RADIATION CHARACTERISTICS OF OVERMODED DIELECTRIC RODS

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ABSTRACT

Radiation characteristics of overmoded dielectric rods have been derived by using the aperture theory as well as the theory based on the Equivalence Principle and have been verified by the experimental results.

Key words: Radiation, overmoded dielectric rods.

1. INTRODUCTION

The paper is a continuation of the study on overmoded dielectric rods[1,2] reported earlier.

Theoretical analysis of the radiation characteristics of dielectric antennas by different authors follow one of the following methods:

(i) Scalar Huyghen's Theory[3,4],

(ii) Schelkunoff's Equivalence Principle[5-17],

(iii) Aperture Theory[18-21],

(iv) Lens Theory[22].

The methods (ii) and (iii) will be referred to as the first and second theory, respectively.

In the scalar Huyghen's theory the vector nature of the field is not revealed and the structure of the radiation pattern and the beam width of the major lobe in the two planes $\phi = 0°$ and $\phi = 90°$ remain the same though the mode of excitation of the aerial, which is the asymmetric HE_{11} mode requires that the structure of the radiation pattern and the beam width be different in different planes.

The vector potential approach with the Equivalence Principle which considers radiation from the surface as well as the end shows a good agreement with the experimental radiation patterns in different planes.

The aperture theory considers the dielectric rod as a transmission line feeding energy to the free-end aperture of the rod and ignores radiation from the surface of the rod.

The operation of a dielectric rod radiator can be considered to be that of a lens having cross-section dimensions of the order of a wavelength. The sides of this lens, as shown by Wilkes[22], is responsible for the concentration of energy and there exists an optimum length at which the energy is maximum. The existence of maximum and minimum in the radiation field is explained by considering the difference in phase velocities of the wave in the media inside and outside the dielectric rod. Expressions for gain and radiation have been derived by using the concept of a lens. The maxima obtained and plotted against the lens aperture show a well-defined minimum at an aperture of one square wavelength, below which the energy increases with decreasing cross-section and above which the energy increases with increasing cross-section. It appears that this method has not been persued by any other author subsequently.

In critically reviewing the two aforementioned vector theoreis, (ii) and (iii), it may be said that the Schelkunoff's Equivalence Principle is identical to the vector Kirchoff's formulation. This has been shown subject to certain limitations, by Fradin[12] and has been proved in a general way by James[11]. It is considered relevant and worthwhile to discuss briefly the equivalence of the two different formulations.

The radiation field for a given aperture is frequently calculated using the Kirchoff's formula which has the following form:

$$\psi_P = \frac{1}{4\pi} \int_S \left[\psi_S \frac{\partial}{\partial n} \left\{ \frac{\exp(-jkr)}{r} \right\} - \frac{\exp(-jkr)}{r} \frac{\partial \psi_S}{\partial n} \right] dS \qquad (1)$$

where ψ is a scalar function characterising the electromagnetic field of the wave. The subscript, P, indicates the value of the function at the point P, for which the value of the function is defined, and S indicates the value of the function at the surface of integration. By successive substitutions of the rectangular components of the electric and magnetic vectors for the function ψ in eq. (1) the following vector formula for the electromagnetic field is obtained:

$$E_P \simeq \int_S \left[E \frac{\partial \phi}{\partial n} - \phi \frac{\partial E}{\partial n} \right] dS \tag{2}$$

$$H_P \simeq \int_S \left[H \frac{\partial \phi}{\partial n} - \phi \frac{\partial H}{\partial n} \right] dS \tag{3}$$

which are exact only in the case of a closed surface, S, and where $\phi = [\exp(-jkr)]/r$. The above vector expressions can be reduced[23] to rectangular components given below:

$$E_{xP} = \frac{1}{4\pi} \int_S \left[E_x \frac{\partial}{\partial n} \left\{ \frac{\exp(-jkr)}{r} \right\} - \frac{\exp(-jkr)}{r} \frac{\partial E_x}{\partial n} \right] dS \tag{4}$$

$$H_{zs} = \frac{1}{4\pi} \int_S \left[H_z \frac{\partial}{\partial n} \left\{ \frac{\exp(-jkr)}{r} \right\} - \frac{\exp(-jkr)}{r} \frac{\partial H_z}{\partial n} \right] dS \tag{5}$$

These have the form of eq. (1).

By considering the surface current densities, J^E and J^H, confined to a surface S (which may be open or closed, connected or disconnected); and relating the fields E and H by the Schelkunoff's Equivalence Principle $J^E = -n \times H, J^H = n \times E$), and using Silver's radiation formula,

$$E_P = -j\omega\mu A^H + \frac{1}{j\omega\epsilon} \text{grad div } A^H - \text{Curl } A^E$$

involving the vector potentials A^E and A^H Stratton and Chu[24] have generalized the vector Kirchoff's formula into the form

$$\begin{aligned} E_P = \frac{1}{4\pi} [& - \iiint_V \{ j\omega\mu J^E \psi - J^H \times \nabla \psi - \left(\frac{\rho}{\epsilon}\right) \nabla \psi \} \, dv \\ & + \iint_S \{ -j\omega\mu (n \times H) \psi - (n \times E) \times \nabla \psi + (n \cdot E) \nabla \psi \} \, da \\ & - (j\omega\epsilon)^{-1} \{ \oint_{S_1} \nabla \psi H \cdot dS + \oint_{S_2} \nabla \psi H \cdot dS + \cdots \\ & + \oint_{S_n} \nabla \psi H \cdot dS \}] \end{aligned} \tag{6}$$

Silver[25] has applied this relation to apertures to obtain a relation for the far field pattern. Bouix[11] has used Kottler's formulation which is equivalent to the above equation, eq. (6). Brown[18], by considering the electric field vector (**E**) as orthogonal to the magnetic field vector (**H**), has simplified the vector Kirchoff's formula, and has used it in his aperture theory for deriving the radiation pattern. This establishes the equivalence between the source field approach and the aperture theory approach. Hence it may be concluded that the vector Kirchoff's formula for the radiation field forms the common basis of all present day techniques for studying the antenna theory.

In overmoded dielectric rods the entire field on the surface of the rod is obtained by the superposition of all the modal fields. The completeness of the Maxwell's field equations' solutions requires that the solution includes not only the surface wave modes which result from the solution of the eigenvalue equation but also the radiation modes. The orthogonality properties of the surface wave modes have been discussed in the previous reports[1, 2]. The structure of the radiation patterns is influenced by the higher order modes existing on the surface of the rod. It is therefore considered worthwhile to discuss the orthogonality properties of the fields in the case of an open waveguide like a dielectric rod.

The orthogonality condition for any two radiation modes is expressed by the relation[26].

$$\tfrac{1}{2}\int_{-\infty}^{\infty}[\hat{E}(\vec{\rho}) \times \hat{H}^*(\rho')] \cdot e_z \, dx = S_\rho \, (\beta^*/|\beta|)\, P\, \delta\,(\rho-\rho') \qquad (7)$$

where the power P carried by the mode is a real, positive quantity and ρ and ρ' are used to label two different modes. The caret on top of the E and H indicates exclusion of propagation vectors exp $(-j\beta z)$. It may, however, be remarked that the inclusion of the propagation vectors will not make any difference when the phase constant, β, is real since the complex conjugate cancels them out for $\rho = \rho'$ and $\rho \neq \rho'$ the integral vanishes. But, when β is imaginary the propagation vector will not cancel out and hence the orthogonality expression would become a function of z. In eq. (7), $S_\rho = 1$ for real β and $S_\rho = -1$ for imaginary β in order to keep ρ positive.

The above condition, eq. (7), holds for any two modes regardless of whether they are both guided modes, both radiation modes or whether one is a guided mode while other is a radiation mode.

For guided modes, the field can be normalized so that the right-hand side of eq. (7) becomes equal to ρ. The function $\delta\,(\rho - \rho')$ represents a delta function when both the modes are labelled as radiation modes. For guided modes, the function $\delta(\rho - \rho')$ is interpreted as the Krönecker delta, δ_{ij}, which is equal to unity for $i = j$ and equal to zero when $i \neq j$.

In the case of dielectric guides, if there are two or more possible solutions of Maxwell's equations describing surface waves (E_{sm}, H_{sm}) which represent the asymptotic solution of the problem, since their amplitudes remain constant (assuming the dielectric rod to be lossless) along the rod, and since the radiation fields (E_R, H_R) decrease, the total fields (E_T, H_T) may be described by the following relations[21]:

$$\begin{aligned} \mathbf{E}_T &= \sum_m A_m \mathbf{E}^+_{sm} + \mathbf{E}_R^+ \\ \mathbf{H}_T &= \sum_m A_m \mathbf{H}^+_{sm} + \mathbf{H}_R^+ \end{aligned} \qquad z > 0$$

$$\begin{aligned} \mathbf{E}_T &= \sum A_m E^-_{sm} + \mathbf{E}_R^- \\ \mathbf{H}_T &= \sum_m A_m H^-_{sm} + \mathbf{H}_R^- . \end{aligned} \qquad z < 0 \tag{8}$$

As the total field and the individual surface waves of unit amplitude are solutions of Maxwell's equations they satisfy the reciproctty relation which states that

$$\oiint_A \left(\mathbf{E}_T \times \mathbf{H}^{+-}_{sm} - \mathbf{E}^{+-}_{m} \times \mathbf{H}_T \right) \cdot d\mathbf{a} = 0 \tag{9}$$

where the positive and negative subscripts indicate the waves in the $+z$ and $-z$ directions. Using the reciprocity theorem and the expressions for the total field, Goubau[27] has shown that the following orthogonality relation between any surface wave modes and the radiation field is satisfied.

$$\oiint_A (\mathbf{E}_R \times \mathbf{H}_m) \cdot d\mathbf{a} = \oiint_A (\mathbf{E}_m \times \mathbf{H}_R) \cdot d\mathbf{a} = 0. \tag{10}$$

It has also been shown by Adler[28] that the following orthogonality relation between any two non-degenerate surface wave modes

$$\oiint_A (\mathbf{E}_{sm} \times \mathbf{H}_{sn}) \cdot d\mathbf{a} = \oiint_A (\mathbf{E}_{sn} \times \mathbf{H}_{sm}) \cdot d\mathbf{a} = 0 \tag{11}$$

is satisfied in closed waveguides and is also valid for open waveguides like dielectric waveguides.

The mode orthogonality relation, eq. (7), with respect to the average power applies only for lossless dielectric waveguides. But if the dielectric constant of the waveguide is complex this relation, eq. (7), is not strictly valid. It will, however, to a good approximation still hold for slightly lossy waveguides, although it will no longer be strictly true.

In this context, it may be emphasized that eq. (10) holds for both dissipative as well as non-dissipative waveguides. When the waveguide is lossless however both the relations defined by eqs. (7) and (10) hold good. Since the condition defined by eq. (10) remains valid more generally it is this relation which acts more effectively as an orthogonality relation. It may be remarked that in the absence of complex conjugation the quantity $E \times H$ does not have a physical meaning and hence the simple interpretation of power orthogonality, in this case, is not valid.

The above resumé of orthogonality relations leads to the conclusion that the relations can be applied to dielectric rods and that they permit the evaluation of the total power flow in an overmoded dielectric rod by a summation of the power carried by each mode individually, provided the modes are non-degenerate.

The object of this paper is to present the derivation of the radiation pattern of overmoded dielectric rods by using both the mathematical approach (ii) and (iii) and to compare them with the experimental results with a view to determine the limits of validity, if any, of the two theories and to study also the effects of the higher order modes on the radiation pattern.

3. FUNDAMENTAL RELATIONS FOR THE FIRST THEORETICAL APPROACH

The electric ($\boldsymbol{J}$) and magnetic ($\boldsymbol{M}$) sheet currents on the surface of the dielectric rod according to the Equivalence Principle are given by

$$\boldsymbol{J} = \boldsymbol{n} \times \boldsymbol{H}^0 \tag{12 a}$$

$$\boldsymbol{M} = -\boldsymbol{n} \times \boldsymbol{E}^0 \tag{12 b}$$

where

$\boldsymbol{n}$ is the unit normal vector directed outwards from the surface S;

$\boldsymbol{E}^0$ is the electric field vector on the surface;

$\boldsymbol{H}^0$ is the magnetic field vector on the surface;

S includes the surfaces S_1 and S_2 shown in Fig. 1 *a* which gives the co-ordinate system employed for the purpose of determining the radiation field and a large sphere S_3 as shown in Fig. 1 *b*. The currents over S_3 are assumed to be negligible;

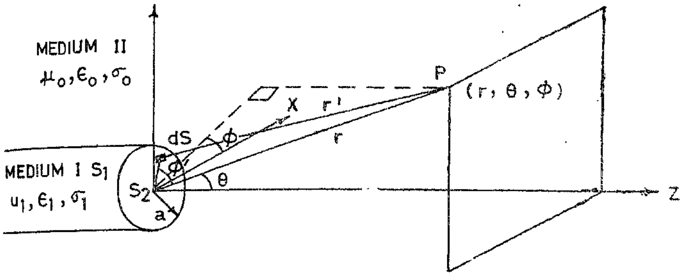

FIG. 1*a* Coordinate system for calculating the radiation pattern by the (I) theory.

