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SLOW STEADY MOTION OF AN ELASTICO-VISCOUS FLUID WITH HEAT TRANSFER IN A WAVY CHANNEL

BY M. N. MATHUR
(*Department of Applied Mathematics, Indian Institute of Science, Bangalore-12*)

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ABSTRACT

The problem of heat transfer due to the slow steadv motion of an elastico-viscous fluid between two wavy walls is considered. Taking the deformation of the boundaries to be small, the equations of continuity, momentum and energy have been solved using the perturbation technique. The solution for the velocity field is then employed to study the nature of the temperature field, under the two types of thermal boundary conditions:

(i) both the walls are at the same constant temperature;

(ii) one wall is at a constant temperature while, the other is heat insulated.

It is found that the velocity profiles are affected by the stress relaxation time only, while the stresses and the temperature distribution are affected by both stress relaxation and strain retardation times. Certain similarities are noticed in the results of the present investigation and that in reference[3].

INTRODUCTION

Using Fourier Transform Technique Citron[1] has studied the slow motion of a viscous incompressible fluid between two rough circular cylinders rotating about their common axis, taking the roughness in the form of a sinusoidal deformation of the boundaries extending up to infinity. His assumption of azimuthal velocity vanishing at infinity is not correct. Bhatnagar (P.L.) and Rao[2] have investigated the flow of a Reiner-Rivlin fluid between two wavy circular cylinders employing the Fourier series instead of Fourier transforms in order to avoid the explicit reference to the conditions at infinity as when the roughness is sinusoidal the vanishing of disturbances at infinity is not possible. Bhatnagar (R. K.) and Mathur[3,4] have investigated the problem of heat transfer in the slow steady motion of a visco-elastic fluid, characterised by Rivlin-Ericksen constitutive equation, between two wavy walls and in a wavy cylindrical tube, the small periodic deformation in the boundaries being represented by a Fourier series in the axial coordinate.

In the present investigation, we discuss the slow steady motion of an elastico-viscous fluid with heat transfer in a wavy channel. The constitutive equation of such a fluid as given by Oldroyd[5] is of the form

$$p_{ik} + \lambda_1[(D/Dt)(p_{ik}) - p_{ij}\, d_{jk} - p_{jk}\, d_{ij}] = 2\mu[d_{ik} + \lambda_2\{(D/Dt)(d_{ik}) - 2\, d_{ij}\, d_{jk}\}],$$

where

$$p_{ik} = t_{ik} + p\delta_{ik},$$

$$d_{ik} = \tfrac{1}{2}(v_{i,k} + v_{k,i}),$$

are respectively the reduced stress tensor and the rate of strain tensor and λ_1, λ_2 and μ are constants characteristing the relaxation time, the retardation time and the static shear viscosity. The material derivative (D/Dt) is defined as

$$(D/Dt)(a_{ij}) = (\partial/\partial t)(a_{ij}) + v_k\, a_{ij,k} + \omega_{ik}\, a_{kj} + \omega_{jk}\, a_{ik}$$

where

$$\omega_{ij} = \tfrac{1}{2}(v_{i,j} - v_{j,i})$$

is the vorticity tensor.

We indicate the flow pattern by drawing the stream lines in Fig. 1. After determining the flow field, we discuss the problem of heat transfer under the following two thermal boundary conditions: (i) both the boundaries are being maintained at the same constant temperature and (ii) one of the boundaries being maintained at a constant temperature while the other is thermally insulated.

We have drawn the isotherms to indicate the temperature distribution in Figs. 2 and 3. We mention that we have obtained the solutions to the first power of the small deformation in the boundaries and correct to the square of the Reynolds number appropriately defined later.

We note that to our approximation, the velocity profiles are affected by the relaxation time, while the stresses and the temperature distribution are affected by both the relaxation and retardation times.

