

Control on TOL and ETOL array systems

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Abstract

Regular, context-free and context-sensitive controls are imposed on the tables of TOLAS and the hierarchy established. The effect of control with appearance checking and minimal table interpretations are investigated over the tables of Part ETOLAS. It is interesting to note that the regular control on the tables of Part DETOLAS will not increase the generative capacity of Part DETOLAS.

Key words: Array systems, Chomsky grammars, string languages, table interpretation.

1. Introduction

L-systems, introduced by Lindenmayer, originally in connection with some problems in theoretical biology, have later stimulated a substantial amount of research^{1,2}. In these systems one does not distinguish between terminals and non-terminals, productions are applied in parallel on a word and the starting string of the system is of length greater than or equal to one.

In trying to extend this concept of parallel rewriting to two-dimensions, we propose OL and TOL array systems where parallel rewriting of every symbol in a rectangular array is considered, each symbol is replaced by an array of the same size or dimension to avoid distortion of rectangular arrays and the axiom is a rectangular array³. The use of non-terminals is a very well established mechanism in formal language theory. In L-systems, this notion is represented by Extended TOL and Extended OL systems. We have investigated ETOL and EOL array systems in detail⁴.

In string languages, various control devices are introduced to increase the generative capacity of given grammars⁵. Once control is introduced, it is meaningful to consider the idea of appearance checking which is exhaustively studied for string languages⁶.

In Chomsky grammars with regulated rewriting, at any step in a derivation of the grammar, the choice of the production to be applied is restricted by a kind of control mechanism. In L-systems with regulated rewriting, in one step of a derivation, a whole subset of productions is applied where the choice of this subset is somehow restricted. Such systems are called Extended partial table OL systems with minimal table interpretation⁶. It is shown that generative capacity of extended partial table OL systems with minimal table interpretation is stronger than that of regular control and appearance checking, whereas it is equal to the combined effect of regular control and appearance checking⁶.

In this paper, we extend these ideas of minimal table interpretation, regular control and appearance checking on partial ETOLAS and we observe that most of the results of Nelsen⁶ will carry over to Part ETOLAS. We also consider in this paper the effect of context-sensitive, context-free and regular control on the tables of TOLAS. We observe that context-free and context-sensitive control on the tables of TOLAS will generate array languages which are not generable by TOLAS with regular control.

2. Control on TOL array systems

First we review some definitions needed for this paper.

Let I be an alphabet—a finite non-empty set of symbols. A matrix M_{mn} (or array, over I) is an $m \times n$ rectangular array of symbols from I ($m, n \geq 1$) and the dimensions of the matrix M_{mn} is denoted by $|M_{mn}| = (m, n)$. The set of all matrices over I (including λ) is denoted by I^{**} and $I^{++} = I^{**} - \{\lambda\}$.

Definition 2.1¹

A tabled OL array system (TOLAS) is a 3-tuple $G = (\Sigma, \mathcal{P}, \omega)$ where Σ is a finite non-empty set (the alphabet, say $\Sigma = \{a_1, \dots, a_k\}$); $\omega \in \Sigma^{++}$ is the axiom; and \mathcal{P} consists of a finite set $\{P_1, \dots, P_f\}$ for $f \geq 1$ and each P_f is a finite subset of $\Sigma \times \Sigma^{**}$ called a table with the following two conditions.

- (i) $(\forall P)_f, (\forall a)_{\Sigma} (\exists a)_{\Sigma^{**}} ((a, a) \in P)$;
- (ii) $(\forall a)_{\Sigma} (\exists (a, a))_f, a$'s are of the same dimension.

