A POTENTIAL ANALOGUE METHOD FOR STUDY OF THE FIELDS AND THE CHARACTERISTIC IMPEDANCE OF BAR LINES

BY K. C. GUPTA

(*Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore*)

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ABSTRACT

A resistance analogue method for studying the characteristic impedance and the field distributions of bar line type slow wave structures is described. The field distributions for the travelling slow wave are obtained by combining those for two standing waves.

INTRODUCTION

The electric and magnetic fields associated with the bar line type slow wave structures (the interdigital line, the meandre line, the ladder line, etc.) are assumed to be stationary in a plane normal to the direction of bars[1]. This suggests the use of potential analogue techniques for the study of these two dimentional fields.

Arnaud[2] has described an analogue method for determining the capacitances between various bars constituting the structure. These capacitance values are employed for the determination of the characteristic impedance of the structure. He has, further, described a method for obtaining the travelling E_z field as

$$E_z(z)_{progr} = \sum_{n=-\infty}^{+\infty} \frac{\partial V(z-np)}{\partial z} e^{-jn\phi} \qquad [1]$$

where $V(z)$ is the potential distribution obtained by connecting the zeroth bar at potential + 1, the others being at ground potential and p is the pitch of the structure.

In this paper, it is shown that the field of a travelling slow wave may be obtained by combining those of two standing waves.

THEORY OF THE METHOD

The distribution of the r.f. fields along a periodic structure is characterised by the voltages on various bars as given by the relation

$$V_n = V_0 e^{-jn\phi} \qquad [2]$$

where V_n V_0 are the voltages on the nth and the zeroth bars respectively. As suggested by Arnaud[2], this condition can be satisfied for applying to successive bars alternating voltages proportional to $\cdots$, $e^{-2j\phi}$, $e^{-j\phi}$, 1, $e^{+j\phi}$, $e^{+2j\phi}$, $\cdots$, etc.

For the method described here equation [2] is written as

$$V_n = V_0 (\cos n\phi + j \sin n\phi) = V_{n1} + jV_{n2} \qquad [3]$$

Because of the linearity of Laplace's equation, the complete solution of V_n can be obtained by combining the contributions of V_{n1} and V_{n2}. The potential distributions corresponding to V_{n1} and V_{n2} are obtained by connecting to the different bars d.c. voltages having magnitudes given by $V_0 \cos n\phi$ and $V_0 \sin n\phi$ respectively as shown in Fig. 1(a) and 1(b).

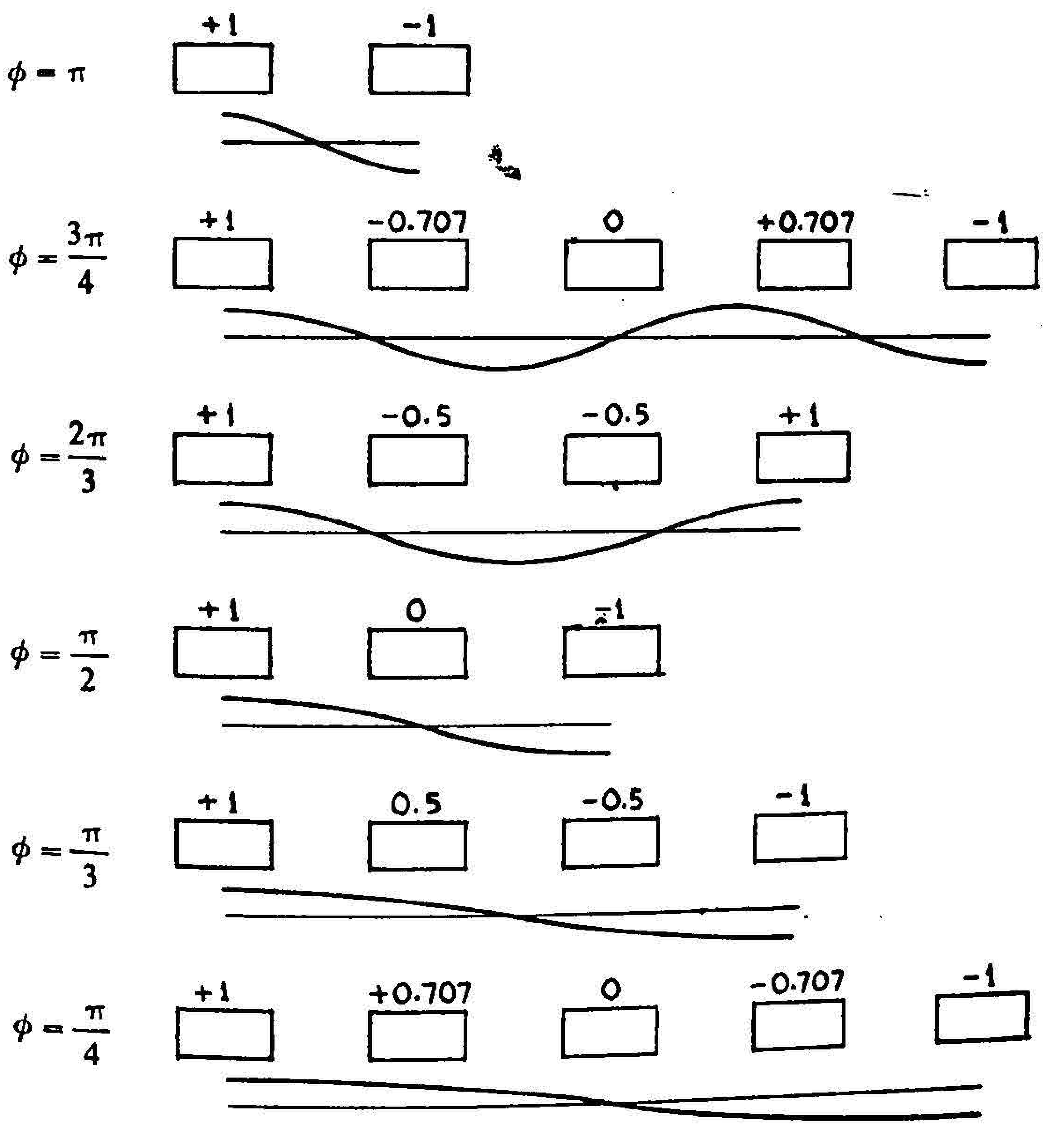

FIG. 1(*a*)

Potential distributions for different modes corresponding to the first term of eq. [3].