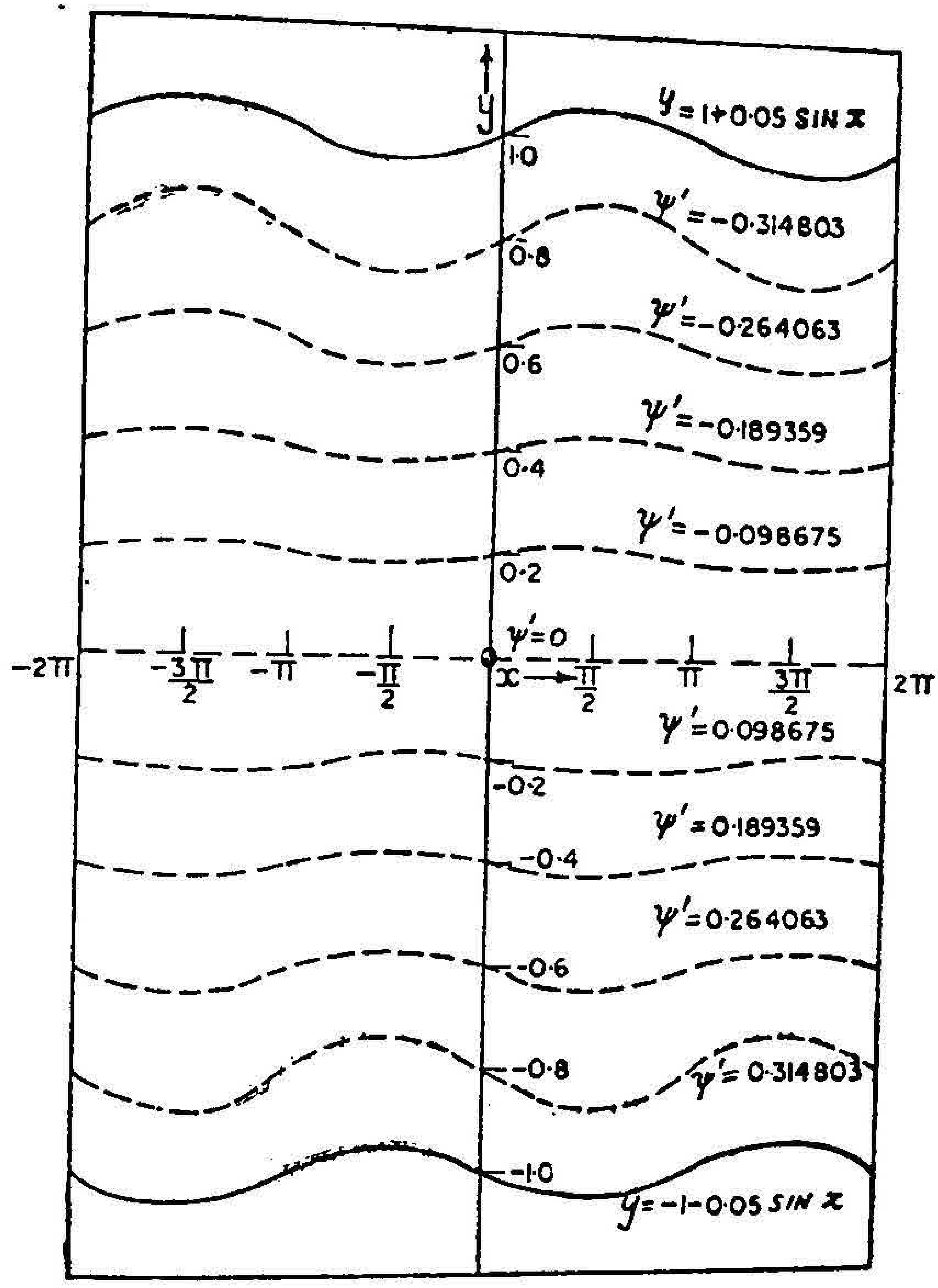

FIG. 1 STREAMLINES FOR PARTICULAR VALUES : $\epsilon = 0{\cdot}05$, $h = 1$, $P_x^{(0)} = 1$, $R = 0{\cdot}1$, $X_1' = 0{\cdot}1$

1. Basic Equations of the Problem

Let a be the average half distance between the walls and U be the velocity at the mid-plane with the neglect of deformation. Introducing the dimentionless parameters through the following relations :

$$x = \frac{x'}{a}, \quad y = \frac{y'}{a}, \quad u = \frac{u'}{U}, \quad v = \frac{v'}{U}, \quad p = \frac{p'}{\rho U^2}, \quad p_{ij} = \frac{p'_{ij}}{\rho U^2}, \quad T = \frac{T'}{T_\infty},$$

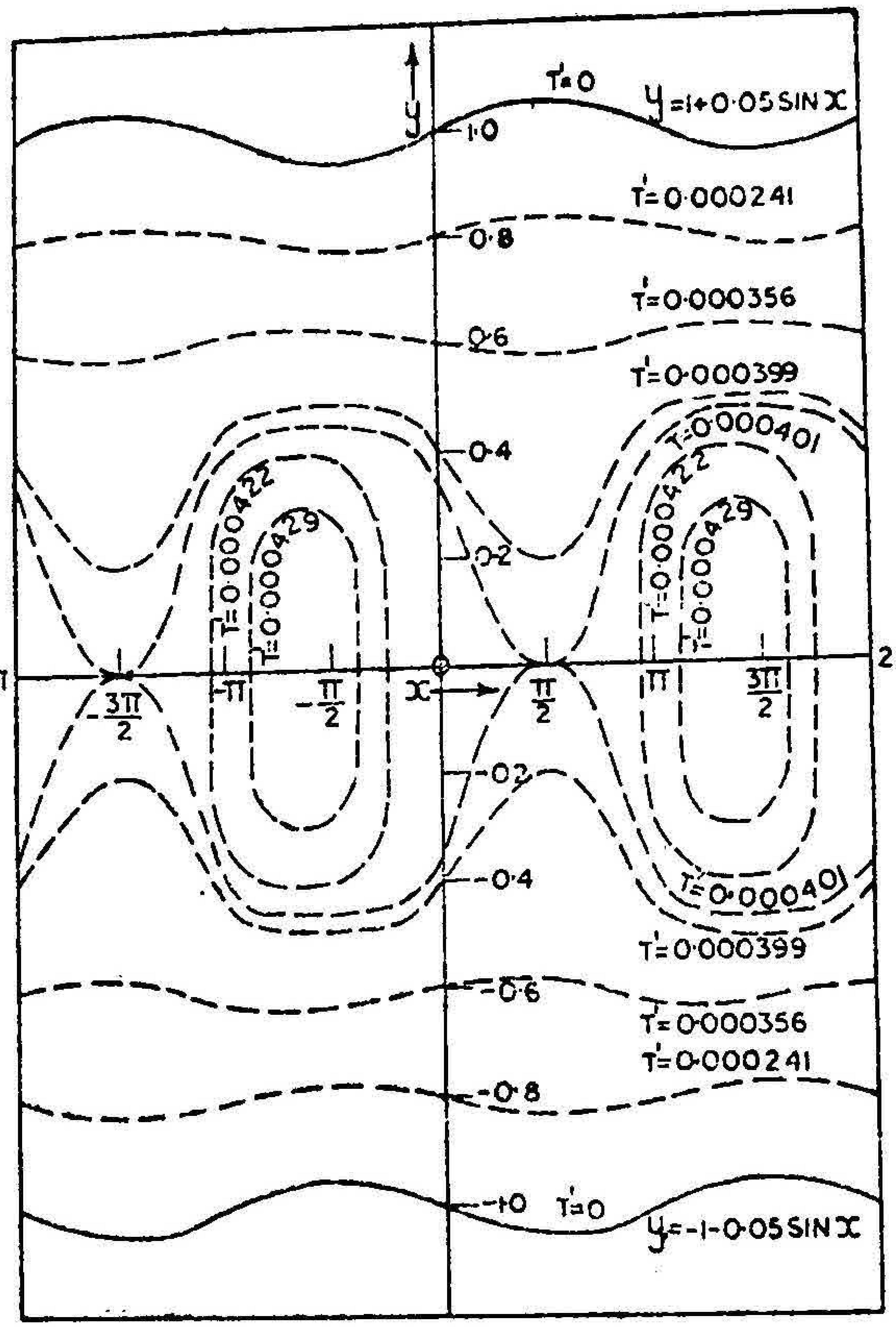

FIG. 2 ISOTHERMS FOR PARTICULAR VALUES FOR CASE (a)

$\epsilon = 0\cdot05,\ h = p_x^{(0)} = \sigma = 1,\ R = 0\cdot1,\ E = 0\cdot5,\ \lambda_1' = 0\cdot1,\ \lambda_2' = 0\cdot011$

the equations of the problem reduce to :

$$p_{xx} + \lambda_1' \left[\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} \right) p_{xx} - 2\frac{\partial u}{\partial x} p_{xx} - 2\frac{\partial u}{\partial y} p_{xy} \right]$$

$$= \frac{2}{R} \cdot \frac{\partial u}{\partial x} + 2\lambda_2' \left[\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} \right) \left(\frac{\partial u}{\partial x} \right) - 2\left(\frac{\partial u}{\partial x} \right)^2 - \frac{\partial u}{\partial y} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right], \quad [1.1]$$

