J. Indian Inst. Sci., 64 (B), Aug. 1983, Pp. 199–210

Source flow between two non-coaxial rotating cylinders

A. RAMACHANDRA RAO AND P. RAGHUPATHI RAO
Department of Applied Mathematics, Indian Institute of Science, Bangalore 560 012, India.

Received on July 31, 1982 ; Revised on January 6, 1983.

Abstract

Two-dimensional flow of an incompressible viscous fluid between two rotating non-coaxial cylinders has been investigated when fluid is injected uniformly through the surface of the inner cylinder and removed through the surface of the outer cylinder. Under the assumption of small eccentricity, solution of the governing Navier–Stokes equations is obtained for the case when the gap between the cylinders is finite. Solutions of the governing equations under the geometrical restriction of narrow gap are also presented. The effect of the radial flow or suction on the transverse velocity is discussed for the narrow gap. The existence of force component in the x-direction on the inner cylinder is due to the radial flow.

Key words: Source flow, Non-coaxial rotating system.

1. Introduction

Two-dimensional flows between moving nearly coaxial cylinders have applications in the design of control mechanisms for aircraft and rockets.

Wood[1] has studied the two-dimensional flow of an incompressible viscous fluid between two non-coaxial rotating cylinders. Using an appropriate conformal transformation to map the non-concentric circular boundaries into concentric ones, he has obtained the solution of the governing Navier–Stokes equations under the assumption of small eccentricity. Segel[2] used the conformal mapping technique to study the unsteady flow between non-coaxial cylinders when the outer cylinder is kept fixed and the inner cylinder is made to rotate or vibrate about a slightly eccentric point. Kulinski and Ostrach[3] have also used the idea of conformal mapping in studying the flow between rotating cylinders with axes slightly apart.

Urban[4] has used a polar coordinate system with origin at the centre of the inner cylinders to study the basic flow between two non-coaxial cylinders when the distance between their axes is small. Writing the boundary conditions at the outer boundary which is not a coordinate curve is difficult, while solving the linearized equations arising in the perturbation method. The principle of transfer of boundary conditions as elucidated by Van Dyke[5] has been used to resolve this difficulty. The principle is to replace the conditions on the actual boundary whose position varies slightly with the perturbation parameter ϵ by the conditions on the basic boundary which corresponds to $\epsilon = 0$. The solutions thus obtained satisfy the boundary conditions on the actual boundary more closely by taking ϵ small and including more number of terms in the perturbation series. Urban[4] has also discussed the relative merits of his method of solution. It is worthwhile commenting here that even the conformal mapping method has similar limitations on eccentricity. Nikitin[6] also used polar coordinates in the study of flow between non-coaxial cylinders.

The aim of the present investigation is to extend the problem considered by Urban[4] in the presence of a radial flow due to a line source along the axis of the inner cylinder. Solution of the governing equations is obtained when the gap between the cylinders is finite under the assumption that the distance between the axes of the cylinders is small. Solutions of the governing equations are also presented under the geometrical restriction of narrow gap. The effect of the radial flow is seen both in the first and second order velocities. In fact, the radial flow induces a force in the x direction on the inner cylinder.

2. Formulation

Let O_1 and O_2 be the centres of the inner and outer cylinders with radii R_1 and R_2 respectively. O_1 is taken as the origin of the cylindrical polar coordinate system (r, θ, z) with z-axis along the axis of the inner cylinder (see fig. 1). The distance O_1O_2 is the eccentricity 'e' of the system. Two non-dimensional parameters ϵ, the eccentricity ratio, and δ, the gap ratio, are defined by

$$\epsilon = \frac{e}{d}, \quad (0 < \epsilon < 1),$$

$$\delta = \frac{d}{R_0}, \quad (0 < \delta < 2)$$

where $d = R_2 - R_1$ and $R_0 = (R_1 + R_2)/2$. Using the cosine rule for the triangle O_1O_2P, the radial distance 'h' which is the variable gap between the cylinders is obtained as

$$h(\theta) = -R_1 + \epsilon d \cos\theta + R_2\left(1 - \frac{4\epsilon^2\delta^2}{(2+\delta)^2}\sin^2\theta\right)^{1/2}. \quad (1)$$