Definition 2.2

Let

$$\begin{array}{rcc}
 & a_{.1} \dots \dots a_{.n} & M_{11} \dots \dots M_{1n} \\
 & \dots \dots \dots & \dots \dots \dots \\
 u = & \dots \dots \dots & \text{and } v = \dots \dots \dots \text{ where } a_{ij} \in \Sigma \\
 & a_{m1} \dots \dots a_{mn} & M_m \dots \dots M_{mn}
 \end{array}$$

$M_{ij} \in \Sigma^{**}$, $1 \leq i \leq m$, $1 \leq j \leq n$. We write $u \Rightarrow v$ if $a_{ij} \rightarrow M_{ij}$ are in table P in \mathcal{P} and that $|M_{ij}|$ is a constant. \Rightarrow^* is the reflexive, transitive closure of \Rightarrow .

Definition 2.3

Let $G = (\Sigma, \mathcal{P}, \omega)$ be a TOLAS where $\mathcal{P} = \{P_1, \dots, P_l\}$. $u \Rightarrow^{l_1} v$ ($u \Rightarrow v(l_1)$), if v is derived from u using table P_1 whose label is l_1 . $u \Rightarrow^* v(w)$ extends $u \Rightarrow v(l_1)$ in the usual manner, where $w \in \{l_1, l_2, \dots, l_l\}^*$, where l_i is the label of the table P_i .

Definition 2.4

A controlled TOLAS is a 3-tuple (G, C, L) where G is a TOLAS, L is the set of labels of the tables of G . C is a language over L . $L(G, C, L) = \{x \in \Sigma^{**} / \omega \Rightarrow^* x, \alpha \in C\}$.

Notation

We denote by $\mathcal{F}(\text{TOLAS}: X)$ the family of languages generated by TOLAS with control language from family X .

Now let us investigate the effect of regular, context-free and context-sensitive controls on the tables of TOLAS. It is interesting to observe that context-free (context-sensitive) control on the tables of TOLAS, will generate array languages which are not generable by regular (context-free) control on the tables of TOLAS.

Theorem 2.1

$$\mathcal{F}\text{TOLAL} \subset \mathcal{F}(\text{TOLAS}: R) \subset \mathcal{F}(\text{TOLAS}: CF) \subset \mathcal{F}(\text{TOLAS}: CS).$$

Proof: Inclusions follow from definitions. Proper inclusions are established by the following examples.

(i) Let $G = (\{., x\}, \{P_1, P_2\}, \begin{matrix} x. \\ x. \end{matrix})$ be a TOLAS where

$$P_1 = \{x \rightarrow x., x \rightarrow xx, \cdot \rightarrow \cdot\cdot, \cdot \rightarrow \cdot x\}, P_2 = \left\{ x \rightarrow \begin{matrix} \cdot \\ x \end{matrix}, x \rightarrow \begin{matrix} x \\ \cdot \end{matrix}, \right.$$

$$\left. \cdot \rightarrow \begin{matrix} \cdot \\ x \end{matrix}, \cdot \rightarrow \cdot \right\}. \text{ Let } C = (P_1^m P_2^n / n, m \geq 1) \text{ be a regular control.}$$

pretation and with regular control. Intuitively, at each step of derivation if the elements of an array is exactly equal to the left hand sides of the rules of a table in \mathcal{P} of part ETOLAS then we say that the system is a partial ETOIAS with minimal table interpretation.

Definition 3.1

Let t be a finite subset of $\Sigma \times \Sigma^{**}$ then $\text{reg}(t)$
 $= \{\sigma \in \Sigma \mid \exists a \in \Sigma^{**}, \sigma \rightarrow a \in t\}.$

If $X \in \Sigma^{++}$, then $\text{Min } X = \{a \mid a \in X\}.$

Definition 3.2

A partial ETOLAS (Part ETOLAS) is an ordered 6-tuple $G = (V, \mathcal{P}, L, L^*, \omega, \Sigma)$, where V is a finite, non-empty set (the alphabet of G), $\Sigma \subset V$, the target alphabet, $\omega \in V^{++}$ is the axiom, \mathcal{P} is a finite, non-empty collection of finite subsets of $\Sigma \times \Sigma^{**}$ such that if $t \in \mathcal{P}$, then $t = \{\Sigma \times \Sigma^{**} \mid \Sigma^{**} \text{ are arrays of the same dimension}\}$. The elements of \mathcal{P} are called tables and the elements of the tables are called productions. L is the set of labels of \mathcal{P} (there is a one-to-one correspondence between the elements of \mathcal{P} and L). L^* is a subset of L .

