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Fluctuating flow of a stratified viscous fluid through a porous medium between two parallel plates

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Abstract

The flow of a stratified viscous fluid through a porous medium between two parallel impermeable plates when the lower plate fluctuates in time about a constant mean and the upper plate moves with a constant velocity is analysed. Various graphs have been drawn to show the effect of the permeability of the porous medium and of the stratification upon the velocity field and upon the amplitudes and the phase leads of the fluctuating parts of the skin-frictions on the plates.

Key words : Fluctuating, stratified, flow, viscous fluid, porous medium.

1. Introduction

Fluid motion influenced by the density and viscosity variations in the fluid characterised as stratified flow is finding application in a large number of technological fields. It is widely applicable to withdraw fluid from a region in which fluid density varies in the vertical direction. This type of flow is of great importance to the petroleum engineer concerned with the movement of oil, gas and water through the reservoir of an oil or gas field as the flow behaviour of the fluids in a petroleum reservoir depends to a large extent on the viscous stratification and also on the porous properties of the rock. Therefore, a technique of such study is needed that can give new or additional information about the characteristics of the rock and flow behaviour through the porous rock.

Yih[6] studied the effects of density variations on the fluid flows. Dore[3] has studied forced oscillations in a viscous stratified fluid in which the density and viscosity vary exponentially with vertical coordinate. Channabasappa and Ranganna[2] studied the flow of viscous stratified fluid past a permeable bed with the anticipation that stratification may provide a technique for studying the pore size in a porous medium. Gupta and Sharma[4] studied the stratified viscous flow to investigate the effects of stratification on the mass flow rate in the channel formed by a moving impermeable plate and a permeable bed.

Flow through porous media depends heavily upon Darcy's law. It is valid under certain limitations. It has been shown that it can be possibly valid in a certain seepage velocity-domain, outside which more general flow equation must be used to describe the flow correctly. The porous medium is, in fact, a non-homogeneous medium but for the sake of analysis, it may be possible to replace it with a homogeneous fluid which has dynamical properties equal to those of non-homogeneous continuum. Thus, one can study the flow of a hypothetical homogeneous fluid under the action of the properly averaged external forces. Thus, a complicated problem of the flow through a porous medium reduces to the flow problem of a homogeneous fluid with some resistance. Recently, Ahmadi and Manvi[1] derived a general equation of flow through porous medium and applied the results obtained to some basic flow problems.

The aim of this paper is to study the fluctuating flow of viscous incompressible fluid of variable density and viscosity through a porous medium between two parallel impermeable plates, the lower one fluctuating in time about a constant mean and the upper one moving with a constant velocity. We have calculated the skin friction at both the plates and have shown graphically the effects of permeability of porous medium, and the stratification parameter upon the velocity field and upon the amplitudes and the phase leads of the fluctuating parts of the skin frictions.

2. Mathematical analysis

Let us consider the flow of a viscous incompressible fluid in which density and viscosity vary exponentially with vertical coordinate, through a porous medium between two parallel infinite plates with a breadth h. Let x-axis and y-axis be taken along and perpendicular to the lower plate. Let μ_0 and ρ_0 be the viscosity and the density of the fluid at the lower plate. Let $b > 0$ be the stratification parameter and k be the permeability of the porous medium.

The governing equations of motion describing the flow of viscous incompressible fluid through porous medium are :

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - \frac{\mu}{k} u \tag{1.1}$$

$$\rho g = -\frac{\partial p}{\partial y} \tag{1.2}$$

along with the equation of continuity

$$\frac{\partial}{\partial x}(\rho u) = 0 \tag{1.3}$$

where

$$\rho = \rho_0 e^{-by}, \ \mu = \mu_0 e^{-by} \text{ and}$$

u is the fluid velocity along x-axis.

The boundary conditions are :

$$\left.\begin{aligned} u &= u_0 \cos nt \ \text{ at } y = 0 \\ u &= u_0 \qquad\quad \text{ at } y = h \end{aligned}\right\} \tag{1.4}$$

where u_0 is the mean velocity and n is the frequency of fluctuations.