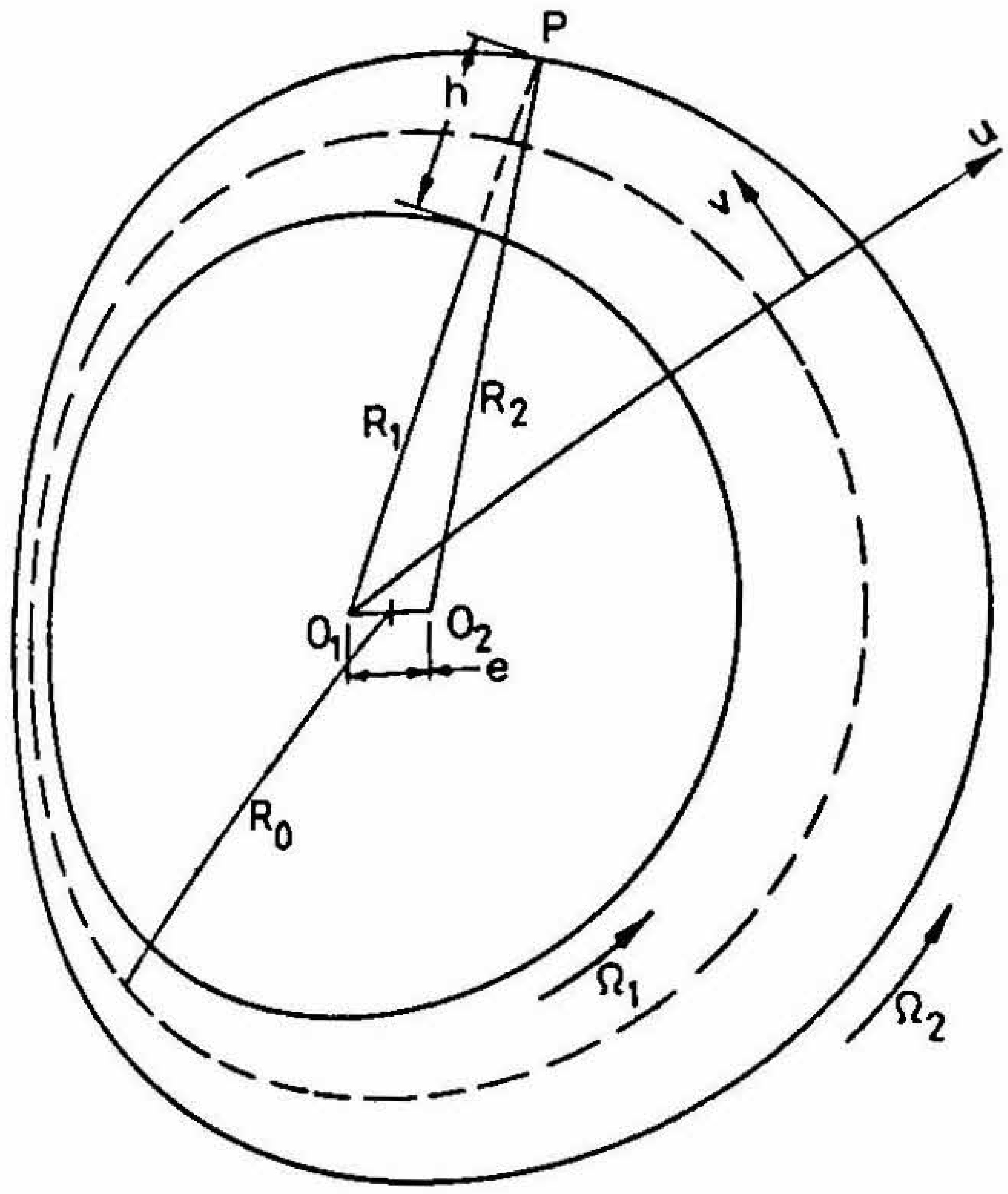

FIG. 1. Non-coaxially rotating cylinders.

Consider the flow problem when the inner and outer cylinders rotate with angular velocities Ω_1 and Ω_2 respectively and when there is a radial source flow due to which the fluid gets radially injected at the inner cylinder and the same amount of fluid is sucked out normally from the surface of the outer cylinder. Assuming that there is no flow in the axial direction of the cylinders, the radial velocity u, the transverse velocity v and pressure p' are functions of r and θ only. The equations governing the flow are the continuity equation and the two-dimensional Navier–Stokes equations in cylindrical polar coordinates. The boundary conditions for the inner cylinder are

$$u = \beta_1,\ v = R_1\Omega_1 \text{ at } r = R_1. \tag{2 a, b}$$

Since we are considering a non-coaxial system of cylinders, the tangential and normal velocities at the outer cylinder are not in the same direction as the radial and transverse velocities. Hence the boundary conditions for the outer cylinder are,

$$u = -\Omega_2 \epsilon d \sin\theta + \frac{\beta}{R_2}\left\{1 - \frac{(\epsilon d)^2}{R_2^2}\sin^2\theta\right\}^{1/2}, \tag{3}$$

$$v = R_2\Omega_2\left\{1 - \frac{(\epsilon d)^2}{R_2^2}\sin^2\theta\right\}^{1/2} + \frac{\beta}{R_2^2}\epsilon d\sin\theta, \tag{4}$$

where

$$\beta = \beta_1 R_1.$$

For the formulated nonlinear boundary value problem, approximate solutions are sought by employing a perturbation method. The velocity field is expanded by a series of the form given by

$$u(r,\theta) = u_0(r) + c u_1(r,\theta) + c^2 u_2(r,\theta) + \ldots \tag{6}$$

$$v(r,\theta) = v_0(r) + c v_1(r,\theta) + c^2 v_2(r,\theta) + \ldots . \tag{7}$$

Using (6) and (7) in the governing equations and equating different powers of c on either side we obtain various order equations which are solved using the corresponding boundary conditions derived from (2a, b), (3) and (4).

3. Solution for the finite gap

The leading terms of the velocity components in (6) and (7) are the exact solutions of the coaxial problem. The equations for radial and transverse velocities are given by

$$r u_{0_r} + u_0 = 0, \tag{8}$$

$$u_0\left(v_{0_r} + \frac{v_0}{r}\right) = \nu\left(v_{0_{rr}} + \frac{1}{r} v_{0_r} - \frac{v_0}{r^2}\right), \tag{9}$$

with the boundary conditions

$$u_0 = \beta_1 \text{ on } r = R_1, \tag{10}$$

$$\left.\begin{aligned} v_0 &= \Omega_1 R_1 \text{ on } r = R_1, \\ v_0 &= \Omega_2 R_2 \text{ on } r = R_2 \end{aligned}\right\} \tag{11}$$

where ν is the kinematic viscosity and the subscripts denote the partial derivatives with respect to the corresponding variables. This notation is used throughout this paper. The solutions of (8) and (9) satisfying (10) and (11) are

$$u_0 = \beta / r, \tag{12}$$

$$v_0 = A r^{s+1} + \frac{B}{r}, \tag{13}$$

where

$$\left.\begin{aligned} A &= \frac{\Omega_1(\eta^2 - \mu) R_2^{-s}}{\eta^{2+s} - 1}, \quad B = \frac{\Omega_1 R_1^2 (\mu \eta^s - 1)}{\eta^{2+s} - 1} \\ \mu &= \frac{\Omega_2}{\Omega_1}, \quad \eta = \frac{R_1}{R_2}, \quad s = \frac{\beta}{\nu}. \end{aligned}\right\} \tag{14}$$

