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New vistas in neutrino astrophysics

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Abstract

Neutrinos produced by thermonuclear reactions in the core of the Sun can get out of it unaffected. Observing these neutrions in Earth-based experiments enables one to study how the Sun generates its energy and verify the theories of neutrino astrophysics. Neutrinos from the collapse of SN1987A are not only important for what they have taught us about them but also for the confirmation of the principal features of the theoretical picture of the star collapse. These observations indicate the possibility that neutrinos may not be exactly zero-mass particles but may possess very small mass and magnetic moments. Extension of standard model is needed to accommodate massive neutrinos with magnetic moment.

Keywords: Supernova, large Megellanic cloud, Milky Way galaxy, SN1987A, FCνSR, SSM, MSW effect.

1. Introduction

Neutrino is one of the elementary particles of nature. It plays an important role in particle physics and astrophysics. Despite their apparently very different objectives, astrophysics—the study of the largest structure of the universe—and particle physics—the study of the smallest– have common ground.

Newton brought about the close connection between physics and astrophysics. He laid the foundation for the field of spectroscopy by analysing sunlight with a simple prism. Spectroscopy not only played an extremely important role in the development of physics—the recognition of the discrete energy states of atoms and molecules, the Bohr's theory of atoms and molecules and the development of quantum mechanics—but also in the field of astrophysics—the physical structure of the stars, how energy is generated and why stars shine.

It is well known that Pauli introduced a hypothetical particle, the neutrino, to save the principles of conservation of angular momentum and energy in neutron decay as well as in radioactive β-decay in general. This hypothetical particle was later detected experimentally in spite of its extremely weak interaction properties. The experimental discovery of the neutrino itself was a big challenge. However, this elusive particle has posed bigger challenges both in the field of physics and astrophysics. The challenges are related to unravelling of some of the very fundamental properties of this particle—the neutrino mass, the neutrino oscillation, the neutrino magnetic moment and the number of flavours of the neutrino. Until recently, spectroscopy was used as a probe to study the internal structure and the constitution of a massive star. Neutrinos produced due to fusion reactions inside stars can more effectively carry information about the physical processes inside the core of the stars, for they are the most weakly interacting particles.

Solar neutrino experiments provide a unique opportunity for studying weak interactions in a realm where new physics may be revealed. At the same time the neutrino carries important information about the physical processes occurring in the otherwise inaccessible core of a main sequence star. The weakly interacting neutrino is the only known particle that can escape almost instantaneously, whereas photons get trapped and have to diffuse out before getting out of the core. Different stages of normal stars and how they collapse at the end have been discussed in detail. The history and the theory of the neutron star, especially 1987A supernova (SN) explosion, are the subject matter of the present review. New physics has emerged from the study of SN1987A. How the data from SN1987A explosion help us to verify our theories concerning astrophysics has been discussed.

After an introduction to the subject of astro-particle physics in Section 2, stellar evolution is thoroughly reviewed. The role of neutrinos and particle physics in general comes in the picture of stellar evolution very frequently. Section 3 presents the Mikheyev–Smirnov–Wolfenstein (MSW) effect[1–3], which describes neutrino oscillation for massive neutrinos. The neutrino mass problem is discussed in Section 4. Neutrino oscillation and magnetic moment are the main subject matter of this review. Experimental results and plausible arguments leading to concrete results are discussed in Section 5. Some recent theoretical works are discussed thoroughly.

2. Stellar evolution

The theory of stellar evolution is essential for understanding the solar neutrino problem. The Sun is assumed to be spherical and to have evolved quasistatically over a period of 5×10^9 years. The evolution is manifested by the loss of photons from the surface of the star, which in turn is balanced by the burning of protons into α particles in the core of the Sun. The overall reaction can be represented symbolically by the reaction

$$4p \rightarrow 2e^+ + 2\nu_e + 25\ \text{MeV} + \alpha.$$

Thus, the thermal energy that is supplied by nuclear fusion ultimately emerges from the surface of the Sun as sunlight. The energy is transported in the deep solar interior mainly by photons. The pressure that supports the Sun is provided largely by the thermal motions of the electrons and ions. The Sun shines by converting protons into α particles. About 600 million tons of hydrogen is burnt every second to supply the solar luminosity.

It is believed[4] that the primary energy source of the Sun and of the other stars, in general, is a series of nuclear fusion reactions occurring inside the star in which the energy is released as four protons and converted into a helium nucleus. In the process, two protons are converted into two neutrons, thereby emitting $2e^+$ and $2\nu_e$. Thus, a star must emit ν_es continuously. As a star continues to generate energy in this way, it accumulates helium ashes in its inner region that eventually become hot enough through gravitational contraction to burn into carbon nucleus by emitting gamma rays. As the star evolves further, the carbon ashes burn into heavier neon nucleus, thus producing more energy. The process goes on until the last cycle of fusion combines silicon nuclei to form iron, especially the common iron isotope ^{56}Fe. Iron is the final stage for spontaneous fusion[5]. ^{56}Fe nucleus with further fusion would absorb, rather than release, energy. The star has an onion-like structure at this stage. A core of iron and related elements is surrounded by a shell of silicon and sulphur and beyond it by shells of oxygen, neon, carbon and helium. The outer envelope is mostly hydrogen. Eventually, when the mass of the ^{56}Fe core exceeds the Chandrashekhar limit ($M_{Ch} > 1.4M_o$), where the fermion pressure cannot any longer overcome the gravitational attraction, the gravitational collapse occurs and the star ends up as a neutron star. A neutron star formed as a result of stellar collapse consists primarily of neutrons and is supported by the degeneracy pressure of neutrons. One of the most spectacular events occurred on 23 February 1987, when light and neutrinos from a supernova explosion in large megellanic cloud (LMC) first reached the Earth[6]. The LMC, a satellite of our Milky Way galaxy, is 170,000 light years away, making the event, code-named SN1987A, the closest visual supernova since Kepler observed one almost 400 years ago. The total light and kinetic energy of this supernova outburst is about 1% of 10^{53} erg energy released. The difference must come out in some invisible form, either neutrino or gravitational waves. Numerous arguments have shown that gravitational radiation can carry at most 1% of this, so that the bulk of energy released is *via* neutrinos. One finds that the observation of neutrinos from the collapse of SN1987A is perhaps the most important event, not for what it has taught us about neutrinos but because it has confirmed the principal feature of the theoretical picture of solar collapse. In fact, SN1987A has provided important check on the theory.

