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GRAVITATIONAL INSTABILITY IN THE PRESENCE OF HALL-CURRENT

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ABSTRACT

The effect of the inclusion of the Hall current due to electron dynamics on the gravitational instability of a finitely conducting medium has been studied using the modifications indicated by Cowling and Lighthill including uniform rotation. It has been found that excepting for a few specal cases, Jeans criterion still determines the gravitational instability The inclusion of Hall current affects the criterion only when the disturbances are propagated transverse to the magnetic field. Numerical estimates of these new dimensions have been obtained and compared with Jeans' result. The conclusions were also justified on the basis of physical arguments.

1. INTRODUCTION

The gravitational instability of an infinite homogeneous medium has been discussed by many authors[1-8] under various conditions. In a fully ionized, low density and high temperature medium the Larmor frequency of the electrons is large compared with the electron collision frequency and hence the Hall-effect has to be taken into account as pointed out by Cowling[9]. In the present note we shall study the gravitational instability of medium like Galactic Halo and H II Clouds taking into account the Hall-current and the Coriolis force in the presence of a uniform external magnetic field. In part A, we consider the gravitational instability of an infinitely conducting medium, while in part B we have studied the influence of finite conductivity on the gravitational instability of this medium.

We note that in the case of infinitely conducting medium, in the presence of magnetic field along the direction of wave-propagation, Jeans criterion[1] for gravitational instability applies for wave-propagation in the direction along or perpendicular to the direction of rotation. The particular case when there is no component of magnetic field along the direction of wave-propagation is of interest, because here due to the inclusion of Hall-current, the instability does not take place in the sense of Jeans but is entirely different. However, the magnitude of the linear-dimension of the galaxies, estimated by taking some standard values for the physical parameters, is of the same order as obtained by Jeans. Also in this case we note that when the wave-propagation is along the direction of rotation the critical wave-number is independent of the magnitude of rotation, while in the case of wave-propagation perpendicular to the direction of rotation the critical wave-number depends on the magnitude of rotation and if Spitzer's condition[6] is satisfied the system is stable for all wave-numbers.

In part B we note that the finite conductivity does not affect the Jeans Criterion when the direction of wave-propagation is along the magnetic filed. When the wave-propagation is perpendicular to the direction of the magnetic field, we find that when the direction of rotation is along the direction of wave-propagation the critical wave-number introduced by Hall-current is removed by the effect of finite conductivity and instability takes place in the sense of Jeans, and when wave-propagation is transverse to the direction of rotation, once again finite conductivity removes the influence of Hall-current and the system is stable for all wave-numbers if Spitzer's condition is satisfied.

We note however, in the cases when wave-propagation is perpendicular to the direction of rotation and magnetic field, on reinterpreting the expression

$$c_s^2 k^2 - 4\pi G\rho_0 + 4\,\Omega_T^2,$$

which determines the stability of the system, that if Jeans criterion holds the stability of the system is assured and the effect of the rotation is to elongate the linear-dimensions of the galaxies.

2. LINEARIZED EQUATIONS AND DISPERSION RELATION

We consider an infinitely extending homogeneous self-gravitating medium of finite conductivity rotating uniformly with an angular velocity $\vec{\Omega}$ embedded in a uniform external magnetic field $\mathbf{H}_0$. The undisturbed state of the medium is taken to be uniform with pressure p_0 and density ρ_0.

Let $\delta\rho$, δp, δU, $\mathbf{v}$, $\mathbf{h}$ be the perturbed density, pressure, gravitational potential, velocity and magnetic field respectively. The linearized hydro-magnetic equations determining these perturbations are following Chandrasekhar[5] and C. Uberoi and C. Devanthan[10]:

Equation of Continuity :

$$\frac{\partial \delta \rho}{\partial t} + \rho_0 \operatorname{div} \mathbf{v} = 0 \qquad [2.1]$$

Equation of Momentum :

$$\rho_0 \left[\frac{\partial \mathbf{v}}{\partial t} + 2\vec{\Omega} \times \mathbf{v}\right] = -\operatorname{grad} \delta p + \rho_0 \nabla \delta U + \frac{1}{4\pi}[(\nabla \times \mathbf{h}) \times \mathbf{H}_0], \qquad [2.2]$$

assuming the effect of the gravitational forces on the density fluctuation to be small.

