J. Indian Inst. Sci. 63 (A), Jan. 1981, Pp. 1–19

Aeroacoustic analysis of straight-through mufflers with simple and extended tube expansion chambers

V. B. PANICKER* AND M. L. MUNJAL
Department of Mechanical Engineering, Indian Institute of Science, Bangalore 560 012, India.

Abstract

Among the various methods for evaluation of the performance of exhaust mufflers, the transfer matrix method is most handy. The transfer matrices for various elements of expansion chamber type of mufflers with simple area changes and extended tubes, relating the aeroacoustic variables across them, are derived from the linearised equations of mass continuity, momentum balance and energy relations. The losses in stagnation enthalpy or pressure occurring at these elements are taken into consideration explicitly. The new transfer matrices are used in the theoretical prediction of noise reduction in experimental mufflers and a good agreement between the predicted and experimental values of noise reduction is observed.

Key words: Aeroacoustic analysis, mufflers, tube expansion chambers, noise reduction, transfer matrix method.

1. Introduction

Reactive mufflers are classified into two categories on the basis of the direction of mean flow of exhaust gases through them : (i) straight-through mufflers, and (ii) reversed-flow mufflers.

Some of the elements used in straight-through reflective mufflers are : (*a*) Sudden contraction, (*b*) Extended outlet, (*c*) Sudden expansion and (*d*) Extended inlet. These muffler elements are illustrated in fig. 1.

In the past, several investigators have tried to analyse the performance characteristics of these muffler elements[1–6].

Alfredson and Davies[6] investigated the effect of sudden area changes on acoustic wave propagation assuming one-dimensional quasi-steady flow conditions and applying linearised quasi-stationary equations of energy, continuity and momentum balance.

* Presently with the Department of Mechanical Engineering, N.S.S. College of Engineering, Palghat 678 008, India.

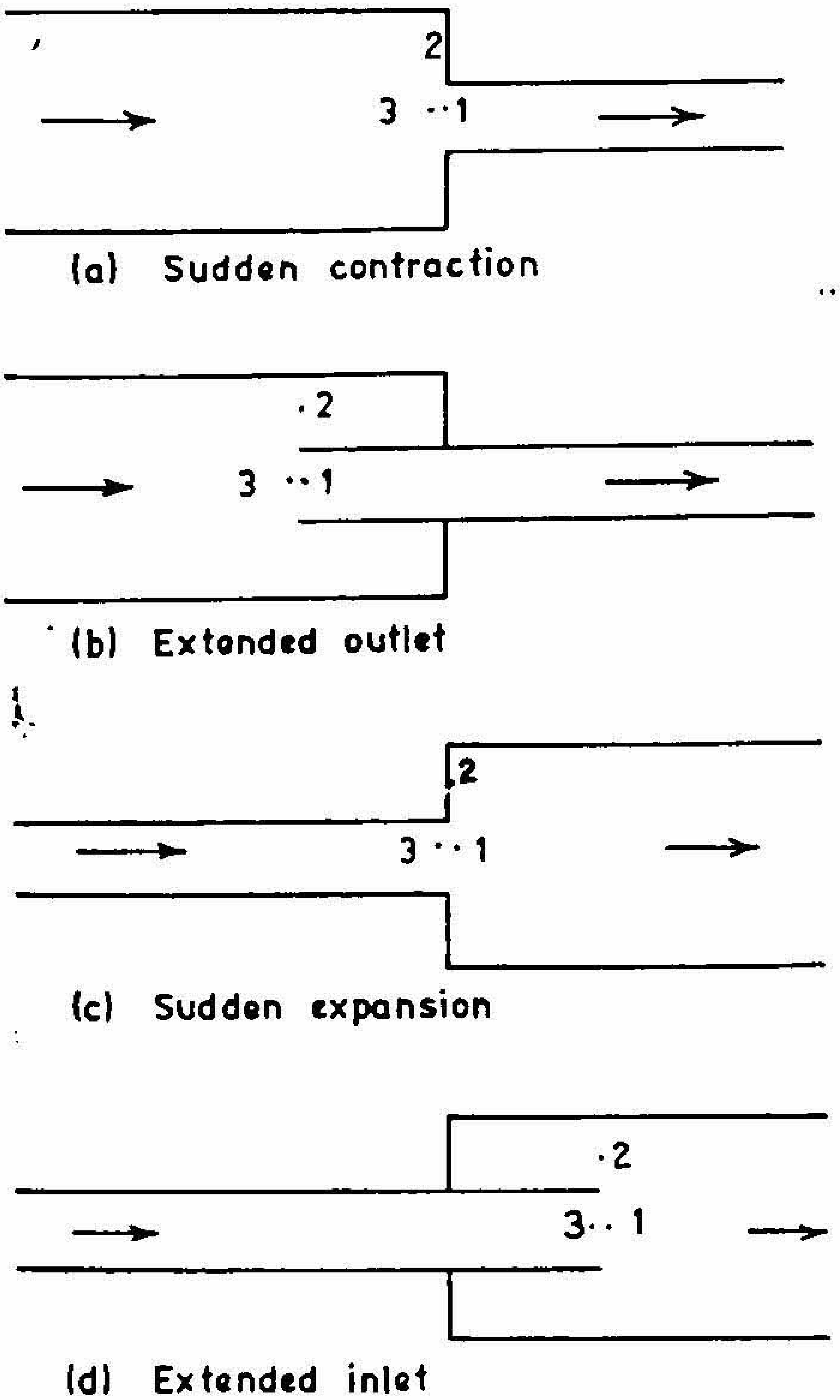

FIG. 1. Muffler elements with abrupt area changes.

They took into consideration the entropy fluctuations downstream of sudden expansion and extended inlet as suggested by Mungur and Gladwell[7].

Munjal[8] proposed the velocity ratio-*cum*-transfer matrix method for the evaluation of the performance of a muffler with mean flow. He suggested that, in the case of mean flow, the classical acoustic variables p and v are to be replaced by what he called aeroacoustic or covective variables p_c and v_c which are the perturbations in total pressure and mass velocity. The acoustic variables p and v are related to the corresponding aeroacoustic variables p_c and v_c by the relations[8]

$$p_c = p + MYv \quad (1)$$

and

$$v_e = v + pM/Y \tag{2}$$

Making use of the formulations by Alfredson and Davies[6] he derived the transfer matrices relating the aeroacoustic state variables across area discontinuities of different muffler elements. Some of those transfer matrices are:

(1) Sudden contraction

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3}$$

(2) Extended outlet

$$\begin{bmatrix} 1 & 0 \\ \dfrac{1}{Z_2} & 1 \end{bmatrix} \tag{4}$$

(3) Sudden expansion

$$\begin{bmatrix} 1 & M_3Y_3(1 - N)^2 \\ 0 & 1 \end{bmatrix} \tag{5}$$

and

(4) Extended inlet

$$\frac{1}{1 + 2NA}\begin{bmatrix} 1 + A & M_3Y_3(1 - 2N + N^2 + N^2 A) \\ \dfrac{1}{Z_2} & 1 + N^2 A \end{bmatrix} \tag{6}$$

where

Z_2 = $- i\, Y_2 \cot kl_2$

Y = c/S

i = $\sqrt{-1}$

c = acoustic velocity

S = area of cross-section

l_2 = length of the annular end-chamber

k = $2\pi f/c$

f = frequency, Hz

A = a parameter $M_3\, Y_3/Z_2$

M = Mach number of meanflow, and

N = S_3/S_1 for sudden expansion and extended inlet

= S_1/S_3 for sudden contraction and extended outlet.

These transfer matrices and the underlying relations[6] imply certain over-simplifying assumptions which are not strictly valid in actual situations. For example, the transfer matrix for sudden contraction presumes that the total pressure and hence the aero-acoustic pressure p_e remains the same on either side of the plane of area discontinuity, which implies that the contraction is loss-free. In actual flow situations, some losses do occur at a sudden contraction. Hence, the transfer matrix (1) has to be modified taking into consideration the effect of flow losses. A similar assumption was made also in the case of an extended outlet. For extended inlet, it was assumed that the pressure in the annular section at the plane of area discontinuity was equal to the static pressure in the upstream tube. In the investigation reported here, these assumptions are avoided and the transfer matrices are derived making use of the information available in the literature regarding the losses in total pressure occurring across simple area changes. The revised transfer matrices are verified for their validity by comparing the noise reduction spectra so predicted with those observed in experimental mufflers as suggested in ref. 9.

2. Extended outlet and sudden contraction

The equations of energy, mass continuity and momentum balance are applied to the sections across an area change relating the conditions at points 1, 2 and 3 as shown in fig. 1 (*b*). The resultant transfer matrix is simplified for application to the element of fig. 1 (*a*).

