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Short Communication

Minimization of TANT networks

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Abstract

This paper presents a method for generating tail factors which realize some principal tail factor of an operational prime permissible implicant (OPPI). Defects of the Gimpel's method and its consequence on the minimization of TANT networks are pointed out. The validity of the new method has been proved.

Key words : TANT networks, Gimpel's algorithm.

1. Introduction

A TANT network is a three-level network composed solely of NAND gates having only uncomplemented (True) inputs. An algorithm for finding for any Boolean function an optimal TANT network was presented by Gimpel[1]. In his algorithm, certain functions realizable by input gates or second-level gates are preselected as candidates for possible use in an optimal network. Candidates with which alone a minimal TANT network can be obtained are called operational prime permissible implicants (OPPI). An extension to a prime implicant table known as CC-table (Cover and Closure Table) had been constructed. This table is used for choosing a subset of OPPI. A network is then obtained by using this subset of OPPI.

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The principal expression for any OPPI is of the form $HT_1' T_2' T_3' \ldots T_n'$ where H, $T_1, T_2, T_3, \ldots, T_n$ are a product of different uncomplemented variables (H may be the Boolean constant 1). H is called the head of the expression and T_i's are called principal tail factors. In the CC-table, there is one column for each principal tail factor of each OPPI and there is one row for every tail factor which realizes some principal tail factor of an OPPI. Therefore, before constructing the CC-table, the tail factors which realize some principal tail factor of an OPPI must be generated.

2. Gimpel's method and its defects

Gimpel[1] has presented a method to generate these tail factors mechanically. It is as follows :

Let T' be a principal tail factor of an OPPI with head H. Let t be a product of complemented variables of all the variables not in H or T. Let $H_1, H_2, \ldots, H_R$ be the set of heads of the fundamental products of Tt. Then the set of tail factors realizing T' is $H_1', H_2', \ldots, H_R'$. Thus the problem reduces to one of finding all the fundamental products of a disjunction of terms of the form Tt.

Let us now analyse this method. Let us consider the OPPI* xzy'. For this example $H = xz$ and $T = y$. w and y are the variables not in H and w, x, and z are the variables not in T. Therefore, the set of variables not in H or T is (w, y, x, z). All the possible products of the form 't' are $w'y'$, $w'x'$, $w'z'$, $y'x'$, $y'z'$, $x'z'$, $w'y'x'$, $y'x'z'$, $w'x'z'$, $w'y'z'$, $w'y'x'z'$. Now if we find the terms Tt using the above products, some of them will become zero if they contain y' as one of the literals. The non-zero terms of the form Tt are $yw'x'$, $yw'z'$, $yx'z'$ and $yw'x'z'$. Let D be the disjunction of these terms. Then,

$$
\begin{aligned}
D &= yw'x' + yw'z' + yx'z' + yw'x'z' \\
&= yw'x'(z + z') + yw'z'(x + x') + yx'z'(w + w') + yw'x'z' \\
&= yzw'x' + yw'x'z' + xyw'z' + ywx'z'
\end{aligned}
$$

Thus, the heads of the fundamental products of Tt are $H_1 = yz$, $H_2 = y$, $H_3 = xy$ and $H_4 = wy$. By the above method, we get the tail factors y', $(yz)'$, $(xy)'$, $(wy)'$. But $(xzy)'$ is also a tail factor which realizes y' and we are not able to generate it. Also, if we replace y' by $(Wy)'$ in the expression for the OPPI considered, the resulting expression will not be an expression for the OPPI. So $(wy)'$ is not a required type of tail factor even though it realizes y'.

We can mathematically prove that the above method will yield such factors like $(wy)'$ whenever there exists one or more variables not in both H and T.

* This is an OPPI for the function considered by him on page 25, fig. 9.

Proof

Let us consider a function of n variables. Let the set of variables be $S = (S_1, S_2, \ldots, S_n)$. Let T be a principal tail factor of an OPPI with head H. Let V_1 be the set of variables not in H or T. Let V_2 be the set of variables in T only.

Let V_3 be the set of variables not in both H and T.