The radial velocity is obtained from the continuity equation and the condition (2a). Substituting (6) and (7) in the governing equation obtained by eliminating pressure and equating the coefficients of ϵ and ϵ^2, one gets two linear boundary value problems for the first and second order velocity components. The first order velocity components are governed by

$$\frac{v_0}{r}(u_{1\theta\theta} - v_{1\theta} - rv_{1r\theta}) + \frac{\beta}{r^2}(-u_{1\theta} + ru_{1r\theta} - rv_{1r} - r^2v_{1rr} + v_1)$$
$$= \nu\left(u_{1rr\theta} + \frac{1}{r^2}u_{1\theta\theta\theta} + \frac{2}{r^2}u_{1\theta} + \frac{1}{r}v_{1r} - 2v_{1rr} - rv_{1rrr} - \frac{1}{r}v_{1r\theta\theta} - \frac{v_1}{r^2}\right). \quad (15)$$

$$u_1 + ru_{1r} + v_{1\theta} = 0, \quad (16)$$

with the boundary conditions

$$\left.\begin{aligned} &u_1 = 0,\ v_1 = 0 \text{ at } r = R_1, \\ &u_1 = -\Omega_2 d\sin\theta + \frac{\beta}{R_2^2}d\cos\theta \text{ at } r = R_2, \\ &v_1 = \frac{\beta d\sin\theta}{R^2} - \left\{(s+1)AR_2^s - \frac{B}{R_2^2}\right\}d\cos\theta \text{ at } r = R_2 \end{aligned}\right\} \quad (17)$$

The second order velocities are governed by

$$\frac{1}{r}v_1u_{1\theta\theta} + u_1u_{1r\theta} - \frac{u_1}{r}u_{1\theta} - v_1u_{1r} - ru_1v_{1rr}$$
$$- v_1v_{1r\theta} - \frac{2}{r}v_1v_{1\theta} - u_1v_{1r} + \frac{v_0}{r}(u_{2\theta\theta} - v_{2\theta} - rv_{2r\theta})$$
$$+ \frac{\beta}{r^2}(-u_{2\theta} + ru_{2r\theta} - rv_{2r} - r^2v_{2rr} + v_2)$$
$$= \nu\left(u_{2rr\theta} + \frac{1}{r^2}u_{2\theta\theta\theta} + \frac{2}{r^2}u_{2\theta} + \frac{1}{r}v_{2r} - 2v_{2rr} - rv_{2rrr} - \frac{1}{r}v_{2r\theta\theta} - \frac{v_2}{r^2}\right), \quad (18)$$

$$u_2 + ru_{2r} + v_{2\theta} = 0. \quad (19)$$

with the boundary conditions

$$u_2 = 0,\ v_2 = 0 \text{ at } r = R_1,$$
$$u_2 = -\frac{\beta d^2}{R_2^3} - d\cos\theta\,(u_{1r})_{r=R_2} \text{ at } r = R_2,$$
$$v_2 = \frac{d^2}{2R_2}\left\{(s+1)AR_2^s - \Omega_2 - \frac{B}{R_2^2}\right\}\sin^2\theta$$
$$+ \frac{d^2}{2}\left\{(s^2+s)AR_2^{s-1} + \frac{2B}{R_2^3}\right\}\cos^2\theta - (v_{1r})_{r=R_2}d\cos\theta, \text{ at } r = R_2. \quad (20)$$

The boundary conditions for the first and second order velocity components at $r = R_2$ are obtained using the transfer of boundary conditions discussed by Van Dyke[5]. The boundary conditions (17) prompt us to take

$$u_1(r, \theta) = \tfrac{1}{2}[U_1(r)\, i\, e^{i\theta} + \text{c.c}]. \quad (21)$$

$$v_1(r, \theta) = \frac{-1}{2}[(rU_{1r} + U_1)\, e^{i\theta} + \text{c.c}] \quad (22)$$

where c.c. denotes the complex conjugate of the quantity preceding it. Substituting (21) and (22) in (15) we get the equation for U_1 as

$$\left(U_{1rrrr} + \frac{6}{r} U_{1rrr} + \frac{3}{r^2} U_{1rr} - \frac{3}{r^3} U_{1r}\right) - \frac{iv_0}{r\nu}\left(U_{1rr} + \frac{3}{r} U_{1r}\right)$$
$$- \frac{s}{r^2}(rU_{1rrr} + 4U_{1rr}) = 0 \quad (23)$$

with the boundary conditions

$$\left.\begin{aligned} &U_1 = 0,\ U_{1r} = 0 \text{ on } r = R_1, \\ &U_1 = \Omega_2 d - id\beta/R_2^2 \text{ on } r = R_2, \\ &U_1 = \{(v_{0r})_{r=R_2} - \Omega_2\} d/R_2 + 2id\beta/R_2^3, \text{ on } r = R_2. \end{aligned}\right\} \quad (24)$$

The solution of (23) satisfying (24) is

$$U_1 = c_1 I_1(r) + c_2 I_2(r) \quad (25)$$

where

$$\left.\begin{aligned}
c_1 &= [(\Omega_2 d - id\beta/R_2^2)\, I_4(R_2) - \{(v_{0r})_{r=R_2} d + id\beta/R_2^2\}\, I_2(R_2)]/\Delta, \\
c_2 &= [\{(v_{0r})_{r=R_2} d + id\beta/R_2^2\}\, I_1(R_2) - (\Omega_2 d - id\beta/R_2^2)\, I_3(R_2)]/\Delta, \\
\Delta &= I_1(R_2)\, I_4(R_2) - I_2(R_2)\, I_3(R_2), \\
I_1(r) &= \int_{R_1}^{r} r^k J_p(\lambda r^{k+1})\, dr - \frac{1}{r^2}\int_{R_1}^{r} r^{k+2} J_p(\lambda r^{k+1})\, dr, \\
I_2(r) &= \int_{R_1}^{r} r^k Y_p(\lambda r^{k+1})\, dr - \frac{1}{r^2}\int_{R_1}^{r} r^{k+2} Y_p(\lambda r^{k+1})\, dr, \\
I_3(r) &= \int_{R_1}^{r} r^k J_p(\lambda r^{k+1})\, dr + \frac{1}{r^2}\int_{R_1}^{r} r^{k+2} J_p(\lambda r^{k+1})\, dr, \\
I_4(r) &= \int_{R_1}^{r} r^k Y_p(\lambda r^{k+1})\, dr + \frac{1}{r^2}\int_{R_1}^{r} r^{k+2} Y_p(\lambda r^{k+1})\, dr
\end{aligned}\right\} \quad (26)$$

and

$$k = \frac{s}{2}, \ \lambda = \left(-\frac{Ai}{\nu}\right)^{1/2}, \ p = \left(\frac{iB}{\nu} + 1 + k^2 - 8k\right)^{1/2} \Big/ (k+1) \tag{27}$$

