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On wave motion due to rolling of a submerged thin vertical plate

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Abstract

The use of Havelock's expansion of water wave potential in the study of wave motion set up due to small rolling oscillations of a thin vertical plate submerged in deep water gives rise to a singular integral equation involving a combination of logarithmic and Cauchy-type kernel in a double interval. Its solution is obtained in a straightforward manner, wherein the Plemelj formula is suitably utilized in the analysis. The amplitude of wave motion at large distances from the plate and the velocity potential are obtained explicitly for this problem.

Keywords: Wave motion, Havelock expansion, water potential, rolling oscillations, submerged plate problem.

1. Introduction

The wave motion due to small rolling oscillations of a thin vertical plate partially immersed in deep water was studied long back by Ursell[1]. He used Havelock's expansion of water wave potential to reduce the problem to the solution of the following singular integral equation of the first kind:

$$\int_0^\infty F(t)G(x,t)\,\mathrm{d}t = 0, \quad x \in L, \tag{1}$$

where

$$G(x,t) = K \ln\left|\frac{x-t}{x+t}\right| + \frac{1}{x-t} + \frac{1}{x+t}, \tag{2}$$

K is a positive constant, L is (a, ∞) and

$$F(t) = \begin{cases} f_0(t), & t \in (0,\infty) - L, \\ g(t), & t \in L. \end{cases} \tag{3}$$

Here $f_0(t)$ is the known horizontal component of velocity on the plate and $g(t)$ is the unknown horizontal component of velocity across the gap and is such that it has integrable singularity at $t = a$. This integral equation was then reduced to another integral equation with Cauchy-type kernel whose solution was known. Utilizing the solution of this integral equation, the amplitude of wave motion at large distances from the plate was obtained. Later, Evans[2] used a tailored version of Green's integral theorem to obtain the amplitude at infinity of the wave motion set up due to a general motion of a partially immersed thin vertical plate. Using this idea of Evans, Mandal[3] obtained the amplitude at infinity of the wave motion generated due to small oscillations of a vertical plate submerged in deep water. He deduced, as a special case, the results for rolling oscillations of the plate. The expressions for the velocity potential were not obtained explicitly both by Evans[2] and by Mandal[3]. Recently, Banerjea and Mandal[4] obtained the closed-form solution of the problem of generation of water waves due to rolling of a vertical plate either partially immersed or completely submerged in deep water. They used Green's integral theorem to reduce the problem to a singular integral equation with a Cauchy kernel, whose solution was obtained by standard techniques. Explicit expression were obtained for the amplitude of wave motion at large distances from the plate. The more general problem of diffraction of water waves by a thin vertical plate submerged in deep water and performing small rolling oscillations with frequency equal to the frequency of the normally incident train of plane waves was studied by Evans[5]. He used the complex-variable theory and introduced the so-called reduced potential which satisfies a Riemann–Hilbert boundary value problem. The solution of this Riemann–Hilbert problem and the Plemelj formula were then utilized to obtain the general solution.

It may be noted that for the submerged-plate problem the integral equation obtained by Banerjea and Mandal[4] was somewhat similar to eqn (1) with $L = (a, b)$, with $g(t)$ denoting the difference of potential across the plate, which as such vanishes at the end points a and b. This property of $g(t)$ vanishing at the end points was exploited to reduce the integral equation to another singular integral equation of the first kind with a Cauchy-type kernel, whose solution was immediate. However, in the present paper, the application of Havelock's expansion of water wave potential to this submerged-plate problem leads to an integral equation similar to eqn (1), with the modification that L now consists of a double interval $(0, a)$ and (b, ∞), where $b - a$ is the vertical length of the submerged plate and $g(t)$ denotes the unknown horizontal component of velocity above and below the plate and satisfies the requirement that it has integrable singularities at a and b. This requirement on $g(t)$ creates hindrance in the possible reduction of the integral eqn (1) (with L consisting of a double interval) to a singular integral equation of the first kind with a Cauchy-type kernel. As such, special attention is needed to study the integral eqn (1) in a double interval.