The gravitational pull of its own mass causes the star to collapse. The collapse is accelerated by the pressure drop due to the rapid capture of free electrons on nuclei and on free protons. Shock waves are produced in the region of severe disturbances propagating at supersonic speeds. They blow off the outer envelope of the star, thereby producing visible fireworks known as supernova.

According to the standard model, in the case of a supernova, it is the electron neutrinos ν_{eL} which are energetic and produced in the inner core in the first 0.03 s of the infall due to the neutronization process $ep \rightarrow n\nu_e$. The neutrinos produced in the initial collapse stage may have energies of the order of 10% of the collapse energy and their spectrum peaks at around a few MeV.

The neutrino magnetic moment[7] has got deep implications for the dynamics of the famous SN1987A, from which the neutrino signal was observed by underground neutrino detectors. The importance of the magnetic moment for supernova neutrinos comes from the fact that the normal ν_{eL}s – which are produced inside the supernova core during the collapse—when scattering incoherently on electrons and protons due to μ_ν (neutrino magnetic moment) can have a flip of their chirality, thereby making these neutrinos sterile with respect to weak interactions. The mean time τ_{LR} in which this process occurs is less than 0.01 s and is thus much less than τ_D of diffusion of ν_{eL}s from the inner core of radius R_c~10 km. The free path with respect to $\nu_L \rightarrow \nu_R$ conversion is 0.1–1.0 × 3000 km, which is larger than R_e so that, the inner core indeed emits right-handed ν_es rather than ν_{eL}s. When the supernova core collapses to form a neutron star, then, under a moderate assumption about the strength of the magnetic field inside the core, the right-handed (RH) ν_es do not leave the star but rather are resonantly converted back to ν_{eL}s at a distance of

$\sim R_c$ from the centre of the core, *i.e.*, still inside the core. Here $R_c \sim 100$ km is the radius of the neutrino sphere, defined as one such that just outside its boundary a thin layer ~ 1 m exists where the difference of the energies of the LH and RH helicity states passes through a zero. The reappeared ν_{eL}s do not escape but are trapped and absorbed in a thin layer (~ 1 m) at a distance $\sim R_c$. As this layer is small, it becomes possible (for $\mu_{\nu_e} = 10^{-11}\,\mu B$) for a magnetic field of 2×10^{12} G to flip the helicity of the ν_{eR}s and transform them into actively interacting ν_{eL}s. The reappeared ν_es are then trapped by the star so that a conventional sphere of radius R 100–200 km is set. The behaviour of the neutrino helicity in a magnetic field B transverse to the neutrino momentum is described by the following equations:

$$i\frac{d}{dr}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix} = \begin{pmatrix}C_L & \mu B\\ \mu B & 0\end{pmatrix}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}, \tag{1}$$

where C_L is the amplitude of the coherent weak interaction of ν_L with the media. In general, ν_Rs do not have any weak interactions in the absence of a magnetic field and, therefore, the ν_R terms should not come in eqn (1) in the absence of the μB term.

Equation (1) becomes

$$i\frac{d}{dr}\nu_L = C_L\nu_L. \tag{1$'$}$$

The process $\nu_L \to \nu_R$ and $\nu_R \to \nu_L$ takes place in the presence of μB in the Hamiltonian. Therefore, the Hamiltonian C_L in eqn (1) is changed to

$$\begin{pmatrix}C_L & \mu B\\ \mu B & 0\end{pmatrix}$$

and the corresponding state vector to

$$\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}.$$

In the case of ν_e the amplitude reads

$$C_L = \frac{G_F\rho}{\sqrt{2}m_p}(3Y_e - 1) \tag{2}$$

where ρ is the density of matter, G_F, the Fermi constant, Y_e, the electron fraction. For any reasonable value of the magnetic field B, both the charged current and the neutral current terms are larger by orders of magnitude than μB, *i.e.*, $C_L \gg \mu$B; hence, eqn (1) implies that the helicity rotation is completely blocked. The remarkable suggestion of Voloshin[7] is that there is definitely a place in the supernova core where $Y_e = 1/3$ and hence C_L vanishes, giving rise to helicity flip in eqn (1), with $\nu_R \to \nu_L$.

It is likely that the critical value $Y_e = 1/3$ occurs at a density of $\rho \simeq \rho_0 \simeq 10^{12}$ g/cm^3 when ν_{eR}s undergo adiabatic resonant conversion into ν_{eL}s at 100 km from the centre of the star. The magnetic moment of the electron neutrino $\mu_{\nu_e} \simeq 10^{-11}\,\mu_B$ is not only responsible for inducing adiabatic resonant helicity flip of the layer of the supernova core but it can also give rise to a rapid energy transfer from the inner core to the outer layers, thereby inducing an outgoing shock wave capable of blowing up the supernova envelope[8].

The prompt ν_{eL} pulse released within a few milliseconds after collapse ($e^- + p \rightarrow \nu_e + n$) according to the standard model carries 10% of the collapse energy and its spectrum is concentrated around a few MeV. Thermal neutrinos carrying 90% of the collapse energy can be created due to plasmons decaying in to $\nu_n + \bar{\nu}_n$ of all species. In the extremely dense concentration of matter having density ~ 10^{14} g/cm^3, even the weakly interacting neutrino cannot escape instantaneously. In the core of the supernova having such high densities, even the neutrinos get trapped and diffuse out in somewhat the same way as do the much more strongly interacting photons.

