J. Indian Inst. Sci. 64 (A), Apri. 1983, Pp. 113-122

Solidification times of hollow plate-shaped castings in sand moulds—a generalised solution

S. SRIDHARA*, MALUR N. SRINIVASAN AND M. R. SESHADRI
Department of Mechanical Engineering, Indian Institute of Science, Bangalore 560 012, India.

Received on September 25, 1982 ; Revised on January 20, 1983.

Abstract

A modified heat balance method was used to determine the solidification times of hollow plate-shaped castings under one-dimensional heat conduction. Solidifiction time values of LM6 (Al-12% Si) hollow plate-shaped castings computed from solutions developed compare favourably with the corresponding experimentally—determined values. Criterion for the applicability of the model based on relative mould surface area and core surface area has been indicated.

Key words : Solidification, hollow castings, heat conduction, LM6 (Al-12% Si), sand moulds, cores.

1. Introduction

It is well recognised that the knowledge of solidification times of castings would be of immense value to foundrymen in the production of sound castings. While experimental determination of solidification times is often convenient for particular conditions, mathematical solutions can be more general and can cover a much wider range of heat transfer conditions.

Mathematical solutions for solidification times of castings are available mainly for solid castings. Most commercial castings, however, have internal cavities formed by placement of cores of different shapes and sizes at desired locations. But even in the simplest case of one-dimensional heat conduction, the solution available for solid castings cannot in general be extended to the hollow castings as the heat extraction rates from the mould and the core can be unequal.

* Present address : Ennore Foundries Ltd., Ennore, Madras 600 057.

It has thus become imperative to consider the heat conduction in hollow castings separately which seems to have attracted little attention in the past. The authors have developed mathematical solutions to determine the solidification times of hollow plate-shaped and cylindrical castings in sand moulds, after analysing the thermal behaviour of the moulds and cores involved, the details of which have been provided elsewhere[1]. The solutions for the solidification times of hollow plate castings are presented and discussed in this paper.

The conventional heat balance method[2-8] has not been applied to determine the solidification time of hollow castings probably due to non-availability of solution to the outward solidification around cores of finite thickness and also the limitations of such an approach to track the movement of solid-liquid interfaces. Lack of generality in solutions and non-inclusion of sensible heat of solidified layer in the absence of knowledge on temperature profile in the solidified region have been the other limitations.

Other approximate analytical methods like integral method, variational technique, perturbation method, embedding technique as also numerical methods have been used to linearize the nonlinear boundary value problems of solidification in general. However, these approaches, often very lengthy and involved, may not be warranted when determination of solidification time is the main objective and not the exact temperature distribution in the solidified region.

While it may not be exactly representative of the actual situation, the assumption of a linear temperature profile in the solidified region has yielded satisfactory results in solidification problems studied earlier[2-11]. Likewise the assumption of a constant metal-sand interface temperature has also led to satisfactory results[6-7].

The only systematic attempt towards the determination of solidification time of hollow plate-shaped castings earlier was by Sciama[12] who used a numerical method. However, his calculations have been very limited due to stability problems of numerical calculations due to long solidification times of Al-13% Si plates and experimental verification of the results is needed.

A detailed account of literature on these aspects is available[1] and will be communicated separately. It is clear, however, that a general solution needs to be developed for better understanding of the solidification of hollow castings and to obtain useful data.

2. Methodology

A modified heat balance method with linear temperature profile in the solidified region has been employed to obtain the required solutions. The principle is to determine the rates of heat extraction by the mould and the core separately and to establish the movement of the solid-liquid interfaces from both sides by considering an overall heat balance The temperature profile was assumed to be linear from solidification temperature to an invariant mould-metal or core-metal interface temperature. A corresponding differential release of sensible heat was incorporated into heat balance equation. This eliminated the need for consideration of a nonlinear partial differential equation which arises due to the location of the solid-liquid interfaces being unknown *a priori* when the heat conduction in metal is considered independently. The present method facilitated the determination of solidification time at the instant at which the solid-liquid interfaces from the mould and the core sides merge.

