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HEAT TRANSFER IN A FLOW OF VISCOUS INCOMPRESSIBLE FLUID IN A CYLINDER AND CYLINDRICAL ANNULUS

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ABSTRACT

The time dependent problems of heat transfer in the laminar flow of viscous incompressible fluid in cylinder and in the cylindrical annulus when the boundaries are kept at temperatures that are functions of time alone are considered taking into consideration the source of heat generation, dissipation and the convective changes in the temperature.

INTRODUCTION

1. Bhatnagar and Tikekar[1] have studied the temperature distribution in an incompressible viscous fluid flowing in the annulus between two coaxial cylinders considering the heat generating source but neglecting the viscous dissipation and convective changes in the temperature. Pai[2] has discussed the laminar flow problem and the steady state temperature distribution in a pipe without considering the heat generating function. Carslaw and Jaeger[3] have discussed the problem of time dependent temperature distribution in a circular cylinder without dissipation and heat generation. In this paper, we have studied the time dependent problem of heat transfer in the laminar flow of viscous incompressible fluid flowing in a doubly infinite circular cylinder taking into account, the source of heat generation. convective changes in the temperature and dissipation. We have also studied the flow problem and the temperature distribution for the same fluid flowing in the annulus between the two coaxial cylinders.

2. In this section we consider the problem of heat transfer in the doubly infinite circular cylinder of radius a for the axisymmetric case. Taking the axis of the cylinder as the z-axis and denoting by r the distance from this axis, the basic equations of the problem are :

Continuity Equation:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \qquad [2.1]$$

Momentum Equations:

$$\rho\left[u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right]=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial^2 u}{\partial r}+\frac{1}{r}\cdot\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right], \qquad [2.1]$$

$$\rho\left[u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right]=-\frac{\partial p}{\partial z}+\mu\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right], \qquad [2.3]$$

Energy Equation:

$$\rho c_v\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right]=\frac{\partial Q}{\partial t}+K\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right]$$

$$+\mu\left[2\left\{\left(\frac{\partial u}{\partial r}\right)^2+\left(\frac{u}{r}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right\}+\left\{\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right\}^2\right], \qquad [2.4]$$

where $(\partial Q/\partial t)$ is a function of time alone representing the rate of heat generation per unit volume per unit time in the fluid and other notations convey their usual meanings.

Initial Conditions:

$$T(r,0)=T_i, \quad Q(0)=Q_0. \qquad [2.5]$$

Boundary Conditions:

(i) For the velocity field : no slip condition at $r=a$ and finite velocity along the axis $r=0$

(ii) For the temperature field :

$$\left.\begin{aligned}&T(0,t)=\text{finite},\\ &T(a,t)=[1+g(t)]\,T_i\end{aligned}\right\}\ t>0 \qquad [2.6]$$

with $g(0)=0$.

Effecting the following transformation:

$$\eta = \frac{r}{a},\ \xi = \frac{z}{a},\ \mathbf{u} = \frac{u}{U},\ \mathbf{w} = \frac{w}{U},\ p_1 = \frac{p}{\rho U^2},$$

$$\tau = \frac{t\mu}{\rho a^2},\ \theta = \frac{T - T_i}{T_i},\ f(\tau) = \frac{\nu Q(t)}{K T_i}, \qquad [2.7]$$

where U is a characteristic velocity, *e.g.* the velocity along the axis $r = 0$, the above equations and the initial and boundary conditions reduce to:

Continuity Equatton:

$$\frac{\partial \mathbf{u}}{\partial \eta} + \frac{\mathbf{u}}{\eta} + \frac{\partial \mathbf{w}}{\partial \xi} = 0, \qquad [2.8]$$