Being trapped in the core by weak scattering on the highly dense matter, νs and $\bar{\nu}$s drift out slowly, keeping all the while in thermal equilibrium with their immediate neighbourhood. At the neutrino sphere, the density is considerably reduced and the matter becomes transparent to neutrinos. Therefore, the neutrinos will stream out, interacting only coherently with matter. The energy flux is equally distributed among the following six species:

$$\nu_{eL}, \nu^C_{eR}, \nu_{\mu L}, \nu^C_{\mu R}, \nu_{\tau L} \text{ and } \nu^C_{\tau R}.$$

The ν_es and ν_e^Cs are trapped a little longer than the ν_μ and ν_τ neutrinos due to their charged-current interaction with electrons. Consequently, the sphere of ν_e is of somewhat bigger radius and lower temperature than the sphere of ν_μ and ν_τ. One can expect only the ν^C_{eR} pulse from SN1987A. Its predicted characteristics are its duration, spectrum and intensity. The duration should be around 10 s, reflecting the long time the neutrinos need to drift out of the core. The spectrum is concentrated around a few MeV, reflecting the temperature of the ν_e sphere. The intensity should correspond to about 15% of the collapse energy. Within the limits of theoretical error and the statistical limitations, all these features were seen.

3. Mikeyev–Smirnov–Wolfenstein (MSW) effect 1

This effect[2, 3] provides an elegant solution to the solar neutrino puzzle, which is nothing but the unexpected difference between the observed and the calculated capture rate of the neutrinos in the ^{37}Cl detector. The reaction that is used to detect solar neutrinos is the inverse of the laboratory decay of ^{36}Ar. The neutrino absorption reaction is $\nu_e + {}^{37}\text{Cl} \rightarrow e^- + {}^{37}\text{Ar}$ ($E_{\text{th}} = 0.8$ MeV). The 0.8 MeV threshold energy permits the detection of all the major solar neutrino sources except the basic pp neutrinos. The fundamental reaction in the solar-energy-generating process is the proto-proton (pp) reaction. In the pp reaction, a proton β-decays in the vicinity of another proton, forming a bound system, deuterium (^{2}H). This reaction produces the great majority of solar neutrinos; however, these pp neutrinos have energies below the detection thresholds for the ^{37}Cl and Kamiokande II experiments: $p + p \rightarrow {}^2\text{H} + e^+ + \nu_e$. The neutrinos on the right-hand side of this equation are known as pp neutrinos. The event rate predicted by the starndard model (solar neutrino unit SNU = 10^{-36} capture per target particle per second) for a ^{37}Cl detector is 7.9 SNU and the observed rate is 2.1 SNU. This discrepancy between calculation and observation can be explained by the MSW effect.

In the standard model, there are three distinct flavours of neutrino, ν_e, ν_μ and ν_τ. Perhaps these flavour eigenstate neutrinos are superpositions of mass eigenstate neutrinos ν_1, ν_2 and ν_3. Beta decay always makes an e^+ and a ν_e but now the ν_es would, on an average, turn out to be the superposition of three separate massive particles. Quantum-mechanically there would be a certain amplitude in the decay for each of the mass eigenstates. A detector that relies on inverse β decay (such as Cl detector) is sensitive only to ν_es. When a Cl nucleus is struck by a physical neutrino, the probability for electron flavour interaction is diminished because the physical neutrino wave function does not have exclusive electron flavour. Consequently, this kind of mixing reduces the rate of neutrino detection. Beta decay launches into space a particular superposition of the wave functions of the three physical neutrinos. The initial individual amplitude and phases of the three waves are such that the superposition corresponds to a pure ν_e. As the wave propagates through space, the slightly different masses of the components cause their phases to get out of step. Therefore, depending upon exactly where one detects the wave, it may be in its ν_e, ν_μ or ν_τ phase alignment or more generally in between. If physical neutrinos are equal mixtures of all three flavours, the vacuum oscillation scenario predicts that the measured number of ν_es would be one-third of that emitted by the Sun. This is just the required reduction needed to explain the experimental result but the chance of neutrinos being equal mixtures of three flavours seems improbable.

All neutrino flavours have the same neutral current (Z exchange) amplitudes from all targets but ν_es have an additional W^+ exchange amplitude for elastic scattering from electrons. The effect of the index of refraction of the medium on the phases is equivalent to adding a term to mass that depends on the electron density. Thus, in the medium the mass and the mixing angle may be considered as functions of electron density. There is a linear relation of mass eigenstates ν_1 and ν_2, and weak eigenstates ν_e and ν_μ, with the mixing angle θ_v such that

$$| \nu_e \rangle_t = \cos \theta_v \, | \nu_1 \rangle_t + \sin \theta_v \, | \nu_2 \rangle_t$$

and

$$| \nu_\mu \rangle_t = \sin \theta_v \, | \nu_1 \rangle_t + \cos \theta_v \, | \nu_2 \rangle_t .$$

For the large-density characteristic of the solar centre, the index of refraction dominates the MSW effect. Masses of free particles, although quite different *in vacuo*, may become equal in certain parts of the Sun. At that point, the oscillation probability becomes resonant and the total conversion of one flavour into another can occur for small intrinsic mixing. Thus, the MSW effect offers a possible explanation of the solar neutrino puzzle.

For the sake of simplicity, two types of neutrinos are considered and there is a linear relation between weak eigenstates ν_e and ν_μ and mass eigenstates (ν_1 and ν_2), so that

$$| \nu_e \rangle_{t=0} = \cos \theta \, | \nu_1 \rangle + \sin \theta \, | \nu_2 \rangle$$

and

$$|\nu_\mu\rangle_{t=0} = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle, \tag{3}$$

where θ is the vacuum mixing angle along the path of the moving neutrino. As a function of time and distance, the state evolves as

$$|\nu_e\rangle_t = \cos\theta\,|\nu_1\rangle\,e^{-iE_1t} + \sin\theta\,|\nu_2\rangle e^{-iE_2t} \tag{4}$$

and

$$|\nu_\mu\rangle_t = -\sin\theta\,|\nu_1\rangle\,e^{-iE_1t} + \cos\theta\,|\nu_2\rangle e^{-iE_2t}$$

and, therefore,

$$|\langle\nu_\mu(t)\,|\,\nu_e(t)\rangle|^2 = \sin^2 2\theta\sin^2(E_2 - E_1)\,t/2. \tag{5}$$