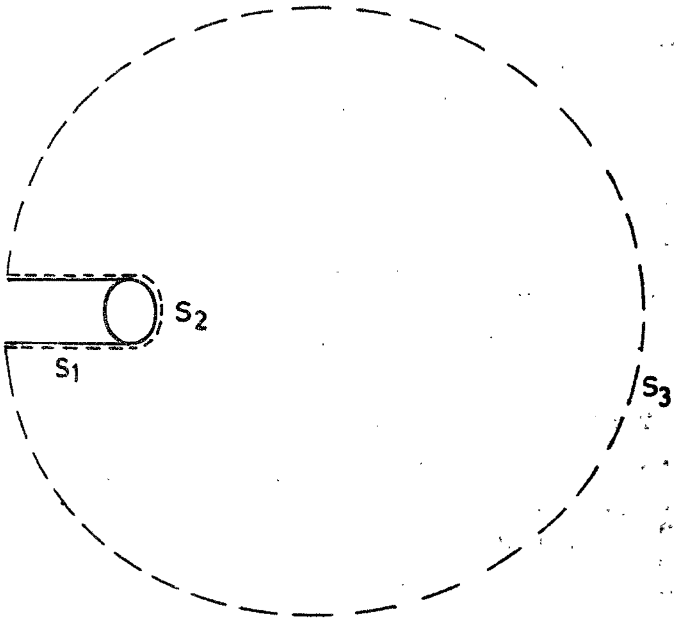

FIG. 1*b* Surface of Integration $S = S_1 + S_2 + S_3$ for finding the Radiation Field.

S_1 is the cylindrical surface of the dielectric rod aerial;

S_2 is the free end surface of the dielectric rod aerial;

dS is an element of area on the dielectric surface;

r' is the distance of dS from the distant point, P, at which the radiation field is to be determined $= r - a \sin\theta \cos(\phi' - \phi) - z\cos\theta$;

r is the distance of the origin of the coordinate system to the point P;

The electric (A^E) and magnetic (A^H) vector potentials at a point $P(r, \theta, \phi)$ with S are given by

$$A^H = \frac{1}{4\pi} \int_S \frac{J \exp(j\omega t - kr')}{r'} dS \tag{13 a}$$

$$A^E = \frac{1}{4\pi} \int_S M \frac{\exp(j\omega t - kr')}{r'} dS \tag{13 b}$$

where k is the free space wave number. The radiation field in the (θ, ϕ) plane at a distant point $P(r, \theta, \phi)$ is given by

$$E = -j\omega\mu_0 A^H + \frac{1}{j\omega\epsilon_0} \text{grad div } A^H - \text{curl } A^E . \tag{14}$$

The transformation of coordinates from the cylindrical system (ρ, ϕ', z) describing the surface of the rod to the spherical polar coordinates (r, θ, ϕ) describing a distant point in space is made according to Table I.

2·1. *Radiated Field: Surface Radiation*

The sheet currents J and M are given by

$$\begin{aligned} J &= -\vec{\phi}' H_z + z H_\phi , \\ &= -\vec{\phi}' A_1 \cos\phi' e^{-j\beta_m z} + z \sin\phi' e^{-j\beta_m z} \end{aligned} \tag{15}$$

TABLE I

Transformation of coordinates from the cylindrical coordinate system into the spherical polar coordinate system

	$\vec{r}$	$\vec{\theta}$	$\vec{\phi}$
$\vec{\rho}$	$\sin\theta\cos(\phi' - \phi)$	$\cos\theta\cos(\phi' - \phi)$	$\sin(\phi' - \phi)$
$\vec{\phi}'$	$-\sin\theta\sin(\phi' - \phi)$	$-\cos\theta\sin(\phi' - \phi)$	$\cos(\phi' - \phi)$
$\vec{z}$	$\cos\theta$	$-\sin\theta$	0

where

$$A_1 = - B_m \frac{k^2_{1m}}{j\omega\mu_0} J_1 (k_{1m} a) \tag{15 a}$$

$$A_2 = - B_m \left[\frac{1}{a} \frac{\beta_m}{\omega\mu_0} J_1 (k_{1m} a) + \frac{b_m}{B_m} k_{1m} J_1' (k_{1m} a)\right] \tag{15 b}$$

The expression for J in the spherical polar coordinate system is obtained from eq. (15) by using the transformations in Table I. Its components are given below:

$$\begin{aligned} J_r &= [A_1 \sin\theta \sin(\phi' - \phi) \cos\phi' + A_2 \cos\theta \sin\phi'] \, e^{-j\beta_m z} \\ J_\theta &= [A_1 \cos\theta \sin(\phi' - \phi) \cos\phi' - A_2 \sin\theta \sin\phi'] \, e^{-j\beta_m z} \\ J_\phi &= [- A_1 \cos(\phi' - \phi) \cos\phi'] \, e^{-j\beta_m z} \end{aligned} \tag{16}$$

$$\begin{aligned} M &= - z \, E_{\phi'} + \vec{\phi}' E_z \\ &= \vec{\phi}' A_3 \sin\phi' \, e^{-j\beta_m z} - z \, A_4 \cos\phi' \, e^{-j\beta_m z} \end{aligned} \tag{17}$$

where

$$A_3 = B_m \frac{b_m}{B_m} \frac{k^2_{1m}}{j\omega\epsilon_1} J_1 (k_{1m} a) \tag{17 a}$$

and

$$A_4 = - B_m \left[k_{1m} \; J_1' \; (k_{1m}a) + \frac{b_m}{B_m} \frac{1}{a} \frac{\beta_m}{\omega\epsilon_1} \; J_1 (k_{1m}a)\right] \tag{17 b}$$

In polar coordinates the components of M are given by

$$\begin{aligned} M_r &= [- A_3 \sin\theta \sin(\phi' - \phi) \sin\phi' - A_4 \cos\theta \cos\phi] \, e^{-j\beta_m z} \\ M_\theta &= [- A_3 \cos\theta \sin(\phi' - \phi) \sin\phi' + A_4 \sin\theta \cos\phi'] \, e^{-j\beta_m z} \\ M_\phi &= [A_3 \cos(\phi' - \phi) \sin\phi'] \, e^{-j\beta_m z} \end{aligned} \tag{18}$$

Hence,

$$\begin{aligned} dA^H = \frac{e^{-jkr}}{4\pi r} \{ & r \, [A_1 \sin\theta \sin(\phi' - \phi) \cos\phi' + A_2 \cos\theta \sin\phi'] \\ & + \vec{\theta} \; [A_1 \cos\theta \sin(\phi' - \phi) \cos\phi' - A_2 \sin\theta \sin\phi'] \\ & + \vec{\phi} \, [- A_1 \cos\phi' \cos(\phi' - \phi)] \}. \\ & \times \exp[jka \sin\theta \cos(\phi' - \phi) + jkz \cos\theta - j\beta_m z] \, dS \end{aligned} \tag{19}$$

$$dA^E = \frac{e^{-jkr}}{4\pi r} \{r [- A_3 \sin\theta \sin(\phi' - \phi) \sin\phi' - A_4 \cos\theta \cos\phi']$$

$$+ \vec{\theta} [- A_3 \cos\theta \sin(\phi' - \phi) \sin\phi' + A_4 \sin\theta \cos\phi']$$

$$+ \vec{\phi} [A_3 \cos(\phi' - \phi) \sin\phi']\}$$

$$\times \exp [jka \sin\theta \cos(\phi' - \phi) + jkz \cos\theta - j\beta_m z] \, dS. \quad (20)$$

Evaluating grad div dA^H, curl dA^E and using eq. (14), the radiated field E_{PS} from the cylindrical surface of the rod is given by

$$E_{PS} = - \frac{jae^{-jkr}}{4\pi r} \int_{z=-L}^{0} \int_{\phi'=0}^{\pi} \{[\vec{\theta} (\omega\mu_0 J_\theta + kM_\phi)$$

$$+ \vec{\phi} (\omega\mu_0 J_\phi - kM_\theta)]$$

$$\times \exp [jka \sin\theta \cos(\phi' - \phi) + jkz \cos\theta]\} \, d\phi' \, dz$$

$$= - j \frac{a}{4\pi} \frac{e^{-jkr}}{r} \int_{z=-L}^{0} \int_{\phi'=0}^{2\pi} [\vec{\theta} \omega\mu_0 \{A_1 \cos\theta \sin(\phi' - \phi)$$

$$\times \cos\phi' - A_2 \sin\theta \sin\phi'\} + \vec{\theta} k \{A_3 \cos(\phi' - \phi) \sin\phi'\}$$

$$+ \vec{\phi} \omega\mu_0 \{- A_1 \cos\phi' \cos(\phi' - \phi)\} - \vec{\phi} k \{- A_3 \cos\theta$$

$$\times \sin(\phi' - \phi) \sin\phi' + A_4 \sin\theta \cos\phi'\}]$$

$$\times \exp [jka \sin\theta \cos(\phi' - \phi) + jkz \cos\theta - j\beta_m z]\} \, d\phi' \, dz \quad (21)$$

which in the $\phi = 0°$ plane reduces to

$$E_{PS} = - j \frac{a}{4\pi} \frac{e^{-jkr}}{r} \int_{z=-L}^{0} \exp [- j(\beta_m - k\cos\theta) z] \, dz$$

$$\times \vec{\theta} \Big[\Big(\frac{\omega\mu_0 A_1}{2} \cos\theta + \frac{kA_3}{2} \int_{\phi'=0}^{2\pi} \sin 2\phi' \exp(jka \sin\theta \cos\phi') \, d\phi'$$

$$- (\omega\mu_0 A_2 \sin\theta) \int_{\phi'=0}^{2\pi} \sin\phi' \exp(jka \sin\theta \cos\phi') \, d\phi' \Big]$$

$$+ \vec{\phi} \Big[- \Big(A_1 \frac{\omega\mu_0}{2} + \frac{A_3 k}{2} \cos\theta \Big) \int_{\phi'=0}^{2\pi} \cos 2\phi'$$

$$\times \exp (jka \sin \theta \cos \phi') \, d\phi'$$

$$+ \frac{k \cos \theta \, A_3}{2} - A_1 \frac{\omega\mu_0}{2} \int_{\phi'=0}^{2\pi} \exp (jka \sin \theta \cos \phi') \, d\phi'$$

$$- (A_4 \, k \sin \theta) \int_{\phi'=0}^{2\pi} \cos \phi' \exp (jka \sin \theta \cos \phi') \, d\phi' \Big] \Big\} . \qquad (22)$$

Evaluating the integrals, the expression for the radiation field due to the surface of the rod for each mode, identified by the subscript m, is given by the following equation:

$$E_{Psm} = \vec{\phi} \frac{e^{-jkr}}{r} q_{sm} \sin [(\beta_m - k \cos \theta] L/2] \qquad (23)$$

where

$$q_{sm} = B_1 \begin{Bmatrix} B_m \, (m > 1) \\ B_1 \, (m = 1) \end{Bmatrix} \frac{a}{2\pi} \frac{\exp [j (\beta_m - k \cos \theta) L/2]}{(\beta_m - k \cos \theta)}$$

$$\times \{\pi k^2_{1m} J_1 (k_{1m} a) [J_2 (ka \sin \theta) - J_0 (ka \sin \theta)]$$

$$- \frac{\pi k \, k^2_{1m}}{\omega \epsilon_1} \frac{b_m}{B_m} J_1 (k_{1m} a) \cos \theta \, [J_2 (ka \sin \theta) + J_0 (ka \sin \theta)]$$

$$+ 2\pi k \Big[k_{1m} J_1' (k_{1m} a) + \frac{b_m}{B_m} \frac{1}{a} \frac{\beta_m}{\omega \epsilon_1} J_1 (k_{1m} a) \Big]$$

$$\times \sin \theta \, J_1 (ka \sin \theta)\}. \qquad (23\,a)$$

The above eq. (23) representing the radiation field due to the surface only can be written in the following functional form

$$E_{Psm} = \vec{\phi} B_1 \frac{e^{-jkr}}{r} f_m (A) f_{1m} (d) f_m (d, L) \exp (j (\beta_m - k \cos \theta) L/2] \qquad (24)$$

where

$$f_m (A) = \frac{B_m \, (m > 1)}{B_1 \, (m = 1)} = \text{relative amplitude constant} \qquad (24\,a)$$

$$f_{1m} (d) = \frac{d}{4\pi (\beta_m - k \cos \theta)} \Big\{ \pi k^2_{1m} J_1 \Big(\frac{k_{1m} d}{2} \Big) \Big[J_2 \Big(\frac{kd}{2} \sin \theta \Big)$$

$$- J_0 \Big(\frac{kd}{2} \sin \theta \Big) \Big]$$

$$- \frac{\pi k \, k^2_{1m}}{\omega \epsilon_1} \frac{b_m}{B_m} J_1 \Big(\frac{k_{1m} d}{2} \Big) \cos \theta \Big[J_2 \Big(\frac{kd \sin \theta}{2} \Big)$$

$$+ J_0\left(\frac{kd}{2}\sin\theta\right)\Big] + 2\pi k\Big[k_{1m}\, J_1'\left(\frac{k_{1m}d}{2}\right)$$

$$+ \frac{b_m}{B_m}\frac{2}{d}\frac{\beta_m}{\omega\epsilon_1} J_1\left(\frac{k_{1m}d}{2}\right)\Big]\sin\theta J_1\left(\frac{kd}{2}\sin\theta\right)\Big\} \tag{24 b}$$

$$f_m(d, L) = \sin\left[(\beta_m - k\cos\theta)\, L/2\right] \tag{24 c}$$

since the radiated field at $P(r, \theta, \phi)$ depends on the diameter and length of the rod and the relative amplitude constant of a particular mode.