Considering the scheme shown in fig. 1, half the region of hollow plate extending from a to b in the positive X-direction is taken for analysis, owing to geometric symmetry. The initial and boundary conditions assumed are

$$u = u_0 \qquad a \lesssim x \lesssim b,\ t = 0$$

$$u = u_i \qquad x = a,\ t > 0$$

$$u = u_i \qquad x = b,\ t > 0$$

$$u = u_0 \qquad x \to \infty.$$

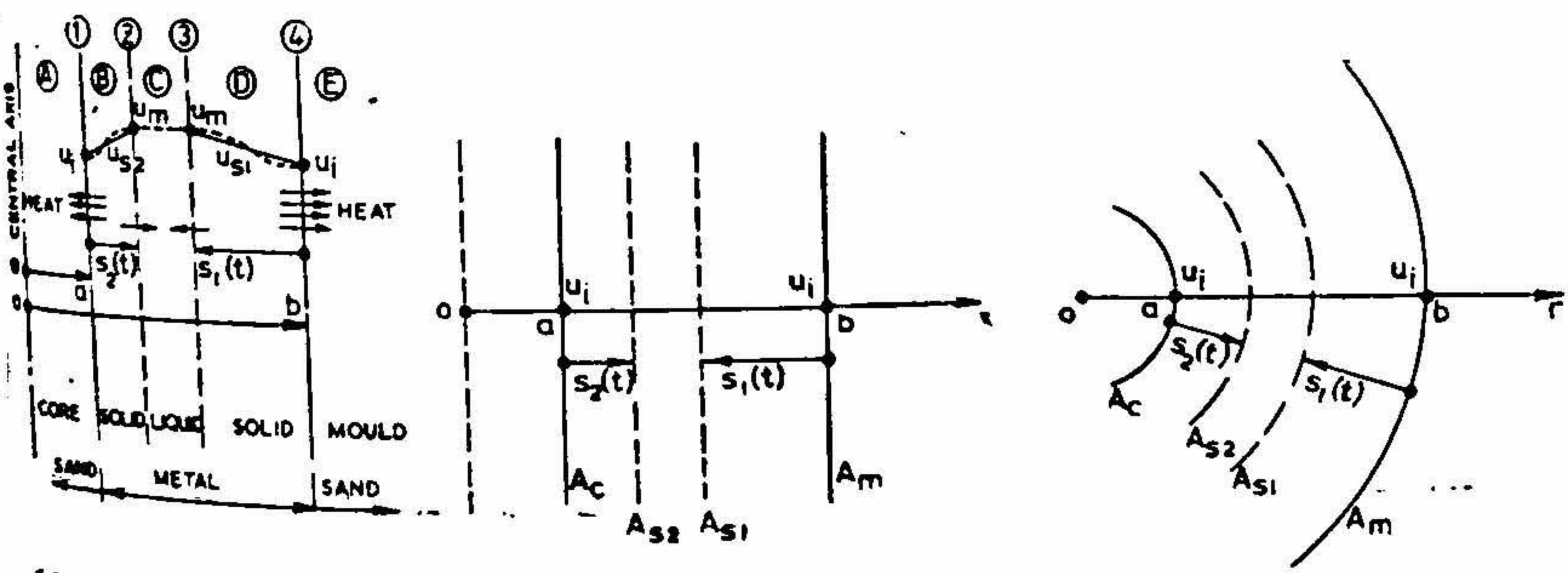
The temperature gradient within the liquid metal is neglected and the superheat is assumed to be liberated at u_m so that it can be merged with the latent heat L.

The governing heat balance equation for the solidification from mould side is

$$L' \rho A_{s1} \frac{ds_1}{dt} + \frac{dH_{s1}}{dt} = - A_m K_1 \frac{\partial u_1}{\partial x}\bigg|_{x = b} \tag{1}$$

The term $\frac{\partial u_1}{\partial x}\Big|_{x = b}$ is evaluated from the well-known temperature distribution equation in plane moulds

$$u_1 = u_0 + (u_i = u_0) \operatorname{erfc} \frac{x - b}{2\sqrt{a_1 t}}\,. \tag{2}$$



(a) General (b) Hollow plate (c) Hollow cylinders

FIG. 1. Solidification of hollow castings.