Momentum Equations:

$$\mathbf{u}\frac{\partial \mathbf{u}}{\partial \eta} + \mathbf{w}\frac{\partial \mathbf{u}}{\partial \xi} = -\frac{\partial p_1}{\partial \eta} + \frac{1}{R}\left[\frac{\partial^2 \mathbf{u}}{\partial \eta^2} + \frac{1}{\eta}\cdot\frac{\partial \mathbf{u}}{\partial \eta} + \frac{\partial^2 \mathbf{u}}{\partial \xi^2} - \frac{\mathbf{u}}{\eta^2}\right], \qquad [2.9]$$

$$\mathbf{u}\frac{\partial \mathbf{w}}{\partial \eta} + \mathbf{w}\frac{\partial \mathbf{w}}{\partial \xi} = -\frac{\partial p_1}{\partial \xi} + \frac{1}{R}\left[\frac{\partial^2 \mathbf{w}}{\partial \eta^2} + \frac{1}{\eta}\cdot\frac{\partial \mathbf{w}}{\partial \eta} + \frac{\partial^2 \mathbf{w}}{\partial \xi^2}\right], \qquad [2.10]$$

Energy Equation:

$$\frac{1}{R}\cdot\frac{\partial \theta}{\partial \tau} + \mathbf{u}\frac{\partial \theta}{\partial \eta} + \mathbf{w}\frac{\partial \theta}{\partial \xi} = \frac{1}{R\sigma}\left[\frac{\partial f}{\partial \tau} + \frac{\partial^2 \theta}{\partial \eta^2} + \frac{1}{\eta}\cdot\frac{\partial \theta}{\partial \eta} + \frac{\partial^2 \theta}{\partial \xi^2}\right]$$

$$+ \frac{E}{R}\left[2\left\{\left(\frac{\partial \mathbf{u}}{\partial \eta}\right)^2 + \left(\frac{\mathbf{u}}{\eta}\right)^2 + \left(\frac{\partial \mathbf{w}}{\partial \eta}\right)^2\right\} + \left\{\frac{\partial \mathbf{u}}{\partial \xi} + \frac{\partial \mathbf{w}}{\partial \eta}\right\}^2\right], \qquad [2.11]$$

where

$$R = \frac{\rho U a}{\mu}, \text{ Reynolds number,}$$

$$E = \frac{U^2}{c_v T_i}, \text{ Eckert number,}$$

$$\text{and } \sigma = \frac{\mu c_v}{K}, \text{ Prandtl number.} \qquad [2.12]$$

Initial Conditions:

$$\theta(\eta, 0) = 0,\ f(0) = f_0, \qquad [2.13]$$

Boundary Conditions:

$u = 0,\ w = 0$ at $\eta = 1$ and finite at $\eta = 0$,

$$\left.\begin{array}{l}\theta(0, \tau) = \text{finite} \\ \theta(1, \tau) = g(\tau)\end{array}\right\} \tau > 0 . \qquad [2.14]$$

Taking the velocity field, compatible with the Continuity equation, to be

$$u = 0,\ w = w(\eta) \qquad [2.15]$$

We have[2]

$$w = -(P/4)(1 - \eta^2), \qquad [2.16]$$

where

$$P = R(\partial p_1/\partial \xi) = \text{constant}. \qquad [2.17]$$

Now the cylinder being doubly infinite, θ becomes independent of ξ and hence the energy equation becomes

$$\frac{\partial^2\theta}{\partial\eta^2} + \frac{1}{\eta}\cdot\frac{\partial\theta}{\partial\eta} - \sigma\frac{\partial\theta}{\partial\tau} = -\frac{\partial f}{\partial\tau} - \frac{E\sigma P^2}{4}\eta^2 \qquad [2.18]$$

We now adopt the usual Laplace transform technique assuming that $\theta(\eta, \tau)$ and $f(\tau)$ remain bounded as $t \to \infty$.