3.2. *Radiated Field: End Radiation*

In this case

$$\boldsymbol{J} = \vec{\phi}'\, H_\rho - \vec{\rho}\, H_{\phi'} \tag{25}$$

and

$$\boldsymbol{M} = -\vec{\phi}'\, E_\rho + \vec{\rho}\, E_{\phi'}. \tag{26}$$

Using the appropriate field components

$$J = \vec{\phi}'\left[X_1 J_1'(k_{1m}\rho) + X_2 \frac{J_1(k_{1m}\rho)}{\rho}\right]\cos\phi'$$

$$-\vec{\rho}\left[X_3 \frac{J_1(k_{1m}\rho)}{\rho} + X_4 J_1'(k_{1m}\rho)\right]\sin\phi' \tag{27}$$

here

$$X_1 = B_m \frac{\beta_m k_{1m}}{\omega\mu_0} \tag{27 a}$$

$$X_2 = B_m \frac{b_m}{B_m} \tag{27 b}$$

$$X_3 = -B_m \frac{\beta_m}{\omega\mu_0} \tag{27 c}$$

$$X_4 = -B_m \frac{b_m}{B_m} k_{1m} \tag{27 d}$$

and

$$M = -\phi'\left\{-B_m\left[\frac{1}{\rho} J_1(k_{1m}\rho) + \frac{b_m}{B_m}\frac{\beta_m k_{1m}}{\omega\,\epsilon_1} J_1'(k_{1m}\rho)\right]\right\}\sin\phi'$$

$$+\vec{\rho}\left\{-B_m\left[k_{1m} J_1'(k_{1m}\rho) + \frac{b_m}{B_m}\frac{\beta_m}{\omega\epsilon_1}\frac{J_1(k_{1m}\rho)}{\rho}\right]\right\}\cos\phi' \tag{28}$$

Let

$$Y_1 = -B_m \quad (28\,a)$$

$$Y_2 = -B_m \frac{b_m}{B_m} \frac{\beta_m k_{1m}}{\omega\epsilon_1} \quad (28\,b)$$

$$Y_3 = -B_m\, k_{1m} \quad (28\,c)$$

$$Y_4 = -B_m \frac{b_m}{B_m} \frac{\beta_m}{\omega\epsilon_1}\,. \quad (28\,d)$$

The above two expressions for $\boldsymbol{J}$ and $\boldsymbol{M}$ in polar coordinates become

$$\begin{aligned}
\boldsymbol{J} = \boldsymbol{r} \Big\{ &-\sin\theta\sin(\phi'-\phi)\cos\phi'\Big[X_1 J_1'(k_{1m}\rho) + X_2 \frac{J_1(k_{1m}\rho)}{\rho}\Big] \\
&-\sin\theta\cos(\phi'-\phi)\sin\phi'\Big[X_3 \frac{J_1(k_{1m}\rho)}{\rho} + X_4 J_1'(k_{1m}\rho)\Big]\Big\} \\
&+\vec{\theta}\Big\{-\cos\theta\sin(\phi'-\phi)\cos\phi'\Big[X_1 J_1'(k_{1m}\rho) + X_2 \frac{J_1(k_{1m}\rho)}{\rho}\Big] \\
&-\cos\theta\cos(\phi'-\phi)\sin\phi'\Big[X_3 \frac{J_k(k_{1m}\rho)}{\rho} + X_4 J_1'(k_{1m}\rho)\Big]\Big\} \\
&+\vec{\phi}\Big\{\cos(\phi'-\phi)\cos\phi'\Big[X_1 J_1'(k_{1m}\rho) + X_2 \frac{J_1(k_{1m}\rho)}{\rho}\Big] \\
&-\sin(\phi'-\phi)\sin\phi'\Big[X_3 \frac{J_1(k_{1m}\rho)}{\rho} + X_4 J_1'(k_{1m}\rho)\Big]\Big\} \qquad (29)
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{M} = \boldsymbol{r} \Big\{ &\sin\theta\sin(\phi'-\phi)\sin\phi'\Big[Y_1 \frac{J_1(k_{1m}\rho)}{\rho} + Y_2 J_1'(k_{1m}\rho)\Big] \\
&+\sin\theta\cos(\phi'-\phi)\cos\phi'\Big[Y_3 J_1'(k_{1m}\rho) + Y_4 \frac{J_1(k_{1m}\rho)}{\rho}\Big]\Big\} \\
&+\vec{\theta}\Big\{\cos\theta\sin(\phi'-\phi)\sin\phi'\Big[Y_1 \frac{J_1(k_{1m}\rho)}{\rho} + Y_2 J_1'(k_{1m}\rho)\Big] \\
&+\cos\theta\cos(\phi'-\phi)\cos\phi'\Big[Y_3 J_1'(k_{1m}\rho) + Y_4 \frac{J_1(k_{1m}\rho)}{\rho}\Big]\Big\} \\
&+\vec{\phi}\Big\{-\cos(\phi'-\phi)\sin\phi'\Big[Y_1 \frac{J_1(k_{1m}\rho)}{\rho} + Y_2 J_1'(k_{1m}\rho)\Big] \\
&+\sin(\phi'-\phi)\cos\phi'\Big[Y_3 J_1'(k_{1m}\rho) + Y_4 \frac{J_1(k_{1m}\rho)}{\rho}\Big]\Big\} \qquad (30)
\end{aligned}$$

which yields the components J_r, J_θ, J_ϕ and M_r, M_θ, M_ϕ respectively.

The magnetic and electric vector potentials are

$$dA^H = \frac{e^{-jkr}}{4\pi r}\Big\{-\boldsymbol{r}\Big[\sin\theta\sin(\phi'-\phi)\cos\phi'\Big\{X_1 J_1'(k_{1m}\rho) + X_2\frac{J_1(k_{1m}\rho)}{\rho}\Big\} + \sin\theta\cos(\phi'-\phi)\sin\phi'\Big\{X_3\frac{J_1(k_{1m}\rho)}{\rho} + X_4 J_1'(k_{1m}\rho)\Big\}\Big] - \vec{\theta}\Big[\cos\theta\sin(\phi'-\phi)\cos\phi' \times\Big\{X_1 J_1'(k_{1m}\rho) + X_2\frac{J_1(k_{1m}\rho)}{\rho}\Big\} + \cos\theta\cos(\phi'-\phi)\sin\phi'\Big\{X_3\frac{J_1(k_{1m}\rho)}{\rho} + X_4 J_1'(k_{1m}\rho)\Big\}\Big] + \vec{\phi}\Big[\cos(\phi'-\phi)\cos\phi'\Big\{X_1 J_1'(k_{1m}\rho) + X_2\frac{J_1(k_{1m}\rho)}{\rho}\Big\} - \sin(\phi'-\phi)\sin\phi'\Big\{X_3\frac{J_1(k_{1m}\rho)}{\rho} + X_4 J_1'(k_{1m}\rho)\Big]\Big\} \times\exp[jk\rho\sin\theta\cos(\phi'-\phi)] \tag{31}$$

$$dA^E = \frac{e^{-jkr}}{4\pi r}\Big\{\boldsymbol{r}\Big[\sin\theta\sin(\phi'-\phi)\sin\phi'\Big\{Y_1\frac{J_1(k_{1m}\rho)}{\rho} + Y_2 J_1'(k_{1m}\rho)\Big\} + \sin\theta\cos(\phi'-\phi)\cos\phi' \times\Big\{Y_3 J_1'(k_{1m}\rho) + Y_4\frac{J_1(k_{1m}\rho)}{\rho}\Big\}\Big] + \vec{\theta}\Big[\cos\theta\sin(\phi'-\phi)\sin\phi'\Big\{Y_1\frac{J_1(k_{1m}\rho)}{\rho} + Y_2 J'_1(k_{1m}\rho)\Big\} + \cos\theta\cos(\phi'-\phi)\cos\phi'\Big\{Y_3 J'_1(k_{1m}\rho) + Y_4\frac{J_1(k_{1m}\rho)}{\rho}\Big\}\Big] + \vec{\phi}\Big[-\cos(\phi'-\phi)\sin\phi'\Big\{Y_1\frac{J_1(k_{1m}\rho)}{\rho} + Y_2 J_1'(k_{1m}\rho)\Big\} + \sin(\phi'-\phi)\cos\phi'\Big\{Y_3 J_1'(k_{1m}\rho) + Y_4\frac{J_1(k_{1m}\rho)}{\rho}\Big\}\Big]\Big\} \times\exp[jk\rho\sin\theta\cos(\phi'-\phi)] \tag{32}$$

with

$$r' = r - \rho \sin\theta \cos(\phi' - \phi).$$

Evaluating grad div dA^H and curl dA^E in the radiation formula, eq. (14), and substituting for J_θ, J_ϕ, M_θ and M_Φ appropriately from eqs. (29 and 30), the radiation from the end is given by

$$E_{Pe} = \frac{-j\,e^{-jkr}}{4\pi r} \int_{\rho=0}^{a} \int_{\phi'=0}^{2\pi} \Big\{ -\vec{\theta}\,\omega\mu_0 \Big[\cos\theta \sin(\phi'-\phi)\cos\phi'$$

$$\times \Big\{ X_1 J_1'(k_{1m}\rho) + X_2 \frac{J_1(k_{1m}\rho)}{\rho} \Big\}$$

$$+ \cos\theta \cos(\phi'-\phi)\sin\phi' \Big\{ X_3 \frac{J_1(k_{1m}\rho)}{\rho} + X_4 J_1'(k_{1m}\rho) \Big\} \Big]$$

$$+ \vec{\theta}\, k \Big[-\cos(\phi'-\phi)\sin\phi' \Big\{ Y_1 \frac{J_1(k_{1m}\rho)}{\rho} + Y_2 J_1'(k_{1m}\rho) \Big\}$$

$$+ \sin(\phi'-\phi)\cos\phi' \Big\{ Y_3 J_1'(k_{1m}\rho) + Y_4 \frac{J_1(k_{1m}\rho)}{\rho} \Big\} \Big]$$

$$+ \vec{\phi}\,\omega\mu_0 \Big[\cos(\phi'-\phi)\cos\phi' \Big\{ X_1 J_1'(k_{1m}\rho) + X_2 \frac{J_1(k_{1m}\rho)}{\rho} \Big\}$$

$$- \sin(\phi'-\phi)\sin\phi' \Big\{ X_3 \frac{J_1(k_{1m}\rho)}{\rho} + X_4 J_1'(k_{1m}\rho) \Big\} \Big]$$

$$+ \vec{\phi}\, k \Big[\cos\theta \sin(\phi'-\phi)\sin\phi' \Big\{ Y_1 \frac{J_1(k_{1m}\rho)}{\rho} + Y_2 J_1'(k_{1m}\rho) \Big\}$$

$$+ \cos\theta \cos(\phi'-\phi)\cos\phi' \Big\{ Y_3 J_1'(k_{1m}\rho) + Y_4 \frac{J_1(k_{1m}\rho)}{\rho} \Big\} \Big] \Big\}$$

$$\times \exp[jk\rho \sin\theta \cos(\phi'-\phi)]\, \rho\, d\rho\, d\phi'. \tag{33}$$