Neutrino oscillations may be viewed as precession in flavour space. If the neutrino mass is much less than its energy then the time dependence is replaced by $\sin^2(\pi l/l_\nu)$, where l is the distance between the source and the detector and

$$l_\nu = \frac{4\pi p}{\Delta m^2}$$

where

$$\Delta m^2 = m_2^2 - m_1^2 \tag{6}$$

is called the vacuum oscillation length. For a neutrino momentum p of 1 MeV/c and mass difference Δm^2 of 1 eV^2, the oscillation length is 2.5 m. Here θ should be in degrees. Typical experiments provide an upper limit on Δm of about 0.2 eV. If $m_2 \gg m_1$, this corresponds to a limit of 0.5 eV, where a natural assumption is that ν_2 is mainly ν_μ. Neutrinos passing through matter have an index of refraction n given by $1 - n = \sqrt{2}\,G_F n_e$ where n_e is the electron density in the medium. The time development of ν_e and ν_μ in matter is governed by a 2×2 Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix}|\nu_e(t)\rangle\\ |\nu_\mu(t)\rangle\end{pmatrix} = H_{\text{matter}}\begin{pmatrix}|\nu_e(t)\rangle\\ |\nu_\mu(t)\rangle\end{pmatrix}, \tag{7}$$

where

$$H_{\text{matter}} = H_{\text{vacuum}} + \begin{pmatrix}\sqrt{2}G_F n_e & 0\\ 0 & 0\end{pmatrix}. \tag{8}$$

The new term is the effect of the coherent forward scattering for $\nu_e\,e^- \to \nu_e\,e^-$ with the charged current. The effect of neutral current scattering has been neglected because it is the same for ν_e and ν_μ and, therefore, contribute only to the overall phase. For the large-density characteristic of the solar centre, the index of refraction effect dominates the MSW effect, so that ν_e is primarily in the upper state with $\theta(n_e)$ close to 90°. At the solar surface, where the electron density is zero, ν_e is mainly in the lower state ν_1 and the mixing angle $\theta(n_e)$ is equal to its normal vacuum value θ_ν, which is assumed to be fairly small. Figure 1 illustrates the mass eigenvalue and eigenfunctions as a function of N_e[2,3]. The neutrinos produced at the centre of the Sun are of ν_e type and are primarily in the

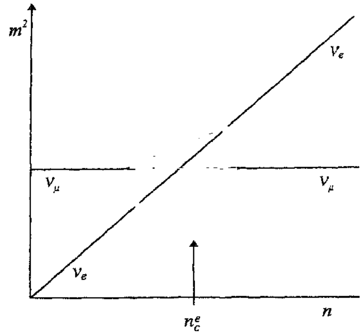

FIG. 1.

upper state ν_2 and there is only a small probability $\cos^2\theta(n_e)$ of their being in the ν_1 state. As neutrinos move outward from the centre of the Sun, there is some probability of a transition to the state ν_1 in the region where ν_1 and ν_2 are close together. When the neutrinos reach the solar surface, they are still primarily in the mass eigenstate ν_2, which is now mainly composed of the flavour eigenstate ν_μ. This happens when all ν_e particles get converted into ν_μ at a critical density. The two levels cross at this critical value of n_e, for which the diagonal values of the effective neutrinos' mass matrix are equal. Eventually, it is necessary that the value of Δm^2 be small enough so that the index of refraction effect will dominate at the solar centre. For ^{8}B (boron) neutrinos, this yields the requirements $\Delta m^2 < 10^{-4}$ eV^2 and $\sin\theta \geq 3 \times 10^{-8}$ eV$^2/\Delta m^2$. For θ_V of about 0.1° and assuming $m_{\nu_2} \gg m_{\nu_1}$ the mass m_{ν_2} must be between 10^{-2} and 5×10^{-4} eV. Thus, the MSW effect offers a possible explanation of the solar neutrino puzzle, and the particles produced at the core of the Sun at $n_e > n_e^{\text{critical}}$ move out of the centre of the Sun to regions of lesser density and then get totally converted into other neutrinos. Then the only particles which survive to be detected in experiments performed on the surface of the Earth are ν_es with sufficiently small energy.

4. Neutrino mass problem

Neutrinos occur in one helicity state, *i.e.*, left-handed neutrinos and right-handed antineutrinos. They are distinguished by assigning them to three different flavours according to charged particles which take part in the creation and absorption of neutrinos of the three kinds (however, from cosmological arguments, at most one or two additional flavours are expected). In the minimal version of the standard theory of unified electromagnetic and weak interactions, neutrinos are strictly massless. This has not been contradicted so far by any confirmed experimental result. It is very difficult to understand the smallness of neutrino mass compared to that of other charged fermions (say), quarks and leptons. Majorana masses for the neutrinos have, therefore, been favoured over Dirac masses provided the lepton number is not conserved. Experimentally[8–10], the limits of the neutrino masses are given by

$$m_{\nu_e} < 18 \text{ eV}, \; m_{\nu_\mu} < 0.25 \text{ MeV and } m_{\nu_\tau} < 35.0 \text{ MeV}. \tag{9}$$

Also, the mass of ν_e is 13.4 eV (C.L. 95%) obtained from the decay of molecular tritium. The magnetic moment, of ν_e, ν_μ and ν_τ are given by $1.5 \times 10^{-10}\,\mu_B$, $1.2 \times 10^{-9}\,\mu_B$ and $1.1 \times 10^{-11}\,\mu_B$, respectively, with $\mu_B = e/2m_e$. These are the upper limits. The neutrino mass is of great importance for astrophysics and cosmology. Masses of the 10 eV order of could account for the dark matter of the universe[11], while masses of $< 10^{-2}$ eV could resolve the solar neutrino problem. Mass limits on ν_τ, ν_μ and ν_e follow essentially from some kinematical constraints involving decays where neutrinos are involved in the final state[12].

The best bound on m_{ν_τ} has been obtained by the ARGUS collaboration[13]. They have studied the decay of τ leptons $\tau \to \nu_\tau + 5\pi$ and filled the invariant mass spectrum near the τ mass to obtain a bound on m_{ν_τ}. The limit reported by ARGUS at the Munich conference
at 95% C.L. is 35 MeV. The best limit on m_{ν_μ} comes from studying the two-body decay $\pi^- \to \mu^- + \bar{\nu}_\mu$ and the limit[14] one obtains at 90% C.L. is $m_{\nu_\mu} < 250$ keV. The best limit on m_{ν_e} comes from the end point spectrum of tritium β decay

$$^3\text{H} \to {}^3\text{He} + e^- + \bar{\nu}_e. \tag{10}$$

Here one observes $m_{\nu_e} < 35$ eV.

