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Scattering of surface waves by a submerged circular cylinder in a fluid of finite depth

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Abstract

When a wave train is normally incident on a submerged infinitely long cylinder with horizontal axis in a fluid of infinite depth, it is well known that it passes over the cylinder with a change of phase but without any change of amplitude and experiences no reflection. But when the depth of fluid is taken into account it is shown here that the normally incident wave train does experience reflection, and the reflection coefficient can be asymptotically evaluated for large depth 'h' of the fluid as an algebraic series in powers of a/h, starting with $(a/h)^2$, 'a' being the radius of the cylinder. For particular values of the wave number and depth of the axis of the cylinder below the mean free surface, numerical values of the reflection coefficient are obtained for different values of h/a.

Key words: Scattering, submerged cylinder, reflection and transmission coefficients, Green's function, fluid of finite depth.

1. Introduction

Most of the problems associated with surface wave scattering by obstacles present in a fluid of either infinite or finite depth do not admit of an exact solution except perhaps when the obstacles are in the form of fixed vertical barriers (*cf.* Ursell[1]), although an integral equation formulation is always possible by an appropriate use of the Green's integral theorem in the fluid region. For normal incidence of surface waves on a fixed vertical barrier involving fluid of infinite depth the corresponding integral equation has an exact solution (*cf.* Goswami[2] and others). In general, the integral equations obtained in these problems cannot be solved in closed forms and can be solved only approximately by appropriate techniques involving some non-dimensional parameter. A few problems involving finite depth of fluid have been considered by Mei and Black[3], Packham and Williams[4], and others. Macaskill[5] has given a numerical method which encompasses different types of vertical barriers in fluids of both infinite and finite depth.

A train of surface waves normally incident on a completely submerged infinitely long horizontal circular cylinder in fluid of infinite depth is known to experience no reflection by the cylinder (*cf.* Dean[6], Ursell[7], Levine[8]). In the present paper this problem is generalised to include the case of finite depth of fluid, and it is shown that the normally incident surface wave now does experience reflection by the submerged cylinder. By an appropriate use of Green's integral theorem, the problem is reduced to the solution of an integral equation of the second kind in the scattered potential on the contour of the cylinder. When this potential is replaced by its equivalent general Fourier series in the angular co-ordinate with origin at the centre of the circular cross-section, two linear infinite systems are obtained. The reflection coefficient (complex) is seen to vanish identically when these two linear systems become identical which happens only when the fluid depth is infinite. These two linear systems can be solved approximately. As a first approximation, all the unknown coefficients except the first ones in these systems are equated to zero, and then approximations are made again for large h/a, h/f. 'a' being the radius of the cylinder and 'f' being the depth of its axis below the mean free surface. It is then seen that the reflection coefficient can be asymptotically expressed as an algebraic series in powers of a/h commencing with $(a/h)^2$. To illustrate the method, numerical values of the reflection coefficient (real) are calculated for different values of h/a and fixed Ka and a/f, 'K' being the wave number.

2. Formulation of the problem

A rectangular cartesian co-ordinate system is used with origin at the centre of a fixed submerged circular cylinder the generators of which are horizontal and oriented along the z-axis, the y-axis is taken vertically downwards and the x-axis is horizontal. Let 'a' be the radius of the cylinder, 'f' ($> a$) the depth of the axis of the cylinder below the mean free surface, and 'h' ($>> f$) the depth of the fluid. The fluid is assumed to be ideal and under the action of gravity only, and the effect of viscosity is neglected. A harmonically time dependent train of surface waves is normally incident on the fixed cylinder from the negative x direction. The problem is two-dimensional in nature and is independent of z. The motion is irrotational and can be described by a velocity potential. Let the incident wave field be represented by $\mathrm{Re}\{\phi_0(x, y)e^{-i\sigma t}\}$ where σ is the angular frequency. Then within the framework of linearised theory of fluid. the velocity potential can be represented by $\mathrm{Re}\{\Phi(x, y)\, e^{-i\sigma t}\}$ where the time-independent complex valued function $\Phi(x, y)$ satisfies the Laplace's equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \Phi(x, y) = 0 \quad \text{in the fluid region,} \tag{2.1}$$