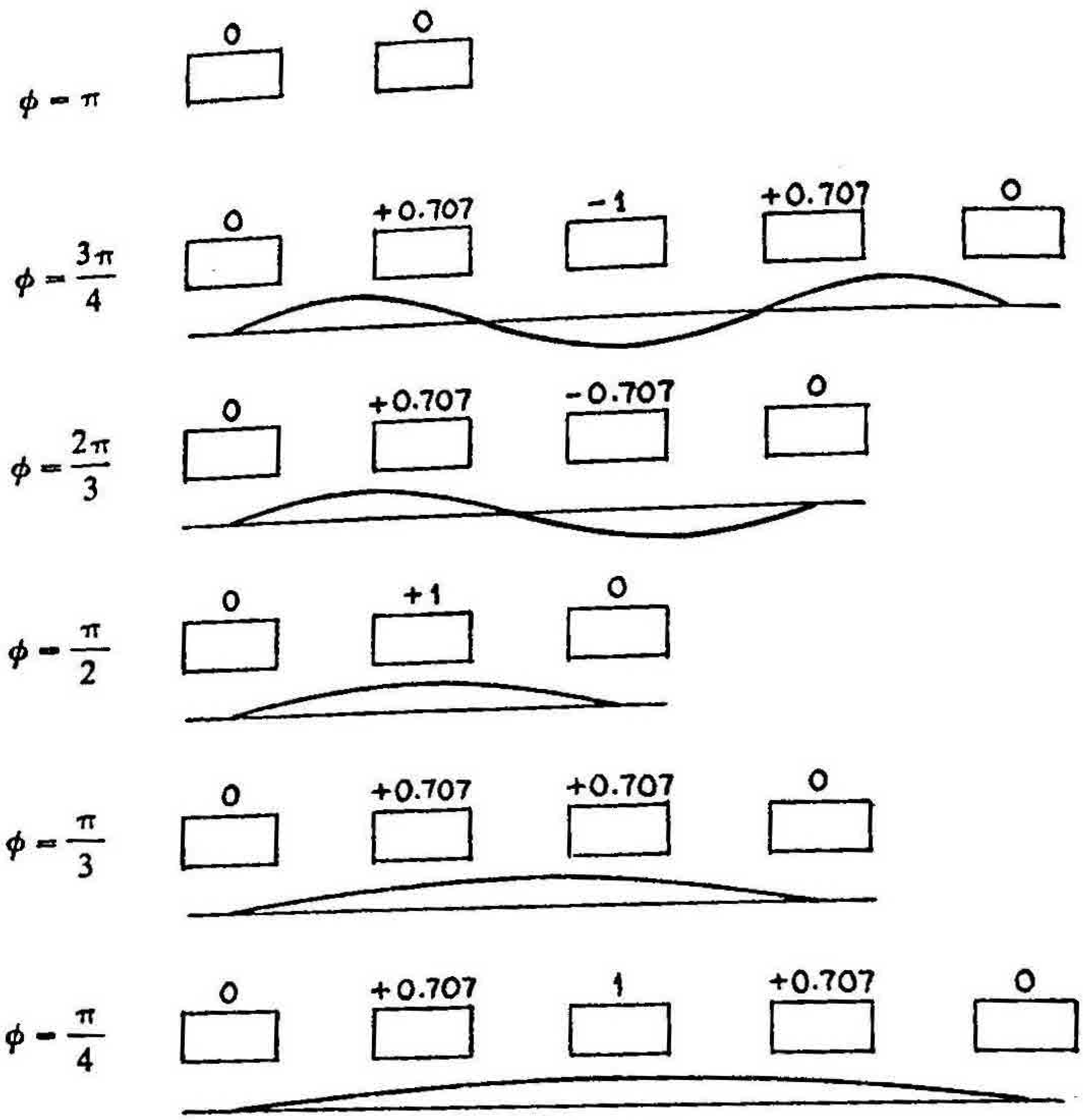

FIG. I (*b*)

Potential distributions for different modes corresponding to the second term of eq. [3]

The characteristic admittance of the nth bar for a periodic array of bars may be written as

$$Y(\phi) = c \sum_m \gamma_{m,n} \, e^{j(m-n)\phi} \tag{4}$$

where c is the velocity of light and $\gamma_{m,n}$ is the capacity coefficient between the mth and the nth bars. On the analogue $Y(\phi)$ may be computed by measuring the current flowing in the nth bar when the voltages on the various bars are in accordance with relation [3].

If i_{n1} and i_{n2} denote the values of current of the nth bar for distributions corresponding to the two terms on the right hand side of equation [3] and

$$|i_n| = (i_{n1}^2 + i_{n2}^2)^{\frac{1}{2}} \tag{5}$$

$$|V_n| = (V_{n1}^2 + V_{n2}^2)^{\frac{1}{2}} \tag{6}$$

and the characteristic admittance may be written as

$$Y(\phi) = | i_n | / | V_n | \tag{7}$$

Experimental Details

Measurements of the E_z field and the characteristic admittance have been carried out for the bar line structures shown in figures II (a, b and c). These structures consist of infinitely thin tapes of equal width placed parallel to each other in a plane with spacings as shown. These structures have been selected because for these cases, it is possible to estimate the field variations in the gaps theoretically[3, 4]. In the case of the structure $T-1$, with gap equal to

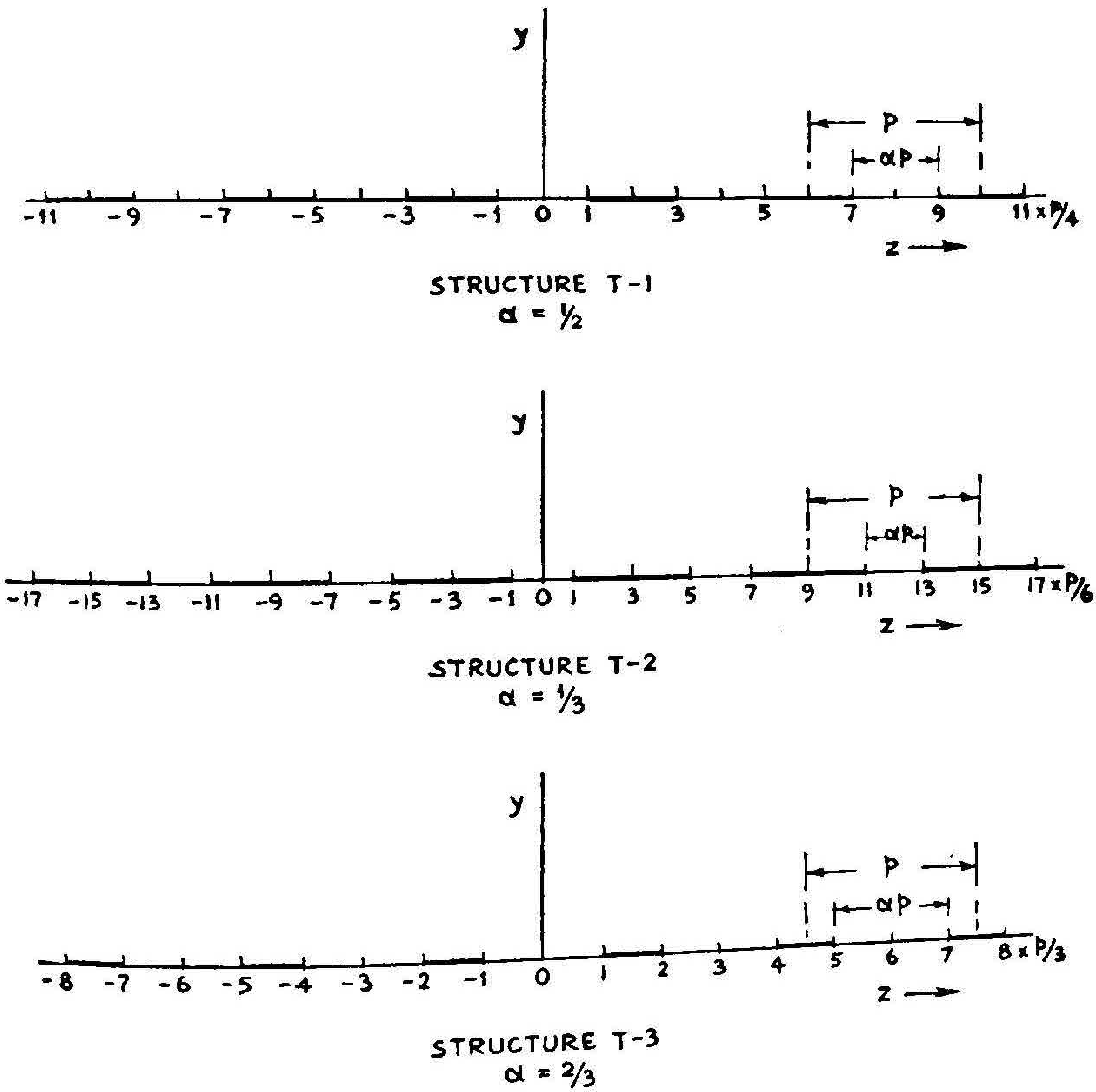

Fig. II

Tape structure.

the width of tapes, the theoretical values of characteristic admittance[3, 4] have been evaluated according to the formulæ

$$Y(\phi) = \sqrt{\frac{\epsilon_0}{\mu_0}} \cdot \left| 4 \sin \frac{\phi}{2} \right| \qquad [8]$$

The potential analogue used for measurement is a resistance net work employing approximately 400 resistors (3.3 K, 1%) (Figure III). The voltage measurements have been carried out by balancing the voltage to be measured against the output of a standard voltage divider with the help af a precision galvanometer. The arrangements for the measurements of the characteristic admittance and the potential distributions are shown in Figure IV and V respectively. Theoretically, the analogue should extend infinitely in the y direction but this dimention has been limited by the use of a terminating network[5] which is shown in Figure VI.