Equation of State :

$$\delta p = c_s^2 \delta \rho, \quad c_s = \sqrt{\frac{\gamma p_0}{\rho_0}}, \qquad [2.3]$$

the sound speed corresponding to the undisturbed medium.

Poison's Equation :

$$\nabla^2 \delta U = -4\pi G \delta \rho \qquad [2.4]$$

Maxwell's Equations : (e.m.u.)

$$\operatorname{div} \mathbf{h} = 0 \qquad [2.5]$$

$$\frac{\partial \mathbf{h}}{\partial t} = \operatorname{curl}(\mathbf{v} \times \mathbf{H}_0) + \eta \nabla^2 \mathbf{h} - \frac{c}{4\pi ne} \operatorname{curl}(\operatorname{curl} \mathbf{h} \times \mathbf{H}_0) + \frac{c^2}{\omega_p^2} \cdot \frac{\partial}{\partial t}(\nabla^2 \mathbf{h}), \qquad [2.6]$$

where

$\eta = \dfrac{c^2}{4\pi\sigma}$, σ being the electrical conductivity and c the velocity of light,

and

$$\omega_p = \left(\frac{4\pi ne^2}{m}\right)^{\frac{1}{2}},$$

the plasma frequency ; e, n, m are the charge, particle density and mass for the electron gas.

The equation [2.6] is obtained by linearizing the induction equation given by Spitzer[11]. The electron pressure gradient term vanishes as the temperature of the medium is taken to be uniform[10].

We shall suppose that orientation of the co-ordinate axes are so chosen that

$$\mathbf{H}_0 = (H_x, 0, H_z) \text{ while } \vec{\Omega} = (\Omega_x, \Omega_y, \Omega_z).$$

We shall seek solutions which are independent of x and y co-ordinates and correspond to the propagation of waves in the z-direction. Hence we assume that all the perturbations vary as $\exp[i(\omega t + kz)]$.

From equation [2.5] we have $h_z = 0$. Substituting this in equation [2.6] and eliminating $\delta\rho$, δp, δU from equation [2.2] with the help of equations [2.1], [2.3] and [2.4] respectively we get the following equations determining v_x, v_y, v_z, h_x and h_y:

$$\left.\begin{aligned}
&i\omega\, v_x + 2\Omega_y v_z - 2\Omega_z v_y - \frac{ikH_z}{4\pi\rho_0} h_x = 0\\
&2\Omega_z v_x + i\omega\, v_y - 2\Omega_x v_z - \frac{ikH_z}{4\pi\rho_0} h_y = 0\\
&-2\Omega_y v_x + 2\Omega_x v_y + v_z\left(i\omega - c_s^2\frac{ik^2}{\omega} + i\frac{4\pi G\rho_0}{\omega}\right) + \frac{ik\,H_x}{4\pi\rho_0} h_x = 0\\
&-ik\,H_z v_x + ik\,H_x v_z + h_x\left(i\omega + \eta k^2 + \frac{k^2c^2\, i\omega}{\omega_p^2}\right) + \frac{ck^2\,H_z}{4\pi ne} h_y = 0\\
&-ik\,H_z v_y - \frac{ck^2\,H_z}{4\pi ne} h_x + h_y\left(i\omega + \eta k^2 + \frac{k^2c^2\, i\omega}{\omega_p^2}\right) = 0.
\end{aligned}\right\} \quad [2.7]$$