Simplification of equation (1) after introducing appropriate dimensionless terms leads to the equation

$$K^*\left(L^* + \frac{1}{2}\right) \int_0^{\tau_1} d\xi_1 = \frac{u_i^*}{\sqrt{\pi/a^*}} \int_0^{\tau_1} \frac{d\tau_1}{\sqrt{\tau_1}} \tag{3}$$

integration of which in turn gives, since $\xi_1\ (0) = 0$,

$$\xi_1\ (\tau_1) = \frac{2u_1^* \sqrt{a^*/\pi}}{K^*\ (L^* + \frac{1}{2})} \sqrt{\tau_1}. \tag{4}$$

Equation (4) is the general solution for movement of solid-liquid interface from plane mould surface and can be plotted for various values of system parameters term

$$\frac{2u_i^* \sqrt{a^*/\pi}}{K^*\ (L^* + \frac{1}{2})}.$$

In fig. 2 is shown the plot corresponding to LM6.

Likewise, the governing heat balance equation for the core side is

$$L^1 \rho A_{s^2} \frac{ds_2}{dt} + \frac{dH_{s^2}}{dt} = A_a K_1 \cdot \frac{\partial u_1}{\partial x}\bigg|_{x = a} \tag{5}$$

simplification and non-dimensionalising which leads to

$$K^*\left(L^* + \frac{1}{2}\right) \cdot \frac{d\xi_2}{d\tau_2} = \frac{\partial u_2^*}{\partial x^*}\bigg|_{x^* = 1}. \tag{6}$$

Unlike the mould, which is assumed to be semi-infinite the core is of finite thickness and thus the well-known error function solution is inapplicable to determine the tempe-