2.1. *The energy relation*

The one-dimensional equation for energy perturbation[6, 7] leads to the following relation in terms of the aeroacoustic variables[8]

$$p_{e,3} = p_{e,1} - \frac{\delta}{\gamma - 1}. \tag{7}$$

The dissipation parameter δ is, therefore, given by

$$\delta = (\gamma - 1)(p_{e,1} - p_{e,3}). \tag{8}$$

The energy equation can also be written taking into consideration the losses occurring at the area transition between points 3 and 1.

For an incompressible flow ($M_1 < 0{\cdot}25$) as obtaining in automotive mufflers, $M_1^2 \ll 1{\cdot}0$. This relation is made use of throughout this analysis. Expressed in terms of the dynamic head of the fluid in the smaller tube, the losses at the transition would be

$$\text{Loss} = K_c \{\tfrac{1}{2} \rho_0 U_{1,0}^2\} \tag{9}$$

where K_c is the head loss coefficient for contraction (elements (*a*) and (*b*) in fig. 1).

The energy relations without and with acoustic perturbation can be written respectively as

$$p_{2,0} + \tfrac{1}{2}\rho_0 U_{2,0}^2 = p_{1,0} + \tfrac{1}{2}\rho_0 U_{1,0}^2 + K_e\{\tfrac{1}{2}\rho_0 U_{1,0}^2\} \tag{10}$$

and

$$(p_{2,0} + p_2) + \tfrac{1}{2}\rho_0 (U_{2,0} + u_2)^2 = (p_{1,0} + p_1) + \tfrac{1}{2}\rho_0 (U_{1,0} + u_1)^2 + K_e\{\tfrac{1}{2}\rho_0 (U_{1,0} + u_1)^2\}. \tag{11}$$

Subtracting eqn. (10) from eqn. (11), neglecting terms of second and higher order in acoustic variables, p, v and M and using eqn. (1) one gets

$$p_{c,2} = p_{c,1} + K_e M_1 Y_1 v_1. \tag{12}$$

Comparing eqns. (7) and (12) the dissipation parameter can be written as

$$\delta = -(\gamma - 1) K_e M_1 Y_1 v_1. \tag{13}$$

Equations (1) and (2) yield the relation

$$v_1 = \left(v_{c,1} - \frac{p_{c,1} M_1}{Y_1}\right) \Big/ (1 - M_1^2). \tag{14}$$

Since $M_1^2 < 1$, eqn. (14) reduces to

$$v_1 = v_{c,1} - p_{c,1} M_1 / Y_1. \tag{15}$$

Equations (12) and (15) lead to the following energy relation in terms of aeroacoustic variables

$$p_{c,2} = p_{c,1}(1 - K_e M_1^2) + K_e M_1 Y_1 v_{c,1} \tag{16}$$

$$\simeq p_{c,1} + K_e M_1 Y_1 v_{c,1}. \tag{17}$$

2.2. *Continuity equation*

The equations of mass continuity without and with acoustic perturbations, along with the relations[6]

$$\rho_2 = \frac{p_2}{c^2} \tag{18}$$

and

$$\rho_1 = \frac{p_1 + \delta}{c^2}, \tag{19}$$

yields[8]

$$v_{c,2} = v_{c,1} + v_2 + \delta \frac{M_1}{Y_1}. \tag{20}$$

2.3. *Momentum equation*

The momentum equation without and with acoustic perturbation can be rearranged to yield[8]

$$S_3(p_{c,3} + M_3 Y_3 v_{c,3}) - p_2(S_3 - S_1) = S_1(p_{c,1} + M_1 Y_1 v_{c,1} + \delta M_1^2). \tag{21}$$

2.4. *Transfer matrices*

In eqn. (21), the pressure in the annular section at the plane of the area change has been denoted by p_2. The same has been assumed to act on the solid area of the inner tube end (corresponding to its thickness).

p_2 can be expressed as

$$p_2 = v_2 Z_2 \tag{22}$$

where Z_2 is the equivalent impedance of the annular cavity. For rigid end walls

$$Z_2 = -iY_2 \cot kl_2 \tag{23}$$

$$\frac{M_3}{M_1} = \frac{S_1}{S_3} = \frac{Y_3}{Y_1} = N. \tag{24}$$

Assuming $M_1^2 \ll 1$ and using eqns. (13), (15), (19) and (22), eqn. (21) can be simplified as

$$v_{c,3} = \frac{1}{Z_2(1 - N - AN^2)} \{(1 - N) p_{c,1} + Z_2(1 - N + A(K_c - N v_{c,1})\} \tag{25}$$

where

$$A = \frac{M_1 Y_1}{Z_2}. \tag{26}$$

The required transfer matrix can be obained from eqns. (17) and (25) as

$$\begin{bmatrix} p_{c,3} \\ v_{c,3} \end{bmatrix} = \begin{bmatrix} 1 & K_c M_1 Y_1 \\ \dfrac{1 - N}{Z_2(1 - N - AN^2)} & \dfrac{1 - N + A(K_c - N)}{1 - N - AN^2} \end{bmatrix} \begin{bmatrix} p_{c,1} \\ v_{c,1} \end{bmatrix}. \tag{27}$$

If $AN^2 \ll 1$, the transfer matrix in eqn. (27) would become

$$\begin{bmatrix} 1 & K_c M_1 Y_1 \\ \dfrac{1}{Z_2} & 1 + \dfrac{A(K_c - N)}{1 - N} \end{bmatrix}. \tag{28}$$

If the entrance to the outlet tube is somehow made loss-free, *i.e.*, $K_c = 0$, the transfer matrix (28) would further simplify to

$$\begin{bmatrix} 1 & 0 \\ \dfrac{1}{Z_2} & 1 - \dfrac{AN}{I - N} \end{bmatrix}. \tag{29}$$

Comparing the matrices (4), (28) and (29), it is clear that the discrepancy in the first row second column term is due to the assumption made in refs. 6 and 8 that the losses at the extended outlet are zero (that leads to $p_{e,1}$ being equal to $p_{e,3}$). The extra term in the second row second column position of eqn. (29), '$AN/(1-N)$', is the consequence of p_2 in the annular chamber not being equal to $p_{e,3}$ and hence $p_{e,1}$—another assumption made in refs. 6 and 8. In fact, from the momentum equation without acoustic perturbation one gets

$$p_{2,0}(S_3 - S_1) = p_{3,0}S_3 + \rho_0 S_3 U_{3,0}^2 - (p_{1,0}S_1 + \rho_0 S_1 U_{1,0}^2). \tag{30}$$

If p_t is the total pressure, eqn. (30) can be written as

$$p_{2,0}(1 - N) = p_{t,3,0} + \tfrac{1}{2}\rho_0 U_{3,0}^2 - N(p_{t,1,0} + \tfrac{1}{2}\rho_0 U_{1,0}^2). \tag{31}$$

Since $U_{3,0} \approx NU_{1,0}$ eqn. (31) can be re-arranged as

$$p_{2,0} = p_{t,3,0} - \left(1 - \frac{K_e}{1 - N}\right) N(\tfrac{1}{2}\rho_0 U_{1,0}^2). \tag{32}$$

As $K_e = (1 - N)/2$ (as shown below) relation (32) clearly shows that $p_{2,0}$ is less than $p_{t,3,0}$ and not equal to it as assumed by Alfredson and Davies[6] and Munjal[8]. Thus, their assumptions

$$p_{e,1} = p_{e,3} \tag{33}$$

and

$$p_2 = p_{e,3} \tag{34}$$

are incorrect. More correctly

$$p_{e,1} < p_{e,3} \tag{35}$$

and

$$p_2 < p_{e,3}. \tag{36}$$

For the case of sudden contraction (fig. 1 (*a*))

$l_2 = 0$ and hence $Z_2 \to \infty$, and consequently,

$\dfrac{1}{Z_2} \to 0$ and $A \to 0$.

Transfer matrix (28) yields the desired relation for sudden contraction

$$\begin{bmatrix} 1 & K_e M_1 Y_1 \\ 0 & 1 \end{bmatrix}. \tag{37}$$

Comparing this with matrix (3) derived by Munjal[8] from the equations of Alfredson and Davies[6], it can be seen that the first-row second-column element is not equal to zero in the present matrix. Evidently, this means that $p_{e,1} < p_{e,2}$ which the earlier investigators assumed to be equal.

The value of the head loss coefficient K_c for the sudden contraction can be had from several text books (*e.g.*, refs. 10, 11). Figure 2 shows a plot of K_c *vs* area ratio. An approximate empirical expression for K_c is given by

$$K_c \simeq \frac{1-N}{2}. \tag{38}$$

In the absence of any published data, this value of K_c is adopted for the case of extended outlet also because the presence of the annular buffer would not change very much the flow convergence pattern. This is in fact amply borne out by the experimental data plotted in fig. 5 of the following paper[15], where it is shown that even for contraction with reversal, the head loss coefficient happens to be as in eqn. (38) for $N < 0{\cdot}2$, as is generally the case. It may be seen that for stationary medium, where $M = 0$, the transfer matrices (28) and (37) would reduce to eqns. (4) and (3) respectively.