In the $\phi = 0°$ the field E_{Pem} radiated by the free end by the m-th mode is given by

$$E_{Pem} = -j \frac{e^{-jkr}}{4\pi r} \int_{\rho=0}^{a} \int_{\phi'=0}^{2\pi} \Big\{ -\vec{\theta} \frac{\omega\mu_0}{2} \cos\theta \sin 2\phi' \Big[X_1 J_1'(k_{1m}\rho)$$

$$\pm \frac{X_2 J_1(k_{1m}\rho)}{\rho} + X_3 \frac{J_1(k_{1m}\rho)}{\rho} + X_4 J_1'(k_{1m}\rho) \Big]$$

$$- \vec{\theta} \frac{k}{2} \sin 2\phi' \Big[Y_1 \frac{J_1(k_{1m}\rho)}{\rho} + Y_2 J_1'(k_{1m}\rho) - Y_3 J_1'(k_{1m}\rho)$$

$$- Y_4 \frac{J_1(k_{1m}\rho)}{\rho}\Bigg]$$

$$+ \vec{\phi}\,\omega\mu_0 \Big[\cos^2\phi' \left\{X_1 J_1'(k_{1m}\rho) + X_2 \frac{J_1(k_{1m}\rho)}{\rho}\right\}$$

$$- \sin^2\phi' \left\{X_3 \frac{J_1(k_{1m}\rho)}{\rho} + X_4 J_1'(k_{1m}\rho)\right\}\Big]$$

$$- \vec{\phi}\, k\cos\theta \Big[\sin^2\phi' \left\{Y_1 \frac{J_1(k_{1m}\rho)}{\rho} + Y_2 J_1'(k_{1m}\rho)\right\}$$

$$+ \cos^2\phi' \left\{Y_3 J_1'(k_{1m}\rho) + Y_4 \frac{J_1(k_{1m}\rho)}{\rho}\right\}\Big]\Big\}\cdot$$

$$\times \exp[jk\rho\sin\theta\cos\phi']\,\rho\, d\rho\, d\phi' \tag{34}$$

which when evaluated yields

$$E_{Pem} = \vec{\phi}\,\frac{e^{-jkr}}{r}\, j\, q_{em} \tag{35}$$

where

$$q_{em} = B_1 \begin{Bmatrix} B_m\,(m>1) \\ B_1\,(m=1) \end{Bmatrix}$$

$$\times \Big\{ -\Big[\frac{k_{1m}}{4}\Big(\frac{b_m}{B_m}\,\omega\mu_0 - \beta_m\Big) + \frac{kk_{1m}}{4}\Big(\frac{b_m}{B_m}\frac{\beta_m}{\omega\epsilon_1} - 1\Big)\cos\theta\Big] F_1$$

$$- \Big[\frac{1}{2}\Big(\beta_m - \frac{b_m}{B_m}\,\omega\mu_0\Big) + \frac{k}{2}\Big(\frac{b_m}{B_m}\frac{\beta_m}{\omega\epsilon_1} - 1\Big)\cos\theta\Big] F_2$$

$$- \Big[\frac{k_{1m}}{4}\Big(\beta_m + \frac{b_m}{B_m}\,\omega\mu_0\Big) - kk_{1m}\Big(\frac{b_m}{B_m}\frac{\beta_m}{\omega\epsilon_1} + 1\Big)\cos\theta\Big]\Big\} F_3 \tag{35a}$$

and

$$F_1 = \int_{\rho=0}^{a} \rho\, J_0(k_{1m}\rho)\, J_2(k\rho\sin\theta)\, d\rho$$

$$F_2 = \int_{\rho=0}^{a} J_1(k_{1m}\rho)\, J_2(k\rho\sin\theta)\, d\rho$$

$$F_3 = \int_{\rho=0}^{a} \rho J_0(k_{1m}\rho)\, J_0(k\rho\sin\theta)\, d\rho \tag{35b}$$

Equation (35) can be written in a functional form as follows:

$$E_{Pem} = \vec{\phi}\,\frac{e^{-jkr}}{r}\, B_1\, j f_m(A)\, f_{2m}(d) \tag{36}$$

$$f_m(A) = \frac{B_m\,(m > 1)}{B_1\,(M = 1)} \tag{36 a}$$

$$\begin{aligned} f_{3m}(d) = &-\left[\frac{k_{1m}}{4}\left(\frac{\beta_m}{B_m}\,\omega\mu_0 - \beta_m\right) + kk_{1m}\left(\frac{b_m}{B_m}\,\frac{\beta_m}{\omega\epsilon_1} - 1\right)\cos\theta\right] F_1 \\ &-\left[\frac{1}{2}\left(\beta_m - \frac{b_m}{B_m}\,\omega\mu_0\right) + \frac{k}{2}\left(\frac{b_m}{B_m}\,\frac{\beta_m}{\omega\epsilon_1} - 1\right)\cos\theta\right] F_2 \\ &-\left[\frac{k_1 m}{4}\left(\beta_m + \frac{b_m}{B_m}\,\omega\mu_0\right) - kk_{1m}\left(\frac{b_m}{B_m}\,\frac{\beta_m}{\omega\epsilon_1} + 1\right)\cos\theta\right] F_3 \end{aligned} \tag{36 b}$$

2.3. *Total Radiated Field*: *Surface* plus *End Radiation*

The total radiation E_{pst} due to the cylindrical surface only for the m modes will be

$$E_{Pst} = \sum_m E_{Psm} \tag{37}$$

and the total radiation due to the end only for m modes will be

$$E_{PeT} = \sum_m E_{Pem}. \tag{38}$$

The total radiation due to the surface and end for the m-th mode will be

$$E_{Pm} = E_{Psm} + E_{Pem}. \tag{39}$$

The total radiation (end *plus* surface) E_{Ptm} for all the m modes will therefore be given by

$$E_{Ptm} = \sum_m (E_{Pm})$$

$$\therefore\; E_{Ptm} = \sum_m (E_{Psm} + E_{Pem})$$

i.e.,

$$\begin{aligned} E_{Ptm} = \vec{\phi}\,\frac{e^{-jkr}}{r}\,B_1 \sum_m \{f_m(A)\,[\,f_{1m}(d) f_m(d, L) \\ \times \exp\{j(\beta_m - k\cos\theta)\,L/2\} + j f_{2m}(d)]\}. \end{aligned} \tag{40}$$

3. Radiated Field: Second Theory

In the first theoretical approach it is assumed that the fictitious sources of electric and magnetic current densities are distributed in space in an arbitrary manner and all the field quantities satisfy Maxwell's equations over the volume of the entire space. When the electric and magnetic current densities are confined to a surface S, which may be open or closed, connected or disconnected, it is usual to refer to the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ on the surface rather than the surface densities $\boldsymbol{J}$ and $\boldsymbol{M}$. The surface fields and surface current densities are related by the Schelkunoff's Equivalence Principle.

3.1. *Simplified Form of Vector Kirchoff's Formula*

When $\boldsymbol{J}$ and $\boldsymbol{M}$ are not restricted to surface distributions, the radiation formula must also include the volume sources of current. In such a case the field at a point in space lying outside a surface that encloses all the sources of the field can be expressed in terms of the integrals of the field vectors over the surface. This is given by the vector Kirchoff integral relation which is as follows:

$$\begin{aligned} 4\pi \boldsymbol{E}_p = & - \iiint_V [j\omega\mu\, \boldsymbol{J}\psi - \boldsymbol{M} X \nabla \psi - \left(\frac{\rho}{\epsilon}\right) \nabla \psi]\, dv \\ & + \int\int_S [-j\omega\mu (\boldsymbol{n} \times \boldsymbol{H})\psi - (\boldsymbol{n} \times \boldsymbol{E}) \times \nabla \psi + (\boldsymbol{n} \cdot \boldsymbol{E}) \nabla \psi]\, da \\ & - \left(\frac{1}{j\omega\epsilon}\right) [\oint_{S_1} \nabla \psi \boldsymbol{H} \cdot dS + \oint_{S_2} \nabla \psi \boldsymbol{H} \cdot dS + \cdots \\ & + \oint_{S_n} \nabla \psi \boldsymbol{H} \cdot dS] \end{aligned} \tag{41}$$

where $S_1, S_2, \cdots, S_n$ are the surfaces bounded by the volume V.

$$\psi = \frac{\exp(-jkr')}{r'}$$

r' = Distance from P to any other point in the volume.

The expression for the $\boldsymbol{H}$ field is similarly expressed.

The method followed here to obtain the radiation due to the apertures is the same as that outlined by James[19]. The radiation field at a distant point can be determined by using Kirchoff's formula, eq. (41). This equation is transformed into a more usable form with the help of the system of coordinates, and vectors shown in Fig. 2, where A_d, A'_d, A_g A'_g are the various apertures associated with the dielectric rod radiator.

$\vec{\rho}'_1$ Vector from the origin of the coordinate system to the element da of the aperture area.

r_1 Unit vector from the origin to the field point in the direction (θ, ϕ).

r'_1 Unit vector from da to the field point.

$\vec{\rho_1} r_1 = z \cos\theta + \rho \sin\theta \cos(\phi' - \phi)$.

The far field pattern is then given by the following expression:

$$E_p = \frac{-jk}{4\pi r} [\exp(-jkr)] \, r_1 \iint_S \{[(n \times E) - \eta_0 r_1 \times (n \times H)] \times \exp(jk\vec{\rho} \cdot r_1)\} \, da \tag{42}$$

The simplified radiation formula for the far field can then be written as,

$$E_{p\theta} = k\,[(L_x{}^E \sin\theta + L_y{}^E \cos\phi) + \eta_0 \cos(L_x{}^H \cos\phi - L_y{}^H \sin\phi)] \tag{43 a}$$

$$E_{p\phi} = -k\,[(L_x{}^E \cos\phi - L_y{}^E \sin\phi)\cos\theta - (\eta_0)(L_x{}^H \sin\phi + L_y{}^H \cos\phi)] \tag{43 b}$$

where L^E and L^H are integrals involving E and H.

$$L^E = \iint_S E\,[\exp(jk\vec{\rho_1} \cdot r_1)]\, da \tag{44 a}$$

$$L^H = \iint_S H\,[\exp(jk\vec{\rho_1} \cdot r_1)]\, da \tag{44 b}$$

and

$\eta_0 =$ Free space intrinsic impedance $= 376 \cdot 7$.

Here terms which are not functions of θ and ϕ have been omitted.

3.2. *Radiating Apertures*

The hollow metallic, cylindrical waveguide supporting the unperturbed H_{11} wave excites the HE_{11} mode in and on the dielectric rod. The dielectric rod transports this surfaces wave from the launcher at the aperture A_g to the end of the rod, aperture A_d (the apertures are shown in Fig. 2).

The larger cylinder of radius a' indicates the approximate bounds of the surface wave, which in theory tend to zero at infinity and the end surfaces to this are A'_g and A'_d. The actual radiating aperture at the end of the rod is A'_d. The total radiation is the vectorial sum of the radiation from the apertures A_g, A_d and $(A'_d - A_d)$. The following assumptions have been made:

(i) Since the dielectric rod is tapered to a point inside the H_{11} metal guide for the purpose of matching (tapering not shown in Fig. 2) the reflection coefficient Γ_T at the free end of the rod, being very small, can also be ignored.

(ii) The metal waveguide at the feed end of the radiator is considered invisible to the aperture radiation.

(iii) The fictitious sources in the free-end aperture are generated entirely from the surface waves guided by the rod.

(iv) Radiation from the mouth of the metal guide A_g is assumed to be into free space[22] and the presence of the dielectric rod is ignored.

(v) Fresnel interaction between the apertures is considered to be negligible irrespective of the distance between A_g and A'_d [29–31].