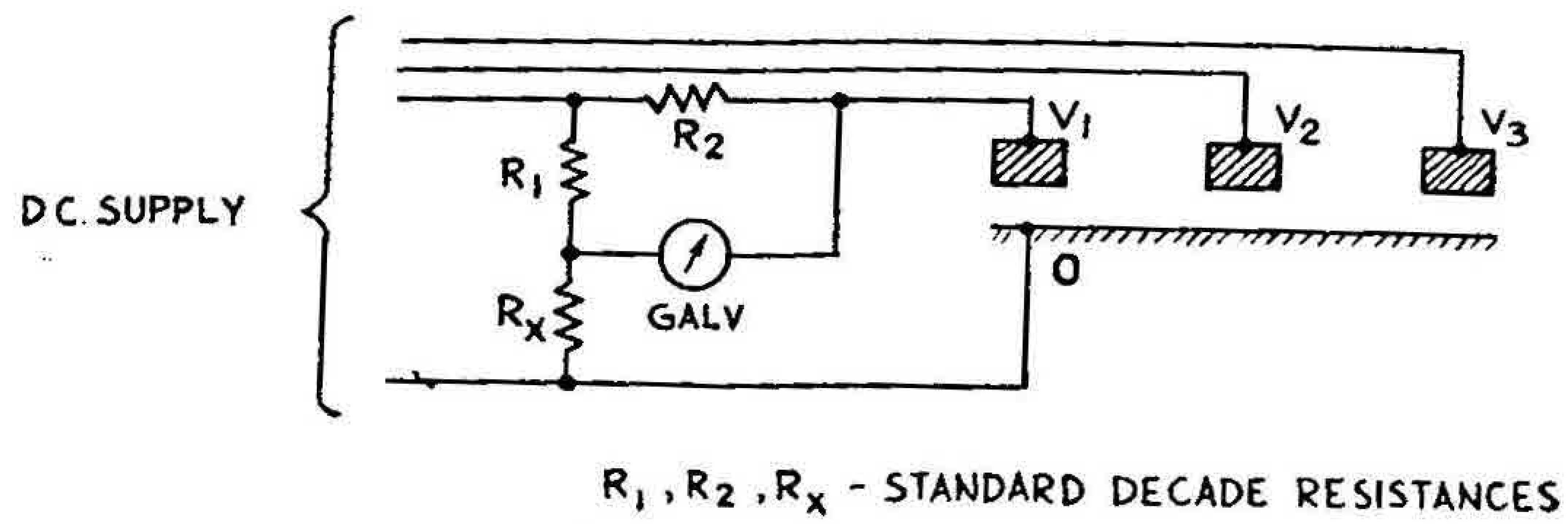

FIG. III

Resistance network with connections made for π Mode of structure T-1.

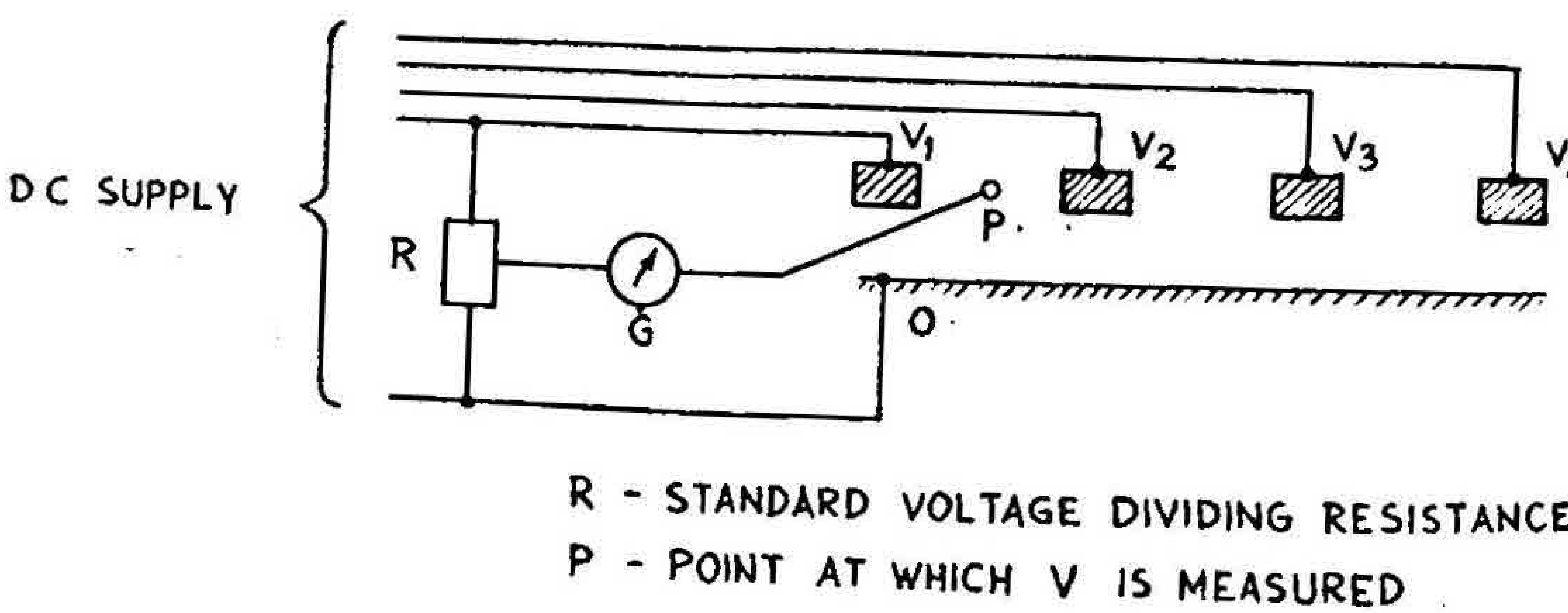

FIG. IV

Arrangement for measurement of characteristic impedance.