The condition that the set [2.7] of linear homogeneous equations admits non-trivial solution leads to the dispersion relation,

$$\begin{aligned}
&\omega^6 - i2\,\Omega_C\,\omega^5 - \omega^4\left(\Omega_C^2 + 2\,\Omega_A^2 + \Omega_B^2 + \Omega_H^2 + \Omega_J^2 + 4\Omega^2\right)\\
&\quad + i\Omega_C\omega^3\left(2\Omega_A^2 + \Omega_B^2 + 2\Omega_J^2 + 8\Omega^2\right) + \omega^2\left[\Omega_A^4 + \Omega_A^2\,\Omega_B^2\right.\\
&\quad + \Omega_J^2\left(\Omega_C^2 + 2\Omega_A^2 + \Omega_H^2 + 4\Omega_z^2\right) + 4\Omega^2\left(\Omega_C^2 + \Omega_H^2\right) + 4\Omega_y^2\,\Omega_A^2\\
&\quad \left.+ 4\Omega_z\,\Omega_H\,\Omega_A^2 + 4\Omega_x\,\Omega_H\,\Omega_A\,\Omega_B + 4(\Omega_x\,\Omega_A - \Omega_z\,\Omega_B)^2\right]\\
&\quad - i\Omega_C\,\omega\left[2\,\Omega_J^2\left(\Omega_A^2 + 4\,\Omega_z^2\right) + 4\,\Omega_y^2\,\Omega_A^2 + 4\left(\Omega_x\,\Omega_A - \Omega_z\,\Omega_B\right)^2\right]\\
&\quad - \Omega_J^2\left[\Omega_A^4 + 4\Omega_z^2\left(\Omega_H^2 + \Omega_C^2\right) + 4\,\Omega_z\,\Omega_H\,\Omega_A^2\right] = 0,
\end{aligned} \qquad [2.8]$$

where

$$\Omega_C = \frac{\eta k^2}{\lambda^2},\quad \Omega_A^2 = \frac{k^2 H_z^2}{\lambda^2\, 4\pi\rho_0},\quad \Omega_B^2 = \frac{k^2 H_x^2}{\lambda^2\, 4\pi\rho_0},\quad \Omega_H = \frac{ck^2 H_z}{\lambda^2\, 4\pi\, ne},$$

$$\lambda^2 = \left(1 + \frac{c^2k^2}{\omega_p^2}\right),\quad \Omega_J^2 = \left(c_s^2k^2 - 4\pi\, G\rho_0\right),\ \Omega^2 = |\vec{\Omega}|^2.$$

3. Discussion of the Dispersion Relation

PART A

We shall discuss the following special cases:

Case I: $\eta = 0,\ H_z \neq 0,\ H_x = 0,\ \Omega_z \neq 0,\ \Omega_x = \Omega_y = 0.$

In this case the dispersion relation [2.8] splits into two factors.

$$\omega^2 - \Omega_J^2 = 0 \qquad [3.1]$$

and

$$\omega^4 - \omega^2 \left(2\,\Omega_A^2 + \Omega_H^2 + 4\,\Omega_z^2\right) + \left(\Omega_A^4 + 4\,\Omega_z^2\,\Omega_H^2 + 4\,\Omega_z\,\Omega_H\,\Omega_A^2\right) = 0. \qquad [3.2]$$

Considering the equation [3.1] we note that there is a mode of wave-propagation which will become unstable when $\Omega_J^2 < 0$, *i.e.*, when

$$c_s^2\,k^2 < 4\pi\,G\rho_0$$

and this is precisely the Jeans condition for gravitational instability.

The equation [3.2] gives two stable modes of wave-propagation

$$\omega_{1,2}^2 = \tfrac{1}{2}\left(2\,\Omega_A^2 + \Omega_H^2 + 4\Omega_z^2\right) \pm \tfrac{1}{2}\left[\left(\Omega_H^2 - 4\,\Omega_z^2\right)^2 + 4\,\Omega_A^2\left(\Omega_H - 2\,\Omega_z\right)^2\right]^{\frac{1}{2}}.$$

These are four Alfven waves each two running in opposite directions with velocity modified by rotation and Hall effect.

Hence Jeans criterion for gravitational instability is unaffected by the magnetic field, and the Coriolis force when the direction of wave-propagation is along the magnetic field and direction of rotation, even in the presence of Hall-current. This result agrees with that obtained by Chandrasekhar[5].