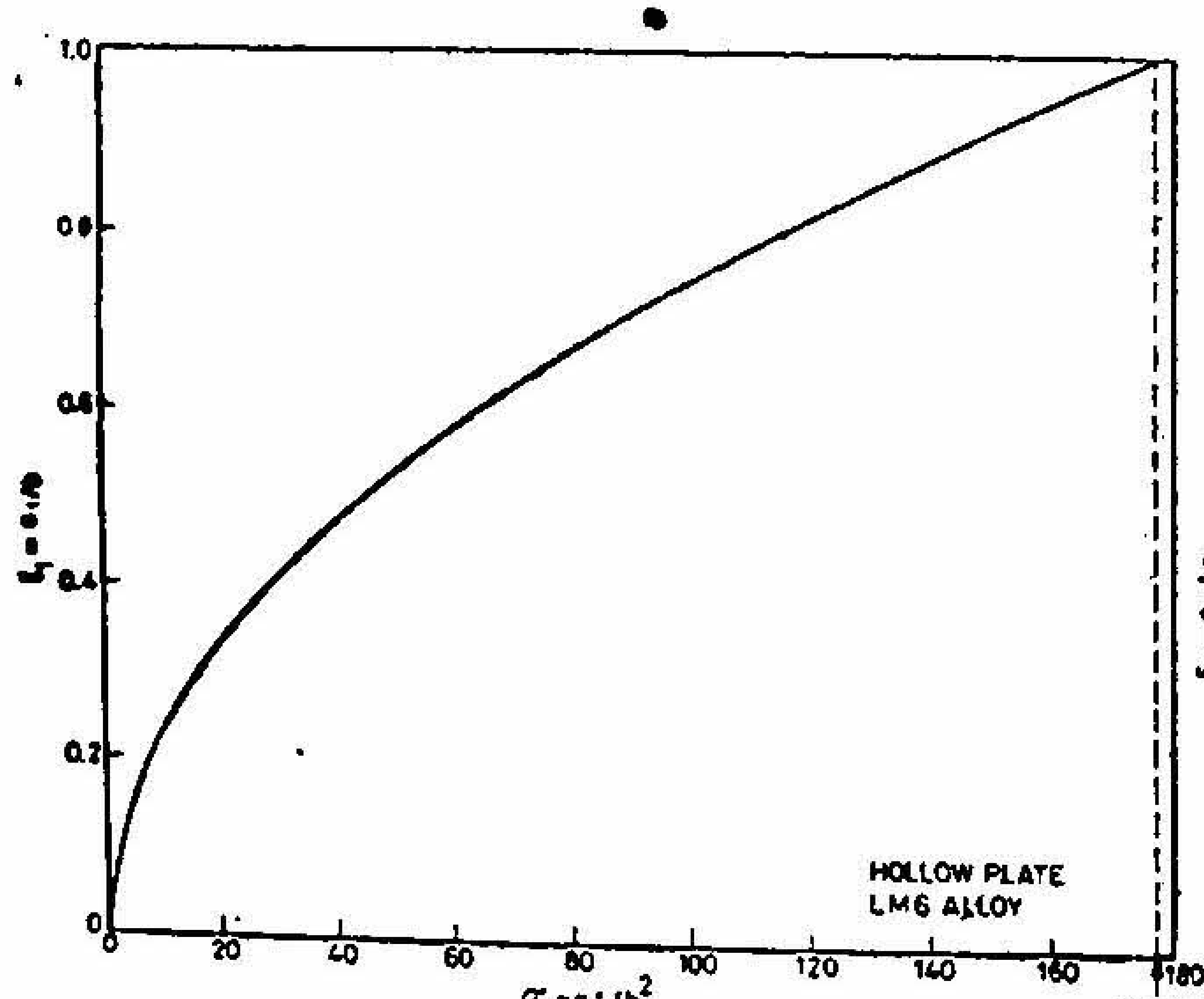


FIG. 2. Movement of solid-liquid interface from mould surface—generalised.