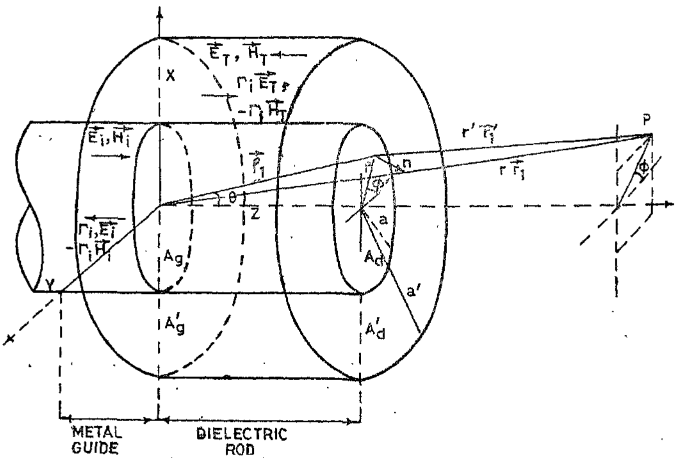

FIG. 2. Coordinate system for calculating the Radiation

3.3. *Radiation Field due to the Dielectric Apertures*

3.3.1. *Field Components*: It is a condition of the vector Kirchoff's formula that the integrands given in eqs. (44 *a* and *b*) must be in the Cartesian form. Hence, with reference to Fig. 2, the field components in the Cartesian coordinate system are given as follows (see Appendix 1). Inside the rod, $\rho \leqslant a$,

$$E_x = B_m \frac{k_{1m}}{2} \left\{ \left(1 + \frac{b_m}{B_m} \frac{\gamma_m}{j\omega\epsilon_1} \right) J_0(k_{1m}\rho) + \left(1 - \frac{b_m}{B_m} \frac{\gamma_m}{j\omega\epsilon_1} \right) J_2(k_{1m}\rho) \cos 2\phi' \right\} e^{-\gamma_m z}$$

$$E_y = B_m \frac{k_{1m}}{2} \left(1 - \frac{b_m}{B_m} \frac{\gamma_m}{j\omega\epsilon_1} \right) J_2(k_{1m}\rho) \sin 2\phi' \, e^{-\gamma_m z}$$

$$H_x = B_m \frac{k_{1m}}{2} \left(\frac{b_m}{B_m} - \frac{\gamma_m}{j\omega\mu_1} \right) J_2(k_{1m}\rho) \sin 2\phi' \, e^{-\gamma_m z}$$

$$H_y = B_m \frac{k_{1m}}{2} \left\{ \left(\frac{b_m}{B_m} + \frac{\gamma_m}{j\omega\mu_1} \right) J_0(k_{1m}\rho) - \left(\frac{b_m}{B_m} - \frac{\gamma_m}{j\omega\mu_1} \right) J_2(k_{1m}\rho) \cos 2\phi' \right\} e^{-\gamma_m z} \tag{45}$$

3.3.2. *Field due to the aperture* A_d.:L^E and L^H are obtained by eqs. (44 *a* and *b*) and substituted into eqs. (43 *a* and *b*) to obtain the radiation field at a distant point due to the aperture A_d.

$$L_x^E = \int_{\rho=0}^{a} \int_{\phi'=0}^{2\pi} E_x \exp\left[jk\rho \sin\theta \cos(\phi' - \phi) + jkL\cos\theta\right] \rho \, d\rho \, d\phi'.$$

Substituting for E_x from eq. (45),

$$L_x^E = \int_{\rho=0}^{a} \int_{\phi'=0}^{2\pi} \left\{ B_m \frac{k_{1m}}{2} \left(1 + \frac{b_m}{B_m} \frac{\gamma_m}{j\omega\epsilon_1} \right) J_0(k_{1m}\rho) \times \exp\left[jk\rho \sin\theta \cos(\phi' - \phi) + jkL\cos\theta - j\beta_m L \right\} \rho \, d\rho \, d\phi'$$

$$+ \int_{\rho=0}^{a} \int_{\phi'=0}^{2\pi} \left\{ B_m \frac{k_{1m}}{2} \left(1 - \frac{b_m}{B_m} \frac{\gamma_m}{j\omega\epsilon_1} \right) J_2(k_{1m}\rho) \cos 2\phi' \cdots \times \exp\left[jk\rho \sin\theta \cos(\phi' - \phi) + jkL\cos\theta - j\beta_m L\right] \right\} \rho \, d\rho \, d\phi'$$

The integral with respect to ϕ' is simplified and the following expression in the $\phi = 0°$ plane is then obtained:

$$L_x^E = \Big[B_m \frac{k_{1m}}{2} \left(1 + \frac{b_m}{B_m} \frac{\gamma_m}{j\omega\epsilon_1}\right) 2\pi \int_{\rho=0}^{a} \rho\, J_0(k_{1m}\rho)$$

$$\times J_0(k\rho \sin\theta)\, d\rho$$

$$- B_m \frac{k_{1m}}{2} \left(1 - \frac{b_m}{B_m} \frac{\gamma_m}{j\omega\epsilon_1}\right) 2\pi \int_{\rho=0}^{a} \rho J_2(k_{1m}\rho) J_2(k\rho \sin\theta) d\rho\Big]$$

$$\times \exp[-j(\beta_m - k\cos\theta) L]$$

or

$$L_x^E = B_m\, k_{1m}\, \pi \Big[\left(1 + \frac{b_m}{B_m} \frac{\gamma_m}{j\omega\epsilon_1}\right) I_{od} - \left(1 - \frac{b_m}{B_m} \frac{\gamma_m}{j\omega\epsilon_1}\right) I_{2d}\Big]$$

$$\times \exp[-j(\beta_m - k\cos\theta) L] \tag{46}$$

where

$$I_{od} = \int_{\rho=0}^{a} \rho\, J_0(k_{1m}\rho)\, J_0(k\rho\sin\theta)\, d\rho \tag{46 a}$$

$$I_{2d} = \int_{\rho=0}^{a} \rho\, J_2(k_{1m}\rho)\, J_2(k\rho\sin\theta)\, d\rho \tag{46 b}$$

Similarly,

$$L_y^E = \int_{\rho=0}^{a} \int_{\phi'=0}^{2\pi} E_y \exp[jk\rho\sin\theta\cos(\phi' - \phi) + jkL\cos\theta]\, \rho\, d\rho\, d\phi'$$

which, after substituting for E_y from eq. (45), in the $\phi = 0°$ plane reduces to

$$L_y^E = \int_{\rho=0}^{a} \int_{\phi'=0}^{2\pi} B_m \frac{k_{1m}}{2} \left(1 - \frac{b_m}{B_m} \frac{\gamma_m}{j\omega\epsilon_1}\right) J_2(k_{1m}\rho) \sin 2\phi'$$

$$\times \exp[jk\rho\sin\theta\cos\phi' + jkL\cos\theta - j\beta_m L]\, \rho\, d\rho\, d\phi'$$

$$= 0 \text{ (after simplification).} \tag{47}$$

Again

$$L_x^E = \int_{\rho=0}^{a} \int_{\phi'=0}^{2\pi} H_x \exp[jk\rho\sin\theta\cos(\phi' - \phi) + jkz\cos\theta]\, \rho\, d\rho\, d\phi'$$

which in the $\phi = 0°$ plane becomes

$$L_x^H = \int_{\rho=0}^{a} \int_{\phi'=0}^{2\pi} B_m \frac{k_{1m}}{2} \left(\frac{b_m}{B_m} - \frac{\gamma_m}{j\omega\epsilon_1}\right) J_2(k_{1m}\rho) \sin 2\phi'$$

$$\times \exp[jk\rho \sin\theta \cos(\phi' - \phi) + jkz\cos\theta - j\beta_m L]\, \rho\, d\rho\, d\phi'$$

$$= 0 \text{ on simplifying.} \tag{48}$$

Also

$$L_y^H = \int_{\rho=0}^{a} \int_{\phi'=0}^{2\pi} H_y \exp[jk\rho\sin\theta\cos(\phi' - \phi) + jkz\cos\theta]\, \rho\, d\rho\, d\phi'$$

which in the $\phi = 0°$ plane becomes

$$L_y^H = \int_{\rho=0}^{a} \int_{\phi'=0}^{2\pi} \left\{B_m \frac{k_{1m}}{2} \left(\frac{b_m}{B_m} + \frac{\gamma_m}{j\omega\mu_1}\right) J_0(k_{1m}\rho)\right.$$

$$\left. \times \exp[jk\rho\sin\theta\cos(\phi' - \phi') + jkz\cos\theta - j\beta_m L]\right\} \rho\, d\rho\, d\phi'$$

$$- \int_{\rho=0}^{a} \int_{\phi'=0}^{2\pi} \left\{B_m \frac{k_{1m}}{2} \left(\frac{b_m}{B_m} - \frac{\gamma_m}{j\omega\mu_1}\right) J_2(k_{1m}\rho)\cos 2\phi'\right.$$

$$\times \exp[jk\rho\sin\theta\cos(\phi' - \phi) + jkz\cos\theta - j\beta_m L]\, \rho d\rho\, d\phi'.$$

On simplifying the integral with respect to ϕ'

$$L_y^H = B_m\, k_{1m}\, \pi \left[\left(\frac{b_m}{B_m} + \frac{\gamma_m}{j\omega\mu_1}\right) I_{od} + \left(\frac{b_m}{B_m} - \frac{\gamma_m}{j\omega\mu_1}\right) I_{2d}\right]$$

$$\times \exp[-j(\beta_m - k\cos\theta) L]. \tag{49}$$

In the $\phi = 0°$ plane eqs. (43 *a* and 43 *b*) reduce to

$$E_{P\theta} = kL_y^H + \eta_0 k\cos\theta\, L_x^H \tag{50 a}$$

$$E_{P\phi} = -kL_x^E\cos\theta + \eta_0 kL_y^H. \tag{50 b}$$

Substituting for L_x^E, L_y^E, L_x^H and L_y^H from eqs. (46–49), eqs. (50 *a* and *b*) become

$$E_{P\theta d}] = 0 \tag{51}$$

$$E_{P\phi d} = -B_m\, kk_{1m}\, \pi \Big[(I_{od} + I_{2d}) \left(\frac{b_m}{B_m} \frac{\gamma_m}{j\omega\epsilon_1} \cos\theta - \frac{b_m}{B_m} \eta_0 \right)$$

$$+ (I_{od} - I_{2d}) \left(\cos\theta + \frac{\gamma_m}{j\omega\mu_1} \right) \eta_0 \Big]$$

$$+ \exp\, [-j(\beta_m - k\cos\theta)\, L]. \tag{52}$$

The integrals I_{od} and I_{2d} are of the Lommel integral type and their values work out to be

$$I_{od} = \frac{a}{k^2{}_{1m} - k^2 \sin^2\theta} [k_{1m}\, J_0(ka\sin\theta) J_1(k_{1m}a)$$

$$- k\sin\theta\, J_0(k_{1m}a)\, J_1(ka\sin\theta)] \tag{52 a}$$

and

$$I_{2d} = \frac{a}{k^2{}_{1m} - k^2 \sin^2\theta} [k\sin\theta J_1(ka\sin\theta)\, J_2(k_{1m}a)$$

$$- k_{1m}\, J_1(k_{1m}a)\, J_2(ka\sin\theta)]. \tag{52 b}$$

Therefore the radiation field due to the dielectric aperture A_d for the m-th mode is given by the following expression

$$E_{Pdm} = -\vec{\phi}\, \frac{e^{-jkr}}{r}\, E_{p\phi d}$$

that is,

$$E_{Pdm} = -\vec{\phi}\, \frac{e^{-jkr}}{r} B_1 \begin{Bmatrix} B_m\ (m > 1) \\ B_1\ \ (m = 1) \end{Bmatrix} \frac{k\, k_{1m}\, \pi a}{k^2{}_{1m} - k^2 \sin^2\theta}\, \cdot$$

$$\times \exp\,[-j(\beta_m - k\cos\theta)\, L] \left\{ \left[\frac{b_m}{B_m} \frac{\gamma_m}{j\omega\epsilon_1} \cos\theta - \frac{b_m}{B_m} \eta_0 \right] \right.$$

$$\times [k_{1m}\, J_1(k_{1m}\, a) \{J_0(ka\sin\theta) - J_2(ka\sin\theta)\}$$

$$- k\sin\theta\, J_1(ka\sin\theta) \{J_0(k_{1m}\, a) - J_2(k_{1m}a)\}]$$

$$+ \left[\cos\theta - \frac{\gamma_m}{j\omega\mu_1} \eta_0 \right] [k_{1m}\, J_1(k_{1m}a) \{J_0(ka\sin\theta)$$

$$+ J_2(ka\sin\theta)\} - k\sin\theta\, J_1(ka\sin\theta) \{J_0(k_{1m}a)$$

$$\left. + J_2(k_{1m}a)\}] \right\} \tag{53}$$

Since the field E_{pdm} depends on the amplitude of the mode, the diameter $d(=2a)$ and length L of the dielectric rod it may be expressed in a functional form as follows:

$$E_{pdm} = \vec{\phi}\,\frac{e^{-jkr}}{r}\,B_1\,g_m(A)\,g_{1m}(d)\,g_m(d, L) \tag{54}$$

where

$$g_m(A) = \frac{B_m\,(m>1)}{B_1\,(m=1)} \tag{54 a}$$

$$g_m(d, L) = \exp\left[-j\,(\beta_m - k\cos\theta)\,L\right] \tag{54 b}$$

$$\begin{aligned} g_{1m}(d) = &-\frac{kk_{1m}\,\pi d}{2\,(k^2_{1m}-k^2\sin^2\theta)}\left\{\left[\frac{b_m}{B_m}\frac{\gamma_m}{j\omega\epsilon_1}\cos\theta - \frac{b_m}{B_m}\,\eta_0\right]\right.\\ &\times\left[k_{1m}\,J_1\left(\frac{k_{1m}d}{2}\right)\left\{J_0\left(\frac{kd\sin\theta}{2}\right) - J_2\left(\frac{kd\sin\theta}{2}\right)\right\}\right.\\ &\left.- k\sin\theta\,J_1\left(\frac{kd\sin\theta}{2}\right)\left\{J_0\,\frac{k_{1m}\,d}{2} - J_2\left(\frac{k_{1m}\,d}{2}\right)\right\}\right]\\ &+\left[\cos\theta - \frac{\gamma_m}{j\omega\mu_1}\,\eta_0\right]\left[k_{1m}\,J_1\left(\frac{k_{1m}\,d}{2}\right)\left\{J_0\left(\frac{kd\sin\theta}{2}\right)\right.\right.\\ &\left.+ J_2\left(\frac{kd}{2}\sin\theta\right)\right\} - k\sin\theta\,J_1\left(\frac{kd}{2}\sin\theta\right)\\ &\left.\left.\times\left\{J_0\left(\frac{k_{1m}\,d}{2}\right) + J_2\left(\frac{k_{1m}d}{2}\right)\right\}\right]\right\} \end{aligned} \tag{54 c}$$