Case II. $\eta = 0,\ H_z \neq 0,\ H_x = 0,\ \Omega_z = 0,\ \Omega_x \neq 0,\ \Omega_y \neq 0.$

Here the dispersion relation can be written as

$$f(X) = 0,$$

where

$$f(X) \equiv X^3 - X^2\left(2\,\Omega_A^2 + \Omega_H^2 + \Omega_J^2 + 4\,\Omega_T^2\right) + X\left[\Omega_A^4 + \Omega_J^2\left(2\,\Omega_A^2 + \Omega_H^2\right) + 4\,\Omega_T^2\left(\Omega_A^2 + \Omega_H^2\right)\right] - \Omega_J^2\,\Omega_A^4, \qquad [3.3]$$

$$X = \omega^2 \text{ and } \Omega_T^2 = \Omega_x^2 + \Omega_y^2.$$

When $\Omega_J^2 < 0$, equation [3.3] has always one negative root, hence there is always one unstable mode of wave-propagation.

When $\Omega_J^2 > 0$ we have

$$\frac{f(\Omega_A^2)}{\Omega_H^2} = -\frac{f(\Omega_J^2 + 4\,\Omega_T^2)}{4\,\Omega_T^2} = \Omega_A^2\,(\Omega_J^2 + 4\,\Omega_T^2 - \Omega_A^2).$$

Hence the cubic equation [3.3] has three real roots ω_1, ω_2, ω_3 lying in the range

$$0 < \omega_1 < \min\,(\Omega_A^2,\ \Omega_J^2 + 4\,\Omega_T^2) < \omega_2 < max\,(\Omega_A^2,\ \Omega_J^2 + 4\,\Omega_T^2) < \omega_3.$$

For $\Omega_J^2 > 0$, therefore we have three stable modes of wave propagation.

Thus in the case when the wave-propagation is along the direction of magnetie field and perpendicular to the direction of rotation the gravitational instability takes place in the sense of Jeans, whether Hall effect is taken into account or not.

Case III. $\eta = 0,\ H_z = 0,\ H_x \neq 0,\ \Omega_z \neq 0,\ \Omega_x = \Omega_y = 0.$

The dispersion relation reduces to two factors

$$\omega^2 - 4\,\Omega_z^2 = 0 \qquad [3.4]$$

and

$$\omega^2 - (\Omega_B^2 + \Omega_J^2) = 0. \qquad [3.5]$$

The equation [3.4] gives a stable mode of wave-propagation with frequency whose magnitude is twice that of the rotational frequency of the system. From [3.5] we note that there is always an unstable mode of propagation when

$$\Omega_B^2 + \Omega_J^2 < 0$$

i.e.,

$$\frac{k^2\,V_x^2}{1 + c^2k^2/\omega_p^2} + c_s^2 k^2 - 4\,\pi\,G\,\rho_0 < 0, \qquad [3\ 6]$$

where $V_x = \sqrt{(H_x^2/4\,\pi\rho_0)}$, the Alfven velocity for the transverse component of the magnetic field.

The critical wave-number k^* for which the expression [3.6] is negative is determined by the equation

$$(c^2\,c_s^2/\omega_p^2)\,k^4 + k^2\,[(V_x^2 + c_s^2 - 4\pi\,G\rho_0\,(c^2/\omega_p^2)] - 4\,\pi\,G\,\rho_0 = 0. \qquad [3.7]$$

As k, the wave-number is taken to be real we have

$$k^{*2} = \frac{-(c_s^2 + V_x^2 - 4\,\pi\,G\rho_0\,c^2/\omega_p^2)}{2\,c^2\,c_s^2/\omega_p^2} + [(c_s^2 + V_x^2 - 4\pi\,G\rho_0\,c^2/\omega_p^2)^2 + (4c^2\,c_s^2/\omega_p^2)\,4\,\pi\,G\,\rho_0]^{\frac{1}{2}}/2\,c^2\,c_s^2/\omega_p^2. \qquad [3.8]$$

The system is unstable for the wave-numbers $k < k^*$. From the equation [3.8] we note that the critical wave-number k^* depends on the plasma frequency ω_p which is introduced by the inclusion of Hall-current. Hence due to Hall-effect the criterion for gravitational instability is different from that obtained by Jeans. It is also interesting to note that the critical wave-number k^* is independent of the magnitude of the frequency of rotation as is expected since the transverse velocity and magnetic field are decoupled.