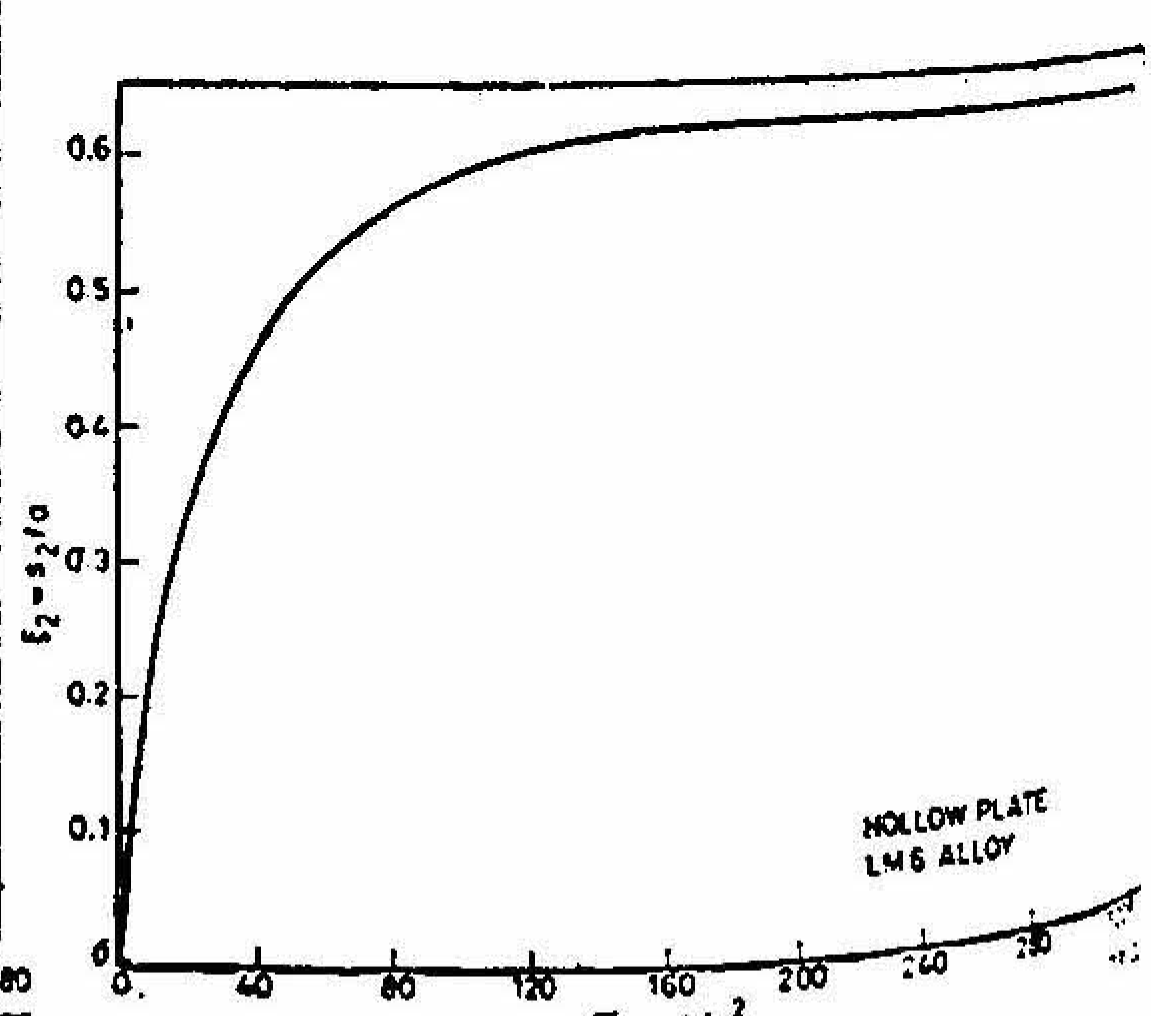


FIG. 3. Movement of solid-liquid interface from core surface—generalised.