3. Extended inlet and sudden expansion

Applying the same procedure as adopted in the case of extended outlet to an extended inlet (fig. 1 (*d*)), the energy relations can be written as

$$p_{e,2} = p_{e,1} - \frac{\delta}{\gamma - 1} \tag{39}$$

$$\rho_2 = \frac{p_2}{c^2} \tag{40}$$

$$\rho_1 = \frac{p_1 + \delta}{c^2} \tag{41}$$

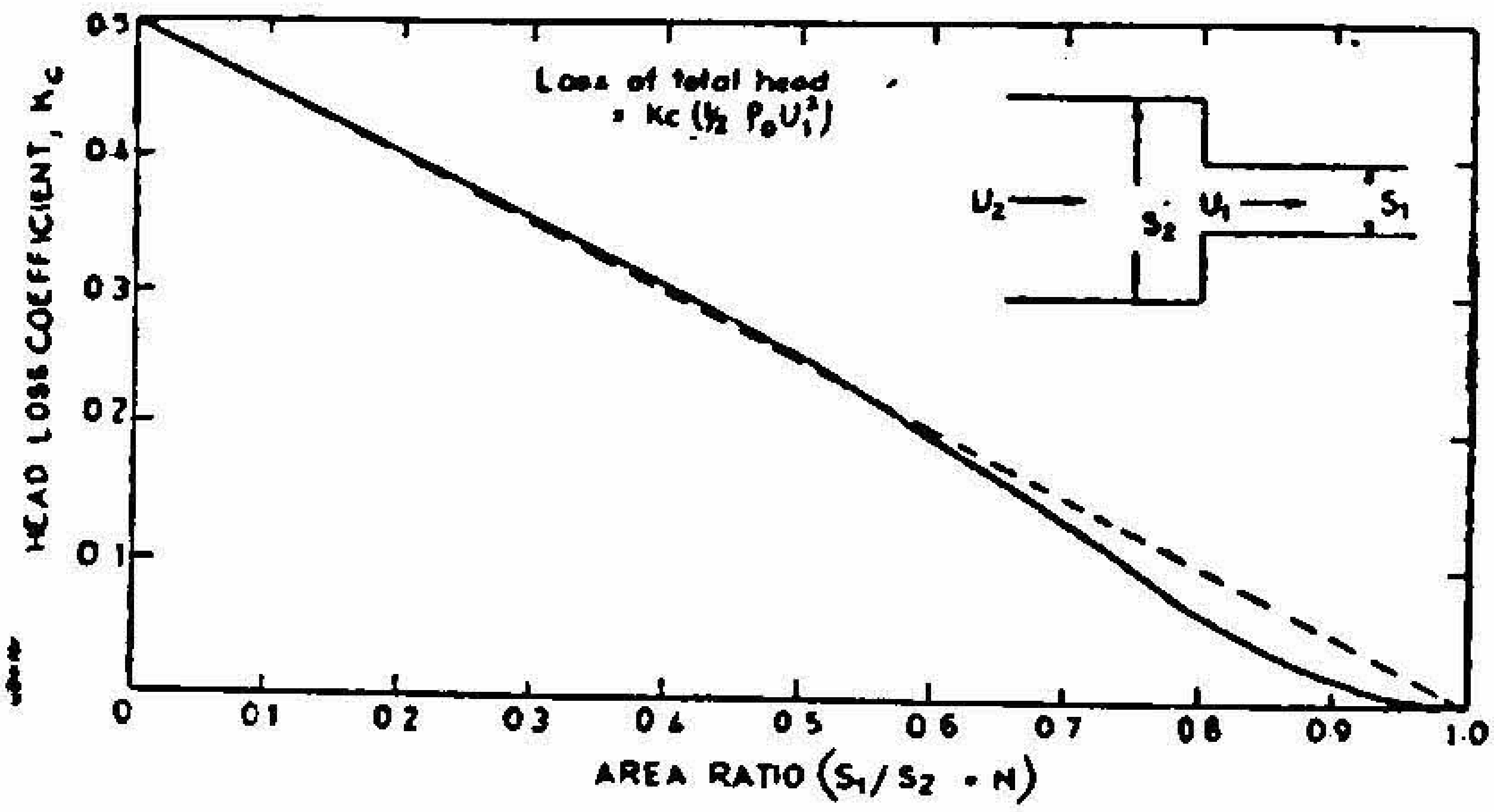

FIG. 2. Head loss coefficient at sudden contraction *vs* area ratio. —— experimental, from reference 10 and 11; - - - empirical, $K_c = (1 - N)/2$.

and

$$p_{e,3} = p_{e,1} + K_e M_3 Y_3 \simeq p_{e,1} + K_e M_3 Y_3 v_{e,3} \tag{42}$$

where K_e is the head loss coefficient at the extended inlet expressed in terms of the dynamic head in the smaller tube.

The continuity equation can be written as

$$v_{e,3} = v_{e,1} + v_2 + \delta \frac{M_1}{Y_1}. \tag{43}$$

The momentum equation is

$$N(p_{e,3} + M_3 Y_3 v_{e,3}) + (1 - N) p_2 = p_{e,1} + N^2 M_3 Y_3 v_{e,1} + \delta N^2 M_3^2 \tag{44}$$

where

$$N = S_3 / S_1. \tag{45}$$

Equation (44) implies the assumption that the pressure on the small solid area of the inner tube wall end section is the same as that inside the annular tube at that plane, *i.e.*, p_2.

Solving eqns. (39)–(44) simultaneously and rearranging, one gets

$$\begin{bmatrix} p_{e,3} \\ v_{e,3} \end{bmatrix} = \frac{1}{1 - N + AN(1 + K_e)} \times \begin{bmatrix} 1 - N + A(N + K_e) & K_e M_3 Y_3 (1 - N + AN^2) \\ \dfrac{1 - N}{Z_2} & 1 - N + AN^2 \end{bmatrix} \begin{bmatrix} p_{e,1} \\ v_{e,1} \end{bmatrix} \tag{46}$$

where

$$A = M_3 Y_3 / Z_2. \tag{47}$$

If $N^2 \ll 1$, the transfer matrix in eqn. (46) would be

$$\frac{1}{1 - N + AN(1 + K_e)} \begin{bmatrix} 1 - N + A(N + K_e) & K_e M_3 Y_3 (1 - N) \\ \dfrac{1 - N}{Z_2} & 1 - N \end{bmatrix}. \tag{48}$$

The following results can be deduced from the transfer matrix (48).

(*a*) For a stationary medium, $M = 0$, $A = 0$, $K_e = 0$

$$p_{e,3} = p_3; \quad v_{e,3} = v_3; \quad p_{e,1} = p_1; \quad v_{e,1} = v_1.$$

Thus one gets the following relation for an extended inlet

$$\begin{bmatrix} p_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_3} & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ v_1 \end{bmatrix}. \tag{49}$$

(*b*) For sudden expansion,

$$l_2 = 0, \quad \frac{1}{Z_3} \to 0, \quad A \to 0.$$

Therefore, eqn. (46) reduces to

$$\begin{bmatrix} p_{c,3} \\ v_{c,3} \end{bmatrix} = \begin{bmatrix} 1 & K_e M_3 Y_3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_{c,1} \\ v_{c,1} \end{bmatrix}. \tag{50}$$

The head loss coefficient K_e for a sudden expansion is given in several text books (*e.g.*, refs. 10 and 11). A plot of K_e *vs* N is given in fig. 3.