Let us introduce non-dimensional quantities

$$u^* = \frac{u}{u_0}, \ y^* = \frac{y}{h}, \ x^* = \frac{x}{h}, \ t^* = \frac{\nu_0 t}{h^2}, \ a = bh, \ k^* = \frac{k}{h^2} \text{ and } n^* = \frac{nh^2}{\nu_0}.$$

Equations (1.1) to (1.3), under these non-dimensional quantities and the constant pressure, give (after dropping stars)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - a\frac{\partial u}{\partial y} - \frac{1}{k}u \tag{1.5}$$

The boundary conditions (1.4) reduce to

$$\left.\begin{aligned} u &= \cos nt = \text{Re } e^{int} \ \text{ at } y = 0 \\ u &= 1 \qquad\qquad\qquad\quad \text{ at } y = 1 \end{aligned}\right\} \tag{1.6}$$

Now putting (Stuart[6])

$$u = f(y) + f_1(y) \text{ Re } e^{int} \tag{1.7}$$

(where Re stands for the real part), in equation (1.5) and separating the harmonic and non-harmonic terms, we obtain

$$D^2 f_0 - aDf_0 - \frac{1}{k} = 0 \tag{1.8}$$

and

$$D^2 f_1 - aDf_1 - \left(\frac{1}{k} + in\right) f_1 = 0 \tag{1.9}$$

where

$$D = \frac{d}{dy}.$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} f_0 = 0,\ f_1 = 1 \text{ at } y = 0 \\ f_0 = 1,\ f_1 = 0 \text{ at } y = 1 \end{aligned}\right\} \qquad (1.10)$$

Now solving (1.8) and (1.9) under (1.10), then substituting for f_0 and f_1 in (1.7) and equating the real part, the velocity field is obtained as

$$u = e^{-a(1-y)/2} \operatorname{cosech} a_0 \sinh a_0 y + (M_r \cos nt - M_i \sin nt)$$

