

Numerical values, computed using methods (i) and (ii), of the dimensionless load capacity W^* and the response time $\bar{t}$ are displayed in Tables I and II for various values of the inertia parameters E and $\bar{e}$, and the curvature parameter B^*. Both W^* and $\bar{t}$ increase with inertia parameters or the curvature parameter. The increases are marked when the upper plate is concave and $B^* > 1$. The increases in W^* and $\bar{t}$ due to inertia are not significant. The values of W^* and $\bar{t}$ obtained by the two methods agree up to at least two significant figures.

Table I

Values of dimensionless load W^* for various values of the curvature parameter B^* and the inertia parameter E by methods (i) and (ii)

B^* \ E	0·0	0·0001		0·01	
	m. (i) & m. (ii)	m. (i)	m. (ii)	m. (i)	m. (ii)
−1·00	0·13348	0·13348	0·13348	0·13369	0·13361
−0·01	0·73533	0·73533	0·73534	0·73533	0·73628
0·01	0·76533	0·76533	0·76534	0·76533	0·76632
1·00	6·86185	6·86192	6·86192	6·86896	6·86897
2·00	84·08933	84·09712	84·08986	84·86810	84·14198

Table II

Values of dimensionless time $\bar{t}$ for various values of the curvature parameter B^* and the inertia parameter $\bar{e}$ by methods (i) and (ii) ; $\bar{h}_2 = \cdot 2$

B^* \ $\bar{e}$	0·0	0·0001		0·01	
	m. (i) & m. (ii)	m. (i)	m. (ii)	m. (i)	m. (ii)
−1·00	1·601704	1·601711	1·601708	1·602466	1·602172
−0·01	8·824000	8·824000	8·824006	8·824000	8·824616
0·01	9·184000	9·184000	9·184006	9·184000	9·184617
1·00	82·342145	82·342147	82·342147	82·342642	82.342640

Dedication

We dedicate this to beloved Master Venugopal, a son of the first author and a promising athlete, who died on 26th August, 1980, at the age of 15, due to leukemia.

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