J_p and Y_p are Bessel functions of order p. The first order radial and transverse velocities are

$$u_1(r,\theta) = \tfrac{1}{2}\left[i\{c_1 I_1(r) + c_2 I_2(r)\} e^{i\theta} + c.c\right], \tag{28}$$

$$v_1(r,\theta) = \tfrac{1}{2}\left[-\{c_1 I_3(r) + c_2 I_4(r)\} e^{i\theta} + c.c\right]. \tag{29}$$

The results given in (26) and (27) in the limit $k \to 0$ coincide with those of Urban[4], corresponding to the case where there is no radial flow.

Following Urban[4] the solution for the narrow gap is obtained by employing asymptotic Missel's series for the Bessel functions $J_p(\lambda r^{k+1})$ and $Y_p(\lambda r^{k+1})$ in (25). The factors r^k and r^{k-1} appearing in the expressions for $I_3(r)$ and $I_4(r)$ in (29) do not offer any difficulty and all the integrations can be performed in a straightforward way except for lengthy algebra. We obtain in the limit of the small gap the expression for the first order transverse velocity $\tilde{v}_1(X,\theta)$ as

$$\begin{aligned} v_1(X,\theta) = {} & [(3\mu + 1) X^2 \Theta_+^2 + \{(3\Theta_+ - 2) + \mu(3\Theta_+ - 4)\} \Theta_- X \\ & + \tfrac{1}{4}\{(3\Theta_+ - 4) + \mu(3\Theta_+ - 8)\} \Theta_-] \cos\theta \\ & + 3\alpha\left[2\Theta_+^2 X^2 + 2\Theta_- X(\Theta_+ - 1) + \left(\frac{\Theta_+^2}{2} - \Theta_+\right)\right] \sin\theta, \end{aligned} \tag{30}$$

where $\alpha = \beta_1/R_0\Omega_1$ is the suction parameter, $\Theta_\pm$ and X are as given in the following section.

4. Solution for the narrow gap

In this section some useful and simple results are derived by considering an extra restriction, namely, the gap between the cylinders is small in addition to the small eccentricity. The idea of narrow gap is introduced in both the governing equations and the boundary conditions and the corresponding solutions are presented.

We introduce a new independent variable X by $r = R_1 + h/2 + hX$ and it has the range $-\frac{1}{2} \le X \le \frac{1}{2}$. Further Θ, Θ_+ and Θ_- are given by $\Theta = X\Theta_- + \epsilon/2 \cos\theta$, $\Theta_\pm = 1 \pm \epsilon\cos\theta$. A point in the fluid domain is now prescribed by X and θ. In our problem the zeroth order transverse velocity satisfying the limiting equation and boundary conditions for small gap is the same as that given by Urban[4] and it is

$$\tilde{v}_0(X) = \left[\frac{1+\mu}{2} - (1-\mu) X\right]. \tag{31}$$

The radial velocity given by (12) under the above restriction becomes constant throughout the gap. The equation corresponding to (23) under the restriction of narrow gap reduces in non-dimensional form to

$$\tilde{U}_{1XXXX} = 0. \tag{32}$$

The corresponding boundary conditions are

$$\left.\begin{aligned} &\tilde{U}_1 = 0,\ \tilde{U}_{1X} = 0 \text{ at } X = -\tfrac{1}{2}, \\ &\tilde{U}_1 = (\mu - i\alpha),\ \tilde{U}_{1X} = (\mu - 1)\,\Theta_+ \text{ at } X = \frac{\Theta_-}{2\Theta_+} \end{aligned}\right\} \tag{33}$$

solving (32) subject to (33), $\tilde{U}_1$ is obtained and from which $\tilde{u}_1(X, \theta)$ and $\tilde{v}_1(X, \theta)$ are written as

$$\begin{aligned} \tilde{u}_1(X, \theta) = {} & [(1 + \mu)\,\Theta_+^3 X^3 + \tfrac{1}{2}\{(3\Theta_+ - 2) + \mu(3\Theta_+ - 4)\}\,\Theta_+^2 X^2 + \tfrac{1}{4}\{(3\theta_+ - 4) \\ & + \mu(3\Theta_+ - 8)\}\,\Theta_+^2 X + \tfrac{1}{8}\{(\Theta_+ - 2) + \mu(\Theta_+ - 4)\}\,\Theta_+^2]\sin\theta \\ & - \alpha\cos\theta\,[2\Theta_+^3 X^3 + (3\Theta_+ - 3)\,\Theta_+^2 X^2 + \tfrac{3}{2}\,\Theta_+ - 3)\,\Theta_+^2 X - \tfrac{1}{4}(\theta_+ + 1)\,\theta_+^2] \end{aligned} \tag{34}$$

$$\begin{aligned} \tilde{v}_1(X, \theta) = {} & [3(\mu + 1)\,\Theta_+^2 X^2 + \{(3\Theta_+ - 2) + \mu(3\Theta_+ - 4)\}\,\Theta_+ X + \tfrac{1}{4}\{(3\theta_+ - 4) \\ & + \mu(3\Theta_+ - 8)\}\,\Theta_+]\cos\theta + 3\alpha(2\Theta_+^2 X^2 + 2(\Theta_+ - 1)\,\Theta_+ X \\ & + \tfrac{1}{2}(\Theta_+ - 2)\,\Theta_+)\sin\theta. \end{aligned} \tag{35}$$

Expression for $\tilde{v}_1(X, \theta)$ given by (35) agrees with the one given in (30) which was obtained from the finite gap solution by an asymptotic analysis. In the limit as $\alpha \to 0$, that is, when there is no radial flow (34) and (35) reduce to the corresponding solutions discussed by Urban[4].