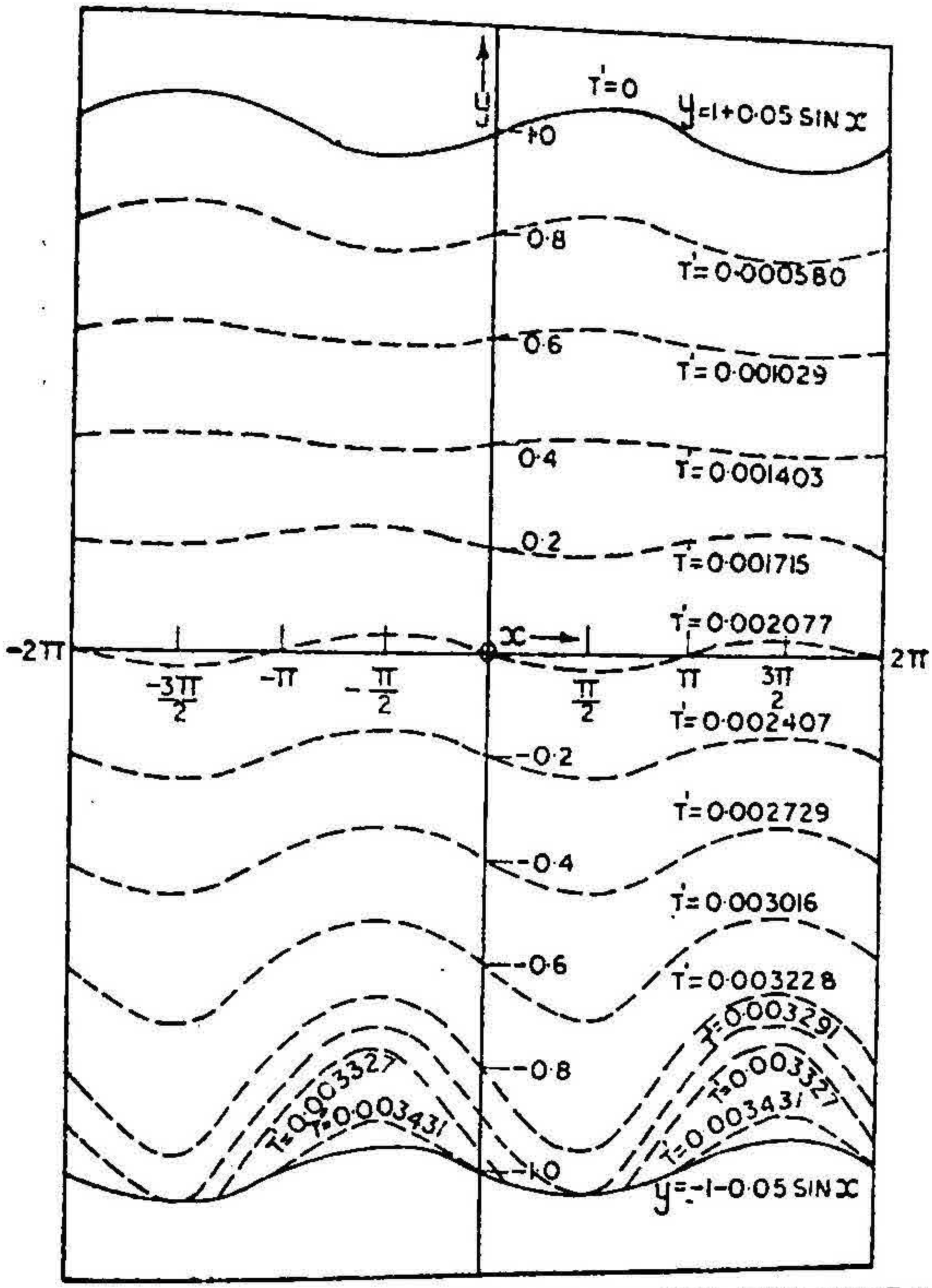

FIG. 3 ISOTHERMS FOR PARTCULAR VALUES FOR CASE (b)

$\epsilon = 0{\cdot}05,\ h \equiv p_x^{(0)} = \sigma = 1,\ R = 0{\cdot}1,\ E = 0{\cdot}5,\ \lambda_1' = 0{\cdot}1,\ \lambda_2' = 0{\cdot}011$

$$p_{xy} + \lambda_1'\left[\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)p_{xy} - \frac{\partial v}{\partial x}p_{xx} - \frac{\partial u}{\partial y}p_{yy}\right] = \frac{1}{R}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$
$$+ \lambda_2'\left[\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - \left(3\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\frac{\partial v}{\partial y} - \left(\frac{\partial u}{\partial y} + 3\frac{\partial v}{\partial x}\right)\frac{\partial u}{\partial x}\right], \quad [1.2]$$

$$p_{yy} + \lambda_1'\left[\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)p_{yy} - 2\frac{\partial v}{\partial x}p_{xy} - 2\frac{\partial v}{\partial y}p_{yy}\right] = \frac{2}{R}\cdot\frac{\partial v}{\partial y}$$
$$+ 2\lambda_2'\left[\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(\frac{\partial v}{\partial y}\right) - 2\left(\frac{\partial v}{\partial y}\right)^2 - \frac{\partial v}{\partial x}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right], \quad [1.3]$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad [1.4]$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial p_{xx}}{\partial x}+\frac{\partial p_{xy}}{\partial y}, \qquad [1.5]$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\partial p_{xy}}{\partial x}+\frac{\partial p_{yy}}{\partial y}, \qquad [1.6]$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{R\sigma}\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right]+E(t_{ik}:d_{ik}), \qquad [1.7]$$

where

(i) p_{xx}, p_{xy}, p_{yy} are components of reduced stress tensor;

(ii) $\lambda_1'=\frac{\lambda_1 U}{a}$, $\lambda_2'=\frac{\lambda_2 \mu}{\rho a^2}$ are the dimentionless parameters characterising respectively the relaxation and retardation time constants, ρ being the density of the fluid;

(iii) $R=\rho aU/\mu$ is the Reynolds number;

(iv) u, v are the dimentionless components of velocity in x and y directions;

(v) $\sigma=\mu c/k$ is the Prandtl number, k and c being respectively the thermal conductivity and the specific heat of the fluid;

(vi) $E=U^2/c\,T_\omega$ is the Eckert number, T_ω being the constant temperature on one of the walls.

