SHORT COMMUNICATION

ON TURAN'S THEOREM

M. R. SRIDHARAN

(*Mathematics Department, Indian Institute of Technology, Kanpur* 208 016)

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ABSTRACT

It is proved by the method of induction that the maximum number of lines among all p *point digraphs with no directed cycle of length three is* $[p^2/2]$ *where* [x] *denotes the greatest integer less than or equal to* x.

We refer to Ref. 2 for basic definitions. Turan[3] has shown that the maximum number of lines among all p point graphs without any triangles is $[p^2/4]$. Generalization to digraphs was done in Ref. 1. Brown and Harary[1] have obtained the maximum number of lines among all p point digraphs with no directed cycle of length three. Here we prove the same result by the method of induction as was done in the case of graphs in Ref. 2.

Theorem: The maximum number of lines among all p point digraphs with no directed cycle of length three is $[p^2/2]$.

Proof: We prove by induction. We consider the case where p is even. A similar argument holds good when p is odd also. Let us assume that the theorem is true for $p \leqslant 2n$. Let D be a digraph without any directed cycle of length three and $p = 2n + 2$. Since D does not consist of isolated points alone, let v_1 and v_2 be two points of D such that v_1 is adjacent to v_2. Consider the digraph D_1 obtained from D by deleting the points v_1 and v_2 and all the lines incident with them. D_1 has $2n$ points. Hence it can have at the most $[p^2/2] = 2n^2$ lines. We will now find the maximum number of lines that D can have. The $2n$ points of D_1 can be adjacent to both v_1 and v_2 and in addition v_1 can be adjacent *from* v_2. So D can have at the most $2n^2 + 2n + 2n + 1 + 1 = 2n^2 + 4n + 2 = [(2n + 2)^2/2]$ lines.

We can show that there exist digraphs with $[p^2/2]$ lines and without any directed cycles of length three for all even p. Let V_1 and V_2 be two sets each with $p/2$ points. Let each point of V_1 be adjacent to and adjacent from every point of V_2. Then we have $2 \cdot p/2 \cdot p/2 = p^2/2$ lines. This digraph has no directed cycle of length three.

REFERENCES

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3. Turan, P. — Eine extremalanfgabe aus der Graphen Theorie. *Mat. Fiz. Lapak*, 1941, **48**, 436–452.