Recently, Banerjea and Mandal[6] obtained the closed-form solution of the integral equation

$$\int g(t)\, G(x, t)\, \mathrm{d}t = e(x), \quad x \in L \tag{4}$$

when L consists of a double interval $(0, a)$ and (b, ∞), $e(x)$ is prescribed and the unknown function $g(t)$ is such that it has integrable singularities at $t = a$ and $t = b$. Therefore, when L consists of $(0, a)$ and (b, ∞), the solution of eqn (1) can be obtained from the solution of eqn (4) by taking

$$e(x) = -\int_a^b f_0(t) G(x, t)\, \mathrm{d}t, \ \ x \in L.$$

However, exploiting this special form of $e(x)$, here we solve the integral eqn (1) for the aforesaid double interval directly, as this facilitates the simplification of the various integrals involved in the later part of the analysis. The solution is obtained in a straightforward manner by utilizing the solution of the integral equation with Cauchy kernel in $(0, \infty)$ and (a, b). The solution in (a, b) is well known in the literature[7] and the solution in $(0, \infty)$ is obtained by function-theoretic method by using the Plemelj formula. Finally, the solution of (1) for $L = (0, a) + (b, \infty)$ is used to solve the rolling problem in closed form. Explicit expression for the amplitude of the wave motion at large distances from the plate is derived. This result agrees fully with that of Banerjea and Mandal[4] but differs by the constant $N(a)\gamma_0/\Delta_0$ (see eqns (56) and (53)) from the result for the rolling problem deduced from Evans[5]. However, a calculation reveals that the constant A in eqn (22) of Evans[5] is actually complex, and not real, as mentioned there. This is evident if one equates the value of the stream function $\psi(0, y)$ on the plate deduced from the expression of the complex potential $\omega(z)$ given by eqn (22) of Evans[5] with eqn (16); there a nonzero value of the imaginary part of A is found. If this is taken into account in Evans[5], then the expression of the wave amplitude for the rolling problem deduced from there coincides with the result obtained here. Also, the expression for the wave amplitude due to rolling of a partially immersed plate (cf. Ursell[1] and Evans[2]) can be deduced from the present result after taking appropriate limit.

2. Statement and formulation of the problem

We consider a thin rigid vertical plate $x = 0$, $a < y < b$, submerged in deep water occupying the region $y \geq 0$, with $y = 0$ as the mean free surface. The plate is hinged at $(0, s)$ $(a < s < b)$ and is forced to perform simple harmonic oscillations of amplitude θ_0 about its mean vertical position. Assuming the motion to be irrotational, it can be described by a velocity potential $\mathrm{Re}[\phi(x, y)\exp(-\mathrm{i}\sigma t)]$, σ being the circular frequency, where ϕ satisfies the following conditions:

$$\nabla^2\phi = 0 \quad \text{in the fluid region;} \tag{5}$$

the linearized free surface condition

$$K\phi + \frac{\partial \phi}{\partial y} = 0 \quad \text{on } y = 0, \tag{6}$$

where $K = \sigma^2/g$, g being the acceleration due to gravity; the condition on the plate,

$$\frac{\partial \phi}{\partial x} = f_0(y) \quad \text{on } x = 0,\ a < y < b, \tag{7}$$

where

$$f_0(y) = \mathrm{i}\sigma\theta_0(y - s); \tag{8}$$

the edge condition,

$$r^{1/2}\,\nabla\phi \text{ is bounded} \quad \text{as } r \to 0, \tag{9}$$

r being the distance from the two sharp edges $(0, a)$ and $(0, b)$ of the plate;
the bottom condition,

$$\nabla\phi \to 0 \quad \text{as } y \to \infty; \tag{10}$$

and the radiation condition,

$$\phi \sim \begin{cases} A\exp(-Ky + \mathrm{i}Kx) & \text{as } x \to \infty, \\ B\exp(-Ky - \mathrm{i}Kx) & \text{as } x \to -\infty, \end{cases} \tag{11}$$

where A and B are the amplitudes (unknown) of the wave motion at large distances (positive and negative infinity, respectively) from the plate.

