MODAL AND RADIATION CHARACTERISTICS OF THE DIELECTRIC SPHERE EXCITED IN TM SYMMETRIC MODE

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Received on March 14, 1977

ABSTRACT

The electromagnetic boundary value problem of the dielectric sphere excited in TM symmetric modes has been solved. Assuming that the fields on a truncated dielectric sphere excited by a coaxial line are the same as those on a complete sphere, the radiation characteristics of such a structure have been derived. Calculated radiation patterns and gains of several structures of varying dimensions have been verified experimentally.

Key words: Modes, Radiation, Dielectric sphere.

1. INTRODUCTION

The dielectric sphere has been studied as a lens by workers like Bekefi and Farwell,[1] Luneberg[2] and Pieles and Coleman.[3]

The resonant properties of the dielectric sphere has been studied by several workers during recent years. Gastine *et al.*[4] have calculated the resonant frequencies of dielectric spheres for the two extreme cases of ka being finite when $\epsilon_1 \rightarrow \infty$ and when $ka \rightarrow \infty$ when $\epsilon_1 \rightarrow \infty$, where $k_1 =$ wave number in the dielectric and 'a' is the radius of the sphere. Sager and Tisi[5] have studied the eigen modes and forced resonant modes of dielectric spheres. Affolter and Eliasson[6] have made a study of electromagnetic resonances and Q factors of lossy dielectric spheres. The resonances of a dielectric resonator of very high permittivity and their excitation have been studied by Van Bladel.[7,8]

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Chatterjee and others have reported theoretical and experimental work on spherical dielectric antennas excited in hybrid[9] and on hemispherical dielectric antennas excited in TM symmetric[10] modes. Neelakantaswamy and Banerjee[11] and Chatterjee and Croswell[12] have reported experimental and theoretical work on waveguide-excited dielectric spheres. Chatterjee[13] has solved the electromagnetic boundary value problem of the dielectric sphere excited by delta-function electric and magnetic sources. In this paper, the modal characteristics of the dielectric sphere excited by a delta-function source in the TM symmetric modes has been studied theoretically and verified experimentally. Assuming that the fields on a truncated dielectric sphere excited by a coaxial line are approximately the same as that on a complete sphere excited in the TM symmetric modes, the radiation characteristics of such truncated dielectric spheres have been derived. The radiation patterns and gain of several such structures of varying dimensions have been calculated and verified by experiment.

2. Electro-Magnetic Boundary Value Problem of Dielectric Sphere Excited in TM Symmetric Modes

Fig. 1 shows the geometry of the structure. Spherical polar coordinates R, θ, ϕ are used. A dielectric sphere of radius $a = D/2$ and constants ϵ_1, μ_1, σ_1 is embedded in another dielectric medium of constants ϵ_0, μ_0, σ_0. This sphere is excited in the TM symmetric modes by an excitation electric field $E_0 \exp(-j\omega t)$ applied uniformly in the z direction on an annular ring of radius $a \sin \theta_1$ and infinitesimal width $da \to 0$. Let E_0 have components $E_{R_0} = E_0 \cos \theta_1$ and $E_{\theta_0} = -E_0 \sin \theta_1$, in the R and θ directions respectively. These components E_{R_0} and E_{θ_0} can be expanded in a series of spherical harmonics as given below:

$$E_{R_0}(R, \theta, \phi) = -\frac{1}{k_1} \sum_{n=0}^{\infty} n(n+1) D_{0n}(R) P_n(\cos \theta) \exp(-j\omega t) \tag{1}$$

$$E_{\theta_0}(R, \theta, \phi) = -\frac{1}{k_1} \sum_{n=0}^{\infty} C_{0n}(R) P_n^1(\cos \theta) \exp(-j\omega t) \tag{2}$$

where

$$C_{0n}(R) = -\frac{k_1(2n+1)}{2\pi n(n+1)} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} E_{\theta_0} P_n^1(\cos \theta) \sin \theta \, d\theta d\phi \tag{3}$$