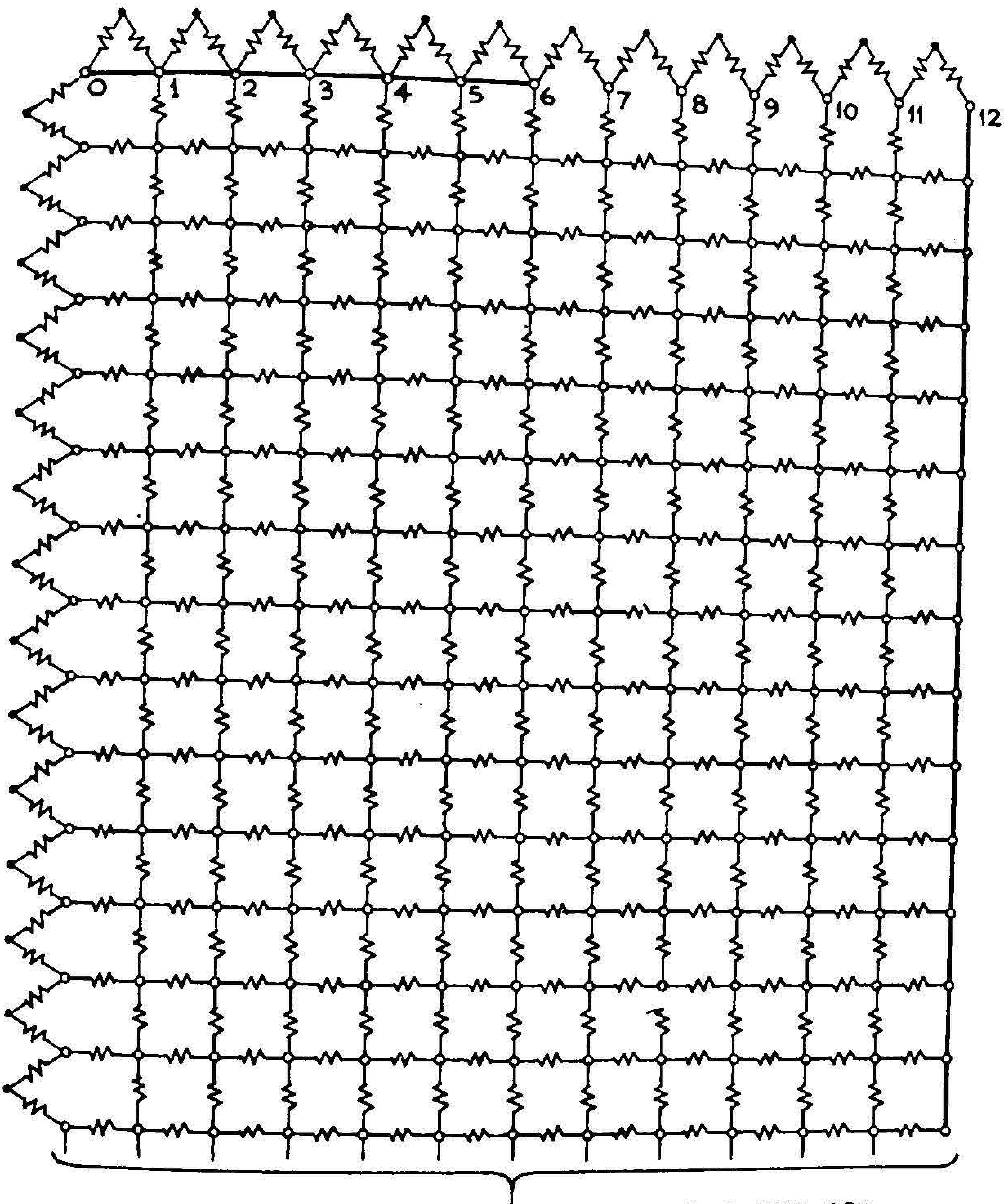

FIG. V

Potential distribution measurement.

The measured values of E_z in the gap are compared with the theoretically calculated values of E_z for the π and $\pi/2$ modes in Figures VII, VIII and IX. The theoretical relation used for calculating E_z is given below [equation 9].

$$E_z(0, z) = \frac{\exp\left[j(\pi - \phi) z/p\right]}{(p/\pi)\left[\sin^2(\pi\alpha/2) - \sin^2(\pi_z/p)\right]^{1/2}} \quad [9]$$

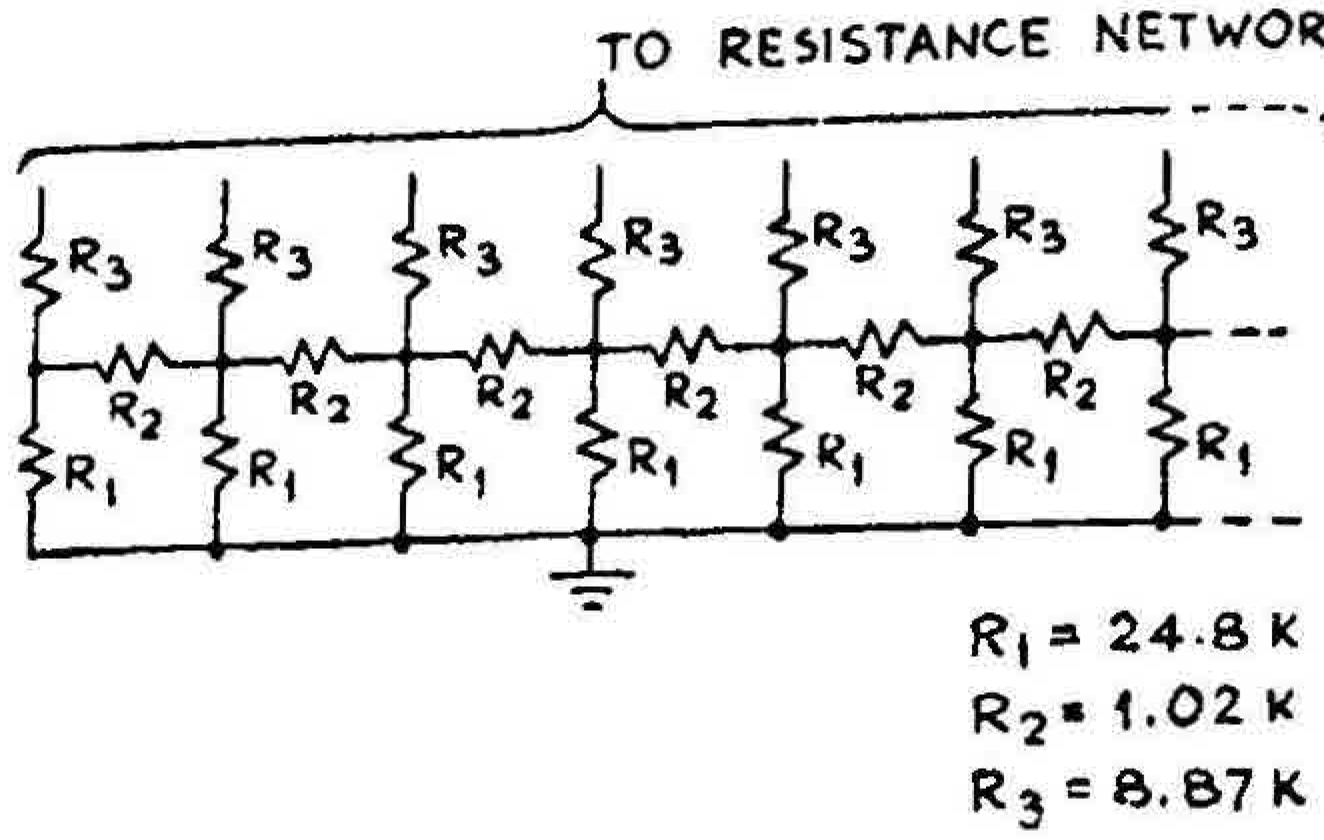

FIG. VI

Terminating network.

where p = pitch, $a\,p$ = gap width and ϕ = phase difference between consecutive bars, the origin $z = 0$ being at the middle point between two adjacent bars[1].

Also the measured values of E_z for $y \neq 0$ near the structure are shown in Figures VII, VIII and IX, because it is in this region that the electron beam interacts with the field of the slow wave. The measured values of E_y for the structure T-1 are shown in Figures XI (a) and X (b).

The measured and theoretical results obtained for the characteristic admittances are presented in Table I.

TABLE I

Values of Characteristic Admittance

Structure	$Y(\phi = \pi)$	$Y(\phi = \pi/2)$
$T-1$ (measured)	4.20	3.16
$T-1$ (theoretical)	4.00	2.88
$T-2$ (measured)	5.40	3.98
$T-3$ (measured)	3.30	2.57

CONCLUDING REMARKS

The method described by Arnaud[2] employs the the summation of series in case of both the characteristic admittance [Eq. 4] and the travelling E_z field [Eq. 1]. Compared to this, the method reported here does not involve expressing the field in the form of series.