We shall compute the value of the wave-length $\lambda_H = (2\pi/k^*)$ with the help of equation [3 8] and $\lambda_J = 2\pi \; (c_s^2/4\pi G\rho_0)^{\frac{1}{2}}$ as obtained by Jeans, by taking the following numerical values for the physical parameters occurring in the equation [3.8]:

$\rho_0 = 1.7 \times 10^{-24}\, gm/cm^3$, $n = 1$ particle$/cm^3$, $V_x^2 = 5 \times 10^{12}\, cm^2/sec^2$,

$c_s^2 = 2.25 \times 10^{12} cm^2/sec^2$, $c^2 = 9 \times 10^{20}\, cm^2/sec^2$, $e = 4.803 \times 10^{-10}$ e.s.u.,

$m = 9.108 \times 10^{-28}\, gm$, $G = 6.658 \times 10^{-8}\, gm^{-1}\, cm^3\, sec^{-2}$.

The numerical values for λ_H and λ_J are:

$$\lambda_H \simeq 1{\cdot}42 \times 10^{22}\, cms., \quad \lambda_J = 7{\cdot}9 \times 10^{21}\, cms.$$

We therefore note that though the Hall-effect changes the instability criterion, the numerical estimate of the linear dimensions of the galaxies is almost of the same order as obtained by Jeans.

Case IV: $\eta = 0,\ H_z = 0,\ H_x \neq 0,\ \Omega_z = 0,\ \Omega_x \neq 0,\ \Omega_y \neq 0.$

In this case the dispersion relation [2.8] reduces to

$$\omega^2 = \Omega_B^2 + \Omega_J^2 + 4\,\Omega_T^2. \qquad [3.9]$$

There is always an unstable mode of propagation when

$$(\Omega_B^2 + \Omega_J^2 + 4\,\Omega_T^2) < 0$$

i.e.

$$\frac{k^2 V_x^2}{1 + c^2 k^2/\omega_p^2} + c_s^2 k^2 - 4\pi G\rho_0 + 4\,\Omega_T^2 < 0. \qquad [3.10]$$

Here we note that when Spitzer's condition $\Omega_T^2 > \pi G \rho_0$, is satisfied the expression [3.10] is always positive and hence the system is stable for all values of the wave-number k.

When Spitzer's condition is not satisfied the system is unstable for all wave-numbers $k < k^*$, where k^* is given as:

$$k^{*2} = [-(V_x^2 + c_s^2 - 4\pi G \rho_0 c^2/\omega_p^2 + c^2 4\Omega_T^2/\omega_p^2)]/[2c^2 c_s^2/\omega_p^2]$$
$$+ [(V_x^2 + c_s^2 - c^2/\omega_p^2 \cdot 4\pi G\rho_0 + c^2/\omega_p^2 \cdot 4\Omega_T^2)^2$$
$$+ (16 c^2 c_s^2/\omega_p^2)(\pi G \rho_0 - \Omega_T^2)]^{1/2}/[2c^2 c_s^2/\omega_p^2]. \quad [3.11]$$

Here we note that unlike the Case III, the critical wave number k^* also depends on the magnitude of the frequency of rotation. Taking the same numerical values as in Case III for the physical parameters and $\Omega_T = 10^{-16}$/sec. we get

$$\lambda_H \simeq 1.44 \times 10^{22} \text{ cms.}, \quad \lambda_J = 7.9 \times 10^{21} \text{ cms.}$$

Thus the effect of the Hall-current is same as in the previous case. But the other point that we note here is that when there is no component of the magnetic field in the direction of wave-propagation and the direction of wave-propagation is perpendicular to the direction of rotation, whether the Hall-current is present or not the system is stable for all wave-numbers if Spitzer's condition

$$\Omega_T^2 > \pi G \rho_0$$

is satisfied. [Refer (3)].