$D_{0n}(R)$

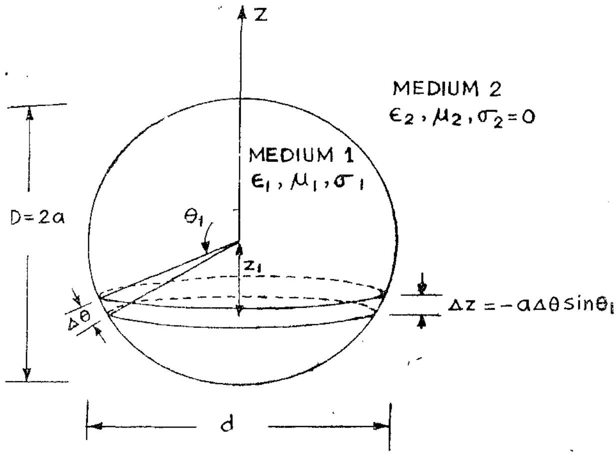

FIG. 1. Geometry of the Structure.

$$= -\frac{k_1(2n+1)}{2\pi n(n+1)} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} E_{R_0} P_n(\cos\theta) \sin\theta \, d\theta d\phi \tag{4}$$

Let E_0 be a delta-function given by

$$E_0 = \frac{-V}{a\triangle\theta\sin\theta_1} \text{ for } \theta_1 - \frac{\triangle\theta_1}{2} < \theta < \theta_1 + \frac{\triangle\theta_1}{2} \tag{5}$$

and $= 0$ for

$$\theta_1 - \frac{\triangle\theta_1}{2} > \theta > \theta_1 + \frac{\triangle\theta_1}{2} . \tag{6}$$

Then

$$C_{0n}(a) = \frac{C_{0n}' [k_1 a j_n(k_1 a)]'}{a}$$

$$= \frac{-V}{a} \frac{k_1(2n+1)}{2n(n+1)} \sin\theta_1 P_n^1(\cos\theta_1) \tag{7}$$

and

$$D_{0n}(a) = D_{0n}' \frac{j_n(k_1 a)}{a}$$

$$= \frac{-V}{a} \frac{k_1 (2n+1)}{2n(n+1)} \cos\theta_1 \, P_n(\cos\theta_1) \tag{8}$$

The field components inside the sphere in the region $0 < r < a$, are

$$E_R{}^i = -\sum_{n=0}^{\infty} n(n+1) A_{0n} P_n(\cos\theta) \frac{j_n(k_1 R)}{k_1 R} \exp(-j\omega t) + E_{R_0} \tag{9}$$

$$E_\theta{}^i = -\sum_{n=0}^{\infty} A_{0n} P_n{}^1(\cos\theta) \frac{[k_1 R j_n(k_1 R)]'}{k_1 R} \exp(-j\omega t) + E_{\theta_0} \tag{10}$$

$$H_\phi{}^i = \sum_{n=0}^{\infty} \frac{k_1}{j\omega\mu_1} A_{0n} P_n{}^1(\cos\theta) j_n(k_1 R) \exp(-j\omega t) \tag{11}$$

and the field components outside the sphere in the region $r > a$ are

$$E_R{}^e = -\sum_{n=0}^{\infty} n(n+1) A_{0n}' P_n(\cos\theta) \frac{h_n{}^{(1)}(k_0 R)}{k_0 R} \exp(-j\omega t) \tag{12}$$

$$E_\theta{}^e = -\sum_{n=0}^{\infty} A_{0n}' P_n{}^1(\cos\theta) \frac{[k_0 R\, h_n{}^{(1)}(k_0 R)]'}{k_0 R} \exp(-j\omega t) \tag{13}$$

$$H_\phi{}^e = \sum_{n=0}^{\infty} \frac{k_0}{j\omega\mu_0} A_{0n}' P_n{}^1(\cos\theta) h_n{}^{(1)}(k_0 R) \exp(-j\omega t) \tag{14}$$

where

$$k_1 = \omega \sqrt{\mu_1 \left(\epsilon_1 + \frac{j\sigma_1}{\omega}\right)}$$

$$k_0 = \omega \sqrt{\mu_0 \left(\epsilon_0 + \frac{j\sigma_0}{\omega}\right)}$$

ω = angular frequency

A_{0n} and A_{0n}' are amplitude coefficients, $P_n(\cos\theta)$ = Legendre function, $j_n(k_1 R)$ and $h_n^{(1)}(k_0 R)$ are the spherical Bessel and Hankel functions respec tively.