The above system of simultaneous partial differential equations [1.1] – [1.7] is to be solved under the following boundary conditions:

Velocity field:

$$u=v=0, \text{ on } y=y_1 \text{ and } y=y_2,$$

where

$$\left.\begin{aligned} y_1&=1+\epsilon\sum_{n=1}^{\infty}(a_n\cos\alpha_n x+b_n\sin\alpha_n x),\\ y_2&=-1+\epsilon\sum_{n=1}^{\infty}(a_n'\cos\alpha_n x+b_n'\sin\alpha_n x),\\ \alpha_n&=(n/h). \end{aligned}\right\} \qquad [1.8]$$

Temperature field:

Case (a) $T=1$, on $y=y_1$ and $y=y_2$,

Case (b) $T=1$, on $y=y_1$,

and $\frac{\partial T}{\partial y}=0$, on $y=y_2$ [1.9]

2. Velocity Field

We define a stream function ψ as

$$u=\psi_y, \quad v=-\psi_x,$$

which satisfies the equation of continuity [1 4] identically. We expand the physical variables in powers of the small deformation ϵ in the boundaries and retain terms upto the first power of ϵ as follows:

$$\psi=\psi^{(0)}+\epsilon\,\psi^{(1)}, \quad p_{xx}=p_{xx}^{(0)}+\epsilon\,p_{xx}^{(1)}, \quad p_{xy}^{(0)}+\epsilon\,p_{xy}^{(1)}=p_{xy}, \quad p_{yy}=p_{yy}^{(0)}+\epsilon\,p_{yy}^{(1)},$$
$$p=p^{(0)}+\epsilon\,p^{(1)}. \qquad [2.1]$$

Solutiom of zero order equations:

The zero order flow is the flow of the fluid under consideration in a channel with plane parallel walls for which $\psi^{(0)}$, $p_{xx}^{(0)}$, $p_{xy}^{(0)}$ and $p_{yy}^{(0)}$ are functions of y alone.

Substituting [2.1] into [1.1] - [1 6] and [1 8] and equating the terms independent of ϵ we obtain the following equations:

$$p_{xx}^{(0)}-2\lambda_1'\,\psi_{yy}^{(0)}\,p_{xy}^{(0)}+2\lambda_2'\,\psi_{yy}^{(0)\,2}=0, \qquad [2.2]$$

$$R\,p_{xy}^{(0)}-\lambda_1'\,R\,\psi_{yy}^{(0)}\,p_{yy}^{(0)}-\psi_{yy}^{(0)}=0, \qquad [2.3]$$

$$p_{yy}^{(0)}=0, \qquad [2.4]$$

$$p_{xy,\,y}^{(0)}=p_x^{(0)}, \qquad [2.5]$$

$$p_{yy,\,y}^{(0)}=p_y^{(0)}, \qquad [2.6]$$

to be solved under the boundary conditions:

$$\psi_y^{(0)}(y=\pm 1)=0. \qquad [2.7]$$

The solution of the equations [2.2] – [2.7] is

$$\psi^{(0)}(y) = \frac{R p_x^{(0)}}{2}\left(\frac{y^3}{3} - y\right) \qquad [2.8]$$

and

$$p_{xy}^{(0)} = y p_x^{(0)}, \qquad p_{xx}^{(0)} = 2 R p_x^{(0)2} (\lambda_1' - \lambda_2' R)\, y^2,$$
$$p_{yy}^{(0)} \equiv 0, \qquad p^{(0)} = x p_x^{(0)} + \text{constant}. \qquad [2.9]$$

From [2.8] and [2.9] we see that the velocity profiles are the same as in the case of Newtonian fluids, while the stresses are affected by relaxation and retardation times.

This result is similar in nature to that obtained in [3] where the velocity profiles are not affected by visco-elasticity and cross-viscosity while the stresses were modified due to these.

First order equations and their solution :

Substituting [2.1], [2 8] and [2.9] into [1 1] – [1.6] and [1.8] and equating the coefficients of ϵ, we obtain the following set of equations :

$$p_{xx}^{(1)} + \lambda_1' R p_x^{(0)} [\tfrac{1}{2} (y^2 - 1) p_{xx,x}^{(1)} - 4 (\lambda_1' - \lambda_2' R)\, y p_x^{(0)} (\psi_x^{(1)} + y \psi_{xy}^{(1)}) - 2y p_{xy}^{(1)}$$
$$- (2y/R)\, \psi_{yy}^{(1)}] = (2/R)\, \psi_{xy}^{(1)} + 2 \lambda_2' R p_x^{(0)} [\tfrac{1}{2} (y^2 - 1)\, \psi_{xxy}^{(1)} - y\, (2 \psi_{yy}^{(1)} - \psi_{xx}^{(1)})], \qquad [2.10]$$

$$p_{xy}^{(1)} + \lambda_1' R p_x^{(0)} [\tfrac{1}{2} (y^2 - 1) p_{xy,x}^{(1)} - (1/R)\, \psi_x^{(1)} + 2 p_x^{(0)} (\lambda_1' - \lambda_2' R)\, y^2 \psi_{xx}^{(1)} - y p_{yy}^{(1)}]$$
$$= (1/R)\, (\psi_{yy}^{(1)} - \psi_{xx}^{(1)}) + \lambda_2' R p_x^{(0)} [\tfrac{1}{2} (y^2 - 1)\, (\psi_{xyy}^{(1)} - \psi_{xxx}^{(1)}) - \psi_x^{(1)} + 2y\, \psi_{xy}^{(1)}], \qquad [2.11]$$

$$p_{yy}^{(1)} + \lambda_1' R p_x^{(0)} [\tfrac{1}{2} (y^2 - 1) p_{yy,x}^{(1)} + (2y/R)\, \psi_{xx}^{(1)}]$$
$$= -(2/R)\, \psi_{xy}^{(1)} + 2 \lambda_2' R p_x^{(0)} [\tfrac{1}{2} (y^2 - 1)\, \psi_{xxy}^{(1)} + y\, \psi_{xx}^{(1)}], \qquad [2.12]$$

$$(R p_x^{(0)}/2)\, (y^2 - 1)\, \psi_{xy}^{(1)} - R p_x^{(0)}\, y\, \psi_x^{(1)} = -p_x^{(1)} + p_{xx,x}^{(1)} + p_{xy,y}^{(1)}, \qquad [2.13]$$

$$-(R p_x^{(0)}/2)\, (y^2 - 1)\, \psi_{xx}^{(1)} = -p_y^{(1)} + p_{xy,x}^{(1)} + p_{yy,y}^{(1)}, \qquad [2.14]$$

to be solved under the following boundary conditions :

$$\left.\begin{aligned} &\psi_x^{(1)}(x, 1) = \psi_x^{(1)}(x, -1) = 0, \\ &\psi_y^{(1)}(x, 1) = -R p_x^{(0)} \sum_{n=1}^{\infty} (a_n \cos \alpha_n x + b_n \sin \alpha_n x), \\ &\psi_y^{(1)}(x, -1) = -R p_x^{(0)} \sum_{n=1}^{\infty} (a_n' \cos \alpha_n x + b_n' \sin \alpha_n x) \end{aligned}\right\} \qquad [2.15]$$