Denoting the Laplace transform of X (t) by X (p) and incorporating the initial and boundary conditions we get

$$\bar{\theta}(\eta, p) = \frac{1}{\sigma p}[p\,f(p) - f_0] + \frac{E P^2 \eta^2}{4 p^2} + \frac{E P^2}{\sigma p^3}$$

$$+ \frac{I_0[\eta\sqrt{(\sigma p)}]}{I_0[\sqrt{(\sigma p)}]}\left[g(p) - \frac{p\,f(p) - f_0}{\sigma p} - \frac{EP^2}{4p^2} - \frac{EP^2}{\sigma p^3}\right]. \qquad [2.19]$$

Since

$$\frac{I_0[\eta\sqrt{(\sigma p)}]}{I_0[\sqrt{(\sigma p)}]} \sim 1 \text{ as } |p| \to 0, \qquad [2.20]$$

$$\sim \eta^{-\frac{1}{2}} e^{-(1-\eta)\sqrt{(\sigma p)}} \text{ as } |p| \to \infty, \qquad [2.21]$$

from [2.20] it is clear that this expresssion has no branch point at $p = 0$, while [2.21] helps us in proving that the integral on the circle $|p| = R_n$, $(1/\sigma)\alpha_n^2 < R_n < (1/\sigma)\alpha_{n+1}^2$, tends to zero as $R_n \to \infty$, $\pm\alpha_n$ being the simple zeros of $J_0(\alpha)$ arranged in the ascending order of magnitude. We can easily show that

$$L^{-1}\left[\frac{I_0[\eta\sqrt{(\sigma p)}]}{I_0[\sqrt{(\sigma p)}]}\right] = \frac{2}{\sigma}\sum_{n=1}^{\infty}\frac{\alpha_n J_0(\alpha_n \eta)}{J_1(\alpha_n)} e^{-\alpha_n^2\tau/\sigma}. \qquad [2.22]$$

Applying the convolution theorem, we get from [2.19] and [2.22] after simplification

$$\theta(\eta, \tau) = \frac{1}{\sigma} f(\tau) - \frac{2f_0}{\sigma} \sum_{n=1}^{\infty} \frac{J_0(\alpha_n \eta)}{\alpha_n J_1(\alpha_n)} e^{-\alpha_n^2 \tau/\sigma}$$

$$+ EP^2\sigma \sum_{n=1}^{\infty} \left(\frac{1}{2\alpha_n^3} - \frac{2}{\alpha_n^5}\right)\left(1 - e^{-\alpha_n^2 \tau/\sigma}\right) \frac{J_0(\alpha_n \eta)}{J_1(\alpha_n)}$$

$$+ \frac{2}{\sigma} \sum_{n=1}^{\infty} \frac{\alpha_n J_0(\alpha_n \eta)}{J_1(\alpha_n)} \int_0^{\tau} \left[g(u) - \frac{f(u)}{\sigma}\right] e^{-\alpha_n^2 (\tau - u)/\sigma} du. \quad [2.23]$$

Knowing that

$$p\, g(p) = g(0) + \text{L.T.}\, [g'(\tau)], \quad [2.24]$$

where dash denotes derivation with respect to the argument and that

$$L^{-1}\left[\frac{I_0[\eta\sqrt{(\sigma p)}]}{p\, I_0[\sqrt{(\sigma p)}]}\right] = 1 + 2 \sum_{n=1}^{\infty} \frac{J_0(\alpha_n \eta)}{\alpha_n J_0'(\alpha_n)}\, e^{-\alpha_n^2 \tau/\sigma} \quad [2.25]$$

we obtain from [2.19], an alternative expression for $\theta(\eta, \tau)$:

$$\theta(\eta, \tau) = g(\tau) + E P^2 \sigma \sum_{n=1}^{\infty} \left(\frac{1}{2\alpha_n^3} - \frac{2}{\alpha_n^5}\right)\left(1 - e^{-\alpha_n^2 \tau/\sigma}\right) \frac{J_0(\alpha_n \eta)}{J_1(\alpha_n)}$$

$$+ 2 \sum_{n=1}^{\infty} \frac{J_0(\alpha_n \eta)}{\alpha_n J_0'(\alpha_n)} \int_0^{\tau} \left[g'(u) - \frac{f'(u)}{\sigma}\right] e^{-\alpha_n^2 (\tau - u)/\sigma} du. \quad [2.26]$$