In order to obtain the second order solutions we define U_2 in a way similar to that of U_1 in (23). In view of the boundary conditions (20) we take U_2 as

$$U_2(r, \theta) = U_2(r)\,e^{2i\theta} + Y(r). \tag{36}$$

Now equation (18) under the restriction of narrow gap reduces in nondimensional form to

$$\tilde{U}_{2XXXX} = 0, \tag{37}$$

$$\tilde{Y}_{XXXX} = 0. \tag{38}$$

The corresponding boundary conditions are

$$\left.\begin{aligned} &\tilde{U}_2 = 0,\ \tilde{U}_{2X} = 0 \text{ at } X = -\tfrac{1}{2} \\ &\tilde{U}_2 = \tfrac{1}{4}(1-\mu) \text{ at } X = \frac{\Theta_-}{2\Theta} \\ &\tilde{U}_{2X} = (\mu+2)\,\Theta \ -3\Theta.\,\alpha \text{ at } X = \frac{\Theta_-}{2\Theta} \end{aligned}\right\} \tag{39}$$

and

$$\left.\begin{aligned} &\tilde{Y} = \tilde{Y}_X = 0 \text{ at } X = -\tfrac{1}{2} \\ &\tilde{Y}_X = (2+\mu)\,\Theta. \text{ at } X = \frac{\Theta_-}{2\Theta}\,. \end{aligned}\right\} \tag{40}$$

The solution of (37) satisfying (39) is

$$\begin{aligned} \tilde{U}_2 = {} & \tfrac{1}{16}\{(3\Theta. - 5) + \mu(3\Theta. - 7)\}\,\Theta_+^2 + \tfrac{1}{8}\{(9\Theta. - 10) + \mu(9\Theta. - 14)\}\,\Theta_+^2 X \\ & + \tfrac{1}{4}\{(9\Theta. - 5) + \mu(9\Theta. - 7)\}\,\Theta_+^2 X^2 + \tfrac{1}{2}(3 + 3\mu)\,\Theta_+^2 X^3 \\ & + 6\alpha\,[\tfrac{1}{16}\{(2 - \Theta.)\,\Theta_+^2\} + \tfrac{1}{8}\{(4 - 3\Theta.)\,\Theta_+^2\}X + \tfrac{1}{4}(2 - 3\Theta.)\,\Theta_+^2 X^2 \\ & - \tfrac{1}{2}\,\Theta_+^3 X^3]. \end{aligned} \tag{41}$$

Solution of (38) satisfying (40) is given by

$$\tilde{Y}(X) = -\frac{(2+\mu)}{(\Theta_+ - 3)}\,\Theta_+^2\,(X^3 + X^2 + \tfrac{1}{4}X). \tag{42}$$

The constant in the solution of (42) is chosen as zero without any loss of generality. From (41) and (42) the solutions for the second order transverse and radial velocities are obtained as

$$\begin{aligned} \tilde{v}_2(X,\theta) = {} & -\cos 2\theta\,[\tfrac{1}{8}\{(9\Theta - 10) + \mu(9\Theta - 14)\}\,\Theta + \tfrac{1}{2}\{(9\Theta - 5) \\ & + \mu(9\Theta - 7)\}\,\Theta.\,X + \tfrac{9}{2}(3 + 3\mu)\,\Theta_+^2 X^2 + 6\alpha\,\{\tfrac{1}{8}(4 - 3\,\Theta.)\,\Theta \\ & + \tfrac{1}{2}(2 - 3\,\Theta.)\,\Theta.\,X - \tfrac{3}{2}\,\Theta_+^2 X^2\}] + \frac{(2+\mu)\,\Theta_+^2}{(\Theta - 3)}(3X^2 + 2X + \tfrac{1}{4}), \end{aligned} \tag{43a}$$

$$\begin{aligned} \tilde{u}_2(X,\theta) = {} & -2\sin 2\theta\,[\tfrac{1}{16}\{(3\,\Theta - 5) + \mu(3\Theta. - 7)\}\,\Theta_+^2 + \tfrac{1}{8}\{(9\Theta - 10) \\ & + \mu(9\Theta_+ - 14)\}\,\Theta_+^2 + \tfrac{1}{4}\{(9\Theta - 5) + \mu(9\Theta_+ - 7)\}\,\Theta_+^2 X^2 \end{aligned}$$

$$+ \tfrac{1}{2}\{(3 + 3\mu)\, \Theta_+^3 X^3\} + 6\alpha\, [\tfrac{1}{16}(2 - \Theta_+)\, \Theta_+^2 + \tfrac{1}{8}(4 - 3\,\Theta_+)\, \Theta_+^2 X + \tfrac{1}{4}(2 - 3\,\Theta_+)\, \Theta_+^2 X^2 - \tfrac{1}{2}\, \Theta_+^3 X^3]. \quad (43b)$$

In the limit $\alpha \to 0$, (43*a*, *b*) reduce to the corresponding results of Urban[4].

5. Discussion of the results for small gap

The physical quantities of interest in this problem are the forces acting on the cylinders. If X' and Y' denote the components of force on the inner cylinder in the x and y directions respectively, then we have for a cylinder of length H, Abbot *et al*[7],

$$X' = R_1 H \int_0^{2\pi} (P_{(rr)} \cos\theta - P_{(r\theta)} \sin\theta)\, d\theta, \quad (44)$$

$$Y' = R_1 H \int_0^{2\pi} (P_{(rr)} \sin\theta + P_{(r\theta)} \cos\theta)\, d\theta, \quad (45)$$

where

$$P_{(rr)} = -P' + 2\rho\nu u_r \text{ and } P_{(r\theta)} = \left(v_r - \frac{v}{r} + \frac{1}{r} u_\theta\right)\rho\nu$$