3. Reduction of the problem to an integral equation and its solution

Let

$$\frac{\partial\phi}{\partial x}(\pm 0, y) = F(y), \quad 0 < y < \infty, \tag{12}$$

then

$$F(y) = \begin{cases} f_0(y) & \text{for } a < y < b, \\ g(y) & \text{for } y \in L, \end{cases} \tag{13}$$

where $f_0(y)$ is given by eqn (7), and $g(y)$ is unknown for $y \in L$, L being the double interval $(0, a)$ and (b, ∞). Moreover, by virtue of eqn (9),

$$g(y) = \begin{cases} 0(|b - y|^{-1/2}) & \text{as } y \to b, \\ 0(|y - a|^{-1/2}) & \text{as } y \to a. \end{cases} \tag{14}$$

Using Havelock's expansion of water wave potential, a suitable representation of $\phi(x, y)$ satisfying eqns (5), (6), (10) and (11) is given by

$$\phi(x, y) = \begin{cases} A\exp(-Ky + \mathrm{i}Kx) + \int_0^\infty C(k)\,M(k, y)\exp(-kx)\,\mathrm{d}k, & a > 0, \\ B\exp(-Ky - \mathrm{i}Kx) + \int_0^\infty D(k)\,M(k, y)\exp(kx)\mathrm{d}k, & x < 0, \end{cases} \tag{15}$$

where

$$M(k, y) = k\cos ky - K\sin ky. \tag{16}$$

Utilizing eqn (15) in eqn (12) and using Havelock's expansion theorem, we find

$$A = -B = -2\mathrm{i}\int_0^\infty F(y)\exp(-Ky)\,\mathrm{d}y \tag{17}$$

and

$$-C(k) = D(k) = \frac{2}{\pi}\frac{1}{k(K^2 + K^2)}\int_0^\infty F(y)\,M(k, y)\,\mathrm{d}y. \tag{18}$$

An integral equation for $g(y)$ is obtained from the fact that $\phi(x, y)$ is continuous below and above the plate so that

$$\phi(+0, y) = \phi(-0, y), \quad y \in L .$$

This gives

$$\frac{\pi}{2} A \exp(-Ky) = \int_0^\infty F(u) \int_0^\infty \frac{M(k, u) M(k, y)}{k(k^2 + K^2)} \, dk \, du, \;\; y \in L. \tag{19}$$

Applying the operator $d/dy + K$ to eqn (19), we obtain

$$\int_0^\infty F(u) \left[K \ln \left| \frac{y-u}{y+u} \right| + \frac{1}{y-u} + \frac{1}{y+u} \right] du = 0, \quad y \in L, \tag{20}$$

which is an integral equation in $g(u)$, $u \in L$.

Let us now define

$$H(u) = \begin{cases} 0 & \text{for } u \in L, \\ h(u) & \text{for } a < u < b, \end{cases} \tag{21}$$

where $h(u)$ is unknown and noting eqn (14) we see that

$$h(u) = \begin{cases} 0(|b-u|^{-1/2}) \;\; \text{as} \;\; u \to b, \\ 0(|u-a|^{-1/2}) \;\; \text{as} \;\; u \to a. \end{cases} \tag{22}$$

Then eqn (20) becomes

$$\int_0^\infty F(u) \left[K \ln \left| \frac{y-u}{y+u} \right| + \frac{1}{y-u} + \frac{1}{y+u} \right] du = H(y), \quad 0 < y < \infty. \tag{23}$$

Writing

$$\lambda(u) = K \int_0^u F(x) \, dx + F(u), \tag{24}$$

eqn (23) reduces to

$$\int_0^\infty \frac{\lambda(u)}{y^2 - u^2} \, du = \frac{H(y)}{2y}, \quad 0 < y < \infty. \tag{25}$$