where

$$M_r = a_1 a_3 - a_2 a_4, \quad M_i = a_2 a_3 + a_1 a_4$$

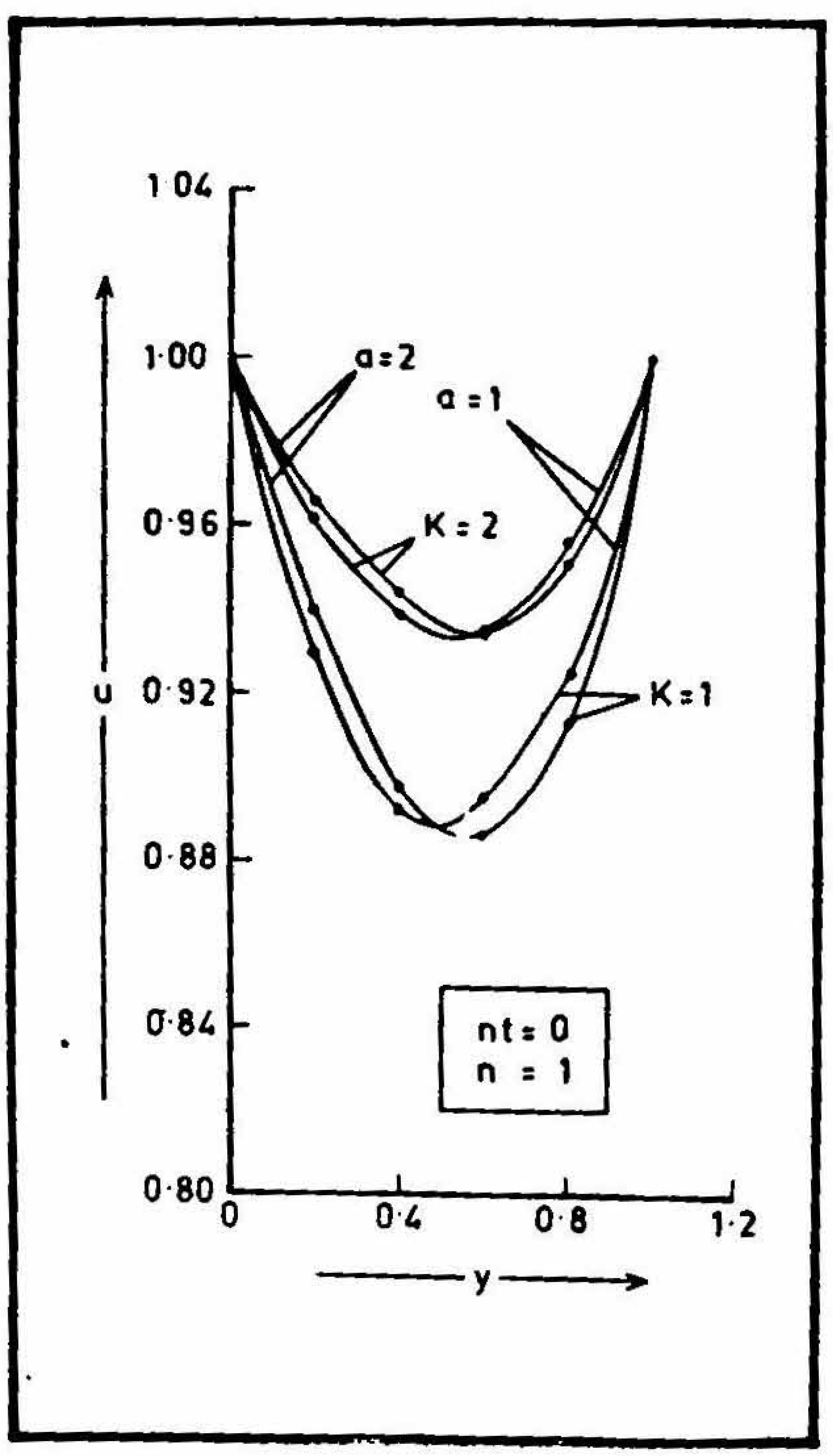

Fig. 1

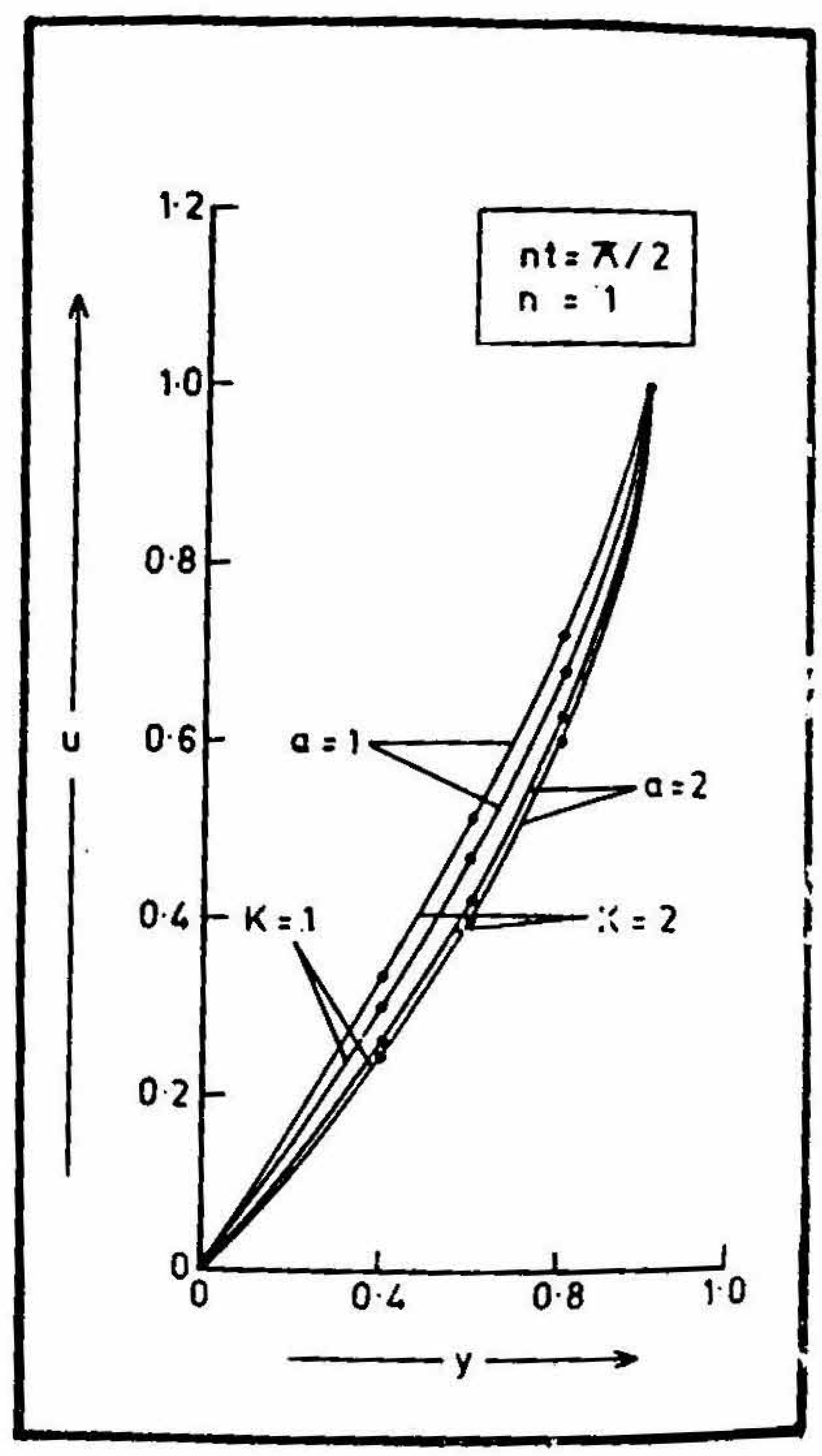