Definition 3.3

Let $G = (V, \mathcal{P}, L, L^*, \omega, \Sigma)$ be a Part ETOLAS and let $X \in V^{++}$, $Y \in V^{**}$. X is said to derive Y directly in G , $X \Rightarrow_G Y$ if and only if

$$(i) \begin{matrix} a_1 \dots a_n & M_{11} \dots M_{1n} \\ X = \dots & \text{and } Y = \dots \\ a_m \dots a_{mn} & M_{m1} \dots M_{mn} \end{matrix} \text{ where } a_{ij} \in V$$

for $1 \leq i \leq m$, $1 \leq j \leq n$, $M_{rs} \in \Sigma^{**}$, $1 \leq r \leq m$, $1 \leq s \leq n$ and M_{rs} are of the same dimension;

(ii) $\mathcal{F}_t \in \mathcal{P}$: $[(a_{ij}, M_{ij}) \in t \text{ for } i = 1, 2, \dots, m, j = 1, 2, \dots, n]$. We also write $X \xRightarrow[l]{G} Y$ where l is the label from L associated with the table t . X is said to derive Y directly under the minimal table interpretation $S \xRightarrow[G]{mt} Y$ iff (i) and (ii) above are satisfied and

(iii) $\text{reg}(t) = \text{Min}(X)$ for $t \in \mathcal{P}$ satisfying (ii). We also write $X \xRightarrow[mt]{G} y(l)$ where l is the

label associated with t . $\Rightarrow^* \left(\xRightarrow[mt]{} \right)$ is the reflexive transitive closure of $\Rightarrow (\xRightarrow[mt]{}).$

Definition 3.4

Let $G = (V, \mathcal{P}, L, L^*, \omega, \Sigma)$ be a Part ETOLAS and let $X \in V^{++}$, $Y \in V^{**}$ and $Z \in L^*$. Then X is said to derive Y with control word Z (under the minimal table interpretation)

$$X \xrightarrow[G]{*} Y(Z) \left(X \xrightarrow[G \text{ } mt]{*} Y(Z) \right) \text{ if and only if,}$$

(i) $Z = l_1 l_2 \dots l_d$ where $l_i \in L$ for $i = 1, 2, \dots, d$;

(ii) there exists arrays M_0, M_1, \dots, M_d from V^{**} such that $M_0 = x$, $M_d = Y$ and $M_{i-1} \Rightarrow M_i(l_i)$ ($M_{i-1} \Rightarrow_{mt} M_i(l_i)$) for every $i = 1, 2, \dots, d$.

X is said to derive Y with control word Z (under minimal table interpretation) with appearance checking. $X \xrightarrow{**} Y(Z) \left(X \xrightarrow[mt]{ac} Y(Z) \right)$ if and only if

(i) $Z = l_1 l_2 \dots l_d$ where $l_i \in L$ for $i = 1, 2, \dots, d$;

(ii) there exists arrays M_0, M_1, \dots, M_d from V^{**} such that $X = M_0$, $Y = M_d$ and for every $i = 1, 2, \dots, d$ (t_i denotes the table associated with the label l_i) if $\text{Min}(M_{i-1}) \subseteq \text{reg}(t_i)$ ($\text{Min}(M_{i-1}) = \text{reg}(t_i)$) then $M_{i-1} \Rightarrow M_i(l_i)$ ($M_{i-1} \Rightarrow_{mt} M_i(l_i)$) otherwise $l_i \in L^*$ and $M_{i-1} = M_i$. The language generated by G is defined as

$L_i^j(G) = \left\{ x \in \Sigma^{**} / \exists Z \in L^* : \omega \xrightarrow[G]{*} x(Z) \right\}$, where i may be the index mt or not and j may be the index ac or not.