Applying the boundary conditions that

$$E_\theta^e = E_\theta^i \quad \text{and} \quad H_\phi^e = H_\phi^i \quad \text{at} \quad R = a,$$

we obtain

$$\frac{A_{0n}}{k_1 a}[k_1 a\, j_n(k_1 a)]' + \frac{C_{0n}(a)}{k_1} = \frac{A_{0n}'}{k_0 a}[k_0 a\, h_n^{(1)}(k_0 a)]' \tag{15}$$

and

$$\frac{k_1}{\mu_1} A_{0n}\, j_n(k_1 a) = \frac{k_0}{\mu_0} A_{0n}'\, h_n^{(1)}(k_0 a) \tag{16}$$

Using equation (7) in equations (15) and (16), the amplitude coefficients A_{0n} and A_{0n} can be uniquely determined for each value of n. The field is thus uniquely determined both inside and outside the dielectric sphere for each value of n. This shows that TM_{0n} modes exist for $n = 1, 2, \cdots$ etc. for this structure.

If the dielectric sphere is excited in the equatorial plane $\theta_1 = \pi/2$, then $P_n^1(0) = 0$ for n even

and

$$= \frac{j^{n-1}\, 2^{1-n}\, n!}{\frac{(n-1)}{2}!} \text{ for } n \text{ odd.} \tag{17}$$

Therefore $C_{0n}(a) = 0$ for n even. Hence only the odd order TM_{0n} modes are excited in the case of equatorial excitation.

3. Theoretical Determination of the Radiation Field of a Truncated Dielectric Sphere Excited in the TM Symmetric Modes

Fig. 2 shows the geometry of a truncated dielectric sphere excited in the TM symmetric mode by a coaxial line. Such a structure may be approximately assumed to be excited by a delta-function source in the form given by equations (5) and (6) in the z direction on an angular ring of radius

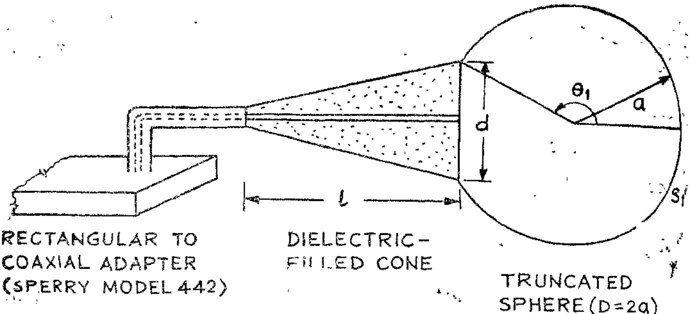

FIG. 2. Geometry of truncated dielectric sphere excited in TM symmetric mode by coaxial line.

$a \sin \theta_1$ and infinitesimal width $da \to 0$. Hence it is assumed that the field components inside the dielectric sphere as well as outside the sphere are approximately given by equations (9) to (14).

In deriving expressions for the radiation field of this structure, Schelkunoff's equivalence principle has been made use of. This principle states that ' a distribution of electric and magnetic currents on a given closed surface S can be found such that outside the surface it produces the same field as that produced by given sources inside S, and also the field inside it is the same as that produced by given sources outside the surface '. These surface electric and magnetic current densities $\boldsymbol{J}$ and $\boldsymbol{M}$ are given by

$$\begin{aligned} \boldsymbol{J} &= \boldsymbol{n} \times \boldsymbol{H}^\circ \\ \boldsymbol{M} &= -\boldsymbol{n} \times \boldsymbol{E}^\circ \end{aligned} \tag{18}$$

where $\boldsymbol{E}^\circ$, $\boldsymbol{H}^\circ$ are the electric and magnetic fields on the surface S and $\boldsymbol{n}$ is the unit normal to the surface S. The radiation field components at the distant point Q in free space of these electric and magnetic current distributions on S are given by

$$E_\theta = \eta_0 H_\phi = \frac{-j}{2\lambda_0 R} [\eta_0 L_\theta{}^m + L_\phi{}^e] \tag{19}$$

$$E_\phi = -\eta_0 H_\theta = \frac{-j}{2\lambda_0 R} [\eta_0 L_\phi{}^m - L_\theta{}^e] \tag{20}$$

where λ_0 = free-space wavelength

$\beta_0 = \frac{2\pi}{\lambda_0} = \frac{\omega}{\sqrt{\mu_0 \epsilon_0}}$, $\eta_0 = \frac{\mu_0}{\epsilon_0}$ = intrinsic impedance of free space.