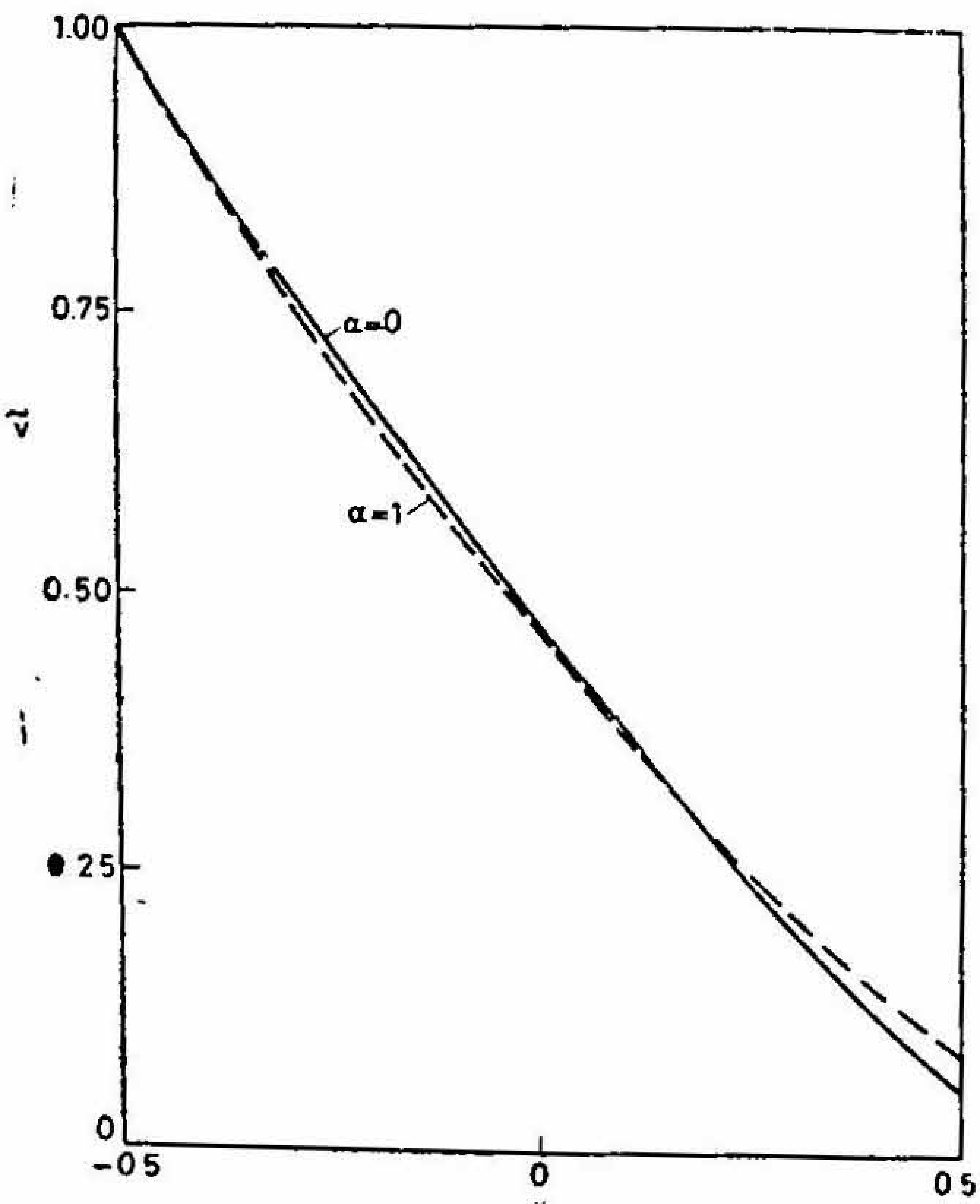

FIG. 2. Transverse velocity profiles at $\theta = 0$ when $\epsilon = 0{\cdot}1$.