It is trivial to get Dirac neutrino masses in the $SU(2) \times U(1)$ standard model. It is enough to add an independent ν_R for each fermion family and fine-tune the corresponding Yukawa coupling to account for the neutrino charged lepton mass difference. In the standard model the Dirac mass is generated by the vacuum expectation value

$$v \simeq \sqrt{2}\,\langle \phi^\circ \rangle \simeq (\sqrt{2}\,G_F)^{-1/2} \simeq 246 \text{ GeV} \tag{11}$$

of the neutral component of the Higgs doublet scalar fields. One has $m_\nu = h_\nu v$ (Dirac mass), where h_ν is the Yukawa coupling of the Lagrangian

$$L \simeq \sqrt{2} h_{\nu_e} (\bar{\nu}_L \bar{e}_L) \begin{pmatrix} \phi^\circ \\ \phi^- \end{pmatrix} N_R = \text{h.c.} \tag{12}$$

of the neutrino to ϕ°, where N_R is the $SU(2) \times U(1)$ singlet right-handed neutrino field. A ν_e mass in the 20 eV range would require an anomalously small Yukawa coupling, $h_{\nu_e} \leq 10^{-10}$. Moreover, the h_{ν_e} would have to be smaller by $m_{\nu_e} / m_e \leq 10^{-4}$ than the analogous Yukawa coupling for the electron. The most general mass lagrangian L_{mass} containing both Dirac and Majorana mass term is

$$L_{\text{mass}} = Y\left(\bar{\nu}_{eL}\, \bar{N}_L^{0C}\right) \begin{pmatrix} m_t & m_D \\ m_D^T & m_s \end{pmatrix} \begin{pmatrix} \nu_R^{0C} \\ N_R^0 \end{pmatrix} + \text{h.c.} \tag{13}$$

where N_R^0 is the new singlet with $\nu_R^{0C} = C\bar{\nu}_R^{0T}$ and $N_L^{0C} = C\bar{N}_L^{0T}$. Here m_t and m_s are the left- and right-handed Majorana mass matrices of dimension 3×3 for ν_L^0 and N_R^0, respectively, whereas m_D and its transpose m_D^T are the 3×3 Dirac mass matrices. If U_L and U_R are 6×6 unitary matrices, then the mass matrix

$$M = \begin{pmatrix} m_t & m_D \\ m_D^T & m_s \end{pmatrix} \tag{14}$$

can be diagonalized in the following way:

$$U_L^\dagger \begin{pmatrix} m_t & m_D \\ m_D^T & m_s \end{pmatrix} U_R = \begin{pmatrix} m_l & 0 \\ 0 & m_h \end{pmatrix}, \tag{15}$$

where m_l and m_h are the masses of the light left-handed and heavy right-handed Majorana neutrinos, respectively. Further, if $m_l = 0$ and $m_s = M_x I$ then the mass eigenvalues of the left-handed Majorana neutrinos are given by $m_{\nu_L} \simeq m_D^2 / m_s$. The scale of M_x is related to that of new physics. In simple grand unified models, one assumes that the scale is a typical GUT unification scale of about 10^{15} GeV. In SO(10) the neutrinos Dirac mass m_{D_i} is the same as m_{ik} where m_i is the 2/3 charged quark mass and $k \simeq 3.0$ represents the running of the Yakawa coupling between GUT scale and low energies[16]. For $\nu_e \simeq 20$ eV then M_x becomes 10 GeV. In turn, it follows that m_{ν_μ} and m_{ν_τ} tend to 1 MeV and 300 MeV, respectively. The latter values are near the levels where today's present experimental bounds for m_{ν_μ} and m_{ν_τ} exist.

5. Neutrino oscillation and magnetic moment

Two experiments are demonstrated below to show how neutrinos oscillate among themselves. Electron antineutrinos from a reactor were observed in a deuterium detector 11 m away[17]. Two antineutrino-induced reactions were studied: $\bar{\nu}_e + d \rightarrow n + n + e^+$ and $\bar{\nu} + d \rightarrow n + p + \bar{\nu}$. The first is a conventional charged-current weak interactions to which only $\bar{\nu}_e$s contribute. But enough energy does not exist for $\bar{\nu}_\mu$s and $\bar{\nu}_\tau$s to make, respectively,[17] μ^+ and τ^+. The second reaction is a neutral current weak interaction unaffected by neutrino oscillations because the neutral currents involve all neutrino species equally.

The ratio of the number of charged-current events to the number of neutral events is, therefore, sensitive to the existence of neutrino oscillations but relatively insensitive to the poorly known flux of antineutrinos from the reactor. This ratio was found to be half of what one would expect with no neutrino oscillations. This suggests considerable neutrino mixing and neutrino mass splittings of at least 0.5 eV although the statistics are not convincing.

Very energetic ν_es and ν_μs of equal number are produced at CERN accelerator. ν_es and ν_μs can interact in the detector to produce readily electrons and muons, respectively. If there are no neutrino oscillations, one should see equal number of electrons and muons in the detector. The CERN bubble chamber sitting in this neutrino beam has so far observed about 20 neutrino-induced events[18]. Only half as many electrons as muons were seen, but the statistics are clearly limited. If the observations are significant, however, a possible explanation is that ν_es mix with ν_τs. This could explain the apparent systematic deficit of the ν_e component in the CERN experiment.