L^m and L^e are the electric and magnetic radiation vectors given by

$$L^e = \int_S e^{j\beta_0\,(PQ)}\, dp^m\, e^{-j\omega t} \tag{21}$$

$$L^m = \int_S e^{j\beta_0\,(PQ)}\, dp^e\, e^{-j\omega t} \tag{22}$$

where dp^e and dp^m are the moments of electric and magnetic current situated at $P(R', \theta', \phi')$ on surface S, and are given by

$$dp^e = (n \times H^\circ)\, da \tag{23}$$

$$dp^m = -(n \times E^\circ)\, da \tag{24}$$

$$PQ = R - R' \cos\theta \cos\theta' - R' \sin\theta \sin\theta^1 \cos(\phi - \phi'). \tag{25}$$

The surface S is selected as a closed surface consisting of the outer surface S_1 of the truncated dielectric sphere, the outer surface of the dielectric filled cone, the outer surface of the coaxial line and the rectangular to coaxial adapter, and an infinitely large sphere to close it. The only currents of importance on this surface are the electric and magnetic currents on surface S_1. These are given by

$$J = u_\theta J_{\theta'} = (n \times u_\phi)\, H_{\phi'} \tag{26}$$

$$M = u_\phi M_{\phi'} = -(n \times u_\theta)\, E_{\theta'} \tag{27}$$

where H_ϕ' and E_θ' are given by

$$H_\phi' = \sum_{n=0}^{\infty} \frac{k_0}{j\omega\mu_0} A_{0n}'\, P_n^1(\cos\theta^1)\, h_n^{(1)}(k_0 a)\, e^{-j\omega t} \tag{28}$$

$$E_\theta' = \sum_{n=0}^{\infty} A_{0n}'\, P_n^1(\cos\theta')\, \frac{[k_0 a\, h_n^1(k_0 a)]'}{k_0 a}\, e^{-j\omega t} \tag{29}$$

Using equations (26) to (29) in equations (19) to (22), we obtain the components of the electric field at the distant point $Q(r, \theta, \phi)$ as

$$E_\theta = \frac{j\pi a^2}{\lambda_0 r} A_{0n}^1 \exp\{j(k_0 R - \omega t)\} \Big[h_n^{(1)}(k_0 a)$$

$$\{ \cos\theta \int_{\theta'=0}^{\theta_1} P_n^1(\cos\theta') \sin\theta' \cos\theta'\, J_1(k_0 a \sin\theta \sin\theta')$$

$$\times \exp(-jk_0 a \cos\theta \cos\theta')\, d\theta'$$

$$+ j \sin\theta \int_{\theta'=0}^{\theta_1} P_n^1(\cos\theta') \sin^2\theta'\, J_0(k_0 a \sin\theta')$$

$$\times \exp(-jk_0 a \cos\theta \cos\theta')\, d\theta\}'$$

$$- j \frac{[k_0 a\, h_n^{(1)}(k_0 a)]'}{k_0 a} \int_{\theta'=0}^{\theta_1} P_n^1(\cos\theta') \sin\theta'\, J_1(k_0 a \sin\theta \sin\theta')$$

$$\times \exp(-jk_0 a \cos\theta \cos\theta')\, d\theta' \Big] \tag{30}$$

$$H_\phi = E_\theta/\eta_0 \tag{31}$$

and

$$E_\phi = H_\theta = 0 \tag{32}$$

The directivity of the antenna is given by

$$D = \frac{\text{Maximum radiation intensity}}{\text{Average radiation intensity}}$$

$$= \frac{4\pi\, |E_\theta|^2_{\max}}{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} |E_\theta|^2 \sin\theta\, d\theta} \tag{33}$$