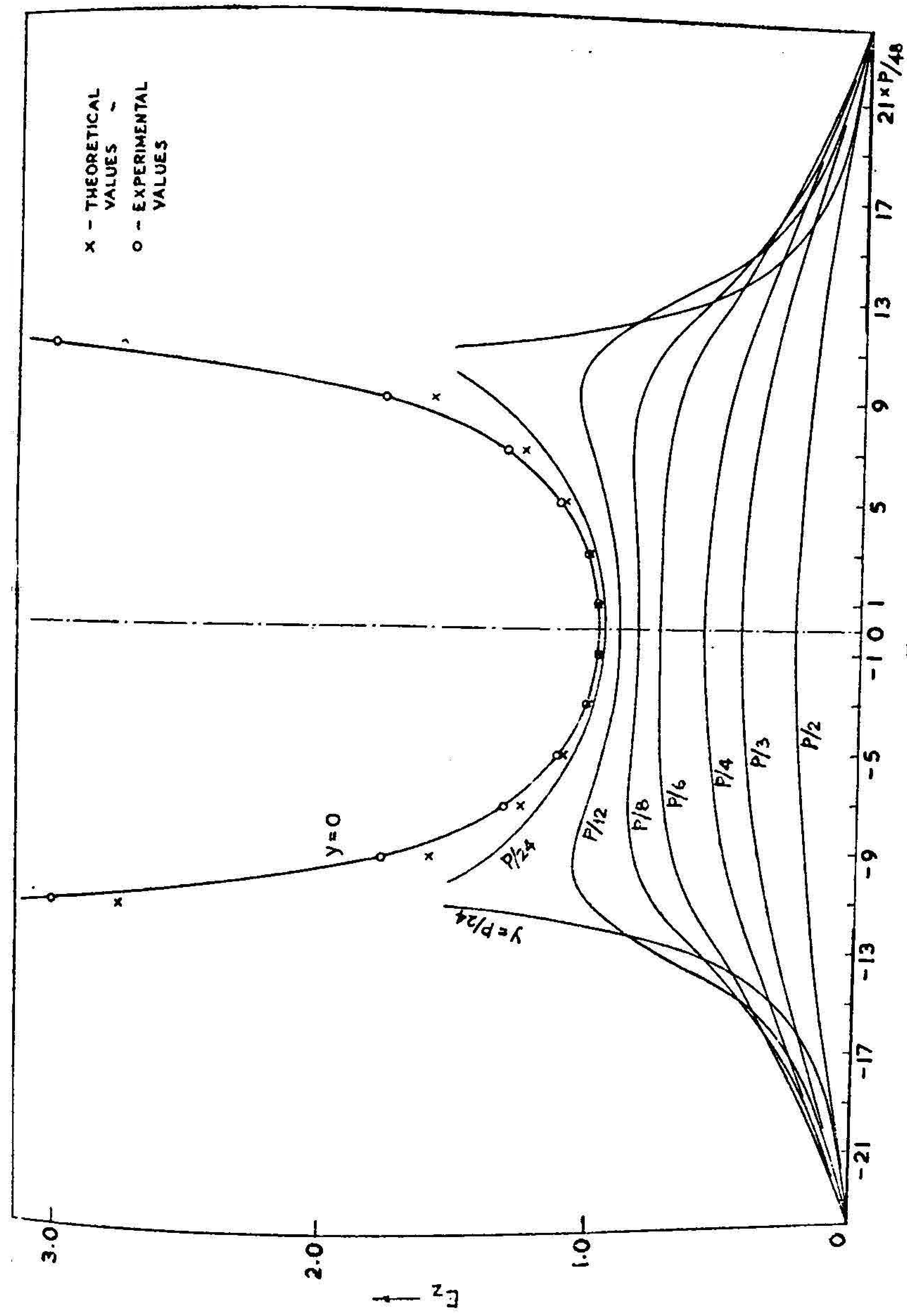

FIG. VII (*a*)
π Mode E_z field distribution for structure *T*-1

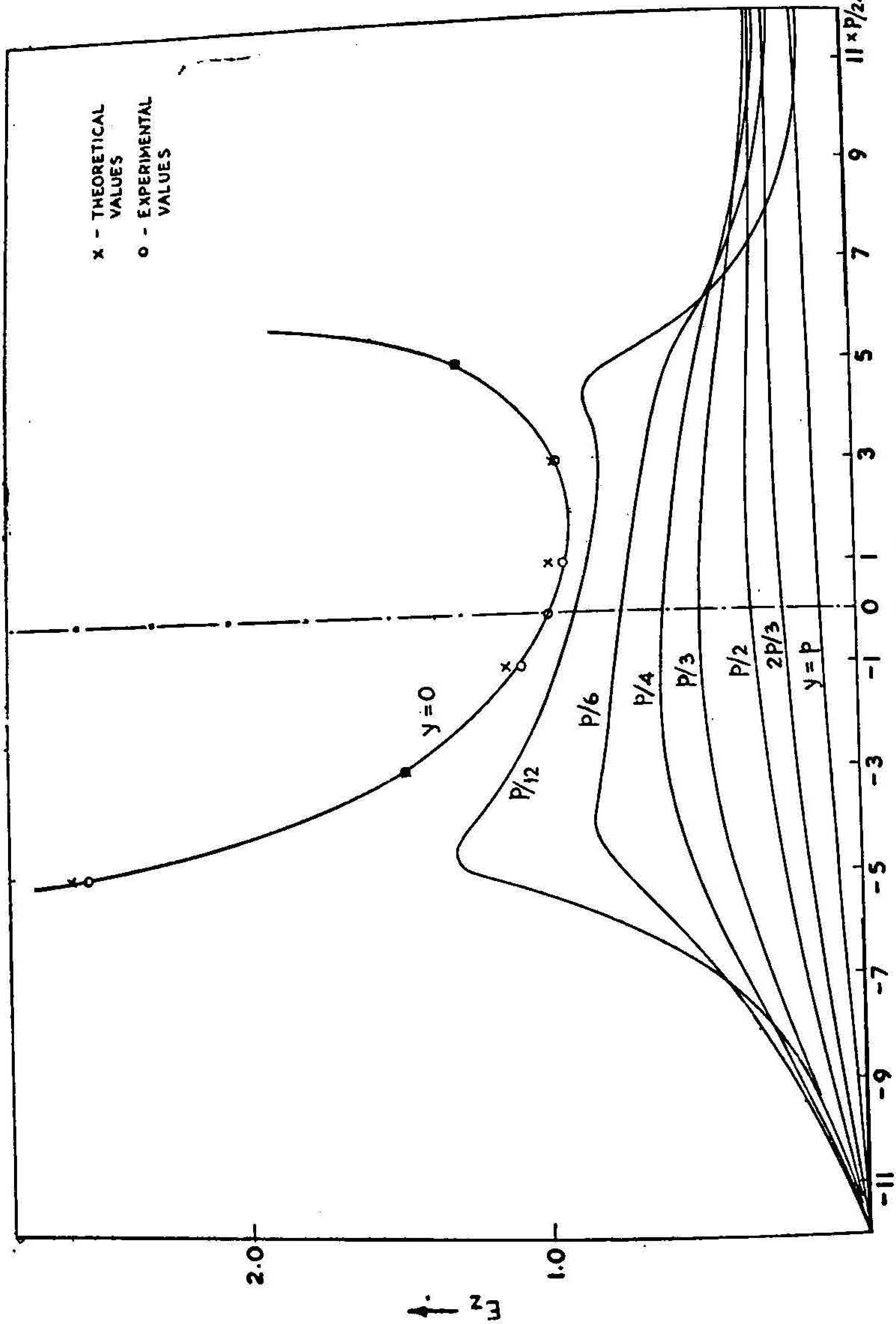

FIG. VII (*b*)

$\frac{\pi}{2}$ Mode E_z field distribution for structure *T*-1.