PART B

Case I. $\eta \neq 0,\ H_z \neq 0,\ H_x = 0,\ \Omega_z \neq 0,\ \Omega_x = \Omega_y = 0.$

In this case the dispersion relation reduces to two factors

$$W^2 + \Omega_J^2 = 0 \quad [3.12]$$

and

$$W^4 - 2\Omega_C W^3 + W^2(\Omega_C^2 + 2\Omega_A^2 + \Omega_H^2 + 4\Omega_z^2) - 2\Omega_C W(\Omega_A^2 + 4\Omega_z^2)$$
$$+ [\Omega_A^4 + 4\Omega_z^2(\Omega_H^2 + \Omega_C^2) + 4\Omega_z \Omega_H \Omega_A^2] = 0, \quad [3.13]$$

where

$$iW = \omega.$$

From equation (3.12) we find that there is always an unstable mode of wave-propagation when $\Omega_J^2 < 0$. The equation [3.13] can have only positive roots or complex conjugate roots. When all the roots are real we have four stable modes of wave-propagation. When W is complex, the equation satisfied by the real part of W is

$$64\,(Re\,W)^6 - 192\,\Omega C\,(Re\,W)^5 + 32\,(7\,\Omega_C^2 + 2\,\Omega_A^2 + \Omega_H^2 + 4\,\Omega_z^2)\,(Re\,W)^4$$
$$- 64\,\Omega_C(2\Omega_C^2 + 2\Omega_A^2 + \Omega_H^2 + 4\Omega_z^2)\,(Re\,W)^3 + 4\,[(\Omega_H^2 - 4\Omega_z^2)^2$$
$$+ 4\Omega_A^2\,(\Omega_H - 2\Omega_z)^2 + \Omega_C^2\,(9\Omega_C^2 + 24\,\Omega_A^2 + 10\,\Omega_H^2 + 40\,\Omega_z^2)]\,(Re\,W)^2$$
$$- 4\,\Omega_C\,[\Omega_C^2\,(\Omega_C^2 + 8\,\Omega_A^2 + 2\,\Omega_H^2 + 8\,\Omega_z^2) + (\Omega_H^2 - 4\Omega_z^2)^2$$
$$+ 4\Omega_A^2\,(\Omega_H - 2\Omega_z)^2]\,(Re\,W) + 4\Omega_A^2\,\Omega_C^2\,[\Omega_C^2 + (\Omega_H - 2\Omega_z)^2] = 0. \qquad [3.14]$$

From equation [3.14] we find that $(Re\,W)$ is never negative. Hence the equation [3.13] has always stable modes of wave-propagation.

Thus the system is unstable when $\Omega_J^2 < 0$ which is just the Jeans criterion.

From the discussion of case I in Part A and of the present case we conclude that the Jeans criterion for gravitational instability is unaffected by finite electrical conductivity when the wave-propagation is along the direction of the magnetic field and rotation even in the presence of Hall-current. This conclusion agrees with the results obtained by Pacholczyk and Stodolkiewicz[7].

Case II: $\eta \neq 0,\ H_z \neq 0,\ H_x = 0,\ \Omega_z = 0,\ \Omega_x \neq 0,\ \Omega_y \neq 0.$