The radiation efficiency η is given by

$$= \frac{\text{Total power radiated}}{\text{Dielectric loss } W_L + \text{Total power radiated}}$$

$$= \frac{\dfrac{\pi}{2\eta_0} \displaystyle\int_{\theta=0}^{\pi} R^2\, |E_\theta|^2 \sin\theta\, d\theta}{\left[\dfrac{\sigma_1}{2} \displaystyle\int_{R=0}^{a} \int_{\theta=0}^{\theta_1} \int_{\phi=0}^{2\pi} \{|E_R^i|^2 + |E_\theta^i|^2\} R^2 \sin\theta\, d\theta\, dR + \dfrac{\pi}{2\eta_0} \displaystyle\int_{\theta=0}^{\pi} R^2\, |E_\theta|^2 \sin\theta\, d\theta\right]}. \tag{34}$$

Hence the gain of the antenna is given by

$$G = \eta D. \tag{35}$$

4. Numerical Calculations and Experimental Verification

The field components on the surface of the antenna have been calculated using equations (28) and (29) for different values of n. The

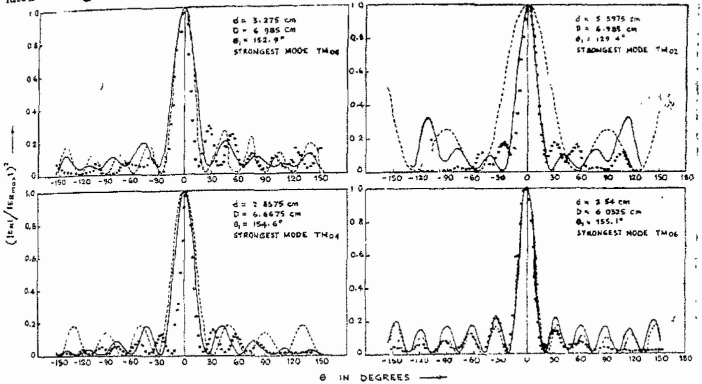

FIG. 3. Normalized $|E_R|^2$ vs θ for truncated spheres on cones. $\varepsilon_r = 2\cdot 56$: $\tan\delta = 0\cdot 005$; $f = 9375$ MHz.
— Six modes combined; - - - - strongest mode; . . . . . Experimental points.

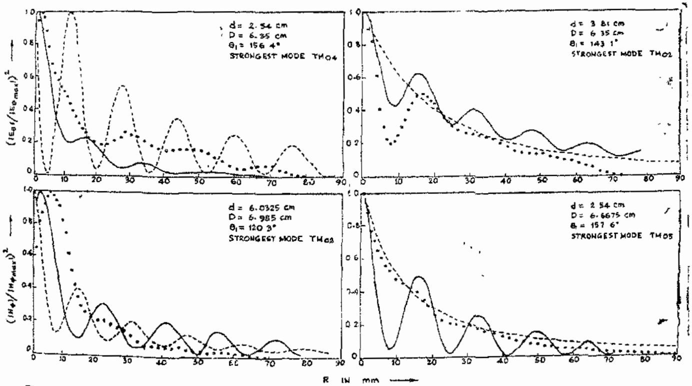

FIG. 4. Normalized $|E_\theta|^2$ and $|H_\phi|^2$ radial decay for truncated spheres on cones. $\varepsilon_r = 2\cdot 56$; $\tan\delta = 0\cdot 005$, $f = 9375$ MHz; $\theta = 45°$.
— Six modes combined - - - - strongest mode; . . . . .Experimental points.

value of n for which the amplitude has the maximum value at the frequency considered is called the strongest mode. Figs. 3 and 4 show typical theoretical and experimental curves of E_R^2, E_θ^2 and H_ϕ^2 as functions of θ and R for the strongest mode as well as for the first six modes combined.

Using expressions (30) to (32) for the radiated field, the radiation patterns have been calculated for different values of n. Figs. 5 and 6 show some typical calculated and experimental radiation patterns.

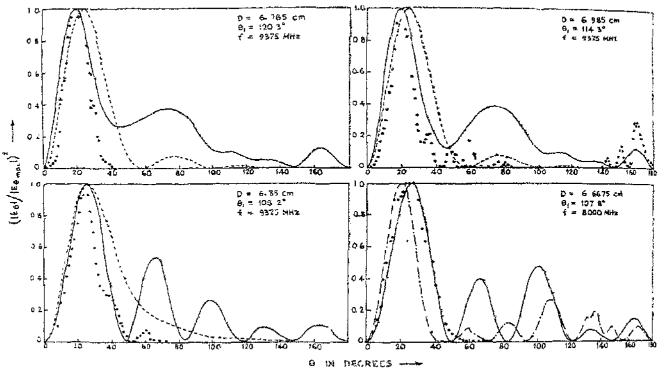

FIG. 5. $\phi = 0°$ Plane radiation pattern for spherical dielectric antenna (Theory and experiment).