A possible explanation for such kind of phenomena suggests that the weak neutrino field ν_e is transformed into another neutrino field of different flavour. The observation that the solar neutrino flux in the chlorine experiment[18] averaged over 15 years indicates a substantial depletion by a factor of 3 to 4 with respect to the standard solar model prediction. Davis has also pointed out that the neutrino capture rate of the ^{37}Cl experiment was inclusively anticorrelated with the sunspot number with an 11-year cycle, which in turn is a manifestation of the solar magnetic activity. A plausible explanation of the time variation of the solar neutrino flux was proposed by Okun[19a], Voloshin[19b] and Vyotsky[19c] (OVV), who revived the idea of Cicernos[20]. According to their scenario, the ν_{eL} undergoes spin precession in the strong magnetic field present in the Sun and so emerges as a ν_{eR} sterile and undetectable. At the time of high sunspot activity, the magnetic field is large and so the probability of precession to a right-handed neutrino is also large. In other words, the probability of detecting a solar neutrino is smaller at the time of high sunspot activity since the left-handed component in the neutrino beam is smaller. This results in an anticorrelation of the neutrino flux with the sunspot number. This is possible if the neutrino possesses the magnetic moment that interacts with the strong magnetic field present in the Sun. Since the neutrino does not possess any charge, it cannot have magnetic moment. This magnetic moment of the charged particle arises when it interacts with the electromagnetic field. The neutrino having no charge cannot interact with the electromagnetic field and, therefore, cannot have magnetic moment in the usual sense but can have magnetic moment through the loop diagram, which will be discussed later. The magnetic moment is an effective neutrino–photon coupling through which the neutrinos can interact with electrons. Laboratory measurements on neutrino–electron scattering put the bound $|\mu_\nu| \leq 10^{-10}\ \mu_B$. For the solar neutrinos, the energy is of the order 10 MeV or less. The magnetic field in the Sun is not well known at all. Educated guesses give $B \sim 10^3$–10^4 G and $\Delta m^2 < 10^{-7}\ \mathrm{eV}^2$. After using these values[18], the mass difference of solar neutrinos should be very small to precess.

The solution to the solar neutrino puzzle that incorporates the anticorrelation of the neutrino flux with the sunspot activity (described below) is consistent with alllaboratory, cosmological and astrophysical bounds. In the presence of matter the flavour-changing neutrino spin rotation (FCνSR) due to the transition magnetic moment interaction of the type $\overline{n}^C_{eL}\sigma_{mn}n_{e_L}F^{mn}$, which connects two different neutrino flavours, both of which participate in the usual weak interaction. Since ν^C_{eL} energy is below the threshold of the weak interaction, ν^C_{eL}s escape detection, thus accounting for the solar neutrino flux deficit. The 1987A supernova results put a severe bound on the neutrino magnetic moment $\mu_\nu < 10^{-12}\ \mu_B$, allowing only values that are too small to account for the solar neutrino flux depletions[21]. The above interaction easily avoids the supernova bounds since ν^C_{eL}s generated become automatically trapped and do not add new channels to the energy loss mechanism. Neutrinos of two different flavours need not be degenerate in mass for the transition magnetic moment mechanism to be relevant to the solar interior and, therefore, their mass difference should satisfy $\Delta m^2 < 10^{-7}$–10^{-8} eV^2. In a magnetic field B, the evolution equation of the system of ν_{eL} and $\nu^C_{\mu R}$ is governed by the equation[22]

$$i\frac{d}{dt}\begin{pmatrix}\nu_{eL}\\ \nu^C_{\mu R}\end{pmatrix}=\begin{pmatrix}a_{\nu_e}-\dfrac{\Delta m^2}{4p} & \mu B_\perp\\ \mu B_\perp & \dfrac{\Delta m^2}{4pa_{\nu_\mu}}\end{pmatrix}. \tag{16}$$

Here

$$a_{\nu_e}=\left(G_F/\sqrt{2}\right)\left(2N_e-N_n\right)$$

and

$$a_{\nu_\mu}=\left(G_F/\sqrt{2}\right)(-N_n), \tag{17}$$

where N_e and N_n are the electron and neutron number densities, respectively, and G_F, the Fermi constant. For simplicity, the mixing angle between ν_{eL} and $\nu^C_{\mu R}$, considered to be very small, is not introduced at all.

One can really obtain the probability of finding $\nu^C_{\mu R}$ at point r provided there are only ν_{eL}s at the origin. Thus,

$$P\left(\nu_e\to\nu^C_{\mu R};r\right)=\sin^2 2\theta\sin^2\frac{\pi r}{l},$$

where the mixing angle θ and the precession length l are given by

$$\tan 2\theta=2\mu B_\perp\Big/\left[\frac{\Delta m^2}{2p}-\sqrt{2}G_F\left(N_e-N_n\right)\right] \tag{18}$$

and

$$l=2\pi\left\{\left(2\mu B_\perp\right)^2+\left[\frac{\Delta m^2}{2p}-\sqrt{2}G_F\left(N_e-N_n\right)\right]^2\right\}^{-1/2}. \tag{19}$$

It follows from eqn (16) that for the resonant amplification of the precession to be possible the following condition[23]

$$\sqrt{2}G_F\left(N_e-N_n\right)=\Delta m^2/2p \tag{20}$$

must be satisfied. The condition on Δm^2 is important for the location of resonance and the condition on the magnetic transition moment determines the resonance adiabaticity so that, on an average, the ν_{eL} flux is suppressed by a factor of 3. In matter the potential energies of interaction with the electrons and nucleons add to the kinetic energies of the neutrinos[24]. Equation (20) explains the resonant phenomenon of the FCνSR, whereas a_{ν_e} and a_{ν_μ} represent the potential of ν_e and a_{ν_μ}, respectively, and $\Delta m^2/2p$ is the difference of the kinetic energies of ν_{eL} and $\nu^C_{\mu R}$. This phenomenon is roughly analogous to the resonant neutrino oscillations, the MSW effect[1]. According to Lim and Marciano[21] and

Akhmedov[22] (LMA) the solar ν_{eL} is rotated into a $\nu^{C}_{\mu R}$ in an MSW fashion. The mass-squared difference is such that in vacuum $\nu^{C}_{\mu R}$ is heavier than ν_{eL} but in the core of the Sun ν_{eL} is heavier than $\nu^{C}_{\mu R}$ due to weak interaction with matter. In the resonance region, where the mass difference changes sign, the interaction of the magnetic fields serves as the mixing term necessary to rotate ν_{eL} into $\nu^{C}_{\mu R}$. The depletion of the electron neutrino flux is correlated with the sunspot activity if the resonance occurs in the convective zone. To obtain such a correlation, LMA requires $\Delta m^2 \leq 10^{-7}$–10^{-8} eV^2 and

$$\mu_\nu \simeq 1\text{–}10 \times 10^{-11}\, \mu_B. \tag{21}$$

The recent work on the transition moment of the ν as an explanation of the solar neutrino puzzle with the help of the Zee model[25] is reproduced here without giving much details of derivations. Not much is known about coupling constants and the Higgs scalar but one gets

$$\mu_{\nu_e} \simeq 8.2 \times 10^{-11}\, \mu_B \tag{22}$$

and

$$\Delta m^2 \simeq 6.0 \times 10^{-8}\ \text{eV}^2.$$

An $SU(2)_H$ symmetry suggested by Voloshin can solve the problem of the neutrino to have a ~ 10–30 eV mass in a natural way and at the same time a magnetic moment ~ 10^{-11}–$10^{-10}\, \mu_B$ for explaining a possible time modulation of the solar neutrino flux. In the case of Majorana neutrinos, the symmetry is simply a horizontal one between two neutrino flavours[26]. The interactions producing the mass term and the magnetic moment may be expressed in terms of a horizontal symmetry connecting the first two lepton generations.