3.3.3. *Field due to the aperture.* $(A'_d - A_d)$: The field due to the aperture $(A'_d - A_d)$ can be obtained by inspection from the expression for the field due to the aperture A_d, eqs. (51 and 52). Hence,

$$E'_{P\theta d} = 0 \tag{55}$$

$$\begin{aligned} E'_{P\phi d} = &-C_m\,kk_{2m}\,\pi\left[(I'_{od} + I'_{2d}\,\frac{c_m}{C_m}\left(\frac{\gamma_m}{j\omega\epsilon_1}\cos\theta - \eta_0\right)\right.\\ &\left.+ (I'_{od} - I'_{2d})\left(\cos\theta - \frac{\gamma_m}{j\omega\mu_1}\,\eta_0\right)\right]\\ &\times(\exp\left[-j\,(\beta_m - k\cos\theta)\,L\right] \end{aligned} \tag{56}$$

where

$$\begin{aligned} I'_{od} = &\frac{a}{k^2_{2m} - k^2\sin^2\theta}\,\left[k_{2m}\,J_0\,(ka\sin\theta)\,H_1^{(1)}\,(k_{2m}a)\right.\\ &\left.- k\sin\theta\,H_0^{(1)}\,(k_{2m}a)\,J_1\,(ka\sin\theta)\right] \end{aligned} \quad (56\ a$$

$$I'_{2d} = \frac{a}{k^2_{2m} - b^2 \sin^2\theta} [k \sin\theta J_1(ka \sin\theta) H_2^{(1)}(k_{2m}a) - k_{2m} H_1^{(1)}(k_{2m}a) J_2(ka \sin\theta)] \tag{56 b}$$

Therefore the radiation field due to the dielectric aperture $(A'_d - A_d)$ for the m-th mode is given by the following expression:

$$E'_{Pdm} = \vec{\phi}\,\frac{e^{-jkr}}{r}\, E'_{Pd\phi}.$$

That is,

$$E'_{Pdm} = -\vec{\phi}\,\frac{e^{-jkr}}{r} B_1 \left\{\frac{B_m\ (m>1)}{B_1\ (m=1)}\right\} \frac{kk_{2m}\,\pi d}{2\,(k^2_{2m} - k^2 \sin^2\theta)} \times \frac{k^2_{1m}}{k^2_{2m}}$$

$$\frac{J_1\left(\frac{k_{1m}\,d}{2}\right)}{H_1^{(1)}\left(\frac{k_{2m}\,d}{2}\right)} \exp[-j(\beta_m - k\cos\theta)\,L].$$

$$\times \left[\frac{\epsilon_0}{\epsilon_1}\frac{b_m}{B_m}\frac{\gamma_m}{j\omega\epsilon_1}\cos\theta\,\eta_0\right]\left[k_{2m} H_1^{(1)}\left(\frac{k_{2m}d}{2}\right)\cdot \times \left\{J_0\left(\frac{kd}{2}\sin\theta\right) - J_2\left(\frac{kd}{2}\sin\theta\right)\right\} - k\sin\theta\, J_1\left(\frac{kd}{2}\sin\theta\right) \times \left\{H_0^{(1)}\left(\frac{k_{2m}d}{2}\right) - H_2^{(1)}\left(\frac{k_{1m}d}{2}\right)\right\}\right] + \left[\cos\theta - \frac{\gamma_m\,\eta_0}{j\omega\mu_1}\right]\cdot \times \left[k_{2m} H_1^{(1)}\left(\frac{k_{2m}\,d}{2}\right)\left\{J_0\left(\frac{kd}{2}\sin\theta\right) + J_2\left(\frac{kd}{2}\sin\theta\right)\right\} - k\sin\theta\, J_1\left(\frac{kd}{2}\sin\theta\right)\left\{H_0^{(1)}\left(\frac{k_{2m}d}{2}\right) + H_2^{(1)}\left(\frac{k_{2m}d}{2}\right)\right\}\right]\Bigg\}. \tag{57}$$

Noting the functional dependence on the amplitude of the mode, the diameter d and length L of the rod, E'_{Pdm} may be expressed as follows:

$$E'_{Pdm} = \vec{\phi}\,\frac{e^{-jkr}}{r} B_1\, g_m(A)\, g_{2m}(d)\, g_m(d, L) \tag{58}$$

where

$$g_m(A) = \frac{B_m\,(m>1)}{B_1\,(m=1)} \tag{58 a}$$

$$g_m(d, L) = \exp[-j(\beta_m - k\cos\theta)L] \tag{58 b}$$

$$g_{2m}(d) = \frac{-k\,k_{2m}\,\pi d}{2(k^2_{2m} - k^2\sin^2\theta)}\,\frac{k^2_{1m}}{k^2_{2m}}\,\frac{J_1\left(\frac{k_{1m}d}{2}\right)}{H_1^{(1)}\left(\frac{k_{2m}d}{2}\right)}$$

$$\times\left\{\left[\frac{\epsilon_0}{\epsilon_1}\frac{\gamma_m}{B_m}\frac{\gamma_m}{j\omega\epsilon_1}\cos\theta - \eta_0\right]\left[k_{2m}\,H_1^{(1)}\left(\frac{k_{2m}\,d}{2}\right)\right.\right.$$

$$\times\left\{J_0\left(\frac{kd}{2}\sin\theta\right) - J_2\left(\frac{kd}{2}\sin\theta\right)\right\} - k\sin\theta\,J_1\left(\frac{kd}{2}\sin\theta\right)$$

$$\left.\times\left\{H_0^{(1)}\left(\frac{k_{2m}d}{2}\right) - H_2^{(1)}\left(\frac{k_{2m}d}{2}\right)\right\}\right]$$

$$+\left[\cos\theta - \frac{\gamma_m}{j\omega\mu_1}\eta_0\right]k_{2m}\,H_1^{(1)}\left(\frac{k_{2m}d}{2}\right)$$

$$\times\left\{J_0\left(\frac{kd}{2}\sin\theta\right) + J_2\left(\frac{kd}{2}\sin\theta\right)\right\}$$

$$-k\sin\theta\,J_1\left(\frac{kd}{2}\sin\theta\right)\left\{H_0^{(1)}\left(\frac{k_{2m}d}{2}\right)\right.$$

$$\left.\left.+H_2^{(1)}\left(\frac{k_{2m}\,d}{2}\right)\right\}\right]\right\}\,. \tag{58 c}$$

3.4. *Radiation Field due to the Metallic Guide Aperture*, $\mathbf{A}_g$

In the $\phi = 0°$ plane, as obtained by James[19] the radiation field due to the metallic guide aperture A_g is given by the following expression:

$$E_{P\theta g} = (1 + \eta_{ag}\cos\theta)\frac{2\pi D}{\sin\theta}J_1(x_3)\,J_1(ka\sin\theta)\sin\phi = 0 \tag{59}$$

$$E_{P\phi g} = -(\cos\theta + \eta_{ag})\frac{2\pi Dka}{\left(1 - \frac{k\sin\theta}{k_g}\right)^2}J_1(x_3)\,J'_1(ka\sin\theta)\cos\phi$$

which reduces to

$$E_{P\phi g} = -(\cos\theta + \eta_{ag})\frac{2\pi Dka}{\left(1 - \frac{k\sin\theta}{k_g}\right)^2}J_1(x_3)\,J'_1(ka\sin\theta) \tag{60}$$

where

$$k = 2\pi/\lambda_0, \qquad x_3 = k_g a = 1\cdot 84, \qquad \eta_{ag} = \lambda_0/\lambda'_g.$$

To evaluate λ'_g, the following expression relating λ'_g to k_g is used:

$$k_g = k(\eta g^2 - \eta ag^2)^{1/2}, \qquad \eta g = (\epsilon_r \mu_r)^{1/2}$$

ϵ_r, μ_r = Relative permittivity and permeability of the rod.

D = Excitation constant.

λ'_g = Wavelength in the metal guide.

To determine the excitation constant D the power flow P_G in the guide is equated to the total power flow in the dielectric rod.

The power flow in the metal guide is given by

$$P_G = \frac{\pi D^2 \eta ag}{4\eta_0} [J_1(x_3)]^2 (x_3^2 - 1) \tag{61}$$

To find D:

$$P_G = P_{tm} = \sum_m P_m$$

where[1]

$$P_{tm} = B_1 B_1^* \sum_m \left[\left\{ \frac{F_m(m>1)}{F_1(m=1)} \right\}^2 \quad \right].$$

Substituting for P_{tm}

$$D = \left[\frac{4\eta_0}{\pi \eta ag (x_3^2 - 1)} \right]^{1/2} J_1(x_3)$$

$$\times \left\{ B_1 B_1^* \sum_m \left[\left\{ \frac{F_m(m>1)}{F_1(m=1)} \right\}^2 P'_m \right] \right\}^{1/2} \tag{62}$$

Hence the field due to the metallic guide aperture is given by the following expression:

$$E_{Pg} = -\vec{\phi} \frac{e^{-jkr}}{r} B_1 (\cos\theta + \eta ag) \frac{2\pi ka}{\left(1 - \frac{k\sin\theta}{kg}\right)^2} [J_1(x_3)]^2$$

$$\times J'_1(ka\sin\theta) \left[\frac{4\eta_0}{\pi \eta ag (x_3^2 - 1)} \right]^{1/2}$$

$$\times \left\{ \sum_m \left[\left\{ \frac{F_m(m>1)}{F_1(m=1} \right\}^2 P'_m \right] \right\}^{1/2}. \tag{63}$$

It may be noted that the excitation constant D depends on the total power flow in the dielectric guide which is the sum of the powers carried by all the modes. Hence the excitation of all the modes has been considered. It may however be emphasised that the radiation from the metallic aperture (*i.e.*, feed-end) is single moded irrespective of the diameter of the dielectric rod, *i.e.*, irrespective of the presence of the higher order modes in the dielectric rod. Hence the radiation field from the dielectric rod alone is considered to be the resultant of the contributions of all the modes present. In a functional form E_{Pg} can be written as follows:

$$E_{Pg} = \vec{\phi}\, \frac{e^{-jkr}}{r} B_1\, g_3(d)\, g(A, d) \tag{64}$$

where

$$g_3(d) = -(\cos\theta + \eta_{ag}) \frac{\pi k d}{\left(1 - \frac{k \sin\theta}{k_g}\right)^2} [J_1(x_3)]^2$$

$$J'_1(ka \sin\theta)\left[\frac{4\eta_0}{\pi\eta_{ag}(x_3^2 - 1)}\right]^{1/2} \tag{64 a}$$

$$g(A, d) = \left\{\sum_m \left[\left\{\frac{F_m(m > 1)}{F_1(m = 1)}\right\}^2 P'_m\right]\right\}^{1/2} \tag{64 b}$$

3.5. *Total Aperture Radiation* (*Dielectric* plus *Metallic Guide Apertures*)

The total aperture radiation at a distant point, P, due to the m-th mode is the sum of the radiation fields, E_{Pdm} and E'_{Pdm} due to the dielectric aperture and the radiation field E_{Pg} due to the metallic aperture. Therefore the total radiation field due to the m-th mode is given by the following relation:

$$E_{Pm} = E_{Pdm} + E'_{Pdm} + E_{Pg}$$

that is,

$$E_{Pm} = \vec{\phi}\, \frac{e^{-jkr}}{r} B_1 [g_m(A)\, g_{1m}(d)\, g_m(d, L)$$

$$+ g_m(A) g_{2m}(d)\, g_m(d, L) + g_3(d)\, g(A, d)]. \tag{65}$$

Hence considering all the modes, the field at a distant point $P(r, \theta, \phi)$ is given by the following expression:

$$E_{Ptm} = \sum_m [E_{Pdm} + E'_{Pdm}] + E_{Pg}.$$

Hence,

$$E_{ptm} = \vec{\phi}\, \frac{e^{-jkr}}{r}\, B_1 \sum_{m} g_m(A)\, g_m(d, L)\, [g_{1m}(d) + g_{2m}(d)]\}$$
$$+ \vec{\phi}\, \frac{e^{-jkr}}{r}\, B_1\, g_3(d)\, g(A, d). \tag{66}$$