An empirical expression for K_e is given by

$$K_e = (1 - N)^2. \tag{51}$$

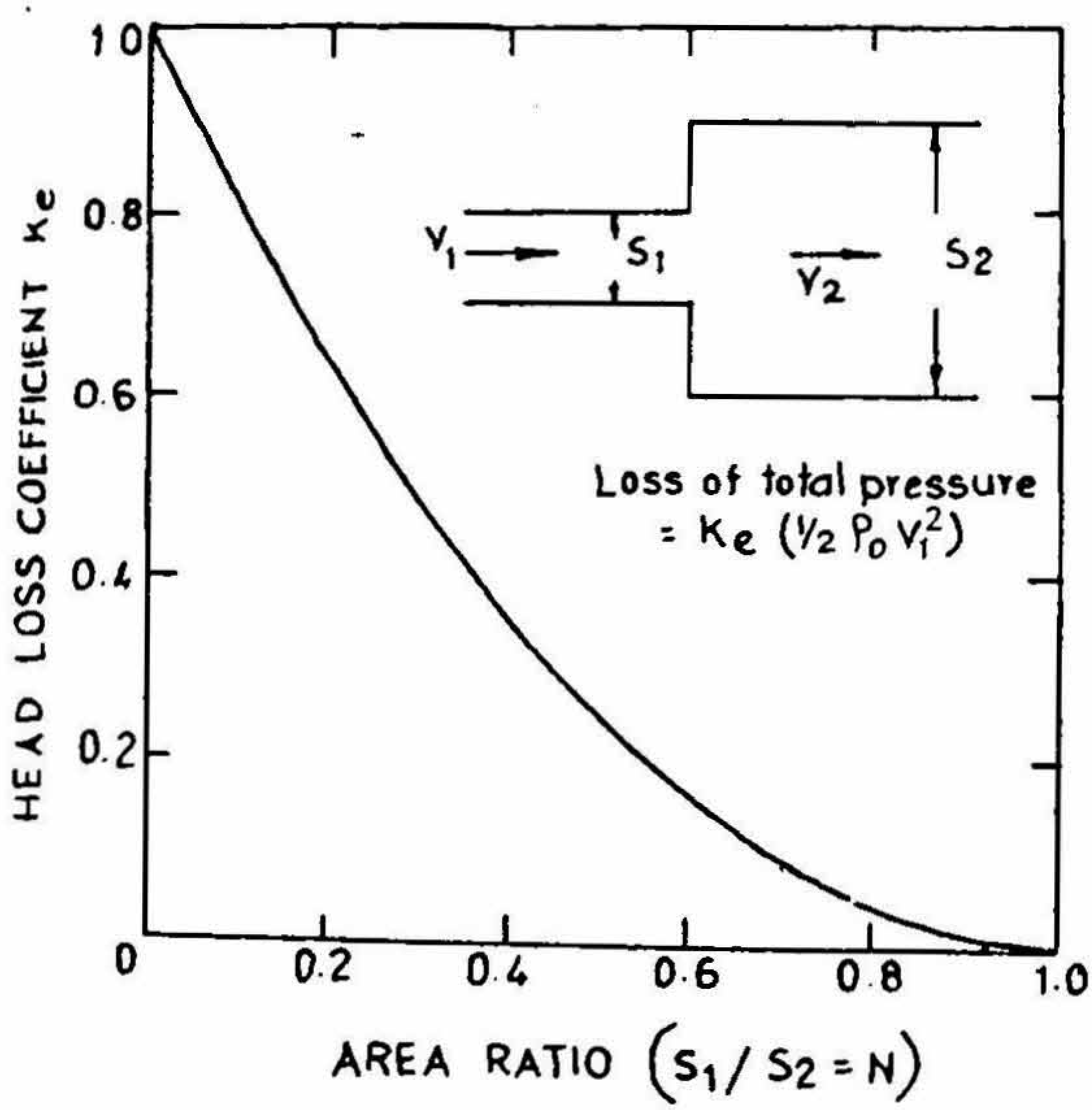

FIG. 3. Head loss coefficient K_e *vs* area ratio at sudden expansion. —— experimental $K_e = (1 - N)$ from references 10 and 11.

For sudden expansion, the transfer matrix in eqn. (50) with $K_e = (1 - N)^2$ is the same as the matrix (5) derived earlier in ref. 8. For extended inlet also, the loss coefficient given by eqn. (51) is adopted for substitution as there is no published data to establish otherwise. This is indirectly borne out by the experimental data plotted in fig. 7 of the following paper[15] where it is shown that even for expansion with reversal, the head loss coefficient happens to be nearly unity, which tallies approximately with eqn. (51) for $N \ll 1$. Further investigations into this would of course be useful. Substituting eqn. (51) in eqn. (48) and simplifying, the transfer matrix for extended inlet becomes

$$\frac{1}{1 + 2AN}\begin{bmatrix} 1 + A & M_3Y_3(1 - 2N) \\ \dfrac{1}{Z_3} & 1 \end{bmatrix}. \tag{52}$$

which is identical to the matrix (6) derived by Munjal[8] for $N^2 \sim AN^2 \ll 1$.

4. Experimental verification of the transfer matrices

A direct experimental verification of various elements constituting the transfer matrix of an element (or a set of elements) can be carried out by the transient testing technique developed by To and Doige[12, 13] which necessitates elaborate experimental set-up and has not yet been tried for moving medium. Another method used by Doige and Thawani[9] is to measure the SPL at any two locations in the muffler across the elements under investigation and compare them with the predicted values using the transfer matrices of these elements. This SPL difference which is known as noise reduction across the element seems to be a suitable method for the verification. The velocity ratio-*cum*-transfer matrix method explained in ref. 8 is made use of in the formulation. This is explained in Appendix I. Noise reduction spectrum across the two points indicated in figs. 7–9 of experimental mufflers was compared with that predicted by the above method by making use of a general FORTRAN program. The program takes into account the radiation impedance and tube attenuation constant as modified by the convective and dissipative effects of mean flow, makes use of the transfer matrices of the constituent elements, and finally makes use of the expression (A-3) of Appendix I. The dimensional details of the two experimental mufflers fabricated in the laboratory are shown in fig. 4.

All the dimensions are in mm. The mufflers were attached to an experimental facility. Air at atmospheric temperature and nearly atmospheric pressure was supplied to the system from an air compressor through a pressure control valve. The air flow rate was measured by an orifice meter which was calibrated separately. The Mach number of the flow was calculated on the basis of the mean flow velocity averaged over the test section. The system was excited by a 10 W loudspeaker housed in a chamber connected to the inlet section of the muffler. One B and K type 1023 beat frequency oscillator was used for the excitation of the loudspeaker at the desired frequency. The

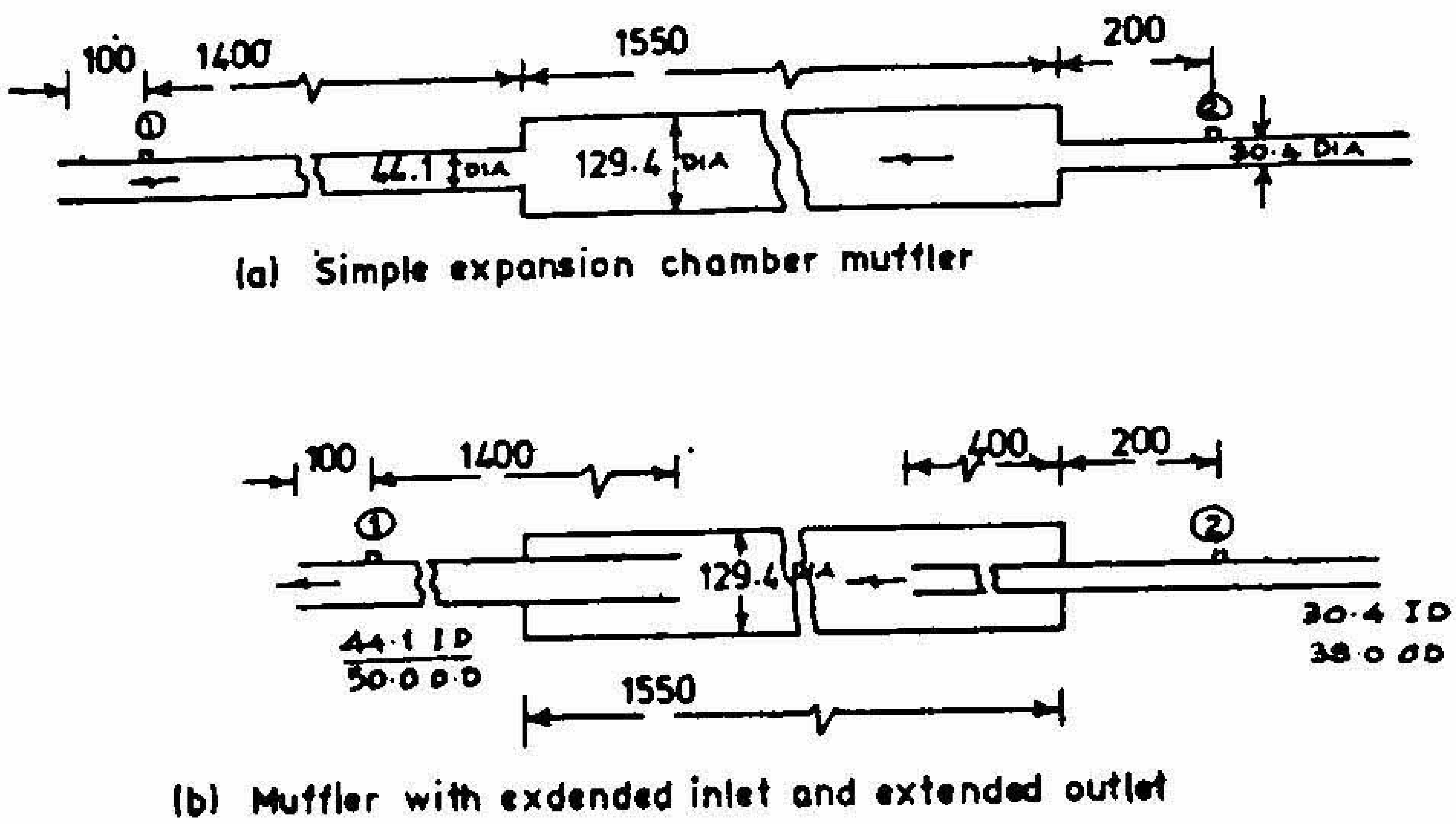

FIG. 4. Experimental mufflers tested.

sound pressure levels at sections 1 and 2 downstream and upstream of the muffler were measured by a 1/4 inch B and K condenser microphone. The microphone was held in a microphone holder which was mounted on connection sockets welded to the tube at the test locations. A probe tube bent in the form of a pitot tube having 2 mm outside diameter communicates the total pressure at the test section to the condenser microphone diaphragm. As pressures on the two sides of the diaphragm are equalized through the holes provided for the purpose, only the perturbations would be picked up as signals. Since the probe facing upstream would pick up the total pressure, this arrangement would be suitable for measuring the perturbations in total pressure p_c.