If V_3 is not a null set, then let $S_1 \in V_3$. Now consider a product t_i of the form ' t ' formed using variables in V_4 except S_1. Let $t_i = s_j' \, s_{j+1} \ldots S_n'$ and let $J = (j, j+1, \ldots, h)$. Since all the variables in t_i are not in V_2, $Tt_i \neq 0$. The heads of fundamental products of Tt_i are of the form TS_R where $S_R \in S$ and $R \notin J$.

Now $S_1 \in S$ and $1 \notin J$, therefore TS_1 is one of the heads of the form TS_R. Hence, the method has yielded $(TS_1)'$ to be a tail factor. If we replace T' by $(TS_1)'$ in the expression $HT_1' \, T_2' \ldots T_n'$ of the OPPI considered, we get $H(TS_1)' \, T'_2 \ldots T_n'$ as the expression for the OPPI. Since $H(TS_i)' T_i' \ldots T_n' = HT' \, T_1' \ldots T_n' + HS_1 ' T_1' \ldots T_n'$ and $HS_i 'T_1' \ldots T_n' \neq 0$, $H(TS_1)' T'_1 \ldots T_n'$ is not an expression for the OPPI considered.

We have observed two things. First, Gimpel's method will not generate the full set of tail factors required and secondly it will sometimes yield odd candidates. Let us next see the effect of this on the minimization of TANT networks.

Every third-level gate in a TANT network realizes some principal tail factor of those OPPI which are selected for forming a minimal network. We have already observed that, in a CC-table, a column is provided for every principal tail factor and a row for every tail factor which realizes some principal tail factor. This is done to minimize the number of third-level gates as follows. If a tail factor T' realizes more than one principal tail factor, instead of generating these principal tail factors separately using different gates, we can replace these gates by a single gate which realizes the tail factor T'.

Let us consider the same example xzy'. We got $(wy)'$ as a tail factor which realizes y'. Let $(wy)'$ realize some of these principal tail factors and y' has not realized any other principal tail factor. Then, the CC-table reduction technique will give $(wy')^1$, as an essential tail factor to be used in the minimal network and this replaces y'. This network will not realize the given function.

On the other hand, if we are not able to generate $(xzy)'$, this may lead to a non-minimal network. If $(xzy)'$ is already selected for forming minimal network, a three-level gate will be required for $(xzy)'$ and another gate for y'. We can replace these two gates by one if we are able to observe that y' may be replaced by $(xzy)'$.

3. New method

This method is based on the Boolean identity

$$x_i x_j' = x_i x_j' + x_j x_j' = x_i \ (x_i x_j)'$$

$(xi\ xj)'$ realizes xj' and the function will not be affected if we replace xj' by $(xixj)$. Let T' be a principal tail factor of an OPPI with head H.

Augment the principal tail factor T' with the head variable or variables. Note that '1' can be thought of as a variable in any head H'. The set of factors obtained by this augmentation is the required set of tail factors. Once the tail factors are known, the TANT network can be synthesized as usual. We next prove that the set of tail factors obtained by this method is the required (full) set of tail factors.

Proof

Let $HT_1'\ T_2' \ldots T_i \ldots T_n'$ be an OPPI. Let it be required to find the tail factors which realize Ti'. Let V be the set containing all the variables in H. Let $H_R \in V$. Let $T_R = H_R Ti$. If we replace Ti' by T_R' the function will not be altered* and T_R realizes T_i'. Therefore, T_R is a required tail factor.

On the other hand, let $H_R \notin V$. Then, if we replace T_i' by T_R', the function will be altered. The terms obtained by augmenting the variable or variables in V and the terms obtained by augmenting the variable or variables not in V together form the full set (R) of terms which realizes the given principal tail factor T_i'.

The elements of R which are obtained by augmenting head variable or variables are alone the required tail factors and no other tail factor which realizes T_i' exists for Ti'.

4. Conclusion

Mere augmentation of the head variable or variables gives the required tail factors instead of the cumbersome methods, leading to more systematic method of minimization of TANT networks.

Reference

1. GIMPEL, J. F. The minimization of TANT networks, *IEEE Trans. Electron. Computers*, 1967, EC-16, No. (1), pp. 18-38.

* This fact is presented by Gimpel on page 31, lemma-4.