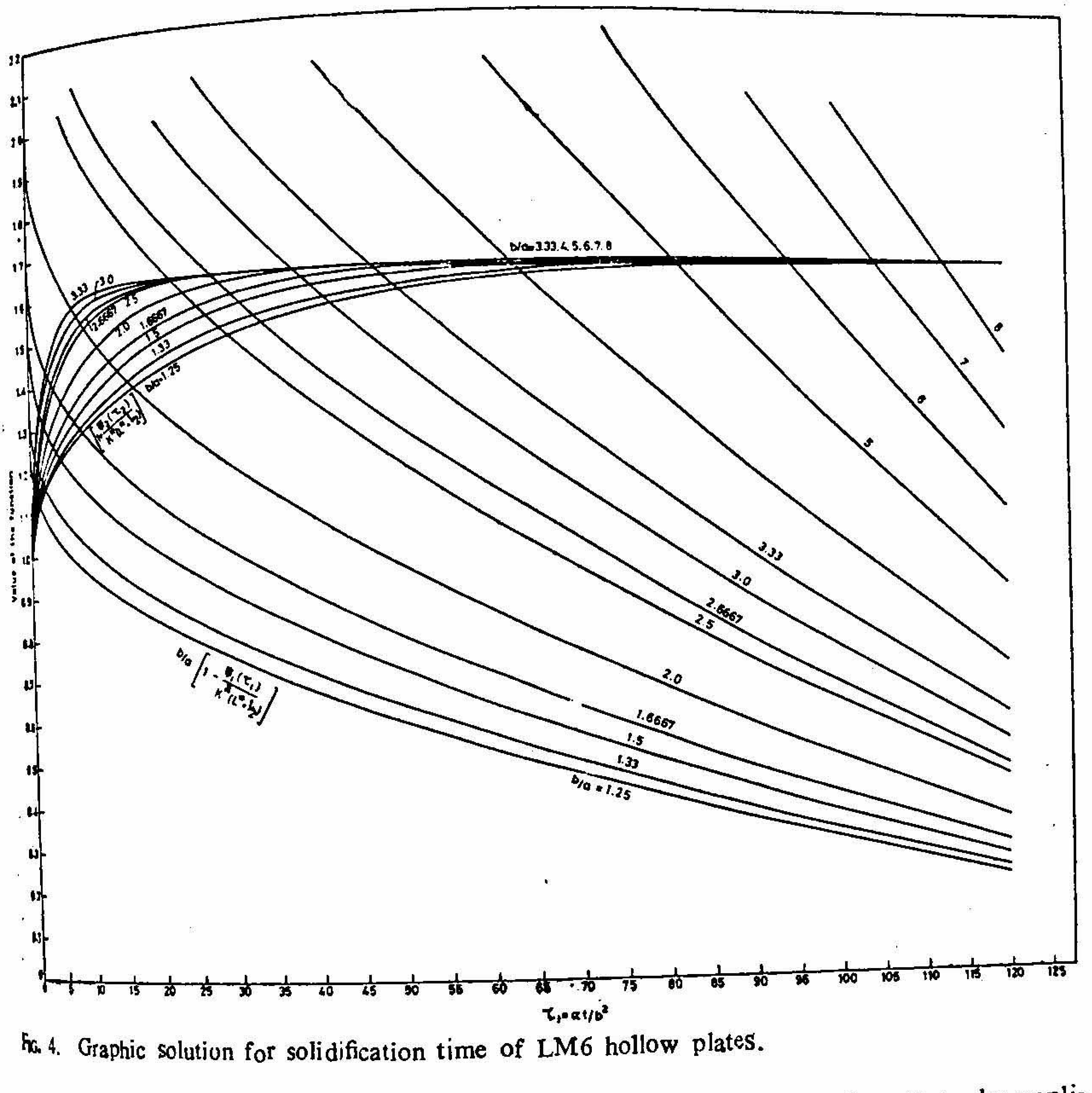


FIG. 4. Graphic solution for solidification time of LM6 hollow plates.

rature distribution in the latter. The following equation has been found[1] to be applicable for the latter :

$$u_2^* = \frac{-4u_i^*}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \exp\left[-(2n+1)^2 \pi^2 \tau_2/4a^*\right] \cdot \cos \frac{2n+1}{2} \pi x^* \tag{7}$$

from which $\frac{\partial u_2^*}{\partial x^*}\Big|_{X^*=1}$ may be obtained and substituting this value in equation (6) and integrating leads to the equation

$$K^*\left(L^* + \frac{1}{2}\right) \cdot \xi_2(\tau_2) := \frac{8u_i^* a^*}{\pi^2}\Bigg[\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} \sin \frac{2n+1}{2}\pi$$

$$- \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} \text{ in } \frac{2n+1}{2} \pi \exp\{-(2n+1)^2 \pi^2 \tau_2/4_{a^*}\}\Big]. \quad (8)$$

Equation (8) is the general solution for the outward solidification around a plane core. Figure 3 represents the solution corresponding to LM6 alloy which indicates the core saturation effect on the advancing solid–liquid interface.

The coupling condition for solidification of hollow plate is

$$\{1 + \xi_2(\tau_2)\} = b/a\ \{1 - \xi_1(\tau_1)\} \quad (9)$$

which follows from $s_1 + s_2 = b - a$. (10)

Equation (9) is the general solution for the solidification time of hollow plate castings. In fig. 4 are plotted the l.h.s. and r.h.s. of equation (9) with respect to τ_1 for different values of b/a, for LM6 alloy. The solidification times, τ_{1s}, obtained from the points of intersection of corresponding curves are plotted as a function of the geometric parameter of b/a in fig. 5, which is the final solution for LM6 hollow plates. In table 1 are given some numerical results along with the corresponding experimentally determined solidification times of hollow plate castings made under conditions closely corresponding to those assumed in the analytical model.

It is observed that the agreement between mathematical and experimental values is quite good in the castings considered ($A_{m1}/A_c \leqslant 1 \cdot 4$). The agreement in the case of larger castings was not good evidently due to pronounced end effects. As expected the agreement was much better when a three-dimensional mathematical model was employed for these castings ; these details will be communicated in a separate paper. Thus, it appears that the heat balance approach can be used even when numerical methods fail due to stability considerations[12].