$\varepsilon_r = 2 \cdot 56$; $\tan \delta = 0 \cdot 005$.

- - - - - TM_{03} ; —·—·— TM_{05} ; ——— TM_{08} ; . . . . Experimental points.

The gain has been calculated for several antennas of varying dimensions using equation (35). Table I gives the theoretical as well as experimental gains of several antennas.

More details are available in Ref. 14.

5. CONCLUSIONS

The following conclusions may be drawn from the present investigations:

(i) The dielectric sphere can be excited in an infinite number of TM symmetric modes by a delta function electric field source.

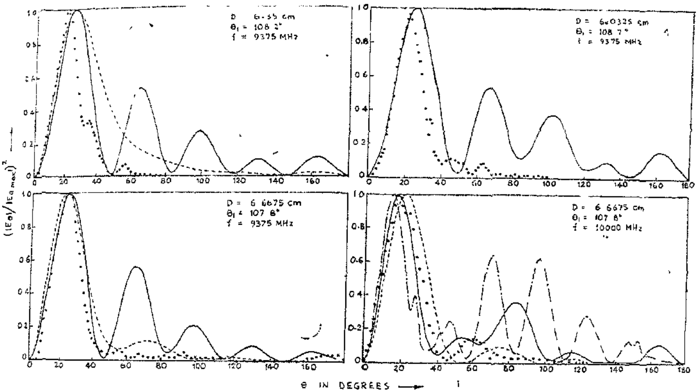

FIG. 6. Normalized radiation patterns in the $\phi = 0°$ plane.
$\varepsilon_r = 2·56$; $\tan \delta = 0·005$.
----TM_{03} ; — — —TM_{05} ; — —TM_{06} ; . . . . Experimental points.

TABLE I

Theoretical and experimental gain of truncated dielectric spherical antennas

$\varepsilon_r = 2·56$; $\tan \delta = 0·005$; $f = 9375$ MHz

'a' in cms.	θ_1 in degrees	Theoretical directivity in db for mode						Experimental gain by comparison method
		TM_{01}	TM_{02}	TM_{03}	TM_{04}	TM_{05}	TM_{06}	
3·4925	120·3	10·51	6·748	12·35	7·966	9·203	9·456	11·25
3·4925	114·3	8·454	7·6354	12·383	6·46	9·602	8·76	9·208
3·33375	107·8	7·994	7·38	11·767	8·03	9·6714	10·65	10·0
3·175	115·8	9·7526	10·257	9·9826	6·52	11·328	8·316	10·0
3·175	108·2	8·77	10·23	10·35	7·85	11·84	7·783	11·6
3·01625	122·65	8·998	10·237	9·523	8·07	8·189	7·973	9·208
3·01625	108·7	9·023	8·958	9·4626	7·425	8·832	10·2	9·3
3·4925	129·4	12·73	8·347	12·5	7·24	7·246	9·2446	9·285
3·175	156·4	9·415	9·625	7·1166	7·8244	9·134	4·81	8·239
3·175	150·0	9·675	8·873	7·935	7·73	9·873	9·83	8·239
3·175	143·1	9·875	10·05	8·876	7·6797	10·385	9·93	11·597
3·175	195·6	9·76	9·179	9·667	7·682	10·24	9·01	7·446
3·175	126·9	9·42	6·382	10·013	7·65	9·339	9·1	10·0
3·01625	151·75	9·35	9·828	9·47	8·2924	9·027	9·3	9·01
3·01625	132·5	9·542	9·874	9·7355	8·2483	9·758	8·5	10·458

(ii) A truncated dielectric sphere excited by a coaxial line has very nearly the same field configuration of one of the TM_{0n} modes.

(iii) Such a structure can be used as an antenna which has a radiation pattern with a null on the axis and two major lobes situated symmetrically with respect to the axis.

ACKNOWLEDGEMENT

The authors are grateful to Dr. S. Dhawan, Director of the Indian Institute of Science, for giving all facilities for the work.

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