with the boundary conditions

$$\frac{\partial \Phi}{\partial y} + K\Phi = 0 \quad \text{on } y = -f,\ |x| < \infty, \tag{2.2}$$

$$\frac{\partial \Phi}{\partial y} = 0 \quad \text{on } y = h - f,\ |x| < \infty, \tag{2.3}$$

$$\frac{\partial \Phi}{\partial r} = 0 \quad \text{on } r = a \tag{2.4}$$

where $x = r \sin\theta$, $y = r\cos\theta$ $(-\pi \leq \theta \leq \pi)$, $K = \sigma^2/g$, g being the acceleration due to gravity. (2.2) is the linearised boundary condition on the mean free surface, (2.3) and (2.4) are the conditions of zero normal velocity on the fluid bottom and the surface of the cylinder respectively. If R and T denote respectively the complex reflection and transmission coefficients corresponding to the incident wave field

$$\phi_0(x, y) = \frac{\cosh k_0 (h - y)}{\cosh k_0 h}\, e^{ik_0 x} \tag{2.5}$$

k_0 being the positive real root of the transcendental equation

$$k \tanh kh = K \tag{2.6}$$

then the far field behaviour of the velocity potential $\Phi(x, y)$ is given by

$$\Phi(x, y) \to T\phi_0(x, y) \text{ as } x \to \infty \tag{2.7}$$

and

$$\Phi(x, y) \to \phi_0(x, y) + R\,\phi_0(-x, y) \text{ as } x \to -\infty \tag{2.8}$$

We choose h to be sufficiently large so as to assume the difference between k_0 and K to be exponentially small.

3. Reduction to two infinitely linear systems

By an appropriate use of Green's integral theorem, at any fluid point (ξ, η) the scattered potential defined by $\phi(\xi, \eta) = \Phi(\xi, \eta) - \phi_0(\xi, \eta)$ can be obtained as

$$2\pi\phi(\xi, \eta) = -\int_{-\pi}^{\pi} \phi(\theta) \left\langle a\frac{\partial}{\partial r} G(x, y, \xi, \eta)\right\rangle d\theta - \int_{-\pi}^{\pi} G(a\sin\theta,\ a\cos\theta,\ \xi,\ \eta)\left\langle a\frac{\partial}{\partial r}\phi_0(x, y)\right\rangle d\theta \tag{3.1}$$

where $\phi(\theta)$ is the unknown scattered potential function on the contour of the cylinder, $G(x, y\,;\,\xi, \eta)$ is the Green's function satisfying (2.1) except at (ξ, η) where it has a logarithmic singularity, with boundary conditions (2.2) and (2.3) and the additional condition that it behaves as a diverging wave at infinity, and the angular bracket denotes the values at $r = a$. Following Thorne[9] $G(x, y\,;\,\xi, \eta)$ can be obtained as

$$G(x, y\,;\,\xi, \eta) = \log\frac{\rho}{\rho'} - 2\int_0^\infty \frac{e^{-kh}\sinh ky_1 \sinh k\eta_1}{k\cosh kh}\cos k(x-\xi)\,dk$$

$$-2\int_0^\infty \frac{\cosh k(h-y_1)\cosh k(h-\eta_1)}{(k\sinh kh - K\cosh kh)\cosh kh}\cos k(x-\xi)\,dk \tag{3.2}$$

where

$$y_1 = y + f,\ \eta_1 = \eta + f,\ \rho = \{(x-\xi)^2 + (y-\eta)^2\}^{1/2},\ \rho' = \{(x-\xi)^2 + (y+\eta_1)^2\}^{1/2},$$