Table I

Solidification times of LM6 hollow plates

Plate thickness cm	b/a ratio	Solidification time, min		% Deviation w.r.t. experimental time, %
		Calculated	Measured	
1·25	1·25	3·66	3·90	—6·2
1·25	1·33	3·24	3·00	8·0
1·25	1·50	3·29	3·00	9·7
1·875	2·00	7·74	7·00	10·6
2·50	1·50	13·16	13·50	—2·5
2·50	1·67	13·25	13·00	1·9
2·50	2·00	13·75	13·00	5·8

Mean quadratic deviation,

$$\epsilon_s = \sqrt{\frac{\Sigma \epsilon^2}{n}} = 7 \cdot 1\%.$$

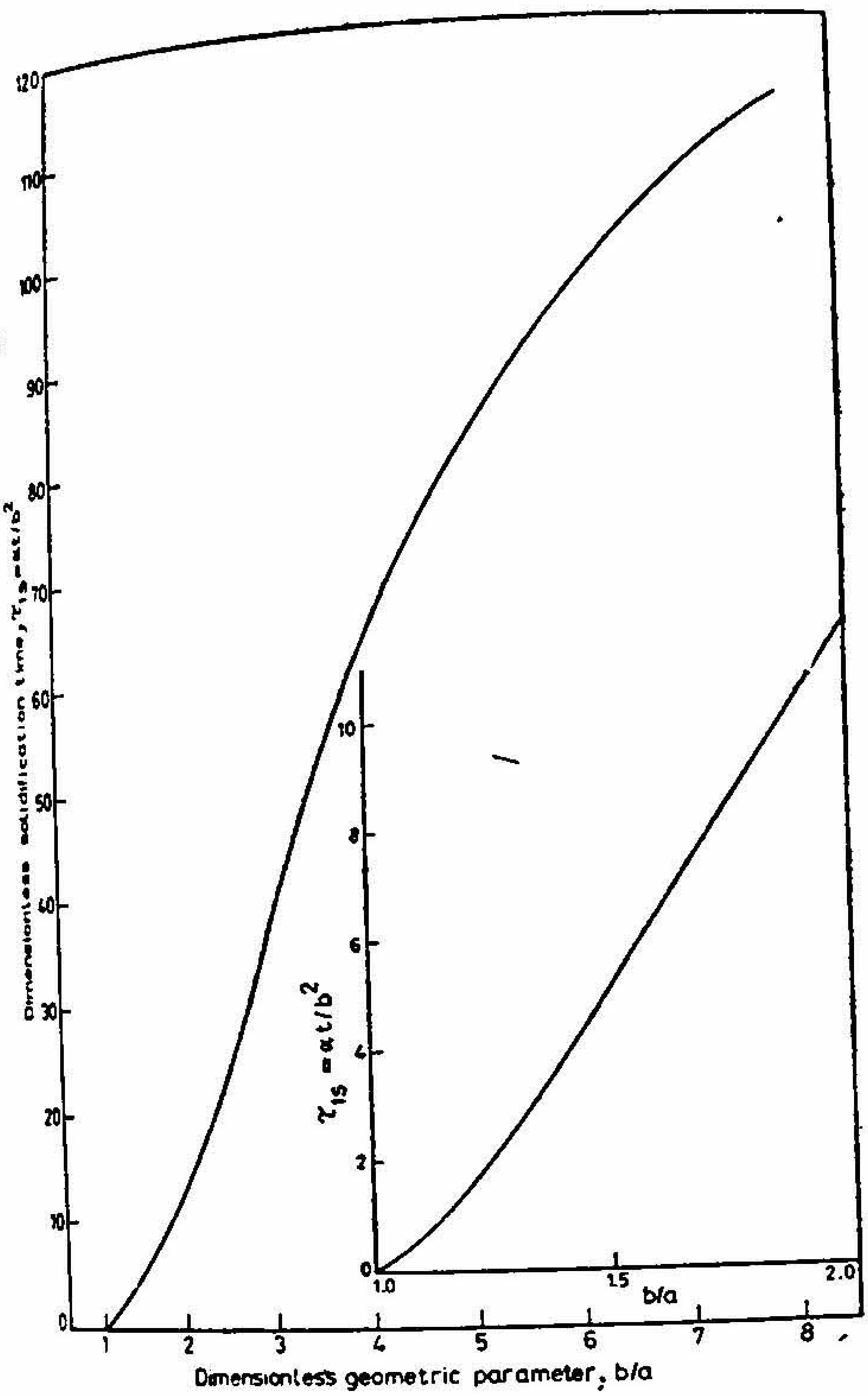


FIG. 5. Solidification time of hollow plates (LM6 alloy)—generalised.