The suppression of the mass of the neutrino in the presence of a finite magnetic moment may be due to $SU(2)_H$ broken symmetry between the neutrino and the antineutrinos of different flavours. The neutrino can precess within the same theoretical framework in a natural way whereas the magnetic moment ~ 10^{-11}–$10^{-10}\, \mu_B$ is needed for explaining a possible time modulation of the solar neutrino flux. The fine-tuning of the order of 10^{-3} is not required to get the neutrino mass in the experimentally allowed eV range when the symmetry is broken in a particular manner. The difference between electron and muon numbers is conserved so that decays like $\mu \to e\gamma$ and $\mu \to eee$ are forbidden. The generalized Zee model in the $SU(2)_H$ framework has been used to obtain μ_ν and m_ν in the allowed region. The left-handed doublet ψ_L and right-singlet ψ_R are given below:

$$\psi_L = \begin{pmatrix} \nu_e & \nu_\mu \\ e & \mu \end{pmatrix}_L, \qquad \psi_R = (e, \mu)_R \tag{23}$$

and under $SU(2)_H$ singlet

$$\psi_L = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}, \qquad \psi_R = \tau_R .$$

Scalar particles are a standard Higgs doublet and two additional multiples S and D:

$$\phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix}$$

$$S = S_{-}S_{+}, \quad D = \begin{pmatrix} D_{-} & D_{+} \\ d_{-} & d_{+} \end{pmatrix}, \tag{24}$$

where the subscripts denote the N_e–N_μ charge. $S_\pm$ and $D_\pm$ have an electromagnetic charge of $+1$, while $d_\pm$ are neutral. The standard Higgs ϕ plays the usual roles in the standard model; it not only develops a VEV breaking the weak guage group to $U(1)_{em}$ and induces masses for the quarks and leptons but also has another role in this model. The lagrangian contains a term $SD^{+}\phi^{*}$ + h.c. when ϕ gets its VEV, the charged scalars of the S multiplet mix with the charged components of the D multiplet. This is essential for the creation of the neutrino magnetic moment. The finite mass $m_\mu \neq m_e$ breaks $SU(2)_H$ to $U(1)_N$ and, therefore, the magnetic moment and mass terms are given below:

$$\mu_\nu \times \left(\overline{\nu}_{eL}\sigma_{\mu\nu}\nu_{\mu R}^{C} + \text{h.c.}\right) \quad m_\nu \times \left(\overline{\nu}_{eL}\nu_{\mu R}^{C} + \text{h.c.}\right). \tag{25}$$

The magnetic moment (μ_ν) transforms as a scalar under $SU(2)_H$, whereas the mass term (m_ν) transforms as the third component of a triplet. The magnetic transition moment is generated *via* loops involving charged physical scalars. The mass correction is given by the same diagram with the photon line removed. The mass and the magnetic moment are proportional to the charged lepton. The τ lepton in the loop diagram is preferable. Since it is a singlet under $SU(2)_H$, the external neutrinos are doublet under $SU(2)_H$ and, therefore, the charged scalar in the loop should also be a doublet. The $SU(2)_L \times U(1)_Y$ representations in the graph suggest that the charged scalar should be a mixture of an $SU(2)_L$ doublet and a singlet. It is worthwhile to note that if N_τ in the loop is conserved, then $S_\pm$ and $D_\pm$ must carry 1 unit of this charge.

Our model[27] is based on generalized Zee model in the framework of the symmetry that contains the following two discrete operations:

$$SU(2)_H \supset \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{26}$$

The first one corresponds to the Zeldovich–Konopinski–Mahmoud (ZKM) lepton number $L_{ZKM} = L_e - L_\mu$. This symmetry is unbroken and it forbids the appearance of the diagonal Majorana neutrino mass operators $\nu_e^T C\nu_e$ and $\nu_\mu^T C\nu_\mu$ as well as flavour nondiagonal decays $\mu \to e\gamma$ and $\mu \to eee$. This symmetry allows nondiagonal Majorana mass term $m_\nu^T \nu_e C\nu_\mu$ where $\nu = \nu_{eL} + \nu_{\mu R}^{C}$. The second symmetry interchanges electron and muon families. It forbids ZKM neutrino mass but allows its magnetic moment. This symmetry is explicitly broken by the difference of electron and muon masses and the ZKM neutrino acquires nonzero mass.

Figure 2 shows the exchange of a charged scalar H, which is approximate singlet with respect to $SU(2)_L$ although it is a mixture of $SU(2)_L$ doublet and a singlet. The scalar H_1 has the following interaction with the leptons:

$$L_{int}^{(1)} = f^{ab}\left(\psi_{aL}^{i} C\psi_{bL}^{j}\right)\varepsilon_{ij}H_1 - f'^{ab}\left(\psi_{aL}^{i} C\psi_{bL}^{j}\right)^{*} H_1^{*}\varepsilon_{ij}^{*} + \text{h.c.} \tag{27}$$

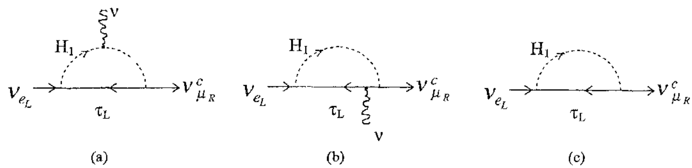

FIG. 2.

where f^{ab} and f'^{ab} are dimensionless coupling constants. It is not difficult to see that apart from the magnetic moment the above interaction also leads to the renormalization of the mass of the neutrino without the photon line. The nature of the mass is the following:

$$m_\nu(\Lambda^2) = (ff'/16\pi^2)\, m_\tau \ln(\Lambda^2/m^2_{H_1}) \tag{28}$$

for $\Lambda^2 \gg m^2_{H_1}$ and also

$$\mu_\nu = (eff'/16\pi^2)\left(\frac{m_\tau}{m^2_{H_1}}\left(\ln\frac{m^2_{H_1}}{m^2_\tau} - 1\right)\right) \tag{29}$$

for $m^2_{H_1} \gg m^2_\tau$.