and the contour of integration in the last integral is indented below the simple pole at $k = k_0$ so as to take into account the diverging type behaviour of $G(x, y\,;\,\xi, \eta)$ as $|x - \xi| \to \infty$. If G_∞ denotes the Green's function for infinite depth of fluid, then it can be shown that

$$G(x, y\,;\,\xi, \eta) = G_\infty(x, y\,;\,\xi, \eta) + G_D(x, y\,;\,\xi, \eta)$$

where

$$G_\infty(x, y\,;\,\xi, \eta) = \log\frac{\rho}{\rho'} - 2\int_0^\infty \frac{e^{-k(y_1+\eta_1)}}{k - K}\cos k(x-\xi)\,dk \tag{3.3}$$

and

$$G_D(x, y\,;\,\xi, \eta)$$

$$= -2\int_0^\infty \frac{e^{-kh}(K\sinh ky_1 - k\cosh ky_1)(K\sinh k\eta_1 - k\cosh k\eta_1)}{k(k-K)(k\sinh kh - K\cosh kh)} \tag{3.4}$$

$$\cdot\cos k(x-\xi)\,dk,$$

the contour of integration in (3.3) being indented below the pole at $k = K$ and that in (3.4) below the poles at $k = K$ and $k = k_0$.

By an use of Green's integral theorem to $\phi(x, y)$ and $G(x, y\,;\,a\sin\alpha, a\cos\alpha)$ in the fluid region with a small indentation at the point $(a\sin\alpha, a\cos\alpha)$ on the circle

$r = a$, $\phi(\theta)$ of (3.1) can be shown to satisfy an integral equation of the second kind given by

$$\pi\phi(\alpha) + \int_{-\pi}^{\pi} \phi(\theta)\left\langle a\frac{\partial}{\partial r} G(r\sin\theta,\ r\cos\theta\ ;\ a\sin\alpha, a\cos\alpha)\right\rangle d\theta$$

$$= -\int_{-\pi}^{\pi} \left\langle a\frac{\partial}{\partial r}\phi_0(x,y)\right\rangle G(\theta\ ;\ \alpha)\, d\theta,\ -\pi < \alpha < \pi \tag{3.5}$$

where $G(\theta;\ \alpha) \equiv G(a\sin\theta,\ a\cos\theta;\ a\sin\alpha,\ a\cos\alpha)$. To solve the integral equation (3.5) the Fourier series expansion of $\phi(\theta)$ given by

$$\phi(\theta) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\theta + b_n \sin n\theta),\ -\pi < \theta < \pi \tag{3.6}$$

is substituted in (3.5) then multiplying both sides of (3.5) by cos $s\alpha$, sin $s\alpha$ respectively and integrating with respect to α from $-\pi$ to π the following two infinite linear systems are obtained

$$\pi^2 a_s + \sum_{n=0}^{\infty} a_n P_{ns}^{(1)} + \sum_{n=1}^{\infty} b_n P_{ns}^{(3)} = V_s^{(1)},\ s = 0,\ 1,\ 2, \tag{3.7}$$

$$\pi^2 b_s + \sum_{n=0}^{\infty} a_n P_{ns}^{(2)} + \sum_{n=1}^{\infty} b_n P_{ns}^{(4)} = V_s^{(2)},\ s = 1,\ 2, \tag{3.8}$$

$$P_{ns}^{(1)} = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \left\langle a\frac{\partial}{\partial r} G(x, y; a\sin\alpha, a\cos\alpha)\right\rangle \cos n\theta \cos s\alpha\, d\theta\, d\alpha \tag{3.9}$$

and $P_{ns}^{(j)}$'s ($j = 2, 3, 4$) are double integrals similar to (3.9) where the subscripts 2, 3, 4 denote the combinations cos $n\theta$ sin $s\alpha$, sin $n\theta$ cos $s\alpha$, sin $n\theta$ sin $s\alpha$ respectively in the integrands ; and

$$V_s^{(1),(2)} = -\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \left\langle a\frac{\partial\phi_0(x,y)}{\partial r}\right\rangle G(\theta\ ;\ \alpha) \begin{matrix}\cos s\alpha \\ \sin s\alpha\end{matrix}\, d\theta\, d\alpha \tag{3.10}$$