Fig. 2

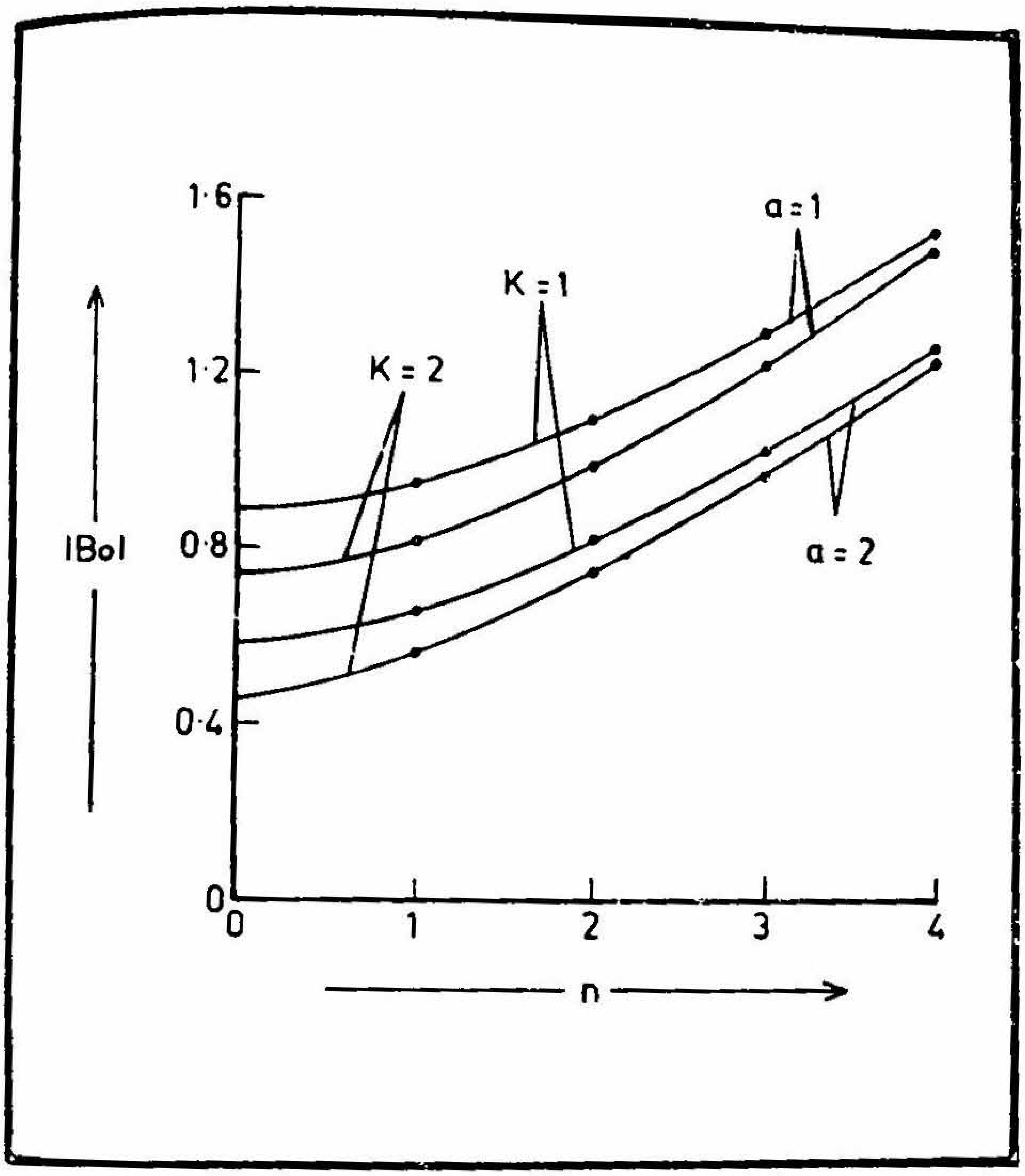

FIG. 3

and

$$2a_0 = (a^2 + 4/k)^{1/2}$$

$$2a_1 = -\, e^{-a/2} \sinh h_1 \cos h_2/(\sinh^2 h_1 \cos^2 h_2 + \cosh^2 h_1 \sin^2 h_2)$$

$$2a_2 = e^{-a/2} \cosh h_1 \sin h_2/(\sinh^2 h_1 \cos^2 h_2 + \cosh^2 h_1 \sin^2 h_2)$$