Substituting $m_{H_1} \simeq 50$ GeV (lower values are forbidden by LEP results). The desired value $\mu_\nu \simeq 10^{-11}\,\mu_B$ is obtained if the value $ff' \simeq 3\times10^{-4}$ is used. The coefficient of $\ln(\Lambda^2/m^2_{H_1})$ in eqn (28) is of the order of 3 keV. To obtain $m_\nu \simeq 3$ eV one needs fine-tuning with an accuracy of 10^{-3}. One can, therefore, try to find the evolution of the mass by restricting the region of the logarithmic evolution of the mass, for instance, by assuming the existence of another scalar, H_2, whose mass is close to that of H_1 but has the opposite sign of the product constants ff' in the second Lagrangian containing H_2. The requirement of the $SU(2)_\nu$ symmetry implies that one has to introduce a second scalar H_2 with the interaction

$$L^{(2)}_{int} = f^{ab}\left(\psi^i_{aL} C \psi^j_{bL}\right)\varepsilon_{ij} H_2 + f'^{ab}\left(\psi^i_{aL} C \psi^j_{bL}\right)^* \varepsilon_{ij} H^*_2 + \text{h.c.}, \tag{30}$$

where H_2 is a linear combination of $SU(2)_L$ doublet and a singlet. Of course, H_2 belongs approximately to a doublet. From eqn (28) it follows that

$$m_\nu \simeq (ff'/16\pi^2) m_\tau\, (\Lambda^2/m^2_{H_2})$$

and

$$\mu_\nu \simeq (eff'/16\pi^2)\left(m_\tau/m^2_{H_2}\right)\left(\ln m^2_{H_2}/m^2_\tau - 1\right). \tag{31}$$

It was mentioned earlier that the different symmetry properties are responsible for generating the mass and magnetic moment of the neutrinos. One uses eqns (27)–(30) to get m_ν and μ_ν:

$$m_\nu \simeq (f\,f'/16\pi^2)\, m_\tau \left(m_{H_2}^2 / m_{H_1}^2\right)$$

and

$$\mu_n \simeq (e f f'/16\pi^2)\, m_\tau \left[\frac{1}{m_{H_2}^2}\left(\ln\frac{m_{H_2}^2}{m_\tau^2}-1\right)+\frac{1}{m_{H_1}^2}\left(\ln\frac{m_{H_1}^2}{m_\tau^2}-1\right)\right]. \quad (32)$$

Thus,

$$m_\nu \simeq (13.5\ \text{eV},\ 27.06\ \text{eV},\ 40.5\ \text{eV}) \quad (33)$$

and

$$\mu_\nu \simeq (1.567\times10^{-11}\,\mu_B,\quad 1.564\times10^{-11}\,\mu_B,\quad 1.5618\times10^{-11}\,\mu_B)$$

for

$$m_{H_2} \simeq (50.1\ \text{GeV},\quad 50.2\ \text{GeV},\quad 50.3\ \text{GeV}),\ \text{respectively.} \quad (34)$$

It has been easy to verify that the contributions of the graphs of Figs 2 and 3 add up to the magnetic moment and of the renormalization neutrino mass substract. This is also obvious from the symmetry considerations discussed above. In this way the one-loop contributions to the mass of the neutrino would strictly be equal to zero if $m_{H_1} = m_{H_2}$ and the value of μ_ν would be finite. It is also clear that with regard to radiative electroweak correction, this situation can only be approximate since the scalar H_1 comes mainly from the singlet contribution with respect to the group $SU(2)_L$ whereas H_2 belongs mainly to a doublet.

6. Conclusion

The number of neutrinos observed in Earth-based detector is much less than the standard solar model theory prediction and, therefore, this aspect of the neutrino problem is far from solved. More observations are needed before a decision can be made as to whether current theories regarding the Sun and other stars are correct. Fortunately, a number of experiments both on neutrino masses and on the solar neutrinos are underway which are likely to give definite answers to questions concerning the basic physics specifying neutrino behaviour in a few year's time. In the meantime, theoretical work can be pursued.

The theoretical problem of constructing a theory of electroweak interactions with massive neutrino is of considerable current interest because one of the suggested

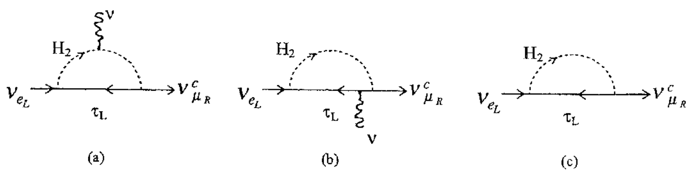

FIG. 3.

elementary particle physics solutions of the solar neutrino puzzle relies on $\mu(\nu_e) \geq 10^{-11}\mu_B$ and a relatively small mass (satisfying the experimental upper limit $m(\nu_e) < 9$ eV). The theory proposed here contains, in particular, two physical charged Higgs particles whose masses coincide up to one part in 10^{-3} but whose couplings to the leptons differ. In other words, the problem of generating appropriate mass and magnetic moment of the neutrino is reduced in this theory to the problem of constructing Higgs sector, which contains two almost degenerate in mass-charged Higgs fields. The problem of proving that this degeneracy is not altered by the higher-order corrections to the Higgs masses is being investigated.

The chlorine experiment[28] is significantly below SSM calculations and is inconsistent with the results of Kamiokande who finds no variations in the solar neutrino flux with time. The overall conclusion is that there is no compelling evidence for a solar neutrino problem nor a need for new physics. The major question whether neutrinos have masses can at best be tackled by using solar neutrinos and hence more experiments are needed to settle the neutrino mass problem.

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