Since the same probe, microphone and holder are used for both the locations, and the SPLs are measured at the same frequencies for both the sections, the effect, if any, of their characteristics would get cancelled out. The signals from the microphone were passed through a B and K type 2020 constant band width heterodyne slave filter and the ouput was measured by a B and K type 2606 measuring amplifier. For measurements, the band width of the filter used was 3·16 Hz. Figure 5 shows the schematic arrangement of the experimental set-up. The noise reduction across the muffler was measured as the difference in SPLs between points 1 and 2 of fig. 5 at various frequencies.

The experiments were performed with and without flow. The theoretical spectra of noise reduction calculated from the transfer matrix method are plotted in figs. 6-9. The measured results are superimposed on these plots.

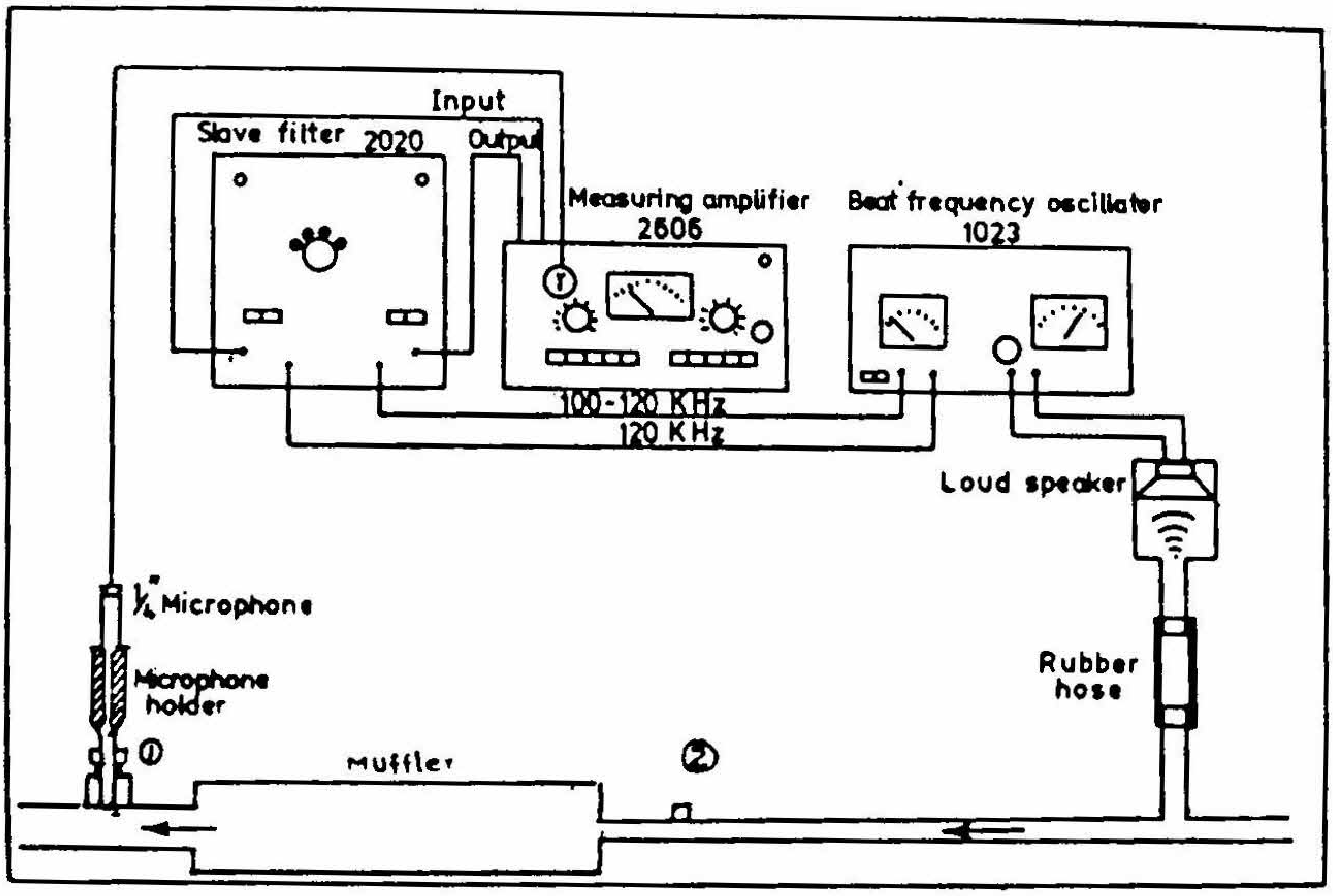

FIG. 5. Schematic arrangement of excitation-*cum*-measurement system.

The experimental results agree quite satisfactorily with the theoretical values of noise reduction especially at low frequencies. The measurement of SPL at frequencies very close to the peaks of the noise reduction curves was difficult and unreliable as the SPL at the downstream location was below the ambient noise level due to flow-turbulence. At frequencies above 800 Hz, they indicate a discrepancy which varies with frequency. This phenomenon is presumably due to the occurrence of three-dimensional effects. In this connection, it may be noted that from the solution of three-dimensional wave equation, Rayleigh[14] showed that the first axisymmetric mode can propagate for