The boundary conditions [2.15] suggest that $\psi^{(1)}_{(x,y)}$ should be chosen in the following form :

$$\psi^{(1)}(x, y) = -R\, p_x^{(0)} \sum_{n=1}^{\infty} [A_n(y) \cos \alpha_n x + B_n(y) \sin \alpha_n x]. \qquad [2.16]$$

In view of [2.16], we choose $p^{(1)}_{xx}$, $p^{(1)}_{xy}$, $p^{(1)}$ in the form :

$$\left.\begin{aligned} p^{(1)}_{xx} &= \sum_{n=1}^{\infty} [C_n(y) \cos \alpha_n x + D_n(y) \sin \alpha_n x], \\ p^{(1)}_{xy} &= \sum_{n=1}^{\infty} [E_n(y) \cos \alpha_n x + F_n(y) \sin \alpha_n x], \\ p^{(1)}_{yy} &= \sum_{n=1}^{\infty} [G_n(y) \cos \alpha_n x + H_n(y) \sin \alpha_n x]. \end{aligned}\right\} \qquad [2.17]$$

In view of [2.16], the boundary conditions [2.15] take the form :

$$\begin{aligned} &A_n(\pm 1) = 0,\ A_n'(1) = a_n,\ A_n'(-1) = a_n', \\ &B_n(\pm 1) = 0,\ B_n'(1) = b_n,\ B_n'(-1) = b_n'. \end{aligned} \qquad [2.18]$$

Eliminating $p^{(1)}$ from [2.13] and [2.14] and substituting [2.16] and [2.17] into [2.10] – [2.14] and equating the coefficients of $\cos \alpha_n x$ and $\sin \alpha_n x$, we obtain the following set of simultaneous ordinary differential equations:

$$\begin{aligned} C_n &+ \lambda_1' R p_x^{(0)} [\tfrac{1}{2}(y^2-1)\alpha_n D_n + 4 R p_x^{(0)2} (\lambda_1' - \lambda_2' R) \\ &\times (B_n + y B_n') \alpha_n y - 2 y E_n + 2 p_x^{(0)} y A_n''] = -2 \alpha_n p_x^{(0)} B_n' \\ &+ 2 \lambda_2' R^2 p_x^{(0)2} [\tfrac{1}{2}(y^2-1) \alpha_n^2 A_n' + (2 A_n'' + \alpha_n^2 A_n) y], \end{aligned} \qquad [2.19]$$

$$\begin{aligned} D_n &+ \lambda_1' R p_x^{(0)} [-\tfrac{1}{2}(y^2-1)\alpha_n C_n - 4 R p_x^{(0)2} (\lambda_1' - \lambda_2' R) \\ &\times (A_n + y A_n') \alpha_n y - 2 y F_n + 2 p_x^{(0)} y B_n''] = 2 \alpha_n p_x^{(0)} A_n' \\ &+ 2 \lambda_2' R^2 p_x^{(0)2} [\tfrac{1}{2}(y^2-1) \alpha_n^9 B_n' + (2 B_n'' + \alpha_n^2 B_n) y], \end{aligned} \qquad [2.20]$$

$$\begin{aligned} E_n &+ \lambda_1' p_x^{(0)} R [\tfrac{1}{2}(y^2-1)\alpha_n F_n + \alpha_n p_x^{(0)} B_n + 2 R p_x^{(0)2} \\ &\times (\lambda_1' - \lambda_2' R) \alpha_n^2 y^2 A_n - y G_n] = -p_x^{(0)} (A_n'' + \alpha_n^2 A_n) \\ &+ \lambda_2' R^2 p_x^{(0)2} \alpha_n [-\tfrac{1}{2}(y^2-1)(B_n'' + \alpha_n^2 B_n) + B_n - 2 y B_n'], \end{aligned} \qquad [2.21]$$

$$\begin{aligned} F_n &+ \lambda_1' R p_x^{(0)} [-\tfrac{1}{2}(y^2-1)\alpha_n E_n - \alpha_n p_x^{(0)} A_n + 2 R p_x^{(0)2} \\ &\times (\lambda_1' - \lambda_2' R) \alpha_n^2 y^2 B_n - y H_n] = -p_x^{(0)} (B_n'' + \alpha_n^2 B_n) \\ &+ \lambda_2' R^2 p_x^{(0)2} \alpha_n [\tfrac{1}{2}(y^2-1)(A_n'' + \alpha_n^2 A_n) - A_n + 2y A_n'], \end{aligned} \qquad [2.22]$$

$$G_n + \lambda_1' R p_x^{(0)} \alpha_n [\tfrac{1}{2}(y^2-1) H_n + 2 \alpha_n p_x^{(0)} y A_n]$$
$$= 2 \alpha_n p_x^{(0)} B_n' + 2 \lambda_2' R^2 p_x^{(0)2} \alpha_n^2 [-\tfrac{1}{2}(y^2-1) A_n' + y A_n], \qquad [2.23]$$

$$H_n + \lambda_1' R p_x^{(0)} [-\tfrac{1}{2}(y^2-1) \alpha_n G_n + 2 \alpha_n^2 p_x^{(0)} y B_n]$$
$$= -2 \alpha_n p_x^{(0)} A_n' + 2 \lambda_2' R^2 p_x^{(0)2} \alpha_n^2 [-\tfrac{1}{2}(y^2-1) B_n' + y B_n] \qquad [2.24]$$

$$\alpha_n (D_n' - H_n') + E_n'' + \alpha_n^2 E_n$$
$$= R^2 p_x^{(0)2} \alpha_n [-\tfrac{1}{2}(y^2-1)(B_n'' - \alpha_n^2 B_n) + B_n], \qquad [2.25]$$

and

$$\alpha_n (G_n' - C_n')) + F_n'' + \alpha_n^2 F_n$$
$$= R^2 p_x^{(0)2} \alpha_n [\tfrac{1}{2}(y^2-1)(A_n'' - \alpha_n^2 A_n) - A_n], \qquad [2.26]$$

where dash denotes differentiation with respeet to y.