$$a_3 = -\, 2e^{a(1+y)/2} \sinh (1 - y)\, h_1 \cos (1 - y)\, h_2$$

$$a_4 = -\, 2e^{a(1+y)/2} \cosh (1 - y)\, h_1 \sin (1 - y)\, h_2$$

$$h_1 = [(a^2 + 4/k) + \{(a^2 + 4/k)^2 + 16n^2\}^{1/2}]^{1/2}/2\,(2)^{1/2}$$

$$h_2 = [-(a^2 + 4/k) + \{(a^2 + 4/k)^2 + 16n^2\}^{1/2}]^{1/2}/2\,(2)^{1/2},$$

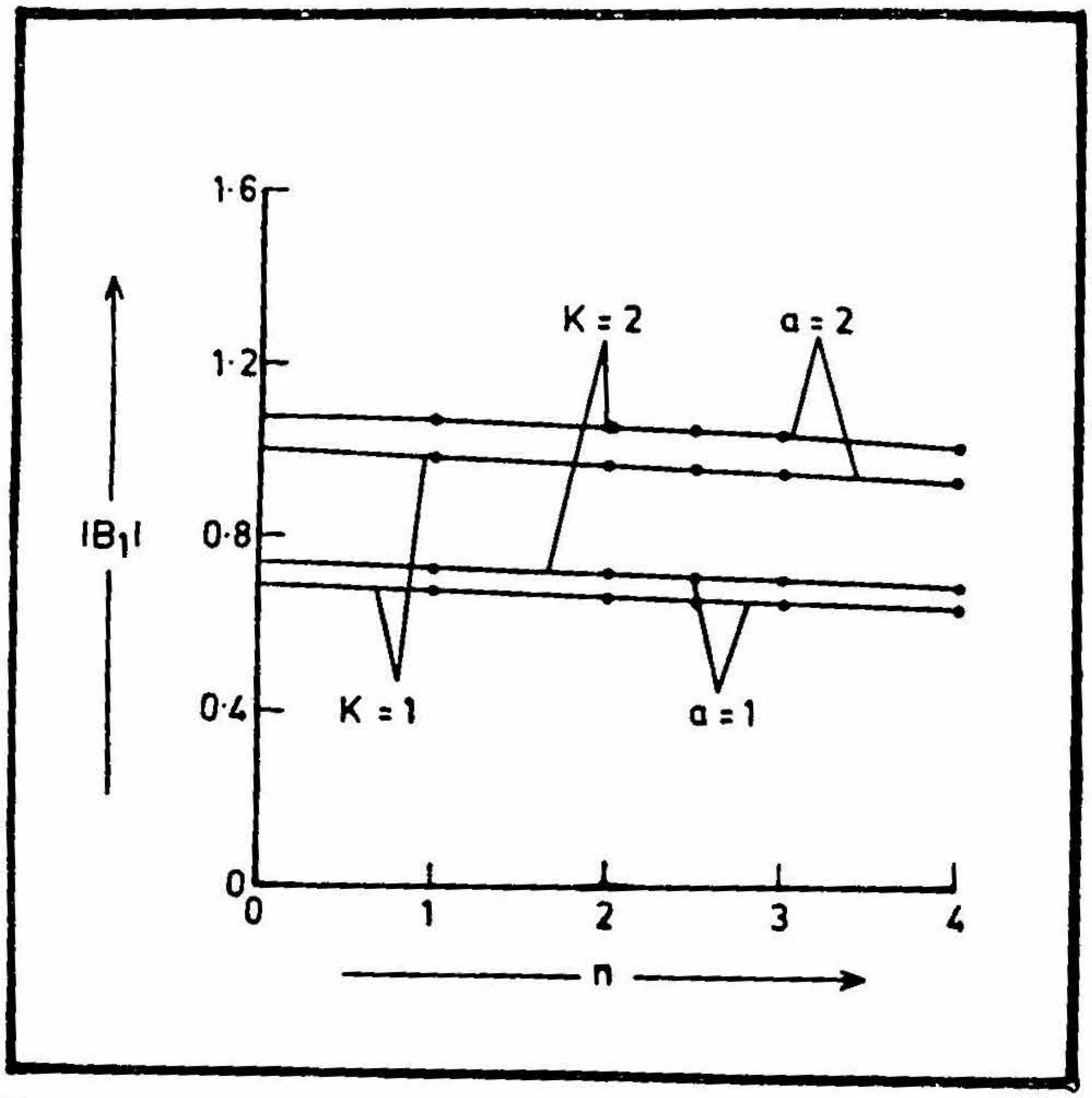

FIG. 4

The non-dimensional skin-friction at the lower plate is

$$\tau_0 = \left.\frac{\partial u}{\partial y}\right|_{y=0} = a_0\, e^{-a/2} \operatorname{cosech} a_0 + |B_0| \cos(nt + \theta_0)$$

where

$$|B_0| = (B_{0r}^2 + B_{0i}^2)^{1/2}, \quad \theta_0 = \tan^{-1}\frac{B_{0i}}{B_{0r}}.$$

$$B_{0r} = e^{a/2}\,[a_1\{2h_1 \cosh h_1 \cos h_2 - 2h_2 \sinh h_1 \sin h_2 - a \sinh h_1 \cos h_2\}$$
$$- a_2\{2h_1 \sinh h_1 \sin h_2 + 2h_2 \cosh h_1 \cos h_2 - a \cosh h_1 \sin h_2\}]$$

$$B_{0i} = e^{a/2}\,[a_1\{2h_1 \sinh h_1 \sin h_2 + 2h_2 \cosh h_1 \cos h_2 - a \cosh h_1 \sin h_2)$$
$$+ a_2\{2h_1 \cosh h_1 \cos h_2 - 2h_2 \sinh h_1 \sin h_2 - a \sinh h_1 \cos h_2\}].$$