Nomenclature

A_m — major area of mould surface in contact with the metal, cm²

A_{m1} — total area of mould surface in contact with the metal, cm²

A_c — area of the core surface in contact with the metal, cm²

A_{i1} — area of the moving solid–liquid interface from the mould surface, cm²

A_{i2} — area of the moving solid-liquid interface from the core surface, cm²

a — half thickness of the core cm

b — half thickness of the mould, cm

c — specific heat of metal, cal/gm°C

H_{i1} — internal energy of the solidified region from the mould surface, cal.

H_{i2} — internal energy of the solidified region from the core surface, cal,

K — thermal conductivity of metal, cal/cm°C sec,

K_1 — mean thermal conductivity of mould and core material, cal/cm°C sec.
L — latent heat of solidification, cal/gm
L^1 — modified latent heat incorporating superheat, cal/gm.
s — thickness of casting, cm.
s_1 — distance moved by solid–liquid interface from mould surface, cm.
s_2 — distance moved by solid–liquid interface from core surface, cm.
t — time, sec.
u — temperature in solidified metal, °C.
u_1 — Temperature in mould, °C
u_2 — temperature in core, °C
u_i — mould-alloy or core-alloy interface temperature, °C (obtained from a modified Reimann equation)
u_m — mean solidification temperature, °C
u_0 — ambient temperature, °C
x — distance, cm
a — thermal diffusivity of the metal, cm²/sec.
a_1 — mean thermal diffusivity of mould and core meterial, cm²/sec.

Dimensionless terms

Dimensionless distances : $x^* = x/a$ or x/b

$$\xi_1 = S_1/b$$

$$\xi_2 = S_2/a$$

Dimensionless temperatures :

$$u^* = (u_i - u_0)/(u_m - u_i)$$

$$u_1^* = (u_1 - u_0)/(u_i - u_0) \text{ or } (u_1 - u_i)/(u_m - u_i)$$

$$u_2^* = (u_2 - u_0)/(u_i - u_0) \text{ or } (u_2 - u_i)/(u_m - u_i)$$

Dimensionless time :

$$\tau = \frac{a_1 t}{a^2} \text{ or } \frac{a_1 t}{b^2}$$

$$\tau_1 = \frac{at}{b^2}$$

$$\tau_2 = \frac{at}{a^2}$$

Dimensionless properties :

$$a^* = a/a_1$$

$$K^* = K/K_1$$

$$L^* = L'/C\,(U_m - U_i)$$

References

1. SRIDHARA, S. — *Thermal considerations in solidification of hollow castings*, Ph.D Thesis, Indian Institute of Science, Bangalore, India, 1980.
2. CHVORINOV, N. — *Giesserei*, 1940, **27**, 177, 201, 222.
3. CHVORINOV, N. — *Hutnicke Listy*, 1951, **6**, 549, 594 and 1953, **8**, 7, 64.
4. RUDDLE, R. W. AND MINSCHER, A. L. — *J. Inst. Metals*, 1949-50, **76**, 43.
5. HALBART, G. — *Theoire Math. de la Fonderie*, H. Vailliant-Carmanne, Liege, 1945.
6. ADAMS, C. M. AND TAYLOR, H. F. — *Trans. Am. Found. Soc.*, 1957, **65**, 170.
7. BERRY, J. T. KONDIC, V. AND MARTIN, G. — *Trans. Am. Found. Soc.*, 1959, **67**, 449.
8. HALBART, G. — *Neue Giesserei Techn. Wiss. Beifefte*, 1949, No. 1, 57.
9. KERN, J. — *Int. J. Heat Mass Transfer*, 1977, **20**, 467.
10. KERN, J. AND WELLS, G. L. — *Met. Trans.*, 1977, **8B**, 99.
11. HRYCAK, P. — *A.I.Ch. E.J.*, 1963, **9**, 585.
12. SCIAMA, G. — *A.F.S. Cast. Met. Res. J.*, 1973, **9**, 67.
13. CARSLAW, H. S. AND JAEGER, J. C. — *Conduction of heat in solids*, 2nd ed., Oxford Univ. Press, London, 1959.