Since we are considering slow motion, we take R to be small. Using this physical fact, we set :

$$\left.\begin{aligned} A_n &= A_{0,n} + R A_{1,n} + R^2 A_{2,n}; & B_n &= B_{0,n} + R B_{1,n} + R^2 B_{2,n}; \\ C_n &= C_{0,n} + R C_{1,n} + R^2 C_{2,n}; & D_n &= D_{0,n} + R D_{1,n} + R^2 D_{2,n}; \\ E_n &= E_{0,n} + R E_{1,n} + R^2 E_{2,n}; & F_n &= F_{0,n} + R F_{1,n} + R^2 F_{2,n}; \\ G_n &= G_{0,n} + R G_{1,n} + R^2 G_{2,n}; & H_n &= H_{0,n} + R H_{1,n} + R^2 H_{2,n}. \end{aligned}\right\} \qquad [2.27]$$

Substituting [2.27] into [2.18] – [2.26] and equating the co-efficients of various powers of R, we obtain the following sets of equations;

$$-C_{0,n} = G_{0,n} = 2 \alpha_n p_x^{(0)} B_{0,n}', \qquad [2.28a]$$

$$D_{0,n} = -H_{0,n} = 2 \alpha_n p_x^{(0)} A_{0,n}', \qquad [2.28b]$$

$$E_{0,n} = -p_x^{(0)} (A_{0,n}'' + \alpha_u^2 A_{0,n}) \qquad [2.28c]$$

$$F_{0,n} = -p_x^{(0)} (B_{0,n}'' + \alpha_n^2 B_{0,n}), \qquad [2.28d]$$

$$\alpha_n (D_{0,n}' - H_{0,n}') + E_{0,n}'' + \alpha_n^2 E_{0,n} = 0, \qquad [2.28e]$$

$$\alpha_n (G_{0,n}' - C_{0,n}') + F_{0,n}'' + \alpha_n^2 F_{0,n} = 0; \qquad [2.28f]$$

$$C_{1,n} + \lambda_1' p_x^{(0)} [\tfrac{1}{2}(y^2-1) \alpha_n D_{0,n} - 2 y E_{0,n} + 2 p_x^{(0)} y A_{0,n}'']$$
$$= -2 \alpha_n p_x^{(0)} B_{1,n}', \qquad [2.29a]$$

$$D_{1,n} + \lambda_1' p_x^{(0)} [-\tfrac{1}{2}(y^2-1) \alpha_n C_{0,n} - 2 y F_{0,n} + 2 p_x^{(0)} y B_{0,n}'']$$
$$= 2 \alpha_n p_x^{(0)} A_{1,n}', \qquad [2.29b]$$

$$E_{1,n}+\lambda_1' p_x^{(0)}\left[\tfrac{1}{2}(y^2-1)\,\alpha_n F_{0,n}+\alpha_n p_x^{(0)} B_{0,n}-y\,G_{0,n}\right]$$
$$=-p_x^{(0)}\left(A''_{1,n}+\alpha_n^2 A_{1,n}\right), \qquad [2..9c]$$

$$F_{1,n}+\lambda_1' p_x^{(0)}\left[-\tfrac{1}{2}(y^2-1)\,\alpha_n E_{0,n}-\alpha_n p_x^{(0)} A_{0,n}-y\,H_{0,n}\right]$$
$$=-p_x^{(0)}\left(B''_{1,n}+\alpha_n^2 B_{1,n}\right), \qquad [2.29d]$$

$$G_{1,n}+\lambda_1' p_x^{(0)}\left[\tfrac{1}{2}(y^2-1)\,\alpha_n H_{0,n}+2\,\alpha_n^2 p_x^{(0)} y\,A_{0,n}\right]$$
$$=2\,\alpha_n p_x^{(0)} B'_{1,n}, \qquad [2.29e]$$

$$H_{1,n}+\lambda_1' p_x^{(0)}\,\alpha_n\left[-\tfrac{1}{2}(y^2-1)\,G_{0,n}+2\,\alpha_n p_x^{(0)} y\,B_{0,n}\right]$$
$$=-2\,\alpha_n p_x^{(0)} A'_{1,n}, \qquad [2.29f]$$

$$\alpha_n\left(D'_{1,n}-H'_{1,n}\right)+E''_{1,n}+\alpha_n^2 E_{1,n}=0, \qquad [2.29g]$$

$$\alpha_n\left(G'_{1,n}-C'_{1,n}\right)+F''_{1,n}+\alpha_n^2 F_{1,n}=0; \qquad [2.29h]$$

$$C_{2,n}+\lambda_1' p_x^{(0)}\left[\tfrac{1}{2}(y^2-1)\,\alpha_n D_{1,n}+4\,\lambda_1' p_x^{(0)2}\,\alpha_n y\right.$$
$$\times\left(B_{0,n}+y\,B'_{0,n}\right)-2\,y\,E_{1,n}+2\,p_x^{(0)} y\,A''_{1,n}\Big]=-2\,\alpha_n p_x^{(0)}$$
$$\times B'_{2,n}+2\,\lambda_2' p_x^{(0)2}\left[\tfrac{1}{2}(y^2-1)\,\alpha_n^2 A'_{0,n}+\left(\alpha_n^2 A_{0,n}+2\,A''_{0,n}\right)y\right], \qquad [2.30a]$$

$$D_{2,n}+\lambda_1' p_x^{(0)}\left[-\tfrac{1}{2}(y^2-1)\,\alpha_n C_{1,n}-4\,\lambda_1' p_x^{(0)2}\,\alpha_n y\right.$$
$$\times\left(A_{0,n}+y\,A'_{0,n}\right)-2\,y\,F_{1,n}+2\,p_x^{(0)} y\,B''_{1,n}\Big]=2\,\alpha_n p_x^{(0)} A'_{2,n}$$
$$+2\,\lambda_2' p_x^{(0)2}\left[\tfrac{1}{2}(y^2-1)\,\alpha_n^2 B'_{0,n}+\left(2B''_{0,n}+\alpha_n^2 B_{0,n}\right)y\right] \qquad [2.30b]$$

$$E_{2,n}+\lambda_1' p_x^{(0)}\left[\tfrac{1}{2}(y^2-1)\,\alpha_n F_{1,n}+\alpha_n p_x^{(0)} B_{1,n}+2\,\lambda_1' p_x^{(0)2}\right.$$
$$\times\,\alpha_n^2 y^2 A_{0,n}-y\,G_{1,n}\Big]=-p_x^{(0)}\left(A''_{2,n}+\alpha_n^2 A_{2,n}\right)$$
$$+\lambda_2' p_x^{(0)2}\alpha_n\left[-\tfrac{1}{2}(y^2-1)\left(B''_{0,n}+\alpha_n^2 B_{0,n}\right)+B_{0,n}-2\,y\,B'_{0,n}\right] \qquad [2.30c]$$

$$F_{2,n}+\lambda_1' p_x^{(0)}\left[-\tfrac{1}{2}(y^2-1)\,\alpha_n E_{1,n}-\alpha_n p_x^{(0)} A_{1,n}+2\lambda_1' p_x^{(0)2}\right.$$
$$\times\,\alpha_n^2 y^2 B_{0,n}-y\,H_{1,n}\Big]=-p_x^{(0)}\left(B''_{2,n}+\alpha_n^2 B_{2,n}\right)$$
$$+\lambda_2' p_x^{(0)2}\,\alpha_n\left[\tfrac{1}{2}(y^2-1)\left(A''_{0,n}+\alpha_n^2 A_{0,n}\right)-A_{0,n}+2\,y\,A'_{0,n}\right], \qquad [2.30d]$$