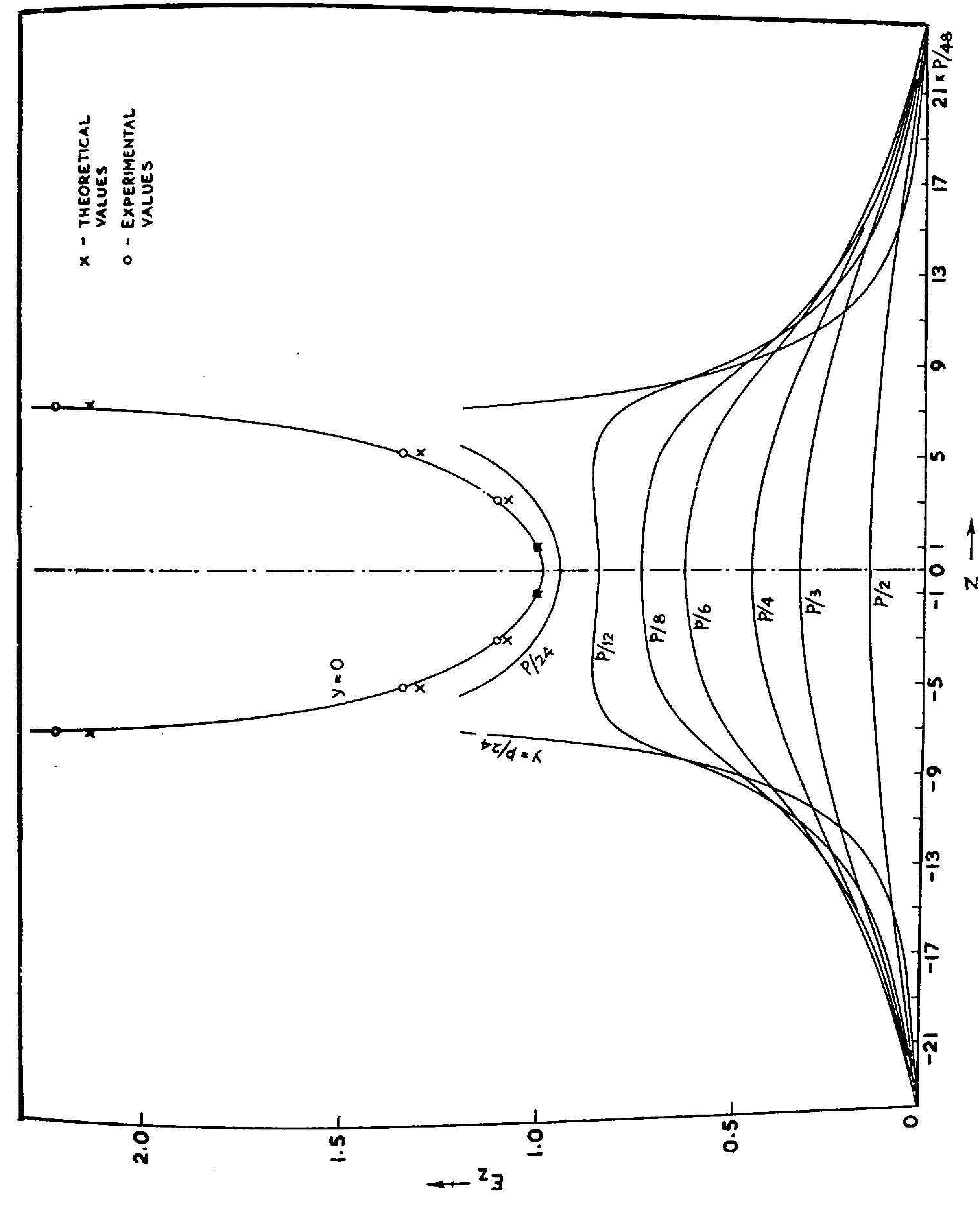

FIG. VIII (*a*)
π Mode E_z field distribution for structure *T*-2.

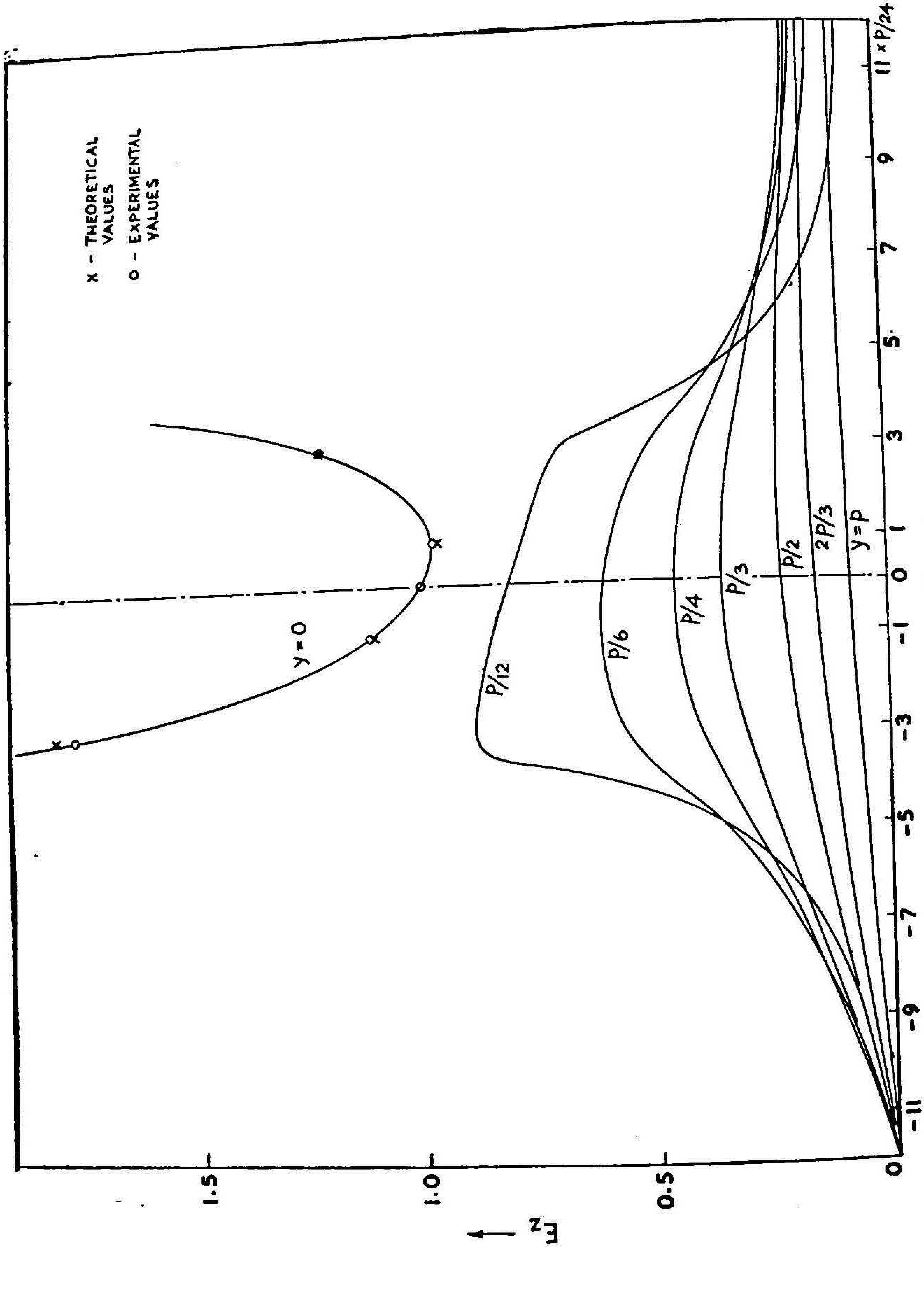

FIG. VIII (b)
π/2 Mode E_z field distribution for structure T-2.