Substituting $y^2 = t$ and $u^2 = v$, eqn (25) becomes

$$\int_0^\infty \frac{\lambda_0(v)}{t - v} \, dv = H_1(t), \quad 0 < t < \infty, \tag{26}$$

where

$$H_1(t) = t^{-1/2} H(t), \qquad \lambda_0(t) = t^{-1/2} \lambda(t) \tag{27}$$

so that

$$\lambda_0(t) = 0(|t|^{-1/2}) \quad \text{as } t \to 0. \tag{28}$$

Let us define a sectionally analytic function

$$\Omega(z) = \frac{1}{2\pi i}\int_0^\infty \frac{\lambda_0(v)}{v-z}\,dv, \quad z = t + i\omega,$$

so that $\Omega(z)$ is analytic in the complex z-plane cut along 0 to ∞ and $|\Omega(z)| < \infty$. By using the Plemelj formula it is easy to show that

$$\begin{aligned}\Omega^+(t) + \Omega^-(t) &= \frac{-H_1(t)}{\pi i} \quad (0 < t < \infty),\\ \Omega^+(t) - \Omega^-(t) &= \lambda_0(t)\end{aligned} \tag{29}$$

where

$$\Omega^{\pm}(t) = \lim_{\omega \to \pm 0} \Omega(z) \quad (t > 0). \tag{30}$$

Equation (29) is a Riemann–Hilbert problem whose solution is given by Muskelishvili[8]:

$$\Omega(z) = z^{-1/2}\left[\frac{S}{2} - \frac{1}{2\pi i}\int_0^\infty t^{1/2}\frac{H_1(t)}{\pi i}\frac{dt}{t-z}\right],$$

where S is an arbitrary constant. Applying the Plemelj–Sokhotskii formula, we find from (30) that

$$\lambda_0(t) = t^{-1/2}\left[S + \frac{1}{\pi^2}\int_0^\infty \frac{v^{1/2}H_1(v)}{v-t}\,dv\right],$$

which on using eqn (27) gives

$$\lambda(y) = \frac{2}{\pi^2}\int_0^\infty \frac{tH(t)}{t^2 - y^2}\,dt + S. \tag{31}$$

Using eqn (24) we now obtain

$$F(x) = \frac{d}{dx}\left[\exp(-Kx)\int_0^x \exp(Ku)\lambda(u)\,du\right], \quad 0 < x < \infty. \tag{32}$$

However, in eqns (32) and (31), $H(t)$ involves the unknown function $h(t)$. An integral equation for $h(t)$ is found from the fact that

$$F(x) = f_0(x), \quad a < x < b,$$

so that

$$\int_0^x \exp(Ku)\left[\frac{2}{\pi^2}\int_0^\infty \frac{tH(t)}{t^2-u^2}\,dt + S\right]du = \exp(Kx)\left[i\sigma\theta_0\left(\frac{x^2}{2} - sx\right) + C\right],$$

where C is an arbitrary constant. This gives

$$\frac{1}{\pi}\int_0^\infty \frac{tH(t)}{t^2-x^2}\,\mathrm{d}t = W + \pi m(x), \quad a<x<b, \tag{33}$$

where $W = \pi(KC - S)$ can be regarded as an arbitrary constant and

$$m(x) = \mathrm{i}\sigma\theta_0\left\{\frac{Kx^2}{2} + (1-Ks)x - s\right\}. \tag{34}$$

An integral equation for $h(x)$ is then obtained as

$$\frac{1}{\pi}\int_a^b \frac{2th(t)}{t^2-x^2}\,\mathrm{d}t = W + \pi m(x), \quad a<x<b. \tag{35}$$

The solution of eqn (35) satisfying eqn (22) is given by Banerjea and Mandal[6]:

$$h(x) = \frac{1}{R_2(x)}\left[W(d^2-x^2) + \int_a^b \frac{2tm(t)}{x^2-t^2}R_2(t)\,\mathrm{d}t\right], \quad a<x<b, \tag{36}$$

where d^2 is an arbitrary constant and

$$R_2(u) = \{(u^2-a^2)\,(b^2-u^2)\}^{1/2}. \tag{37}$$

Now, for $a<x<b$, we have from eqns (13), (32) and (30) that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\exp(-Kx)\left\{\int_0^x \frac{2}{\pi^2}\exp(Ku)\left(\int_a^b \frac{tH(t)}{t^2-u^2}\,\mathrm{d}t\right)\mathrm{d}u\right\} - \frac{S}{K}\right] = f_0(x).$$