$$G_{2,n}+\lambda_1' p_x^{(0)}\,\alpha_n\left[\tfrac{1}{2}(y^2-1)\,H_{1,n}+2\,\alpha_n p_x^{(0)} y\,A_{1,n}\right]$$
$$=2\,\alpha_n p_x^{(0)} B'_{2,n}+2\,\lambda_2' p_x^{(0)2}\,\alpha_n^2\left[-\tfrac{1}{2}(y^2-1)\,A'_{0,n}+y\,A_{0,n}\right] \qquad [2.30e]$$

$$H_{2,n}+\lambda_1' p_x^{(0)}\;\left[-\tfrac{1}{2}(y^2-1)\,G_{1,n}+2\,\alpha_n p_n^{(0)}\,y\,B_{1,n}\right]$$
$$=-2\,\alpha_n p_x^{(0)} A'_{2,n}+2\,\lambda_2'\,\alpha_n^2 p_x^{(0)2}\left[-\tfrac{1}{2}(y^2-1)\,B'_{0,n}+y\,B_{0,n}\right], \qquad [2.30f]$$

$$\alpha_n (D'_{2,n} - H'_{2,n}) + E''_{2,n} + \alpha_n^2 E_{2,n}$$
$$= \alpha_n^2 p_x^{(0)} [-\tfrac{1}{2}(y^2-1)(B''_{0,n} - \alpha_n^2 B_{0,n}) + B_{0,n}], \qquad [2.30g]$$

$$\alpha_n (G'_{2,n} - C'_{2,n}) + F''_{2,n} + \alpha_n^2 F_{2,n}$$
$$= \alpha_n p_x^{(0)2} [\tfrac{1}{2}(y^2-1)(A''_{0,n} - \alpha_n^2 A_{0,n}) - A_{0,n}]; \qquad [2.30h]$$

to be solved under the boundary conditions:

$$\left.\begin{aligned} &A_{0,n}(\pm 1) = 0, \ A'_{0,n}(1) = a_n, \ A'_{0,n}(-1) = a'_n, \\ &B_{0,n}(\pm 1) = 0, \ B'_{0,n}(1) = b_n, \ B'_{0,n}(-1) = b'_n, \end{aligned}\right\} [2.31a]$$

$$\left.\begin{aligned} &A_{1,n}(\pm 1) = A'_{1,n}(\pm 1) = 0, \\ &B_{1,n}(\pm 1) = B'_{1,n}(\pm 1) = 0, \end{aligned}\right\} [2.31b]$$

$$\left.\begin{aligned} &A_{2,n}(\pm 1) = A'_{2,n}(\pm 1) = 0, \\ &B_{2,n}(\pm 1) = B'_{2,n}(\pm 1) = 0. \end{aligned}\right\} [2.31c]$$

Eliminating $C_{0,n}$, $D_{0,n}$, $E_{0,n}$, $F_{0,n}$, $G_{0,n}$ and $H_{0,n}$, from the equations [2.28], $A_{0,n}$ and $B_{0,n}$ are given by

$$(D^4 - 2\alpha_n^2 D^2 + \alpha_n^4)\begin{bmatrix} A_{0,n} \\ B_{0,n} \end{bmatrix} = 0, \qquad [2.32]$$

where $$D \equiv (d/dy).$$

The equations [2.32] and [2.31a] give

$$\left.\begin{aligned} A_{0,n} &= (\alpha_{1,n} + \alpha_{3,n}\, y)\cosh \alpha_n y + (\alpha_{2,n} + \alpha_{4,n}\, y)\sinh \alpha_n y, \\ B_{0,n} &= (\beta_{1,n} + \beta_{3,n}\, y)\cosh \alpha_n y + (\beta_{2,n} + \beta_{4,n}\, y)\sinh \alpha_n y, \end{aligned}\right\} [2.33]$$

where $$\alpha_{1,n} = -\frac{(a_n - a'_n)\sinh 2\alpha_n}{2\alpha_n + \sinh 2\alpha_n}, \qquad [2.34a]$$

$$\alpha_{2,n} = \frac{(a_n + a'_n)\cosh \alpha_n}{2\alpha_n - \sinh 2\alpha_n}, \qquad [2.34b]$$

$$\alpha_{3,n} = -\frac{(a_n + a'_n)\sinh \alpha_n}{2\alpha_n - \sinh 2\alpha_n}, \qquad [2.34c]$$

$$\alpha_{4,n} = \frac{(a_n - a'_n)\cosh \alpha_n}{2\alpha_n + \sinn 2\alpha_n}. \qquad [2.34d]$$

The arbitrary constants $\beta_{1,n}$, $\beta_{2,n}$, $\beta_{3,n}$, and $\beta_{4,n}$ can be obtained from [2.34] by replacing $a_n's$ by $b_n's$. The functions $C_{0,n}$, $D_{0,n}$, $E_{0,n}$, $F_{0,n}$, $G_{0,n}$, and $H_{0,n}$ can be easily obtained from [2.28], [2.33] and [2.34].

Eliminating $C_{1,n}$, $D_{1,n}$, $E_{1,n}$, $F_{1,n}$, $G_{1,n}$ and $H_{1,n}$ from [2.29], $A_{1,n}$ and $B_{1,n}$ are given by

$$(D^4 - 2\alpha_n^2 D^2 + \alpha_n^4)\begin{bmatrix} A_{1,n} \\ B_{1,n} \end{bmatrix} = 0, \qquad [2.35]$$

which along with [2.31*b*] gives

$$A_{1,n} = B_{1,n} \equiv 0. \qquad [2.36]$$

The expressions for $C_{1,n}$, $D_{1,n}$, $E_{1,n}$, $F_{1,n}$, $G_{1,n}$ and $H_{1,n}$ can be obtained from [2.29] using [2.33], [2.34] and [2.36].