The dispersion relation is given as

$$W^6 - 2\,\Omega_C\,W^5 + W^4\,(\Omega_C^2 + 2\,\Omega_A^2 + \Omega_H^2 + \Omega_J^2 + 4\,\Omega_T^2)$$
$$- 2\,\Omega_C(\Omega_A^2 + \Omega_J^2 + 4\Omega_T^2)\,W^3 + [\Omega_A^4 + \Omega_J^2\,(\Omega_C^2 + 2\,\Omega_A^2 + \Omega_H^2)$$
$$+ 4\,\Omega_T^2\,(\Omega_A^2 + \Omega_H^2 + \Omega_C^2)]\,W^2 - 2\,\Omega_C\,(\Omega_J^2 + 4\,\Omega_T^2)\,\Omega_A^2 + \Omega_J^2\,\Omega_A^4 = 0, \qquad [3.15]$$

where $$i\,W = \omega.$$

When $\Omega_J^2 < 0$, the equation [3.15] has always one negative root and hence there is always one unstable mode. When $\Omega_J^2 > 0$ this equation can have positive roots or complex roots. When there are only real roots there is no unstable mode of propagation. Considering the case when there are complex roots we find that the equation satisfied by (ReW) is of 17th degree and has alternatively positive and negative signs for the coefficients. Hence (ReW) is always positive, consequently the system is stable when $\Omega_J^2 > 0$.

Thus we conclude that when wave-propagation is along the direction of magnetic field and at right angles to direction of rotation, Jeans Criterion remains unaffected even in the presence of Hall-current and finite conductivity.

Case III: $\eta \neq 0,\ H_z = 0,\ H_x \neq 0,\ \Omega_z \neq 0,\ \Omega_x = \Omega_y = 0.$

The dispersion relation reduces to three factors

$$(\omega - \Omega_C) = 0 \tag{3.16}$$

$$(W^2 + 4\,\Omega_z^2) = 0 \tag{3.17}$$

and

$$W^3 - \Omega_C W^2 + (\Omega_B^2 + \Omega_J^2)\, W - \Omega_C \Omega_J^2 = 0. \tag{3.18}$$

The equation [3.16] gives a viscous type of damped wave modified by Hall-current with frequency

$$\omega = \frac{i\eta\, k^2}{1 + c^2k^2/\omega_p^2}.$$

The equation [3·17] gives a stable mode of wave-propagation, propagating with the frequency

$$\omega = 2\,\Omega_z.$$

The equation [3.18] has all three positive real roots or a pair of complex conjugate roots and a positive real root when $\Omega_J^2 > 0$. The real roots will give three stable modes. In the case of complex roots we find that the equation satisfied by (ReW) is

$$8\,(Re\,W)^3 - 8\Omega_C\,(Re\,W)^2 + 2\,(\Omega_C^2 + \Omega_B^2 + \Omega_J^2)\,(Re\,W) - \Omega_C\,\Omega_B^2 = 0.$$

Thus (ReW) is always positive. Hence for $\Omega_J^2 > 0$ equation [3.18] gives three stable modes of wave propagation.

When $\Omega_J^2 < 0$ the equation [3.18] always has a negative root, *i.e.* the system is unstable in the sense of Jeans.

From the discussions of case III in Part A and of the present case we note that finite conductivity of the medium removes the influence of Hall-current. Hence the Jeans criterion for the gravitational instability of a finitely conducting medium remains unaffeoted by the transverse magnetic field, Hall-current and rotation with direction parallel to the direction of wave-propagation.

Case IV: $\eta \neq 0,\ H_z = 0,\ H_x \neq 0,\ \Omega_z = 0,\ \Omega_x \neq 0,\ \Omega_y \neq 0.$

In this case dispersion relation reduces to two factors

$$(W - \Omega_C) = 0, \tag{3.19}$$

which gives

$$\omega = \frac{i\,\eta\,k^2}{1 + c^2 k^2/\omega_p^2},$$

a damped wave,

and

$$W^3 - \Omega_C W^2 + (\Omega_B^2 + \Omega_J^2 + 4\Omega_T^2)\,W - \Omega_C(\Omega_J^2 + 4\Omega_T^2) = 0. \qquad [3.20]$$

When $(\Omega_J^2 + 4\Omega_T^2) < 0$ the equation [3.20] will always have an unstable mode. When $(\Omega_J^2 + 4\Omega_T^2) > 0$ it possesses either all real positive roots or one real positive root and a pair of complex conjugate roots. For a real root the mode of wave-propagation is stable. When W is complex we can show that the equation satisfied by $(Re W)$ is

$$8\,(Re\,W)^3 - 8\,\Omega_C\,(Re\,W)^2 + 2\,(\Omega_C^2 + \Omega_B^2 + \Omega_J^2 + 4\,\Omega_T^2)\,(Re\,W) - \Omega_C\,\Omega_B^2 = 0.$$