3. In this section we consider the flow problem and the problem of heat tranrfer in the fluid flowing in the annulus between two coaxial cylinders with radii R_1 and a $(> R_1)$ for the axisymmetric case. The basic equations of the problem are the same as [2.1] to [2.4]. With no slip condition at both the boundaries the velocity field is given by

$$w = \frac{P\eta^2}{4} - \frac{P}{4} + \frac{P(a^2 - R_1^2)}{4a^2\, ln(R_1/a)}\, ln\, \eta. \quad [3.1]$$

We take the initial conditions same as [2.5] in the case of cylinder above in § 2, while we choose the following boundary conditions for the temperature field:

$$\left.\begin{aligned} T(R_1\ t) &= [1 + \theta_1(t)]T_i \\ T(a, t) &= [1 + \theta_2(t)]\ T_i \end{aligned}\right\} t > 0 \quad [3.2]$$

with $\theta_1(0) = \theta_2(0) = 0.$ [3.3]

The tranformation [2.7] reduce the energy equation and the boundary conditions as follows:

Energy Equation:

$$\frac{\partial^2\theta}{\partial\eta^2} + \frac{1}{\eta}\cdot\frac{\partial\theta}{\partial\eta} - \sigma\frac{\partial\theta}{\partial\tau} = -\frac{\partial f}{\partial\tau} - E\sigma\left[\frac{P^2}{4}\eta^2 + \frac{1}{\eta^2}\left\{\frac{P(a^2-R_1^2)}{4a^2\ln(R_1/a)}\right\}^2 + \frac{P^2(a^2-R_1^2)}{4a^2\ln(R_1/a)}\right], \quad [3.4]$$

Boundary Conditions:

$$\left.\begin{aligned}\theta[(R_1/a),\tau] &= \theta_1(\tau)\\ \theta(1,\tau) &= \theta_2(\tau)\end{aligned}\right\}\ \tau>0. \quad [3.5]$$

Taking the Laplace transform, assuming that the temperature and the rate of heat generation remain bounded as $t\to\infty$ and incorporating the the initial and boundary conditions we can easily show that

$$\theta(\eta,\tau) = \frac{1}{4}EP^2\tau\left[\eta^2 - \frac{a^2-R_1^2}{a^2\ln(R_1/a)}\right] + \frac{EP^2}{2\sigma}\tau^2 - \frac{f_0}{\sigma} + \frac{f(\tau)}{\sigma} + \int_0^\tau S_1(u)\,F(t-u)\,du + \int_0^\tau S_2(u)\,G(t-u)\,du + S_3(\tau), \quad [3.6]$$

with

$$S_1(\tau) = L^{-1}\left[\mathrm{S}_1(p) \equiv \bar{\theta}_1(p) - \frac{EP^2R_1^2}{4p^2a^2} + \mathrm{H}(p)\right], \quad [3.7]$$

$$\mathrm{H}(p) = \frac{f_0 - p\,\bar{f}(p)}{\sigma p} + \frac{EP^2(a^2-R_1^2)}{4a^2\ln(R_1/a)}\cdot\frac{1}{p^2} - \frac{EP^2}{\sigma p^3}, \quad [3.8]$$

$$S_2(\tau) = L^{-1}\Bigg[\mathrm{S}_2(p) \equiv \bar{\theta}_2(p) - \frac{EP^2}{4p^2} + \mathrm{H}(p) + \frac{M}{p}\Big\{[K_0\{\sqrt{(\sigma p)}\}\,I_1\{\sqrt{(\sigma p)}\} + I_0\{\sqrt{(\sigma p)}\}\,K_1\{\sqrt{(\sigma p)}\}] - \frac{R_1}{a}\Big\langle K_0\{\sqrt{(\sigma p)}\}\,I_1\Big(\frac{R_1}{a}\sqrt{(\sigma p)}\Big) + I_0\{\sqrt{(\sigma p)}\}\,K_1\Big(\frac{R_1}{a}\sqrt{(\sigma p)}\Big)\Big\rangle\Big\}\Bigg], \quad [3.9]$$