It may be noted that a_0 does not affect the function $\phi(\xi, \eta)$ in (3.1) since

$$\int_{-\pi}^{\pi} \left\langle a\frac{\partial}{\partial r} G(x, y\ ;\ \xi, \eta)\right\rangle d\theta = 0.$$

4. Approximate expressions for $P_{ns}^{(j)}$ $j = 1, 2, 3, 4$ and $V_s^{(j)}$ $j = 1, 2$

Assuming Ka and Kf to be moderate, and following a technique similar to Goswami's, $G_D(\theta, \alpha)$ and $\langle a \frac{\partial}{\partial r} G_D(x, y\,;\, a \sin\alpha,\, a\cos\alpha)\rangle$ can be asymptotically expanded a algebraic series in powers of a/h, and it is seen that both the series start with (a/h). Substituting these series in the double integrals (3.9) and (3.10) and following Lemma, it can be shown that for large h (*i.e.*, large Kh, h/a, h/f)

$$P_{ns}^{(1)} = (-1)^{n+s}\pi^2 \frac{(n+s-1)!}{(n-1)!\,s!}\left(\frac{a}{2f}\right)^{n+s} - 2\pi^2 \frac{(Ka)^{n+s}}{(n-1)!\,s!}\left(\frac{d^{n+s}}{dz^{n+s}}F(z)\right)_{z=2Kf}$$

$$-\frac{1}{2}\frac{\pi^2}{(n-1)!\,s!}\left(\frac{a}{h}\right)^{n+s} S^{(1)}\left(Ka, \frac{f}{a}, \frac{h}{a}\right) \quad (4.1)$$

$P_{ns}^{(2)} = P_{ns}^{(3)} = 0$

$P_{ns}^{(4)}$ is the same expression as $P_{ns}^{(1)}$ with $S_{n+s}^{(1)}$ replaced by $S_{n+s}^{(2)}$, where

$$S_{n+s}^{(1)}, S_{n+s}^{(2)} = \sum_{\lambda=0}^{\infty} \frac{1}{(Ka)^\lambda}\left(\frac{a}{h}\right)^\lambda \left[\sum_{\mu=0}^{\infty} \frac{2^\mu}{\mu!}\left(\frac{f}{a}\right)^\mu \left(\frac{a}{h}\right)^\mu (1 + (-1)^{s+\lambda+\mu})\right.$$

$$\times\left(\alpha_{n+s+\lambda+\mu-1,\lambda+1} + \frac{\alpha_{n+s+\lambda+\mu+1,\lambda+1}}{(Ka)^2}\right) - \frac{2}{Ka}\frac{a}{h}(1-(-1)^{s+\mu})$$

$$\times\,\alpha_{n+s+\lambda+\mu,\lambda+1} \pm ((-1)^n + (-1)^s)\left(\alpha_{n+s+\lambda-1,\lambda+1} + \frac{1}{Ka}\left(\frac{a}{h}\right)^2\right.$$

$$\left.\left.\alpha_{n+s+\lambda+1,\lambda+1}\right)\right], \quad (4.2)$$

the upper and lower sign being for $S_{n+s}^{(1)}$ and $S_{n+s}^{(2)}$ respectively, and

$$\alpha_{nm} = \int_0^\infty u^n (1 - \tanh^m u)\, du, \; n, m \geq 1, \quad (4.3)$$

$$V_s^{(1)} = \frac{\pi^2 e^{-Kf}}{s!}\left[(-1)^s\left\{(Ka)^s - \sum_{n=1}^{\infty} \frac{(Ka)^n (n+s-1)!}{(n-1)!\,n!}\left(\frac{a}{2f}\right)^{n+s}\right\}\right.$$