Definition 3.5

A Part ETOLAS with regular control (RC-Part ETOLAS) is a 7-tuple $G = (V, \mathcal{P}, L, L^*, \omega, \Sigma, R)$ where $G' = (V, \mathcal{P}, L, L^*, \omega, \Sigma)$ is a Part ETOLAS and R is a regular

control over L . Then $L_i^j(G) = \{X \in \Sigma^{**} / \exists Z \in R, \omega \xrightarrow{*} X(z), \text{ where } i \text{ may be the index } mt \text{ or not and } j \text{ may be the index } ac \text{ or not.}$

Definition 3.6

A partial ETOLAS $G = (V, \mathcal{P}, L, L^*, \omega, \Sigma)$ is called an ETOLAS iff for every $t \in \mathcal{P}$ and for every $a \in V$, there is an $M_{ra} \in V^{**}$, r and s are fixed numbers such that $(a, M_{ra}) \in t$. G is called deterministic (Part EDTOLAS) iff for every t and for every $a \in V$ there is at most one $M_{ra} \in V^{**}$, r and s are fixed numbers, such that $(a, M_{ra}) \in t$. G is called propagating (Part EPTOLAS) iff for every $t \in \mathcal{P}$ and for every $a \in V$, $(a, \lambda) \notin t$. The languages generated are denoted by Part EDTOLAL and Part EPTOLAL.

Now let us investigate the relations between the families of DETOLAL, Part DETOLAL $_i^j$, RC-Part ETOLAL $_i^j$ (i may be ac or not and j may be mt or not).

Theorem 3.1

$$\mathcal{F}\text{Part ETOLAL} = \mathcal{F}\text{Part ETOLAL}^{\omega}$$

$$\mathcal{F}\text{Part ETOLAL}_{mt} = \mathcal{F}\text{Part ETOLAL}_{mt}^{\omega}$$

Proof: Proof is similar to theorem 1 of Nielsen⁶.

Theorem 3.2

$$\mathcal{F}\text{ETOLAL} = \mathcal{F}\text{Part ETOLAL}.$$

Proof: Let $G = (V, \mathcal{P}, L, L^{\omega}, \omega, \Sigma)$ be a Part ETOLAS. Let F be a symbol not in V . Define an ETOLAS $G' = (V \cup \{F\}, \mathcal{P}', L, L^{\omega}, \omega, \Sigma)$ where $\mathcal{P}' = \{P' | P \in \mathcal{P}\}$. If $l \in L$ is the label of $P \in \mathcal{P}$ then $P' = P \cup \{\sigma \rightarrow M', \sigma \notin \text{reg } P\} \cup \{F \rightarrow M'\}$, where M' is the rejection array whose dimension is equal to that of the dimension of the right hand side of the rules of P . The label of P' in \mathcal{P}' is l . Hence $L(G) = L(G')$, i.e. $\mathcal{F}\text{Part ETOLAL} \subset \mathcal{F}\text{ETOLAL}$. But by definition $\mathcal{F}\text{ETOLAL}$ is contained in $\mathcal{F}\text{Part ETOLAL}$. Hence the theorem.

Now we state the following theorems without proof as the proofs can be found in Nirmal⁷.

Theorem 3.3

$$\mathcal{F}\text{Part ETOLAL} = \mathcal{F}\text{RC-Part ETOLAL}.$$

Theorem 3.4

$$\mathcal{F}\text{EDTOLAL} = \mathcal{F}\text{Part EDTOLAL}_i^j = \mathcal{F}\text{RC-Part EDTOLAL}_i^j$$

where i (resp. j) is the index mt (resp. ac) or missing.

Remark 3.1

It follows from theorem 3.4 that regular control on the tables of Part DETOLAS will not increase the generative capacity of Part DETOLAS.

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