$$M = \frac{E\sigma P^2 (a^2 - R_1^2)^2}{16 a^4 [ln(R_1/a)]^2}, \tag{3.10}$$

$$S_3(\tau) = L^{-1}\Big[S_3(p) \equiv \frac{M}{p}\Big\{\eta[K_0\{\eta\surd(\sigma p)\} I_1\{\eta\surd(\sigma p)\} + I_0\{\eta\surd(\sigma p)\} K_1\{\eta\surd(\sigma p)\}] - \frac{R_1}{a}\Big\langle K_0\{\eta\surd(\sigma p)\} I_1\Big(\frac{R_1}{a}\surd(\sigma p)\Big) + I_0\{\eta\surd(\sigma p)\} K_1\{\eta\surd(\sigma p)\}\Big\rangle\Big\}\Big], \tag{3.11}$$

$$F(\tau) = L^{-1}[F(p) \equiv \{1/D(p)\}\langle K_0\{\surd(\sigma p)\} I_0\{\eta\surd(\sigma p)\} - I_0\{\surd(\sigma p)\} K_0\{\eta\surd(\sigma p)\}\rangle], \tag{3.12}$$

$$G(\tau) = L^{-1}\Big[G(p) \equiv \frac{1}{D(p)}\Big\{K_0\{\eta\surd(\sigma p)\} I_0\Big(\frac{R_1}{a}\surd(\sigma p)\Big) - K_0\Big(\frac{R_1}{a}\surd\,\sigma p)\Big) I_0\{\eta\surd(\sigma p)\}\Big\}\Big], \tag{3.13}$$

where

$$D(p) = I_0\Big(\frac{R_1}{a}\surd(\sigma p)\Big) K_0\surd(\sigma p) - K_0\Big(\frac{R_1}{a}\surd(\sigma p)\Big) I_0\surd(\sigma p). \tag{3.14}$$

Now we can easily check that

$$S_1(\tau) = \theta_1(\tau) + \frac{j_0 - f(\tau)}{\sigma} + \frac{E P^2 \tau}{4a^2}\Big(\frac{a^2 - R_1^2}{ln(R_1/a)} - R_1^2\Big) - \frac{E P^2 \tau^2}{2\sigma}. \tag{3.15}$$

$S_3(p)$ has a branch point at $p = 0$ so that we use the usual contour that excludes this point along with the entire negative real axis. There are no poles within or on this contour. We can easily show that the integral over the large and the small circles both vanish and we have

$$S_3(\tau) = M\int_0^\infty \frac{e^{-u^2\tau/\sigma}}{u}\Big[\{J_0(\eta u) Y_1(\eta u) - J_1(\eta u) Y_0(\eta u)\} - \frac{R_1}{a}\Big\{J_0(\eta u) Y_1\Big(\frac{R_1}{a}v\Big) - J_1\Big(\frac{R_1}{a}u\Big) Y_0(\eta u)\Big\}\Big] du. \tag{3.16}$$

Similarly we have

$$S_2(\tau) = \theta_2(\tau) + \frac{f_0 - f(\tau)}{\sigma} + \frac{E P^2 \tau}{4}\Big(\frac{a^2 - R_1^2}{a^2\, ln(R_1/a)} - 1\Big) - \frac{E P^2 \tau^2}{2\sigma} + S_2'(\tau), \tag{3.17}$$

where

$$S_2'(\tau) = M\int_0^\infty \frac{e^{-u^2\tau/\sigma}}{u}\Bigg[\{J_0(u)\,Y_1(u) - J_1(u)\,Y_0(u)\}$$

$$-\frac{R_1}{a}\left\{J_0(u)\,Y_1\!\left(\frac{R_1}{a}u\right) - J_1\left(\frac{R_1}{a}u\right)Y_0(u)\right\}\Bigg]\,du \qquad [3.18]$$