$$+ 2\sum_{n=1}^{\infty} \frac{(-1)^n (Ka)^{2n+s}}{n!\,(n-1)!}\left(\frac{d^{n+s}}{dz^{n+s}}F(z)\right)_{z=2Kf}$$

$$\left.+ \frac{1}{2}\sum_{n=1}^{\infty} \frac{(-1)^n (Ka)^n}{n!\,(n-1)!}\left(\frac{a}{h}\right)^{n+s} S_{n+s}^{(1)}\left(Ka, \frac{f}{a}, \frac{a}{h}\right)\right] \quad (4.4)$$

and $V_s^{(2)}$ is a similar expression as in (4.4) multiplied throughout by i and $S_{n+s}^{(1)}$ replaced by $S_{n+s}^{(2)}$. The function $F(z)$ in (4.2) and (4.4) is given by

$$F(z) = \int_0^{\infty} \frac{e^{z\xi}}{\zeta - 1} d\zeta \tag{4.5}$$

where the contour is indented above the simple pole at $\zeta = 1$. Noting that $P_{01}^{(1)} = 0$ and putting

$$\begin{matrix} A_n \\ B_n \end{matrix} = \begin{matrix} a_n \\ -ib_n \end{matrix} + \frac{(-1)^n (Ka)^n e^{-Kf}}{n!} \tag{4.6}$$

the two linear systems (3.7) and (3.8) reduce to

$$sA_s + \sum_{n=1}^{\infty} A_n K_{ns} = 2e^{-Kf} \frac{(-1)^s}{(s-1)!} (Ka)^s, \quad s = 1, 2, \tag{4.7}$$

$$sB_s + \sum_{n=1}^{\infty} B_n L_{ns} = 2e^{-Kf} \frac{(-1)^s}{(s-1)!} (Ka)^s, \quad s = 1, 2, \tag{4.8}$$

where

$$K_{ns} = K_{sn} = \frac{1}{(n-1)!\,(s-1)!} \Big[(-1)^{n+s} (n+s-1)! \left(\frac{a}{2f}\right)^{n+s}$$

$$-2 (Ka)^{n+s} \left(\frac{d^{n+s}}{dz^{n+s}} F(z) \right)_{Z=2Kf} - \frac{1}{2} \left(\frac{a}{h}\right)^{n+s} S_{n+s}^{(1)} \Big], \tag{4.9}$$

and L_{ns} is the same expression as in (4·9) with $S_{n+s}^{(1)}$ replaced by $S_{n+s}^{(2)}$.

It may be noted that for infinite depth of fluid, K_{ns} and L_{ns} coincide by making $a/h \to 0$ in (4.9), so that the two linear systems (4.7) and (4.8) reduce to one which is exactly the same as that obtained by Levine[8].

The two infinite linear systems (4.7) and (4.8) are of the same type given by

$$x_s + \sum_{n=1}^{\infty} x_s \frac{k_{ns}}{s} = l_s, \; s = 1, 2, \ldots.$$

The conditions sufficient for the existence and uniqueness of solution of this linear system (*cf.* Ursell[7]) are

$$\sum_{s=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{s} |k_{ns}| < \infty \text{ and } \det\left(\delta_{sn} + \frac{k_{ns}}{s}\right) \neq 0$$

If $\sum_{s=1}^{\infty} | I_s |$ is convergent, then $\sum_{s=1}^{\infty} x_s$ is also convergent. It is not difficult to show that these conditions are satisfied in our case.