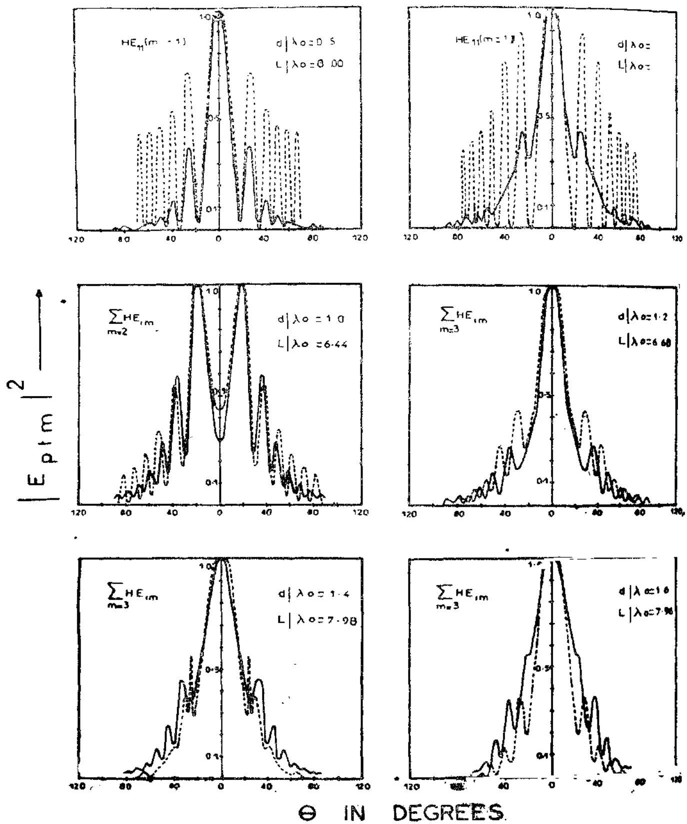

FIG. 3. Comparison of the Theoretical Patterns of the Total Radiation of (1) Single mode rods (2) overmoded rods

Y Axis: —— Dielectric rod as a radiator (Schelkunoff's Equivalence Principle)

– – – – Aperture Theory

It may be worthwhile to mention that both the theories lead to a radiation pattern which is a function of the diameter as well as the length of the rod, although there is some difference in the functional dependence.

4. Discussion

Some of the theoretical radiation patterns computed for both the theories with the aid of an IBM 360 computer are shown in Fig. 3 which exhibits the following interesting points:

(i) The major lobe of an overmoded rod appears to be more irregular as compared to that of a rod which supports only the dipole (HE_{11}) mode.

(ii) The structure of the radiation patterns shows that the number of side lobes is much less for overmoded rods than it is for single moded rods. It is also observed that in general, the side lobes are more suppressed in the case of overmoded rods and their relative intensities compared to the main lobe are much smaller than what it is detected for the single moded rods.

(iii) The appearance of higher order modes seems to affect the beam width of the main lobe. For example, an overmoded rod ($d/\lambda_0 = 1\cdot0$, $L/\lambda_0 = 8\cdot5$) has a beam width of 42° as compared to a beam width **of 28°** for a single moded rod ($d/\lambda_0 = 0\cdot8$, $L/\lambda_0 = 8\cdot35$).

(iv) The side lobe peak appears to shift in the case of overmoded rods. For example the position of the first side lobe for a single moded rod ($d/\lambda_0 = 0\cdot4$, $L/\lambda_0 = 15\cdot84$) is 20° and for an overmoded rod ($d/\lambda_0 = 1\cdot6$, $L/\lambda_0 = 6\cdot19$) is 37°.

(v) As the rod diameter increases the radiation from the free-end of the rod as compared to the surface radiation increases. The gradual increase has been very clearly shown in Fig. 4. This tendency of the rod to behave more like an end-fire radiator is expected since the power flow in the axial direction gets more concentrated inside the rod with increasing diameter. The end and surface radiations, as regards their direction of maxima and their db level as compared to total radiation, have been tabulated in Table II.

As the diameter of the rod is increased, the end radiation becomes more prominent and hence it is more reasonable to consider the dielectric rod as a transmission line feeding energy from the source to the end aperture whose bound is greater than the physical cross-section of the rod because of the nature of the field decay. A study of the radiation pattern obtained by the aperture theory shown in Fig. 3 shows

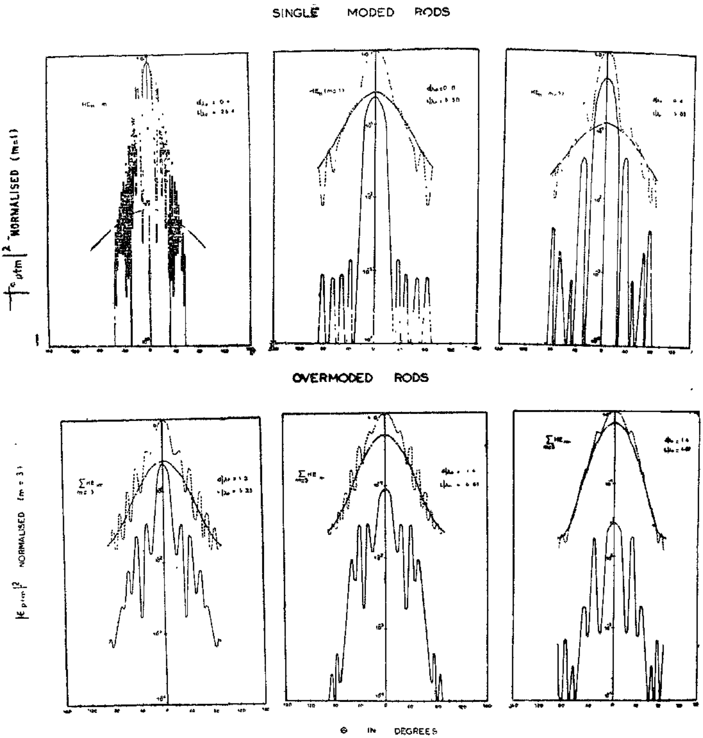

FIG. 4. Comparison of end and surface radiation with respect to the total radiation (end plus surface) for (1) Single moded rods (2) Overmoded rods obtained theoretically using the Schelkunoff's Equivalance Principle

Y - Axis —— Total surface radiation with respect to total end plus surface radiatio
–·— Total end radiation with respect to total end plus surface radiation
- - - - Total end plus surface radiation

TABLE II

Comparison of the end and surface radiation as regards their direction of maximum and their suppression

$d/\lambda_0 = 1 \cdot 6, \quad L/\lambda_0 = 6 \cdot 19, \quad m = 3$

Mode	End radiation/total radiation		Surface radiation/total radiation	
	Direction of maximum	db level	Direction of maximum	db level
HE_{11}	0°	$-2 \cdot 0$	0°	$-16 \cdot 0$
HE_{12}	47°	$-13 \cdot 0$	38°	$-15 \cdot 0$
HE_{13}	0°	$-22 \cdot 0$	0°	$-19 \cdot 0$
$m = 3\ HE_{1m}$	0°	$-1 \cdot 8$	0°	$-13 \cdot 0$

(i) a more regular main lobe than that obtained on the basis of the first theory for the overmoded rods;

(ii) a smaller beam width than that obtained previously;

(iii) a noticeable divergence in the positions of the side lobes of overmoded rods between the two theories although there is a fair agreement in the position of the main lobe;

(iv) a difference in the relative intensity levels of the side lobes between the two theories for both single-moded and overmoded rods.

A comparative study of theory and experiment for overmoded rods leads to the following observations regarding the analysis of the patterns shown in Figs. 5 and 6:

(i) The aperture theory shows a good agreement with experiment regarding the position as well as the beam width of the major lobe where all the modes supported by that particular diameter have been considered; whereas, in the case of the other theory where the main beam is rather irregular the agreement is rather poor. The peak of the main beam however shows agreement with both the theories.

(ii) Though the relative side lobe level differs between experiment and the aperture theory even by considering a combination of the modes

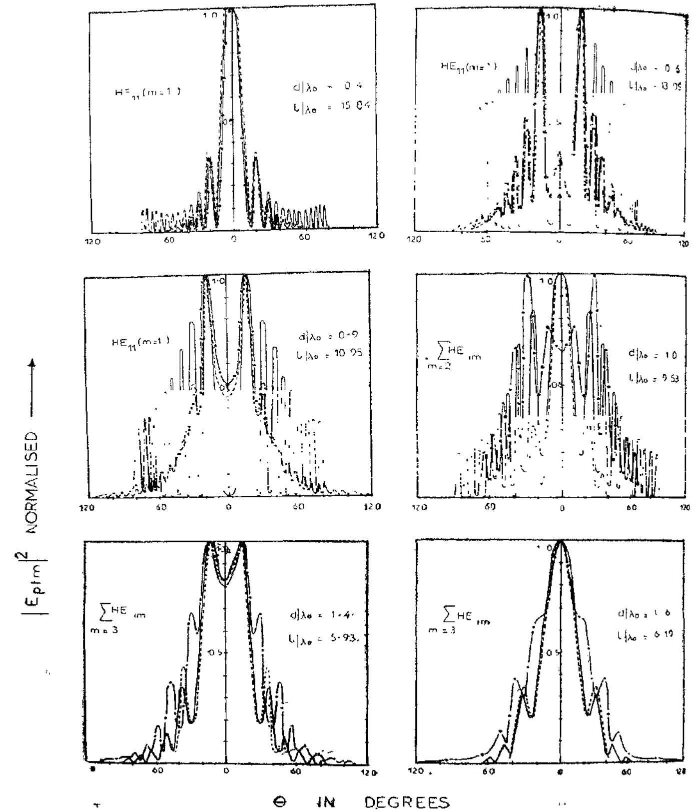

FIG. 5. Comparison of the theoretical patterns of the total radiation obtained by the two theories with the experimental pattern for

(1) Single moded rods

(2) Overmoded rods

Y-Axis —·— Dielectric rod as a radiator (Schelkunoff's Equivalence Principle)

—— Aperture Theory

- - - Experimental

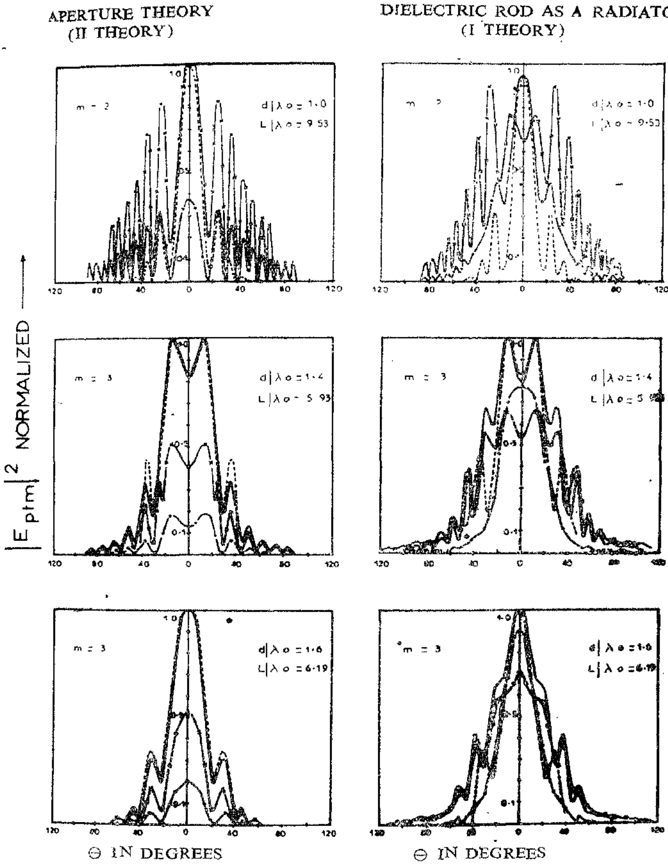

FIG. 6. Comparison of the experimental pattern with the theoretical pattern of the combined modes $(\sum_{m=1,2,3} |E_{ptm}|^2)$ of overmoded rods obtained by the two theories.

Y-Axis: —·— HE_{11}; —×— $HE_{11} + HE_{12}$; ——— $HE_{11} + HE_{12} + HE_{13}$ } Theoretical; - - - - - Experimental there is a very good agreement as regards the position of the minor lobes. The minima do not show nulls but the same is observed for the experimental patterns also. In the case of the first theory, neither the positions of the minor lobes nor the relative side-lobe levels agree very well.