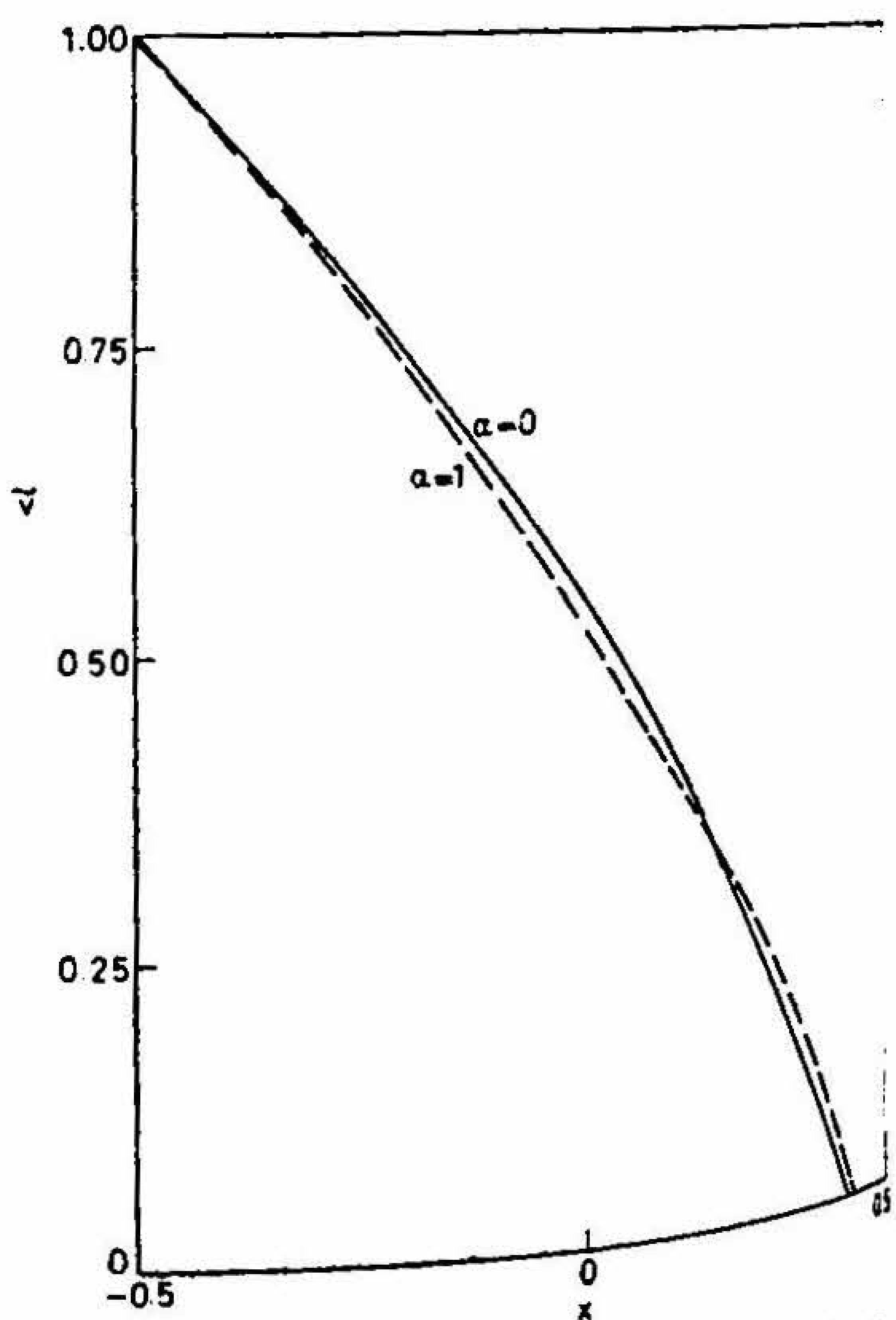

FIG. 3. Transverse velocity profiles at $\theta = \pi$ when $\epsilon = 0{\cdot}1$.

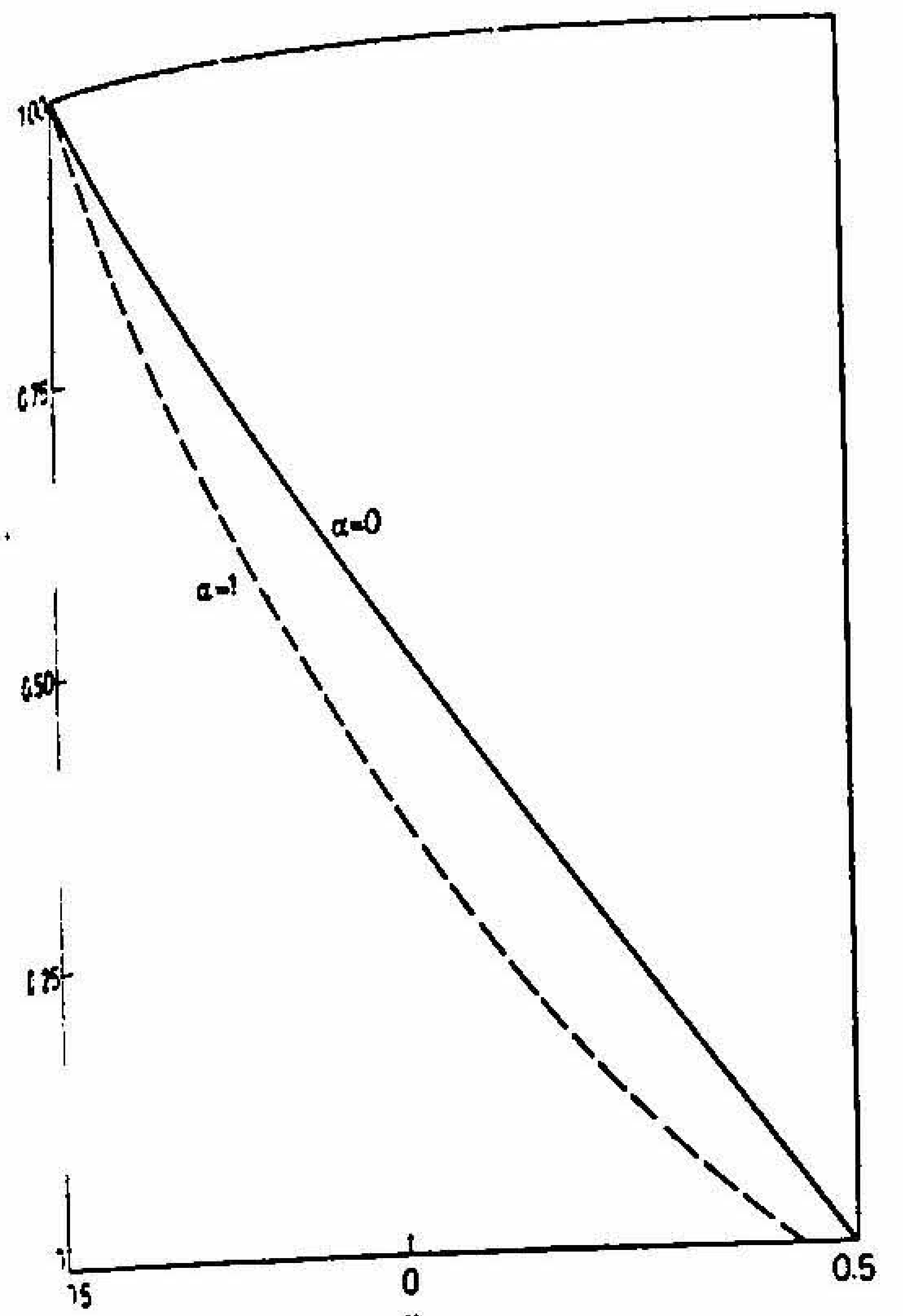

Fig. 4. Transverse velocity profiles at $\theta = \pi/2$ when $\epsilon = 0{\cdot}1$.