5. Reflection and transmission coefficients

By making $\xi \to \mp \infty$ in (3.1) and noting the far field behaviours of $\phi(\xi, \eta)$ and $G(x, y; \xi, \eta)$, the transmission and reflection coefficients can be obtained as

$$T = 1 + \pi i e^{-Kf} \sum_{n=1}^{\infty} \frac{(-1)^n}{(n-1)!} (Ka)^n (A_n + B_n) \tag{5.1}$$

$$R = \pi i e^{-Kf} \sum_{n=1}^{\infty} \frac{(-1)^n}{(n-1)!} (Ka)^n (A_n - B_n) \tag{5.2}$$

after neglecting exponentially small terms for large h. For infinite depth of fluid since A_n and B_n coincide, R vanishes identically. This result has been established earlier by Dean[6], Ursell[7] and Levine[8].

Approximate solution of the linear systems (4.7) and (4.8) can be obtained by truncation. As an illustration, we truncate up to only one term so that we assume A_1, $B_1 \neq 0$, $A_2 = A_3 = \ldots B_2 = B_3 = \ldots = 0$.

Then $R \equiv R^{(1)} = \pi i e^{-Kf} (A_1 - B_1)$, $T \equiv T^{(1)} = 1 - \pi i e^{-Kf} (A_1 + B_1)$

where now

$$A_1 = -2e^{-Kf} Ka/(1 + K_{11}), \quad B_1 = -2e^{-Kf} Ka/(1 + L_{11})$$

Let us write

$$K_{11} = K_{11}^{\infty} + k_{11}, \quad L_{11} = L_{11}^{\infty} + l_{11} = K_{11}^{\infty} + l_{11}$$

where

$$K_{11}^{\infty} = \left(\frac{a}{2f}\right)^2 - 2(Ka)^2 \left(\frac{d^2}{dz^2} F(z)\right)_{z = 2Kf},$$

$$k_{11} = -\frac{1}{2}\left(\frac{a}{h}\right)^2 S_2^{(1)}(Ka, f/a, h/a)$$

$$l_{11} = -\frac{1}{2}\left(\frac{a}{h}\right)^2 S_2^{(2)}(Ka, f/a, h/a).$$

Then

$$A_1^{\infty} = B_1^{\infty} = -2e^{-Kf}\,Ka/(1 + K_{11}^{\infty}) \text{ so that } R_{\infty}^{(1)} = 0,$$

$$T_1^{\infty} = 1 + 4\pi i e^{-Kf}/(1 + K_{11}^{\infty})$$

If terms only up to $(a/h)^2$ are retained, then

$$k_{11}, l_{11} = -\alpha_{11}(1 \pm 1)(a/h)^2$$

so that

$$R^{(1)} = \frac{-4\pi i e^{-2Kf}\alpha_{11}(Ka)^2}{(1 + K_{11}^{\infty})^2}\left(\frac{a}{h}\right)^2 + O\left(\left(\frac{a}{h}\right)^3\right)$$

$$T^{(1)} = 1 + \frac{4\pi i e^{-2Kf}(Ka)^2}{1 + K_{11}^{\infty}}\left\{1 + \left(\frac{a}{h}\right)^2 \frac{\alpha_{11}}{1 + K_{11}^{\infty}}\right\} + O\left(\left(\frac{a}{h}\right)^3\right)$$

and hence $T^{(1)} - T_{\infty}^{(1)} = -R^{(1)}$.

This illustrates the conclusion that the reflection coefficient and the depth correction to the transmission coefficient for large h can be approximated as algebraic series in powers of a/h starting with $(a/h)^2$.

6. Discussion

It may be noted that the approximate results for the complex reflection and transmission coefficients obtained here are valid under the assumption that Ka, $Kf << Kh$. Taking $Ka = 0{\cdot}5$, $f/a = 2{\cdot}0$, numerical values of R are calculated for $h/a = 10$, 20, 30, 40, 50, 60, 70, 80, 90 and 100 correct up to five decimal places and plotted in fig. 1. It is noticed from the graph that as h/a becomes large, R becomes small as should be expected.