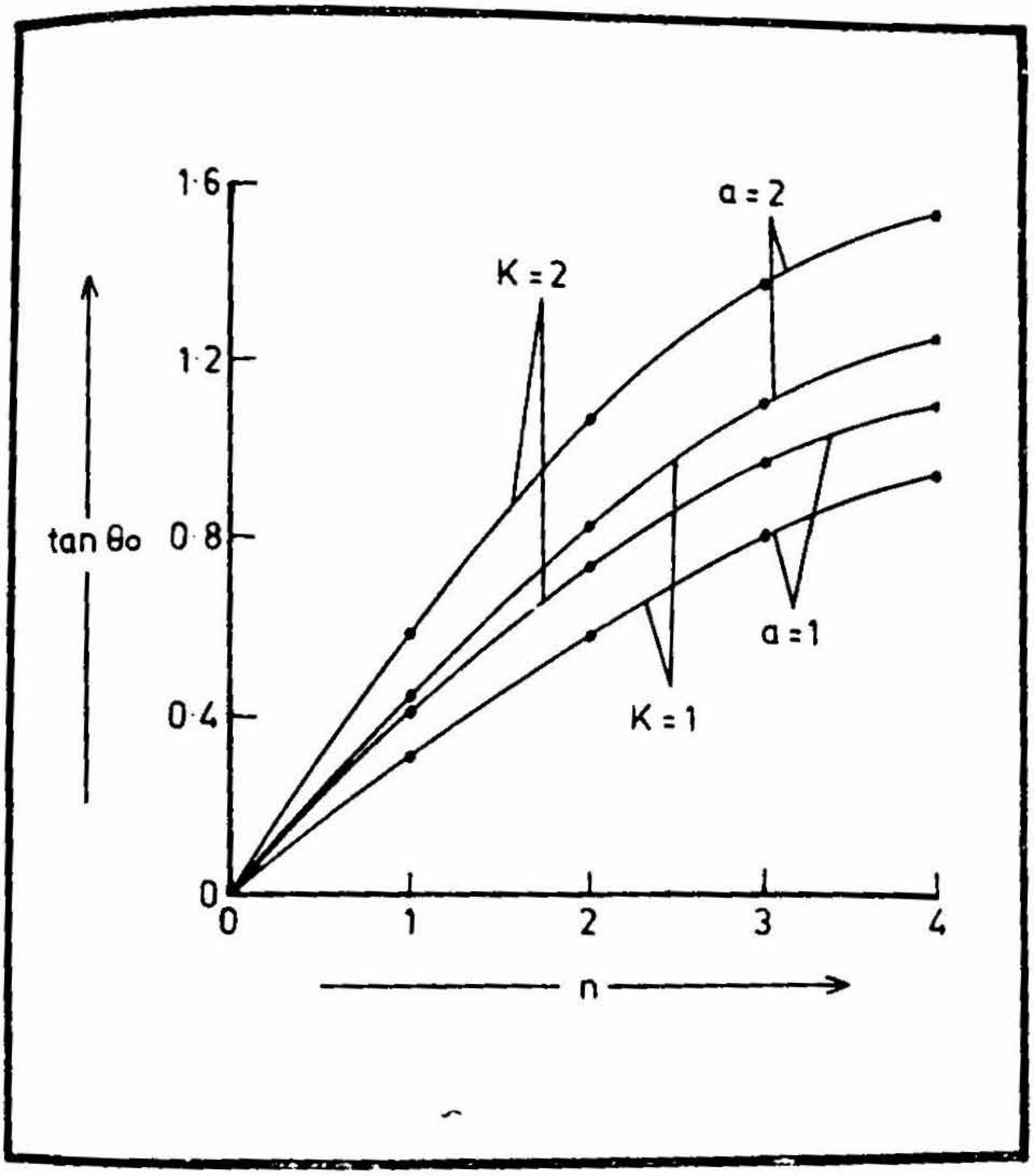

FIG. 5

The non-dimensional skin friction at the upper plate is

$$\tau_1 = \left.\frac{\partial u}{\partial y}\right|_{y=1} = \left(a_0 \coth a_0 + \frac{a}{2}\right) + | B_1 | \cos (nt + \theta_1)$$

where

$$| B_1 | = (B_{1r}^2 + B_{1i}^2)^{1/2}, \ \theta_1 = \tan^{-1} \frac{B_{1r}}{B_{1r}} .$$

$$B_{1r} = e^a \{a_1 h_1 - a_2 h_2\}, \ B_{1i} = e^a \{a_1 h_2 + a_2 h_1\}$$

2.1. *Particular cases*

If we put $a = 0$, the results for homogeneous fluid are immediately obtained. If $k \to \infty$, the results reduce to those of flow in an ordinary medium (of full porosity, *i.e.* = 1).