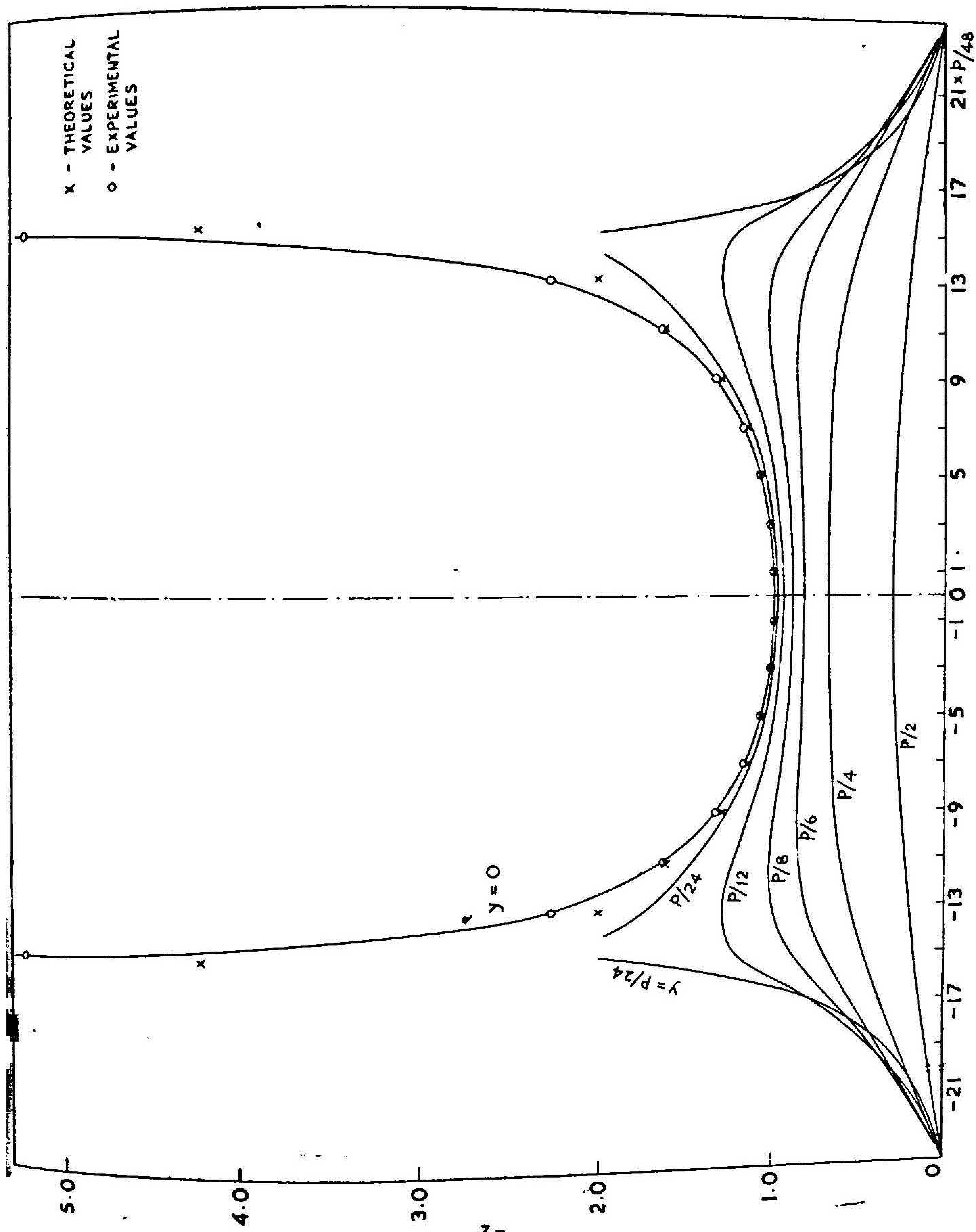

FIG. IX (*a*)
π Mode E_z field distribution for structure *T*-3.

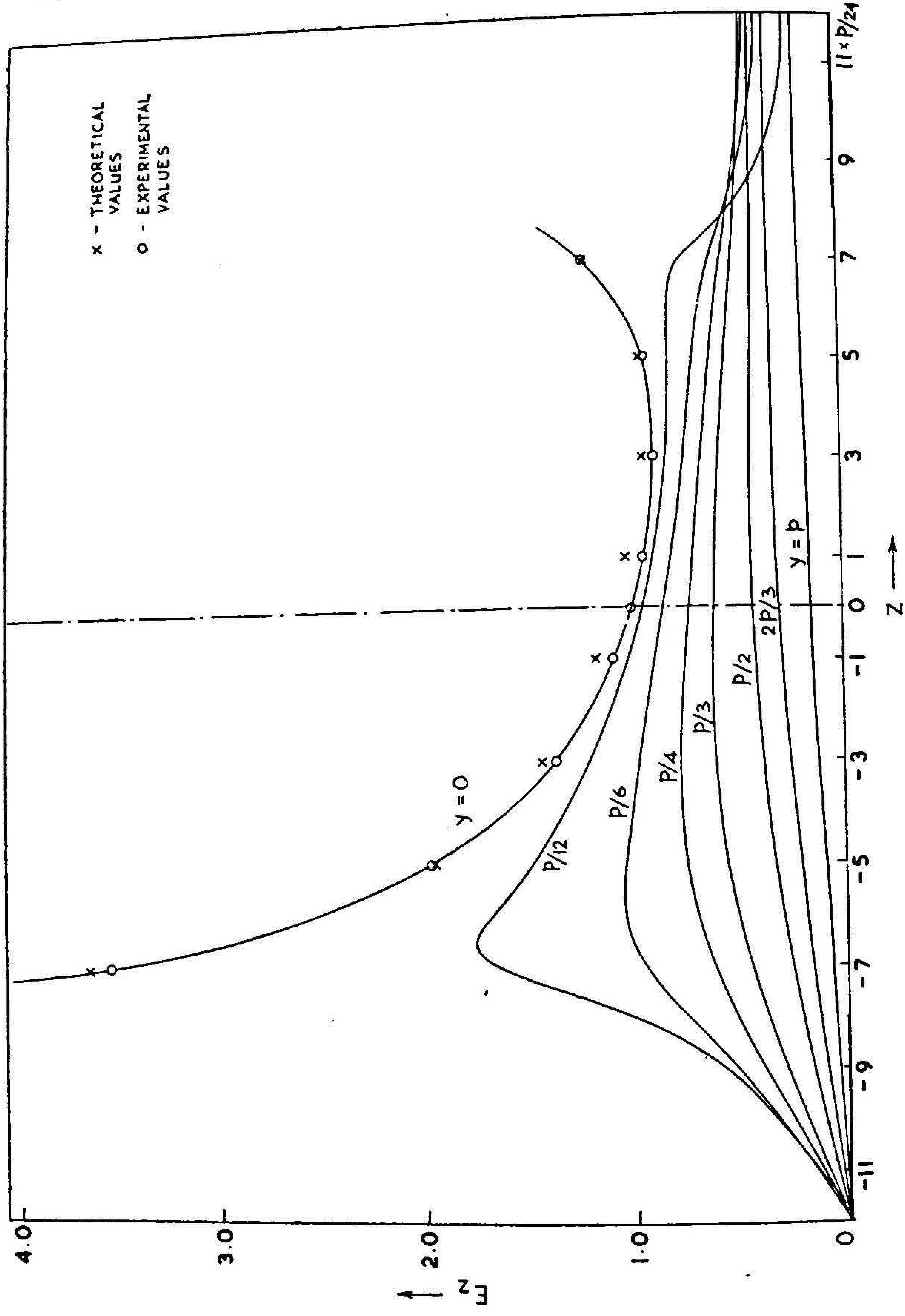

FIG. IX (*b*)

$\pi/2$ Mode E_z field distribution for structure *T*-3.