The u-integral ranging over $(0, x)$ can be subdivided into $(0, a)$ and (a, x) for $a<x<b$. The integral over $(0, a)$ can be simplified by using $h(t)$ from (36). The integral over (a, x) for $a<x<b$ can also be simplified by noting (33). This finally gives

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\exp(-Kx)\left[-C - N(a) + \int_0^a \exp(Ku)\lambda_1(u)\mathrm{d}u\right]\right\} = 0, \quad a<x<b, \tag{38}$$

where C is defined by eqn (33):

$$\lambda_1(u) = \frac{1}{R_1(u)}\left[-W(d^2-u^2) + \frac{2}{\pi}F(a,b,u)\right], \tag{39}$$

$$R_1(u) = \{(a^2-u^2)(b^2-u^2)\}^{1/2},$$

$$F(a,b,u) = \int_a^b \frac{vR_2(v)}{u^2-v^2}\,m(v)\,\mathrm{d}v, \tag{40}$$

$$N(t) = \mathrm{i}\sigma\theta_0\left(\frac{t^2}{2} - ts\right)\exp(Kt). \tag{41}$$

Equation (38) now implies that the expression in the square bracket vanishes. That is,

$$\int_0^a \exp(Ku)\lambda_1(u)\,\mathrm{d}u - C - N(a) = 0. \tag{42}$$

Thus, eqn (42) gives C in terms of W and d^2. Finally, using eqns (32), (36), (37), (42) and (39)–(42), the explicit solution to eqn (20) is given by

$$g(x) = \begin{cases} \dfrac{d}{dx}\left[\exp(-Kx)\left\{N(a) + \int_a^x \exp(Ku)\lambda_1(u)\,du\right\}\right], & x < a, \\ \dfrac{d}{dx}\left[\exp(-Kx)\left\{N(b) + \int_b^x \exp(Ku)\lambda_2(u)\right\}\right], & x > b, \end{cases} \tag{43}$$

where

$$\lambda_2(u) = \frac{1}{R_3(u)}\left[W(d^2 - u^2) - \frac{2}{\pi}F(a,b,u)\right], \tag{44}$$

$$R_3(u) = \{(u^2 - a^2)(u^2 - b^2)\}^{1/2}. \tag{45}$$

It may be noted from eqn (43) that the solution of the integral equation (20) contains two arbitrary constants W and d^2.

4. Solution of the problem

To determine the unknown constants, $F(y)$ is substituted in eqn (19). A considerable manipulation is involved in simplifying the various integrals on the right-hand side of eqn (19). We thus obtain

$$\left[A - \int_a^b \lambda_3(u)\exp(-Ku)\,du\right]\exp(-Ky) = 0 \quad \text{for } y < a, \tag{46}$$

$$\left[A - \int_a^b \lambda_3(u)\{\exp(-Ku) - \exp(Ku)\}\,du\right]\exp(-Ky) = 0 \quad \text{for } y > b, \tag{47}$$

where

$$\lambda_3(u) = \frac{1}{R_2(u)}\left[-W(d^2 - u^2) + \frac{2}{\pi}F(a,b,u)\right], \tag{48}$$

$R_2(u)$ and $F(a, b, u)$ are defined by eqns (37) and (40), respectively. Equations (46) and (47) imply that

$$A = \int_a^b \lambda_3(u)\exp(-Ku)\,du$$

and

$$A = \int_a^b \lambda_3(u)\{\exp(-Ku) - \exp(Ku)\}\,du,$$

so that

$$\int_a^b \lambda_3(u)\exp(Ku)\,du = 0. \tag{49}$$

This gives a relation between W and d^2, so that

$$A = -W\gamma_0 + \gamma_1, \tag{50}$$

where

$$\gamma_0 = \int_a^b \frac{d^2 - u^2}{R_2(u)} \exp(-Ku)\, du, \tag{51}$$

$$\gamma_1 = \frac{2}{\pi}\int_a^b \frac{F(a,b,u)}{R_2(u)} \exp(-Ku)\, du. \tag{52}$$