$$ka > 3 \cdot 832$$

and the first diametral mode for

$$ka > 1 \cdot 84.$$

Thus the plane wave analysis would break down at

$$ka > 1 \cdot 84$$

or

$$f > \frac{1 \cdot 84\, c}{\pi D} \quad (53)$$

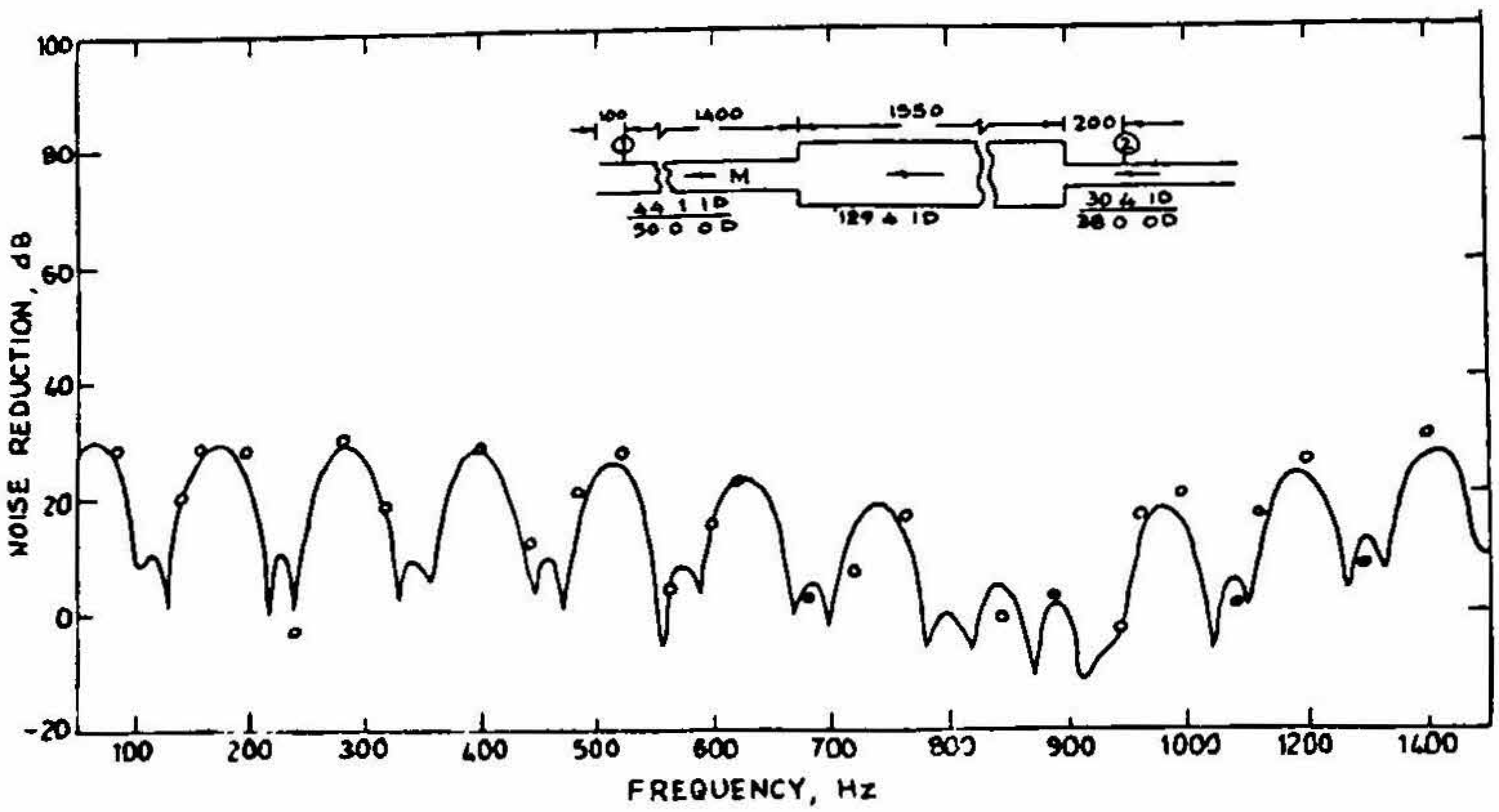

FIG. 6. Noise reduction *vs* frequency for a simple expansion chamber muffler without flow. —— theoretical; o o experimental.

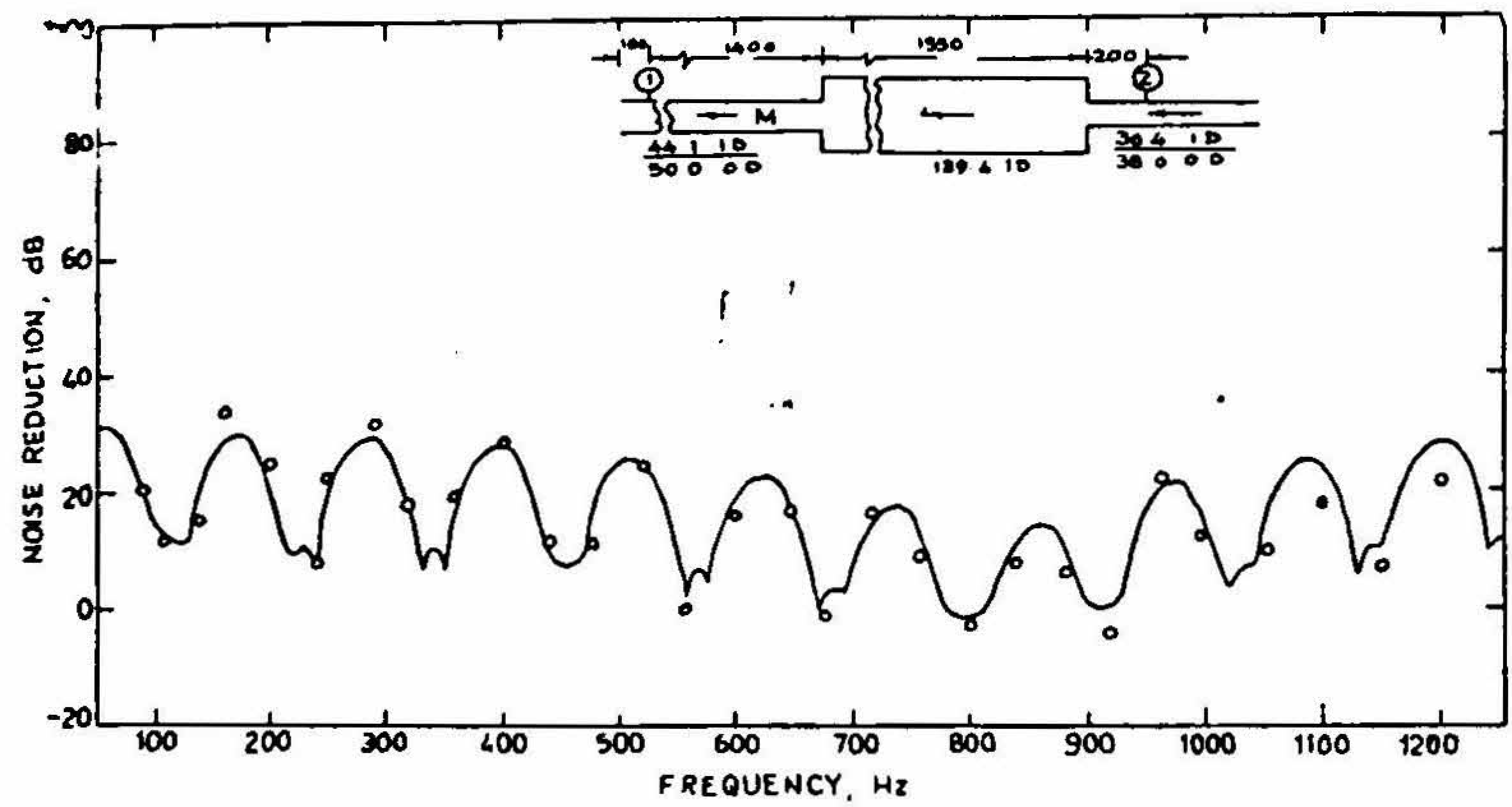

FIG. 7. Noise reduction *vs* frequency for a simple expansion chamber muffler with flow $M = 0{\cdot}06$. —— theoretical; o o experimental.

for stationary medium, and as per Mason[5] at

$$f > \frac{1{\cdot}84c}{\pi D}(1 - M^2) \tag{54}$$

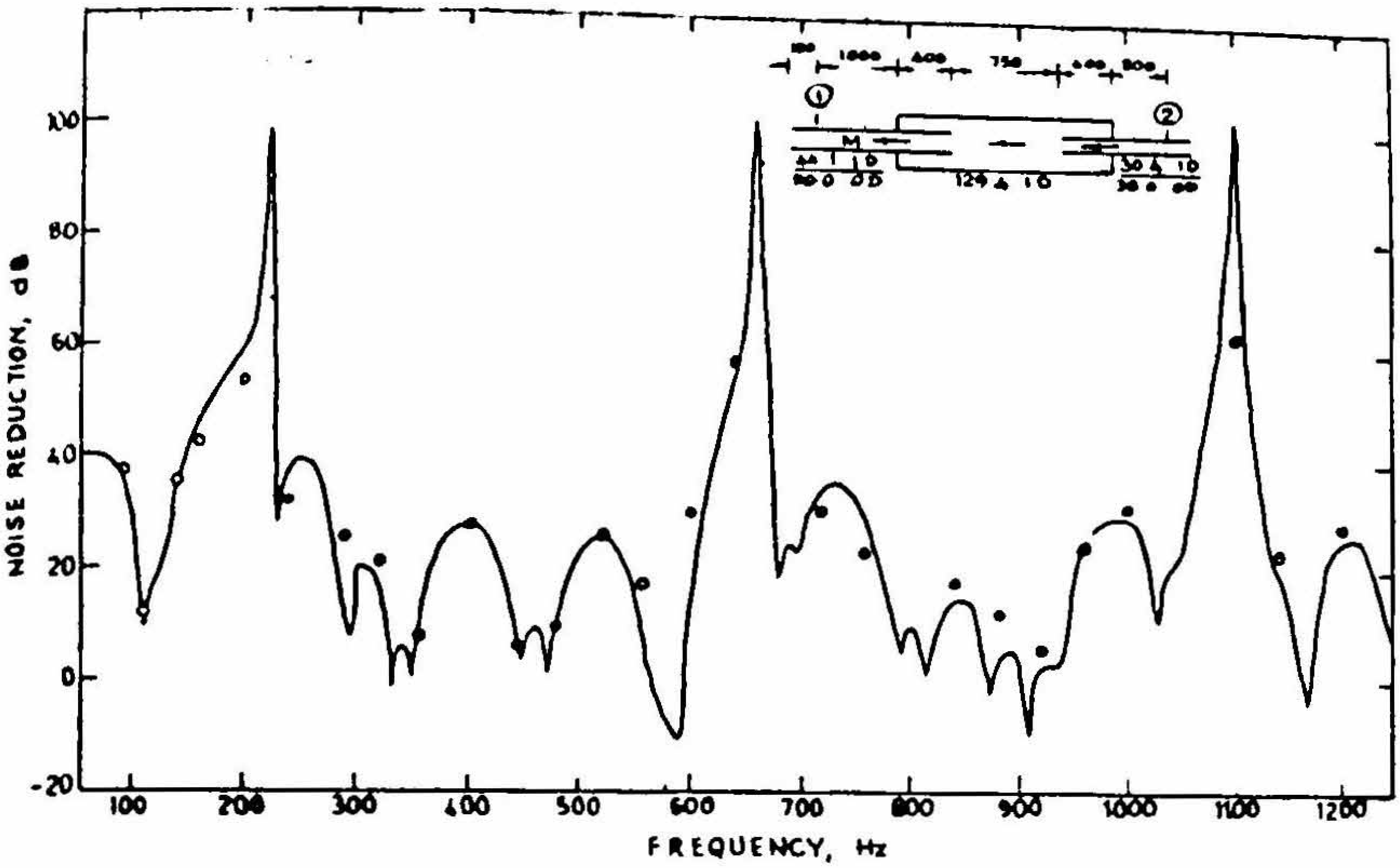

FIG. 8. Noise reduction *vs* frequency for muffler with extended inlet and extended outlet without flow. —— theoretical; o o experimental.