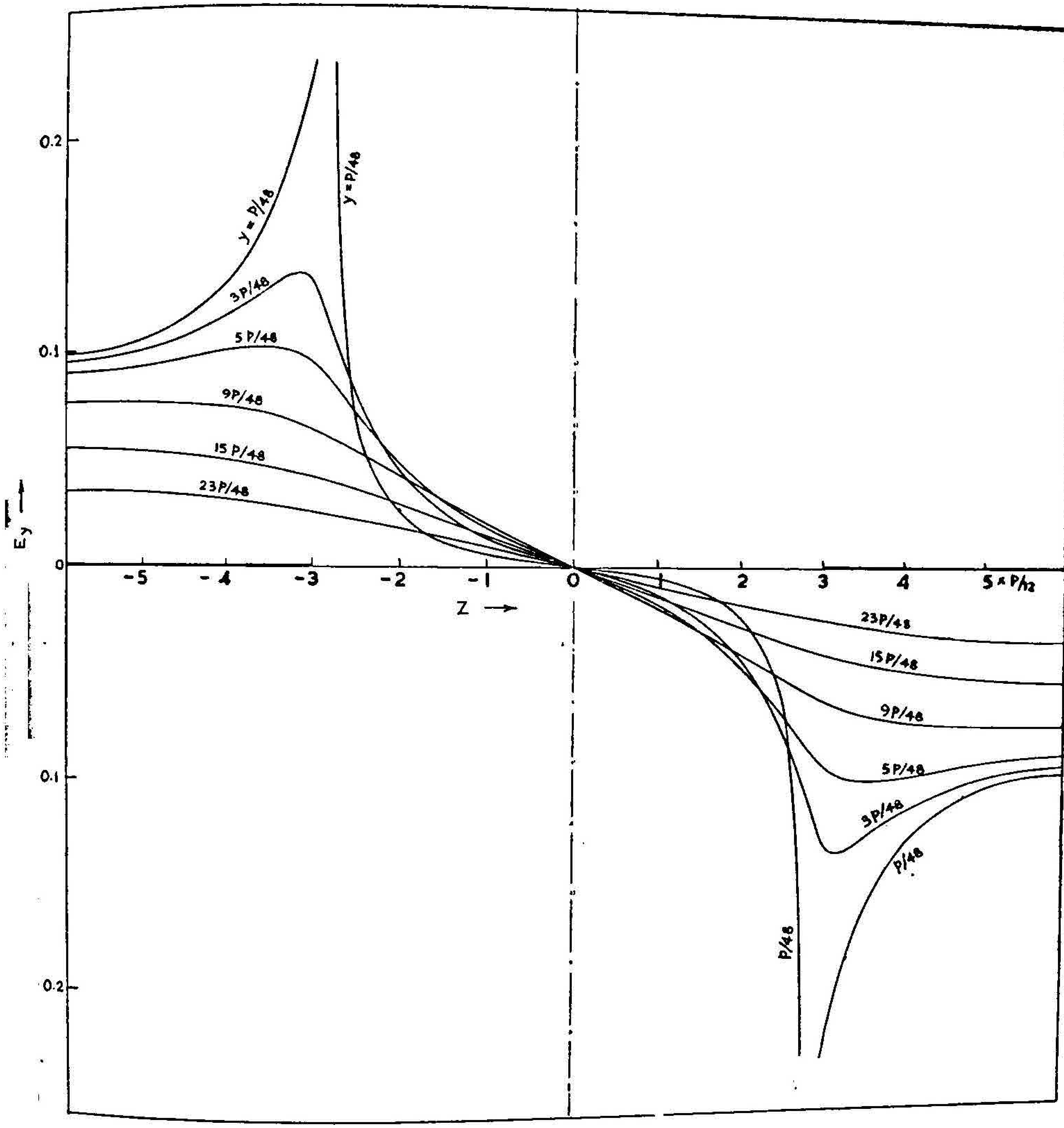

FIG. X (*a*)

π Mode E_y field distribution for structure *T*-1.

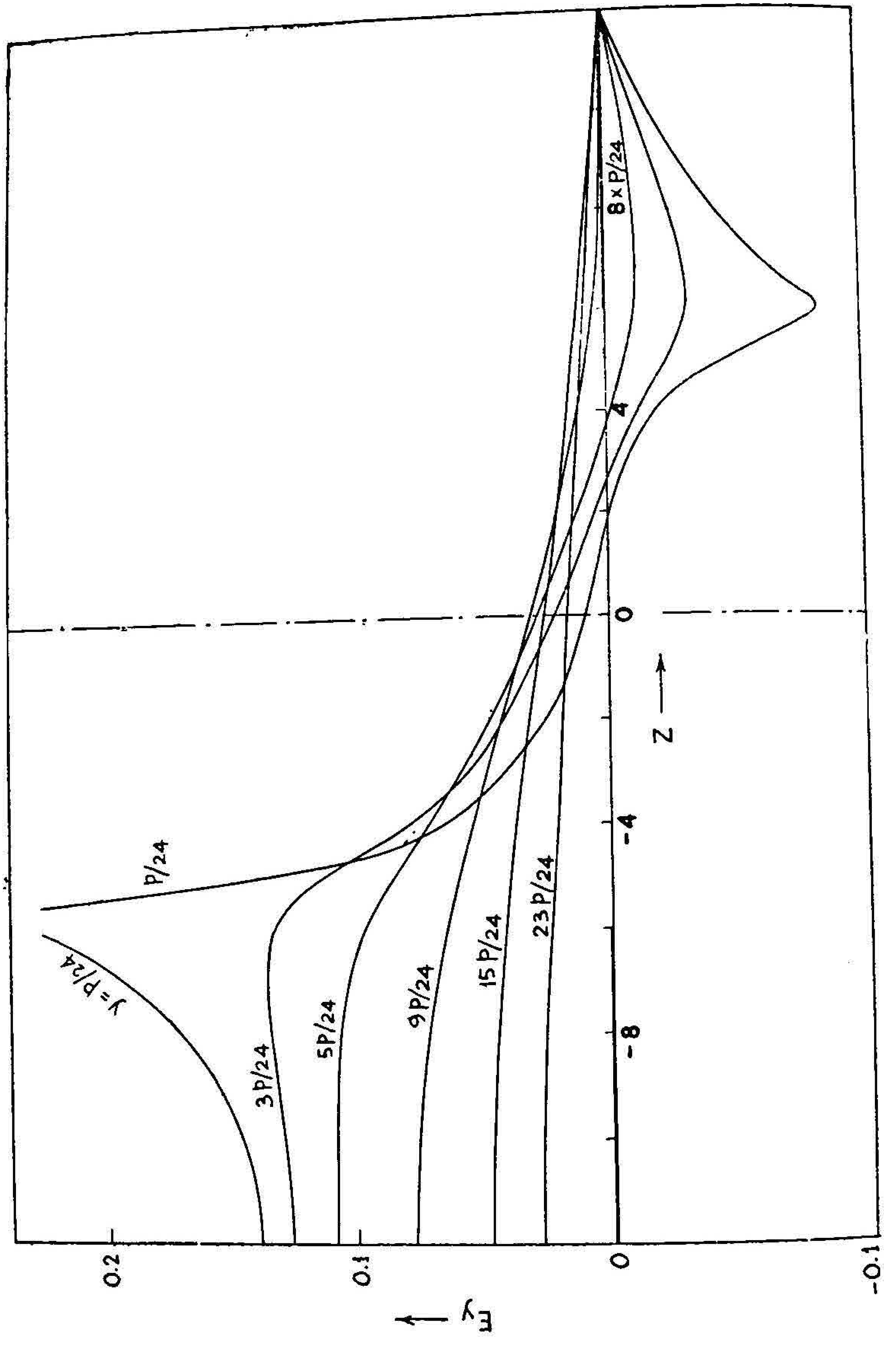

FIG. X (*b*)
$\pi/2$ Mode E_y field distribution for structure *T*-1.

Moreover, since the potential distribution on the analogue in the case of the method described in this paper is periodic, the dimension of the analogue in the z direction may be terminated at half the delayed wavelength or its multiples whichever corresponds to an integral number of structure cells. This results in the economy in the dimension of the analogue in cases where ϕ is a simple fraction of π as indicated in Fig. I.

It is expected that the method described here would prove helpful in study of bar lines.

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