Equation (50) gives a relation between W, d^2 and A. A second relation is obtained by using $g(y)$ from eqn (43) in eqn (17), which is given by

$$A = \mathrm{i}[S(k) + (\alpha_0 - \beta_0)W - \alpha_1 + \beta_1], \tag{53}$$

where

$$S(K) = N(a) - \int_a^b \exp(-Ky)m(y)\, dy,$$

$$\alpha_0 = \int_{-a}^a \frac{d^2 - u^2}{R_1(u)} \exp(-Ku)\, du,$$

$$\alpha_1 = \frac{2}{\pi}\int_{-a}^a \frac{F(a,b,u)}{R_1(u)} \exp(-Ku)\, du,$$

$$\beta_1 = \frac{2}{\pi}\int_b^\infty \frac{F(a,b,u)}{R_3(u)} \exp(-Ku)\, du,$$

$$\beta_0 = \int_b^\infty \frac{d^2 - u^2}{R_3(u)} \exp(-Ku)\, du. \tag{54}$$

Thus, from eqns (50) and (53), we get

$$W = \frac{\Delta_1 - S(K)}{\Delta_0}, \tag{55}$$

where

$$\Delta_1 = \alpha_1 - \beta_1 - \mathrm{i}\gamma_1, \qquad \Delta_0 = \alpha_0 - \beta_0 - \mathrm{i}\gamma_0. \tag{56}$$

Thus, eqn (55) gives another relation between W and d^2. Finally, from eqns (17) and (18),

$$A = -B = [\gamma_0 S(K) - \gamma_0(\alpha_1 - \beta_1) + \gamma_1(\alpha_0 - \beta_0)]\frac{1}{\Delta_0}, \tag{57}$$

$$-C(k) = D(k) = \frac{2}{\pi}\frac{1}{k^2 + K^2}\int_a^b \lambda_3(u)\sin ku\, du. \tag{58}$$

Equation (57) gives the wave amplitude at infinity. The velocity potential is now obtained by using eqns (57) and (58) in eqn (15). These results can be identified with those obtained in Banerjea and Mandal[4]. However, if one deduces the expression for amplitude at infinity of the wave motion generated due to rolling of a submerged plate from Evans[5], then the result coincides with eqn (57) except for the term $N(a)$ in the expression for $S(k)$ (see eqn (54)). This difference apparently occurs due to the fact that the constant A appearing in the expression for the complex potential $\omega(z)$ in eqn (22) of Evans[5] is actually complex, and not real, as has been assumed there. This is evident if one equates the value of the stream function $\psi(0, y)$ on the plate deduced from $\omega(z)$ given in eqn (22) with eqn (16) of Evans[5]. Then it is found that

$$I_m(A) = \sigma\theta_0\left(\frac{a^2}{2} - ac\right)\exp(Ka).$$

With this change, the results derived from Evans[5] now agree with eqn (57). Also, making $\mu\ (= a/b) \to 0$ in eqn (57), one obtains the wave amplitude due to rolling of a partially immersed vertical plate (see Ursell[1]).

5. Discussion

An integral equation formulation based on Havelock's expansion of water wave potential is used to reinvestigate the wave motion due to rolling of a thin vertical plate submerged in deep water. The integral equation which arises in the problem is of the first kind in double interval, having a kernel with logarithmic and Cauchy-type singularity. This integral equation is solved in a straightforward manner utilizing the Plemelj formula and the solution of an appropriate Riemann–Hilbert problem. This solution is then used to determine the velocity potential and the amplitude of the radiated waves at infinity. To the best of our knowledge, this integral equation has not been studied earlier in the literature for closed-form solution and for its application to water waves.

It may be mentioned here that in his analysis, Evans[5] introduced the so-called reduced potential in terms of the complex potential which satisfies a Riemann–Hilbert boundary value problem of the complex-variable theory, whose solution was obtained by standard technique. The solution of this Riemann–Hilbert problem was used to determine the complex potential from which the amplitude of the radiated waves at infinity was obtained using the Plemelj formula at some stage of the analysis. The amplitude of the wave motion obtained in the present paper agrees with that obtained by Banerjea and Mandal[4] and also with the result deduced from Evans[5] after introducing the modification stated earlier.

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