Following Bhatnagar and Tikekar[1], we have

$$F(\tau) = \sum_{n=1}^{\infty} \frac{2\alpha_n}{\rho_n} e^{-\alpha_n^2\tau/\sigma}[J_0(\alpha_n\eta)\,Y_0(\alpha_n) - J_0(\alpha_n)\,Y_0(\alpha_n\eta)] \qquad [3.19]$$

and

$$G(\tau) = \sum_{n=1}^{\infty} \frac{2\alpha_n}{\beta_n} e^{-\alpha_n^2\tau/\sigma}\left[J_0\left(\alpha_n\frac{R_1}{a}\right)Y_0(\alpha_n\eta) - J_0(\alpha_n\eta)\,Y_0\left(\alpha_n\frac{R_1}{a}\right)\right], \qquad [3.20]$$

where

$$\beta_n = \Bigg[J_0\left(\alpha_n\frac{R_1}{a}\right)Y_1(\alpha_n) - J_1(\alpha_n)Y_0\left(\alpha_n\frac{R_1}{a}\right)$$

$$+\frac{R_1}{a}\left\{J_1\left(\alpha_n\frac{R_1}{a}\right)Y_0(\alpha_n) - J_0(\alpha_n)\,Y_1\left(\alpha_n\frac{R_1}{a}\right)\right\}\Bigg] \qquad [3.21]$$

and α_n are the zeros of

$$J_0\left(\alpha\frac{R_1}{a}\right)Y_0(\alpha) - J_0(\alpha)\,Y_0\left(\alpha\frac{R_1}{a}\right) \qquad [3.22]$$

arranged in ascending order of magnitude.

Thus with the help of [3.7] to [3.22] and with the use Fourier-Bessel series for $\frac{1}{2}$ and η^2 we get after simplification

$$\theta(\eta,\tau) = (1/\sigma)f(\tau) + S_3(\tau)$$

$$+\sum_{n=1}^{\infty}\Bigg[\frac{2\alpha_n}{\sigma\beta_n}\{J_0(\alpha_n\eta)\,Y_0(\alpha_n) - J_0(\alpha_n)\,Y_0(\alpha_n\eta)\}\times$$

$$\Bigg\{\int_0^\tau\left\langle\theta_1(u) - \frac{f(u)}{\sigma}\right\rangle e^{-\alpha_n^2(\tau-u)/\sigma}\,du$$

$$- \frac{f_0}{\alpha_n^2} e^{-\alpha_n^2 \tau/\sigma} - \frac{E P^2 \sigma^2}{4 a^2 \alpha_n^4}\left(\frac{a^2 - R_1^2}{ln(R_1/a)} - R_1^2\right)(1 - e^{-\alpha_n^2 \tau/\sigma})$$

$$- \frac{E P^2 \sigma^2}{\alpha_n^6}(1 - e^{-\alpha_n^2 \tau/\sigma})\Big\}\Big]$$

$$+ \sum_{n=1}^{\infty} \left[\frac{2 \alpha_n}{\sigma \beta_n}\left\{J_0\left(\alpha_n \frac{R_1}{a}\right) Y_0(\alpha_n \eta) - J_0(\alpha_n \eta) Y_0\left(\alpha_n \frac{R_1}{a}\right)\right\}\right.$$

$$\times \left\{\int_0^{\tau}\left\langle \theta_2(u) - \frac{f(u)}{\sigma} + S_2'(u)\right\rangle e^{-\alpha_n^2(\tau - u)/\sigma} du\right.$$

$$- \frac{f_0}{\alpha_n^2} e^{-\alpha_n^2 \tau/\sigma} - \frac{E P^2 \sigma^2}{4 \alpha_n^4}\left(\frac{a^2 - R_1^2}{a^2 ln(R_1/a)} - 1\right)(1 - e^{-\alpha_n^2 \tau/\sigma})$$

$$\left. \left. - \frac{E P^2 \sigma^2}{\alpha_n^6}(1 - e^{-\alpha_n^2 \tau/\sigma})\right\}\right]. \qquad [3.23]$$

4. To conclude we note that both in § 2 and § 3 the solution exhibits separately the contributions of the boundary conditions, heat generation and the Eckert and Prandtl numbers. Thus given the boundary conditions and the rate of heat generation, we can specify the solution completely in each case.

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