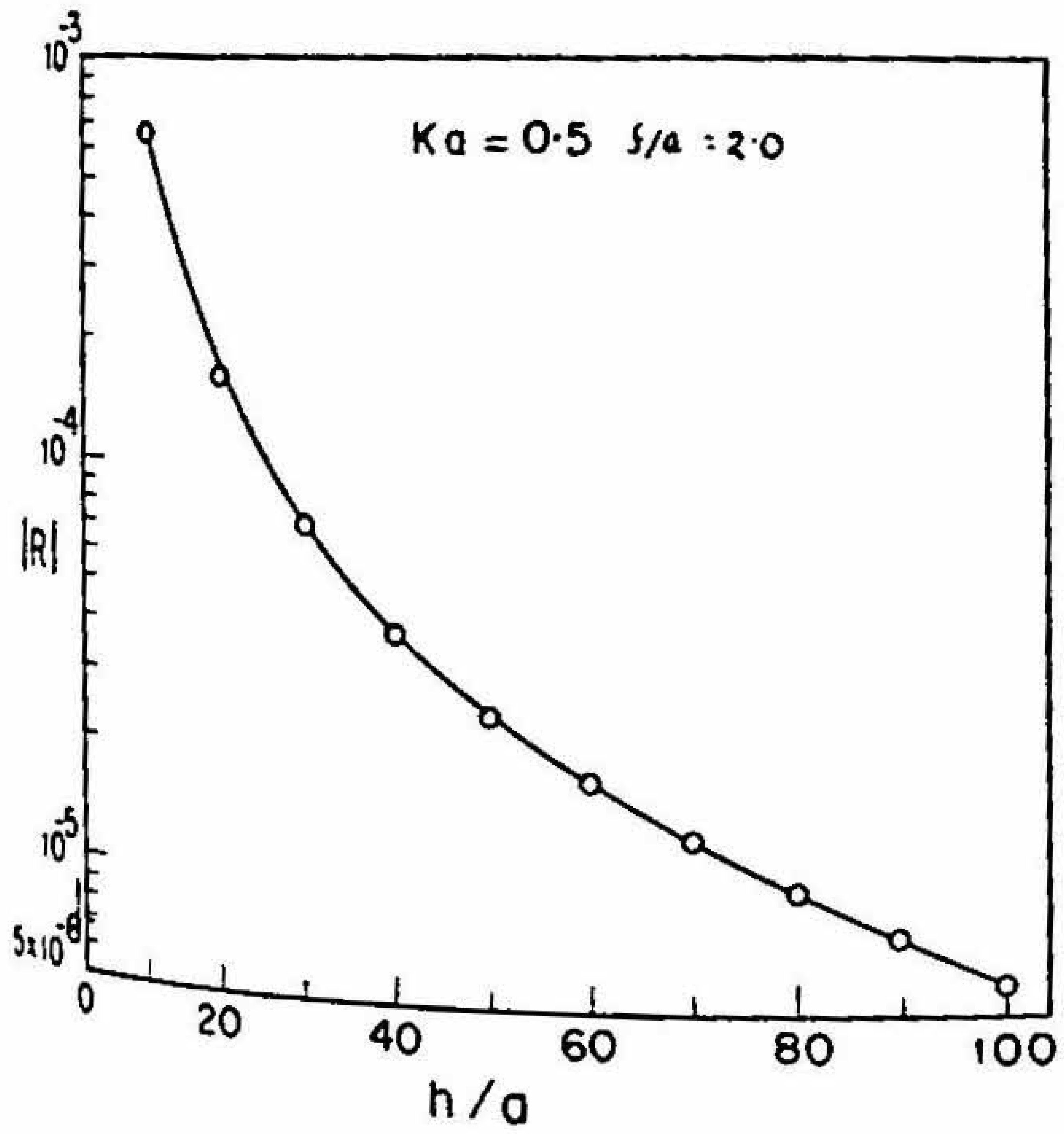

FIG. 1.

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