(iii) In Fig. 6 the radiation patterns have been computed by adding the modes one at a time. It is seen that the experimental pattern agrees best with the pattern obtained by combining all the modes. For example, the radiation pattern of the dielectric rod ($d/\lambda_0 = 1 \cdot 4$) shows that the HE_{11} mode has a maximum in the axial direction whereas the pattern for the combined modes, which agrees with the experimental pattern, shows a dip along the axis and a peak of the main beam in the direction $\theta = 14°$. This is probably due to the effect of the higher order modes. Hence the existence of higher order modes is established (see also ref. 2).

It is worthwhile to compare the radiation pattern of single moded rods ($d/\lambda_0 < 1 \cdot 0$) which support only the HE_{11} mode with the two theories. Fig. 5 shows that

(i) there is a very good agreement between the experimental pattern and the patterns obtained by the first theory;

(ii) the position of the main lobe and its beam width obtained experimentally show very good agreement with the theoretical pattern obtained by both the theories;

(iii) though the positions of the side lobes as regards their maxima and minima show agreement between the two theories and experiment, the relative intensities of the side lobes agree better with the first theory.

From the above discussion on the radiation patterns of single-moded and overmoded rods the validity and limitations of the two theories are obvious. Further, observations with the help of Table III can be made where the experimental results regarding the first side-lobe level with respect to the major lobe level and its position for some values of d/λ_0 and L/λ_0 have been compared with that obtained by the two theories.

From Table II and Fig. 6 it is interesting to note that for rods of diameter $\leqslant 0 \cdot 9 \ \lambda_0$, which support only the HE_{11} mode, the agreement of the first theory with experiment even as regards side-lobe suppression, is much better than it is with the second theory. Whereas, for rods which support the higher order modes (diameter $\geqslant 1 \cdot 0 \ \lambda_0$) the second theory

TABLE III

Comparison of the first side-lobe level suppression and its position of maximum obtained by the two theories with experiment

d/λ_0	L/λ_0	Position of first side lobe			Suppression of first side lobe in db		
		Experi-mental	I theory	II theory	Experi-mental	I theory	II theory
0·4	15·84	20°	20°	20°	—4·4	—5·0	—4·733
0·6	13·05	29°	29°	29°	—3·2	—2·9	—0·719
0·9	10·95	28°	28°	30°	—3·0	—3·3	—1·02
1·0	9·53	28°	49°	24°	—5·0	—1·8	—0·787
1·4	5·925	37°	30°	37°	—3·6	—4·3	—4·62
1·6	6·19	31°	37°	31°	—4·7	—4·1	—5·19

agrees better than the first theory. However a noticeable divergence of both the theories with experiment at $d/\lambda_0 = 1\cdot0$ is observed.

The validity and limitations of the two theories may be attributed to the fact that there is a greater concentration of power inside the rod as d/λ_0 is increased and the surface radiation, which is prominent at small diameters, now becomes insignificant in comparison to the end radiation causing the rod to behave more like a transmission line with the free-end aperture radiating. This greater concentration of the energy towards the axis of the rod is also evident from the radial field decay curves[1], which show a faster decay and hence a higher radial decay coefficient for the overmoded rods.

The set Fig. 7 shows clearly the divergence of the experimental results from the first theory even if any of the possible combinations of radiation from the end or from the surface or the total radiation from the surface and end for individual modes or combined modes are considered.

It seems obvious from the above arguments that the rod behaves more and more as an end-fire radiator as d/λ_0 is increased beyond 0·9.

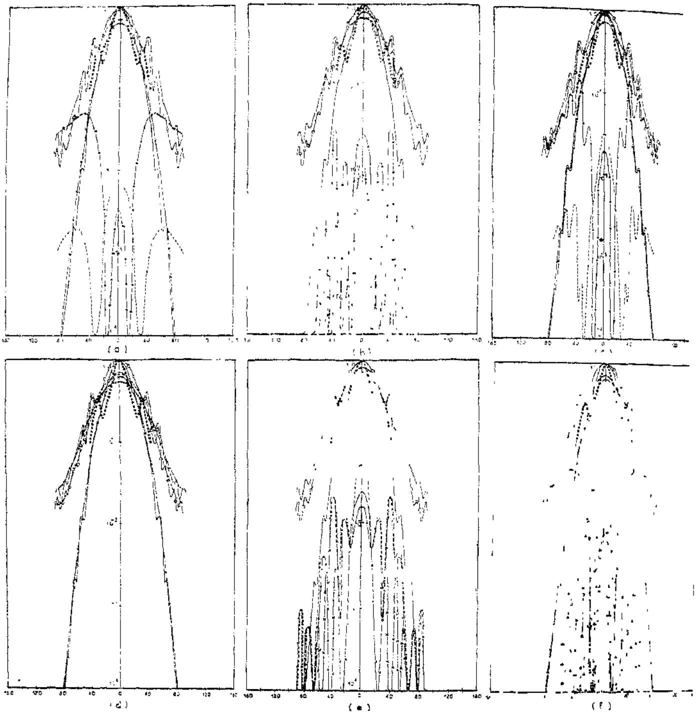

FIG. 7. The end plus surface radiation of the combined modes compared with all the possible combinations of the end and surface radiations of the HE_{11}, HE_{12} and HE_{13} modes of an overmoded rod obtained theoretically by using the Schelkunoff's Equivalence principle $d \mid \lambda o = 1\cdot6$ $L \mid \lambda o = 6\cdot19$

Y-Axis:

(*a*) comparison of experimental pattern (0000) with end plus surface radiation of combined modes and —HE_{11}, - - $HE_{11}+HE_{12}$,—·—$HE_{11}+HE_{12}+HE_{13}$; surface radiation of each mode —x—HE_{11}, —·— HE_{12}, - - - HE_{13}

(*b*) Comparison of experimental pattern (0000) with end plus surface radiation of combined modes and — HE_{11}, - - $HE_{11}+HE_{12}$,—·—$HE_{11}+HE_{12}+HE_{13}$; end radiation of each mode —x—HE_{11}, —·— HE_{12}, - - - HE_{13}

(*c*) Comparison of experimental pattern (0000) with end plus surface radiation of combined modes and —HE_{11}, - - -$HE_{11}+HE_{12}$,—o—$HE_{11}+HE_{12}+HE_{13}$

(*d*) Comparison of experimental pattern (0000) with end plus surface radiation of combined modes and —HE_{11}, - - -HE_{11},+HE_{12}, —o— $HE_{11}\times HE_{12}+HE_{13}$

(*e*) Comparison of experimental pattern (0000) with end plus surface radiation of combined modes and —HE_{11}, - - - $HE_{11}+HE_{12}$,—o—$HE_{11}+HE_{12}+HE_{13}$

(*f*) Comparison of experimental pattern (0000) with end plus surface radiation of combined modes and surface radiation of each mode

end plus surface radiation of each mode —x— HE_{11}, —·— HE_{12} - - - - HE_{13}

end radiation of combined mode —x— HE_{11}, —·— $HE_{11}+HE_{12}$, - -$HE_{11}+HE_{11}+HE_{13}$

surface radiation of combined modes - -x— HE_{11}, —·— $HE_{11}+HE_{12}$, - - - - $HE_{11}+HE_{12}+HE_{13}$

end radiation of each mode and

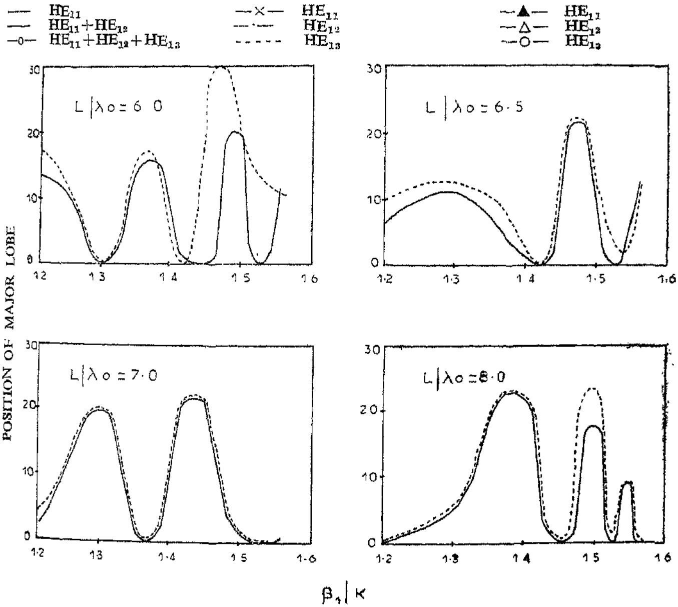

Fig. 8. Position of major lobe vs β_1/k

Y-Axis : —— Aperture Theory

- - - - Dielectric Rod as a Radiator

(Schelkunoff's Equivalence Principle)

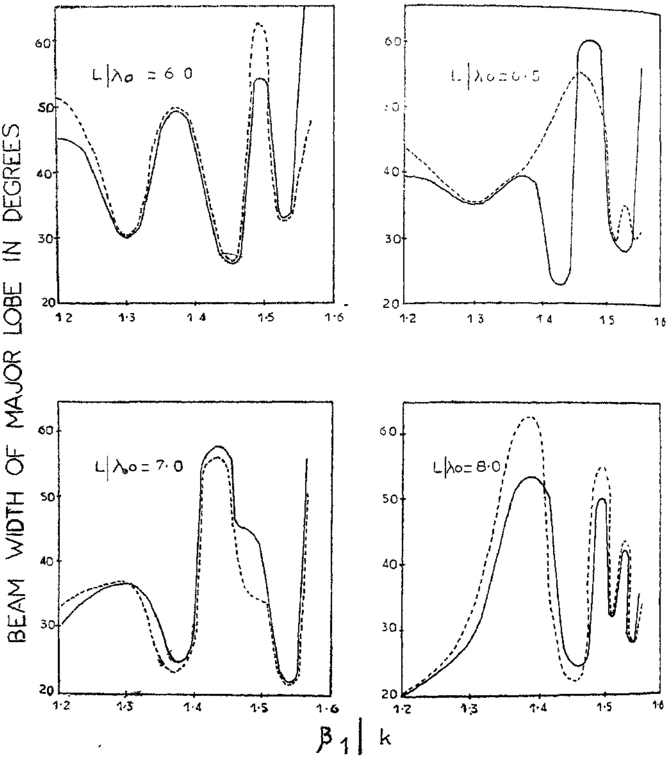

Fig. 9. Beam Width vs β_1/k
Y Axis : - - - - Aperture Theory
—— Dielectric Rod as a Radiator
(Schelkunoff's Equivalence Principle)

The nature of dependence of the far field characteristics such as the position of the major lobe and its beam width can be probably judged from a plot of these factors *vs.* β_1/k where β_1 (ref. 1) is the characteristic of the near field. This has been shown in Figs. 8 and 9.

Appendix

Field components in the Cartesian coordinate system for the HE_{11} mode in the dielectric rod

With reference to Fig. 2 the field components in the cylindrical coordinate system for the HE_{11} mode in the dielectric rod are given by the following relations, $\rho \leqslant a$,

$$E_\rho = B_m \left[\frac{1}{\rho} J_1 (k_{1m}\rho) + \frac{b_m}{B_m} \frac{\gamma_m k_{1m}}{j\omega \epsilon_1} J'_1 (k_{1m}\rho) \right] \cos \phi' \, e^{-\gamma_m z}$$

$$E_{\phi'} = - B_m \left[k_{1m} J'_1 (k_{1m}\rho) + \frac{b_m}{B_m} \frac{1}{\rho} \frac{\gamma_m}{j\omega\epsilon_1} J_1 (k_{1m}\rho) \right] \sin \phi' \, e^{-\gamma_m z}$$

$$H_\rho = B_m \left[\frac{\gamma_m k_{1m}}{j\omega\mu_1} J'_1 (k_{1m}\rho) + \frac{b_m}{B_m} \frac{1}{\rho} J_1 (k_{1m}\rho) \right] \sin \phi \, e^{-\gamma_m z}$$

$$H_{\phi'} = B_m \left[\frac{1}{\rho} \frac{\gamma_m}{j\omega\mu_1} J_1 (k_{1m}\rho) + \frac{b_m}{B_m} k_{1m} J'_1 (k_{1m}\rho) \right] \cos \phi' \, e^{-\gamma_m z} \quad . \tag{A 1}$$

E_x and E_y are related to E_ρ and $E_{\phi'}$ by the expressions

$$E_x = E_\rho \cos \phi' - E_{\phi'} \sin \phi'$$
$$E_y = E_\rho \sin \phi' + E_{\phi'} \cos \phi'. \tag{A 2}$$

Substituting for E_ρ and $E_{\phi'}$ from eq. (A 1) into eq. (A 2) the electric field components in the Cartesian coordinate system are obtained. The magnetic field components are determined similarly. These are given in eq. (45).

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