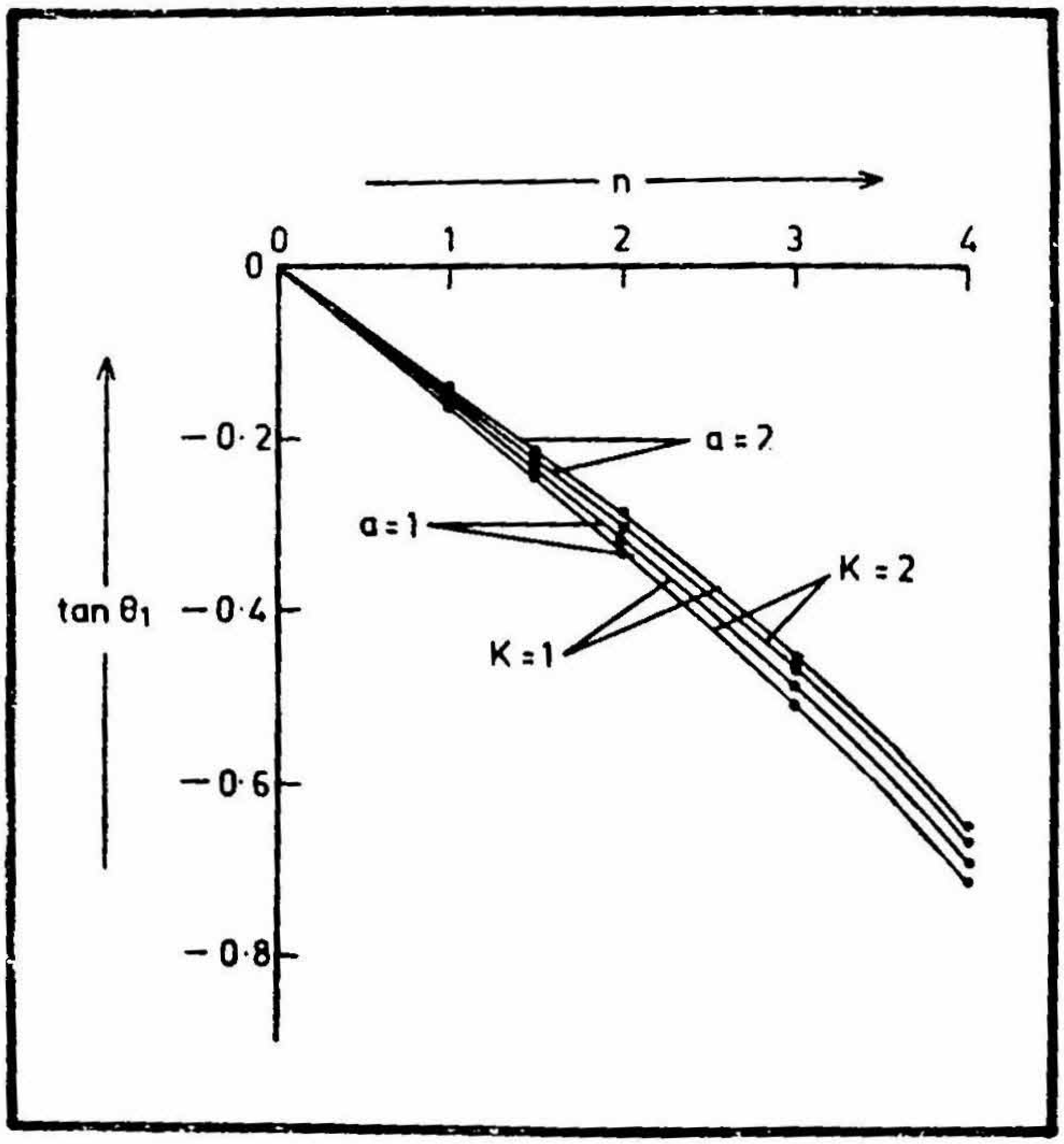

FIG. 6

3. Discussion

In order to study the effects of the permeability (k) of the porous medium and the stratification parameter (a) on the velocity distribution, we have plotted the velocity (u) against y for $n = 1$. In figs. 1 and 2, the velocity is plotted at $nt = 0$ and at $nt = \pi/2$ respectively. We observe that at $nt = 0$, the velocity is maximum near the plates and is minimum in the central plane of the channel (fig. 1), but at $nt = \pi/2$, the velocity is minimum near the lower plate and increases sharply as y increases, making it maximum near the upper plate (fig. 2). It is also evident that for the same value of nt, the increase in k leads to an accelerated flow everywhere but the variation in the stratification parameter (a) leads to different effects at $nt = 0$ and $nt = \pi/2$. Due to the increase in a, the flow is seen accelerated in the lower region and decelerated in the upper region at $nt = 0$, but everywhere decelerated at $nt = \pi/2$. The velocity close to the plates is very less affected by the change in k or a.

In figs. 3 and 4, we have plotted the amplitudes $|B_0|$ and $|B_1|$ of the fluctuations of the skin frictions at the lower and the upper plate respectively, against n. The amplitude $|B_0|$ ($|B_1|$) is seen increasing sharply/decreasing very slowly with the increase in n. $|B_0|$ ($|B_1|$) is more for smaller/larger k and also for smaller/larger a. Thus the stratification decreases/increases and the porous medium increases/decreases the fluctuation of the skin friction at the lower/upper plate.

Figures 5 and 6 show the effect of the permeability and the stratification on the phase leads $\tan\theta_0$ and $\tan\theta_1$ of the fluctuating parts of the skin frictions at the lower and the upper plate respectively. From fig. 5, it can be observed that an increase in k or in a or in n leads to an increase in $\tan\theta_0$. From fig. 6, we observe that $\tan\theta_1$ decreases with the increase in k or in n but increases as a increases.

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