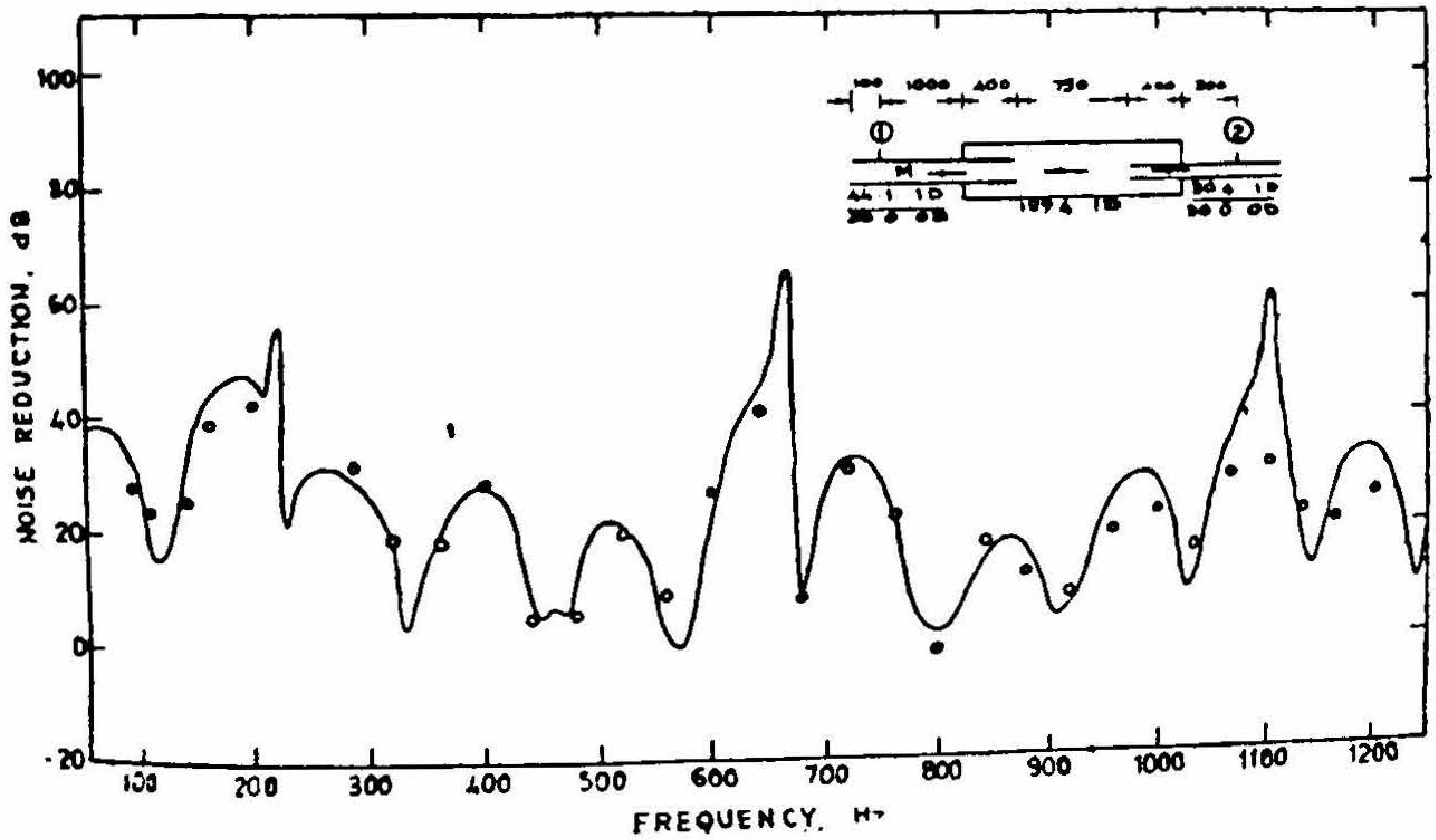

FIG. 9. Noise reduction *vs* frequency for a muffler with extended inlet and extended outlet with flow. $M = 0{\cdot}06$. —— theoretical; o o experimental.

for a moving medium. For the present case, $c = 347$ m/s and the diameter of the larger tube is 0·1294 m, and hence, the cut off frequency would be

$$f_c = 1570 \text{ Hz}.$$

However, downstream of a sudden area discontinuity, the length of the tube required to sufficiently attenuate the higher order modes generated at the discontinuity would be equal to many diameters. For example, Davies and Dwyer[2] observed that the three-dimensional disturbances in the transmitted wave die out in a length of about ten diameters downstream in the case of sudden expansion and four diameters downstream in the case of sudden contraction. In the muffler configuration shown in fig. 4 the tube length after sudden enlargement is only 0·75 m, *i.e.*, about 6D in the case of muffler with extended inlet and extended outlet. This may be one of the reasons why at frequencies as low as one-half of the theoretical cut off frequency, the prediction from one-dimensional wave theory starts deviating from the observed values of noise reduction. Of cour e, there is a need for further investigation into the three-dimensional effects downstream of sudden area discontinuities.

5. Conclusions

It has been found that the changes incorporated into the transfer matrices make only a marginal difference (less than 1 *dB*) in the noise reduction except at peaks and troughs where it is substantial (2 to 3 *dB*). These differences, however, would magnify in commercial mufflers which invariably make use of 2 to 3 chambers and higher Mach Numbers (about 0·25).

The following conclusions can be drawn from the analysis and the results :

(i) The transfer matrices for sudden contraction and extended outlet should take into consideration the flow losses occurring at those sections. The earlier investigators neglected the losses and to that extent, their results were incorrect. Empirically, the loss coefficient can be taken to be $(1 - N)/2$.

(ii) In the case of extended outlet the static pressure in the annular cavity in the plane of area change may be less than the total pressure upstream, while the earlier investigators assumed it to be equal to the latter. This assumption has been avoided in the foregoing analysis.

(iii) The transfer matrices derived for sudden expansion and extended inlet are the same as those established by previous investigators as long as the area ratio is sufficiently small.

(iv) The experimental values of noise reduction are in good agreement with those predicted using the transfer matrices derived in the text. This confirms the validity and accuracy of the transfer matrices.

(v) The effect of mean flow and turbulent friction damping is to flatten the peaks and troughs of the noise reduction spectrum.

Acknowledgements

This work forms part of the thesis " Some studies on the prediction and verification of the aeroacoustic performance of exhaust mufflers " accepted for Ph.D. Degree by the Indian Institute of Science, Bangalore, by the first author, who was then on deputation from N.S.S. College of Engineering, Palghat, Kerala, under the Quality Improvement Programme. The authors gratefully acknowledge the facilities provided by the Indian Institute of Science and the financial assistance extended by the Lal Bahadur Sastri Technological Research Centre, Government of Kerala.