Hence (ReW) is always positive. Consequently the system is stable when

$$\Omega_J^2 + 4\,\Omega_T^2 > 0,$$

i.e.,

$$c_s^2 k^2 - 4\,\pi\,G\rho_0 + 4\,\Omega_T^2 > 0. \qquad [3.21]$$

It is interesting to note from equation [3.21] that when magnetic field and direction of rotation is perpendicular to the direction of rotation even for a finitely conducting medium and in the presence of Hall-current if Spitzer's condition

$$\Omega_T^2 > \pi G\,\rho_0$$

is satisfied the system is stable for all wave-numbers,

When $\Omega_T^2 < \pi\,G\,\rho_0$ the system is unstable for the wave-numbers $k < k^*$, the critical wave-number, given by

$$k^* = 2\left(\frac{\pi\,G\,\rho_0 - \Omega_T^2}{c_s^2}\right)^{1/2}.$$

Comparing with the case IV in Part A we find that here again the finite conductivity removes the effect of Hall-current and magnetic field.

4. Conclusion

From the discussion of the dispersion relation for infinitely conducting medium we find that in the presence of the component of magnetic field in the direction of propagation of perturbation Jeans criterion for gravitational instability applies for wave-propagation in the direction along or at right angles to $\vec{\Omega}$ and these results are independent of the Hall-effect.

When the component of magnetic field in the direction of wave-propagation is absent, we find that due to the inclusion of Hall-current the criterion for gravitational instability differs from that of Jeans. However, in both the cases, when wave-propagation is along or perpendicular to the direction of rotation, the wave-length giving the linear dimension of galaxies for the considered system is of the same order as obtained by Jeans. This is probably due to the fact that for system under consideration the plasma frequency ω_p is much larger than other characteristic frequencies of the system thereby removing the effect of Hall-current as seen from the induction equation [2.6]. Thus the critical wave-numbers k^* obtained in cases II and III for large ω_p reduces to the values given by

$$k^* = \left(\frac{4\pi G \rho_0}{c_s^2 + V_x^2}\right)^{1/2} \quad \text{and} \quad k^* = \left[\frac{4(\pi G \rho_0 - \Omega_T^2)}{(c_s^2 + V_x^2)}\right]^{1/2}$$

respectively.

These are just the critical wave numbers obtained by Jeans where the hydrostatic part of the magnetic pressure is added to the ordinary hydrostatic pressure of the fluid due to the presence of the transverse magnetic field. This explains the order of the wave-length λ_H to be of the same as λ_J. The other fact which can be explained from the above discussion is that if wave-propagation is perpendicular to $\vec{\Omega}$, the system is stable for all wave-numbers if Spitzer's condition is satisfied even in the presence of Hall-current.

The Jeans criterion for gravitational instability remains unaffected when the medium is finitely conducting in the presence of a uniform magnetic field along the direction of the propagation. In the case of transverse magnetic field the effect of Hall-current is removed by the finite conductivity and Jeans criterion holds for the wave-propagation in any direction, except when it is at right angles to $\vec{\Omega}$; in the latter case the system is always stable if Spitzer's condition is satisfied. We also note that in the present form of the induction equation [2.6] the effect of Hall-current can approximately be treated as another dissipative mechanism for magnetic lines of force and hence we expect that except in some particular cases the Hall-effect should not affect the stability criterion. These results are in accordance with the general principle of Jeans[1] that any dissipative mechanism will not affect the instability criterion.

Also we find that the system does not possess any mechanism of overstability as has been indicated by Kalra and Talwar[12].

In a subsequent paper we have considered the effect of viscosity and the combined effects of viscosity and finite conductivity in the presence of Hall-current. We find that results are substantially unaltered.

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