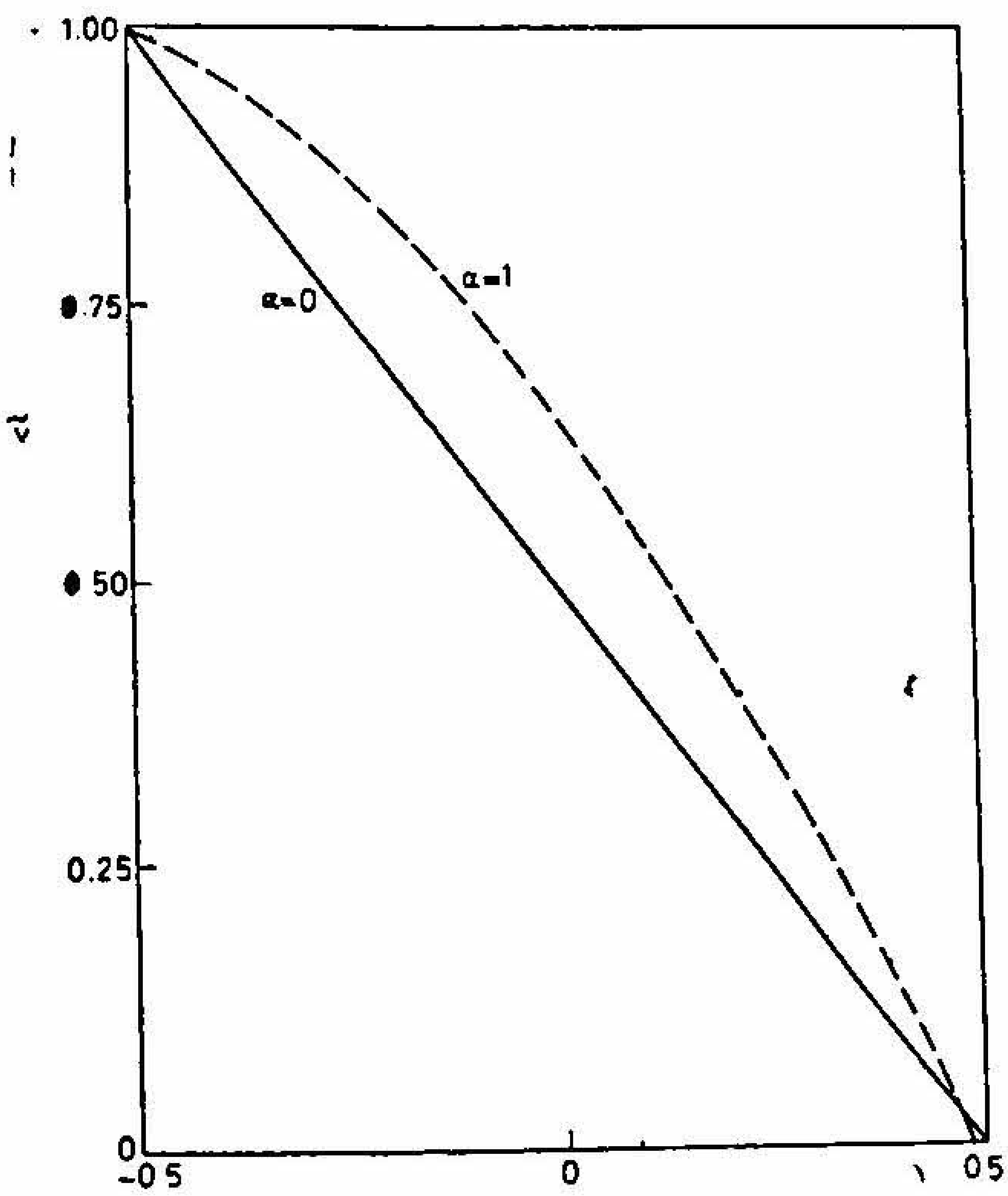

Fig. 5. Transverse velocity profiles at $\theta = 3\pi/2$ when $\epsilon = 0{\cdot}1$.

are the components of stress tensor on the inner cylinder and P' is the pressure. Substituting the expressions for $P_{(rr)}$ and $P_{(r\theta)}$ in terms of nondimensional velocities on the inner cylinder for the narrow gap case, we get the forces as

$$X' = 6R_1 \pi\rho\nu \Omega_1 H \frac{\epsilon}{\delta} \alpha,$$

$$Y' = -2\pi R_1 \rho\nu \Omega_1 H \frac{\epsilon}{\delta}(2\mu + 1) \tag{47}$$

where ρ is the density of the fluid. From (46) we observe that in the limit $\alpha \to 0$ Y' only exists. Thus the force X' is entirely on account of the radial flow due to the source along the axis of the inner cylinder.

The small gap transverse velocity is important in the stability analysis and has been discussed by Urban[4]. Our results coincide with those of Urban[4] when the suction parameter tends to zero. We discuss only the narrow gap transverse velocity profiles when the outer cylinder is at rest ($\mu = 0$), for a given eccentricity ratio which is small and for two prescribed values of the suction parameter. Figure 2 depicts the transverse velocity profiles for $\epsilon = 0 \cdot 1$ and for two values of suction parameter at the location $\theta = 0°$. It is seen that the effect of radial flow is to decrease the transverse velocity near the inner cylinder and to increase it near the outer boundary. The same thing happens, at the location $\theta = \pi$, as seen from fig. 3. Further it is seen that at $\theta = 0$ and π there is no effect of radial flow on the transverse velocity which includes terms up to ϵ order only. The transverse velocity profiles at the locations $\theta = \pi/2$ and $\theta = 3\pi/2$ respectively are shown in figs. 4 and 5 for different suction parameters and for $\epsilon = 0 \cdot 1$. At $\theta = \pi/2$, the effect of the radial flow is to decrease the transverse velocity near the inner cylinder whereas it increases at $\theta = 3\pi/2$.

Acknowledgement

The authors are thankful to the referees for their valuable comments and suggestions.

References

1. WOOD, W. W. *J. Fluid Mech.*, 1957, **3**, 159.
2. SEGEL, L. A. *Q. Appl. Math.*, 1961, **18**, 335.
3. KULINSKI, E. S. AND OSTRACH, S. *J. Appl. Mech.*, 1967, **34**, 16.
4. URBAN, R. L. *Appl. Sci. Res.*, 1971, **24**, 105.
5. VAN DYKE, M. *Perturbation methods in fluid mechanics*, Academic Press, New York, 1964.
6. NIKITIN, A. K. *Inzhenernyi Sbornik*, 1956, **23**, 173.
7. ABBOT, T. N. G., BOWEN, G. W. AND WALTERS, K. *J. Phys. D. Appl. Phys.*, 1971, **4**, 190.