References

1. Davis, D. D., Stokes, G. M., Moore, D. and Stevens, T. L. — *Theoretical and experimental investigation of mufflers with comments on engine exhaust muffler design*, NACA 1954, TN 1192.
2. Davies, P. O. A. L. and Dwyer, M. J. — A simple theory for pressure pulses in exhaust systems, *Proc. Inst. Mech. Engrs.*, (*London*), 1964–65, 179 (Pt. 1, No. 10), 365–375.
3. Trengrouse, G. H. and Soliman, M. M. — Effect of sudden change in flow area, *J. Mech. Engng. Sci.*, 1966, 8 (2), 198–206.
4. Benson, R. S., Garg, R. D. and Woods, W. A. — Unsteady flow in pipes with gradual or sudden area changes, *Proc. Inst. Mech. Engrs.* (*London*), Thermodynamics and fluid mechanics convention, 1963–64, Pt. 31, Paper 9.
5. Mason, V. — Some experiments on the propagation of sound along a cylindrical duct containing flowing air, *J. Sound Vibr.*, 1969, 10, 208–226.
6. Alfredson, R. J. and Davies, P. O. A. L. — Performance of exhaust silencer components, *J. Sound Vibr.*, 1971, 15 (2), 175–196.
7. Mungur, P. and Gladwell, G. M. — Acoustic wave propagation in a sheared fluid contained in a duct, *J. Sound Vibr.*, 1969, 9, 28–48.
8. Munjal, M. L. — Velocity ratio-*cum*-transfer matrix method for evalution of a muffler with mean flow, *J. Sound Vibr.*, 1975, 39 (1), 105–119.
9. Doige, A. G. and Thawani, P. T. — Muffler performance from transmission matrices, *Proc. Noise Conf.*, 1979. Purdue University, W. Lafayette, Indiana, 1979, pp. 245–254.
10. Streeter, V. L. — *Fluid mechanics*, McGraw-Hill Book Co., Second edn., 1958, Ch. 4, Sec. 28.
11. Vennard, J. K. and Street, R. L. — *Elementary fluid mechanics SI version*, John Wiley and Sons, 1976, Fifth edn., Ch. 9.
12. To, C. W. S. and Doige, A. G. — A transient testing technique for the determination of matrix parameters of acoustic system I. Theory and principles, *J. Sound Vibr.*, 1979, 62 (2), 207–222.

13. To, C. W. S. and Doige, A. G. — A transient testing technique for the determination of matrix parameters of acoustic systems II. Experimental procedures and results, *J. Sound Vibr.*, 1979, 62 (2), 223–233.

14. Lord Rayleigh — *Theory of Sound*, Vol. II, Dover Publications, Second edn., 1945' reissue, pp. 297–300.

15. Panicker, V. B. and Munjal, M. L. — Aeroacoustic analysis of mufflers with flow reversals, *J. Indian Inst. Sci.*, 1981, 63 (A), 21–38.

Appendix I

An expression for noise reduction and the error in its estimation if a wall-static pressure probe is used instead of a total pressure probe

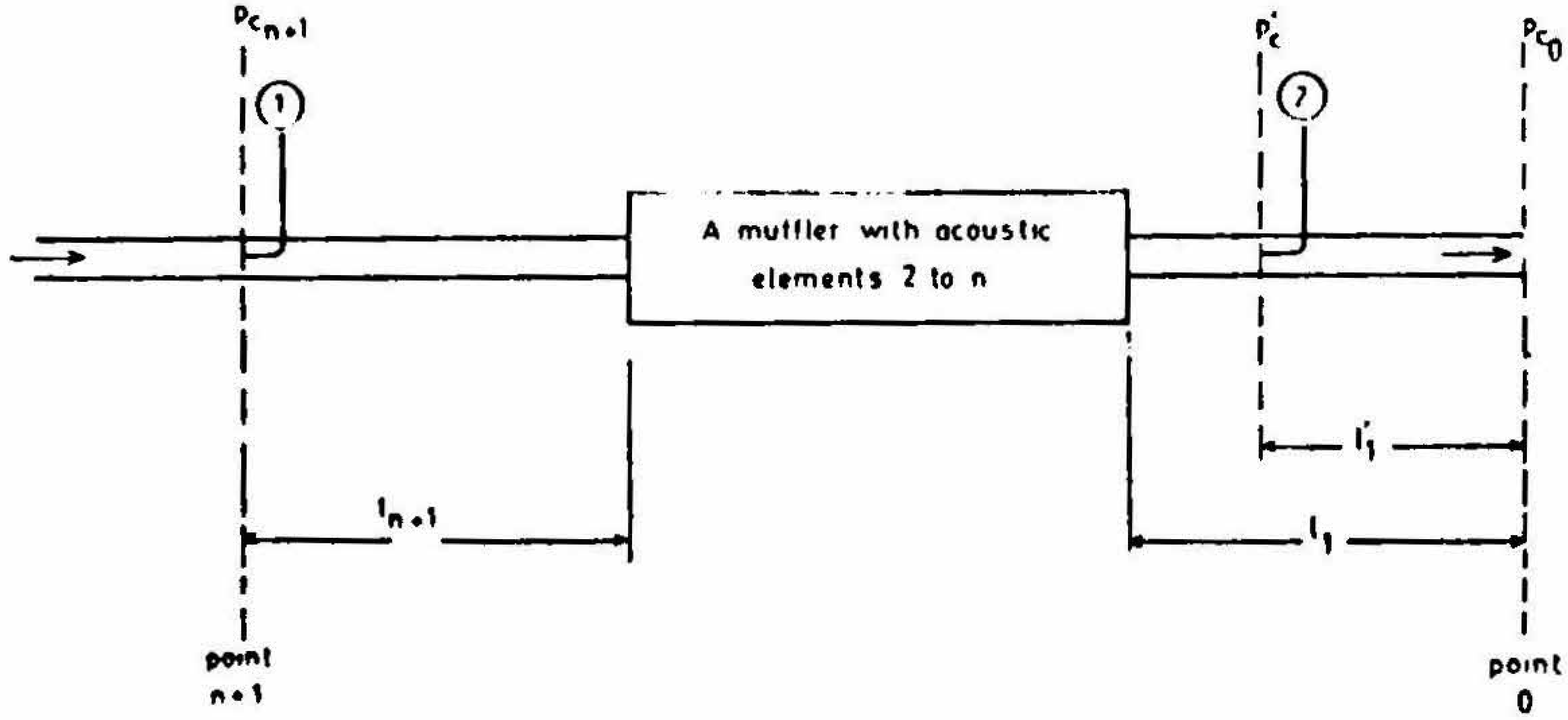

Fig. A-1. Terminology for noise reduction parameter.

Noise reduction is defined here as difference of the convective sound pressure levels (SPL) measured at points (1) and (2) as shown in fig. A-1. In the analysis, the elements are numbered from the radiation end upstream. Thus, connecting points '$n+1$' and '0',

$$\begin{bmatrix} p_{c_{n+1}} \\ v_{c_{n+1}} \end{bmatrix} = [T_{c_{n+1}}][T_{c_n}]\cdots[T_{c_2}][T_{c_1}]\begin{bmatrix} p_{c_0} \\ v_{c_0} \end{bmatrix}$$
$$= [T_{c_{n+1}}][T_{c_n}]\cdots[T_{c_2}][T_{c_1}]\begin{bmatrix} 1 & Z_{c_0} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ v_{c_0} \end{bmatrix}$$
$$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} 0 \\ v_{c_0} \end{bmatrix} \text{ (say)}, \qquad \text{(A-1)}$$

where

$$Z_{c_0} = \frac{p_{c_0}}{v_{c_0}}.$$

And, convective SPLs at points (2) and '0' are related as

$$\begin{bmatrix} p'_c \\ v'_c \end{bmatrix} = [T'_{c_1}] \begin{bmatrix} p_{c_0} \\ v_{c_0} \end{bmatrix}$$

$$= [T'_{c_1}] \begin{bmatrix} 1 & Z_{c_0} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ v_{c_0} \end{bmatrix}$$

$$= \begin{bmatrix} A'_{11} & A'_{12} \\ A'_{21} & A'_{22} \end{bmatrix} \begin{bmatrix} 0 \\ v_{c_0} \end{bmatrix} \text{(say).} \tag{A-2}$$

Now,

$$NR = SPL_{n+1} - SPL'$$

$$= 20 \log_{10} | p_{c_{n+1}}/p'_c |$$

$$= 20 \log_{10} | A_{12}/A'_{12} | \tag{A-3}$$

where SPL_{n+1} and SPL' are the sound pressure levels measured at the upstream and downstream locations respectively at the given frequency.

The wall-static measurements would yield only the acoustic variables p_{n+1} and p' respectively which are related to the corresponding aeroacoustic variables by eqns. (1) and (2). The noise reduction if calculated from the wall static measurements would be

$$NR' = 20 \log_{10} | p_{n+1}/p' |$$

$$= 20 \log_{10} \left| \frac{p_{c,n+1} - M_{n+1} Y_{n+1} v_{c_{n+1}}}{p' - M' Y' v'_c} \right|$$

$$= 20 \log_{10} \left| \frac{A_{12} - M_{n+1} Y_{n+1} A_{22}}{A'_{12} - M' Y' A'_{22}} \right|. \tag{A-4}$$

The error in the estimation of noise reduction would then be

$$NR - NR' = 20 \log_{10} \left| \frac{A_{12} (A'_{12} - M' Y' A'_{22})}{A'_{12} (A_{12} - M_{n+1} Y_{n+1} A_{22})} \right|. \tag{A-5}$$