Eliminating $C_{2,n}$, $D_{2,n}$, $E_{2,n}$, $F_{2,n}$, $G_{2,n}$ and $H_{2,n}$, from [2.30], $A_{2,n}$ and $B_{2,n}$ are given by

$$\begin{aligned}
&(D^4 - 2\alpha_n^2 D^2 + \alpha_n^4) A_{2,n} \\
&= \alpha_n p_x^{(0)} [(\alpha_n \beta_{4,n} y^2 - \beta_{3,n} y - \alpha_n \beta_{4,n} - \beta_{1,n}) \cosh \alpha_n y + (\alpha_n \beta_{3,n} y^2 \\
&\quad - \beta_{4,n} y - \alpha_n \beta_{3,n} - \beta_{2,n}) \sinh \alpha_n y] \\
&\quad + \lambda_1'^2 p_x^{(0)2} \alpha_n^2 [\{\alpha_n^2 \alpha_{3,n} y^3 + (7\alpha_n \alpha_{4,n} - \alpha_n^2 \alpha_{1,n}) y^2 \\
&\quad - 3(\alpha_n^2 + 1) \alpha_{3,n} y - (3\alpha_n \alpha_{4,n} + \alpha_n^2 \alpha_{1,n} + 3\alpha_{1,n})\} \cosh \alpha_n y \\
&\quad + \{\alpha_n^2 \alpha_{4,n} y^3 + (7\alpha_n \alpha_{3,n} - \alpha_n^2 \alpha_{2,n}) y^2 \\
&\quad - 3(\alpha_n^2 + 1)\alpha_{4,n} y - (3\alpha_n \alpha_{3,n} + \alpha_n^2 \alpha_{2,n} + 3\alpha_{2,n})\} \sinh \alpha_n y],
\end{aligned} \qquad [2.37]$$

and

$$\begin{aligned}
&(D^4 - 2\alpha_n^2 D^2 + \alpha_n^4) B_{2,n} \\
&= -\alpha_n p_x^{(0)}[(\alpha_n \alpha_{4,n} y^2 - \alpha_{3,n} y - \alpha_n \alpha_{4,n} - \alpha_{1,n}) \cosh \alpha_n y \\
&\quad + (\alpha_n \alpha_{3,n} y^2 - \alpha_{4,n} y - \alpha_n \alpha_{3,n} - \alpha_{2,n}) \sinh \alpha_n y] \\
&\quad + \lambda_1' \alpha_n^2 p_x^{(0)2} [\{\alpha_n^2 \beta_{3,n} y^3 + (7\alpha_n \beta_{4,n} - \alpha_n^2 \beta_{1,n}) y^2 \\
&\quad - 3(\alpha_n^2 + 1) \beta_{3,n} y - (3\alpha_n \beta_{4,n} + \alpha_n^2 \beta_{1,n} + 3\beta_{1,n})\} \cosh \alpha_n y \\
&\quad + \{\alpha_n^2 \beta_{4,n} y^3 + (7\alpha_n \beta_{3,n} - \alpha_n^2 \beta_{2,n}) y^2 - 3(\alpha_n^2 + 1) \beta_{4,n} y \\
&\quad - (3\alpha_n \beta_{3,n} + \alpha_n^2 \beta_{2,n} + 3\beta_{2,n})\} \sinh \alpha_n y].
\end{aligned} \qquad [2.38]$$

Solving [2.37], [2.38] along with the boundary conditions [2.31c] we get

$$A_{2,n} = (K_{1,n} + K_{3,n}\,y)\cosh\alpha_n y + (K_{2,n} + K_{4,n}\,y)\sinh\alpha_n y$$
$$+ p_x^{(0)}\Bigg[\left\{\frac{\beta_{4,n}}{96}y^4 - \frac{\beta_{3,n}}{8\alpha_n}y^3 + \left(\frac{5\beta_{4,n}}{16\alpha_n^2} - \frac{\beta_{1,n}}{8\alpha_n} - \frac{\beta_{4,n}}{8}\right)y^2\right\}\cosh\alpha_n y$$
$$+ \left\{\frac{\beta_{3,n}}{96}y^4 - \frac{\beta_{4,n}}{8\alpha_n}y^3 + \left(\frac{5\beta_{3,n}}{16\alpha_n^2} - \frac{\beta_{2,n}}{8\alpha_n} - \frac{\beta_{3,n}}{8}\right)y^2\right\}\sinh\alpha_n y\Bigg]$$
$$+ \lambda_1'^2 p_x^{(0)2}\Bigg[\left\{\frac{\alpha_n^2\alpha_{3,n}}{80}y^5 + \left(\frac{\alpha_n\alpha_{4,n}}{96} - \frac{\alpha_n^2\alpha_{1,n}}{96}\right)y^4\right.$$
$$+ \left(\frac{\alpha_n\alpha_{2,n}}{12} - \frac{25\alpha_{3,n}}{48} - \frac{\alpha_n^2\alpha_{3,n}}{8}\right)y^3 + \left(\frac{21\alpha_{4,n}}{16\alpha_n} - \frac{9\alpha_{1,n}}{16}\right.$$
$$\left.\left. - \frac{\alpha_n^2\alpha_{1,n}}{8}\right)y^2\right\}\cosh\alpha_n y + \left\{\frac{\alpha_n^2\alpha_{4,n}}{80}y^5 + \left(\frac{\alpha_n\alpha_{3,n}}{96} - \frac{\alpha_n^2\alpha_{2,n}}{96}\right)y^4\right.$$
$$+ \left(\frac{\alpha_n\alpha_{1,n}}{12} - \frac{25\alpha_{4,n}}{48} - \frac{\alpha_n^2\alpha_{4,n}}{8}\right)y^3 + \left(\frac{21\alpha_{3,n}}{16\alpha_n} - \frac{9\alpha_{2,n}}{16}\right.$$
$$\left.\left. - \frac{\alpha_n^2\alpha_{2,n}}{8}\right)y^2\right\}\sinh\alpha_n y\Bigg], \qquad [2.39]$$

where

$$K_{1,n} = \frac{p_x^{(0)}}{2\alpha_n + \sinh 2\alpha_n}\Bigg[\left\{\frac{\beta_{1,n}}{4} + \beta_{4,n}\left(\frac{11}{48} - \frac{5}{8\alpha_n}\right)\right\}\cosh^2\alpha_n$$
$$+ \left\{\beta_{4,n}\left(\frac{1}{8\alpha_n} - \frac{11\alpha_n}{48}\right) - \frac{\beta_{1,n}}{4}\right\}\sinh^2\alpha_n$$
$$+ \left\{\beta_{4,n}\left(\frac{5}{16\alpha_n^2} + \frac{1}{32}\right) - \frac{\beta_{1,n}}{8\alpha_n}\right\}\sinh 2\alpha_n\Bigg]$$
$$+ \frac{\lambda_1'^2 p_x^{(0)2}}{2\alpha_n + \sinh 2\alpha_n}\Bigg[\left\{\alpha_{1,n}\left(\frac{9\alpha_n}{8} + \frac{13\alpha_n^3}{48}\right) - \alpha_{4,n}\left(\frac{21}{8} + \frac{\alpha_n^2}{8}\right)\right\}$$
$$\times\cosh^2\alpha_n + \left\{\alpha_{4,n}\left(\frac{13}{24} - \frac{91\alpha_n^2}{240}\right) - \alpha_{1,n}\left(\frac{19\alpha_n}{24} + \frac{13\alpha_n^3}{48}\right)\right\}$$
$$\times\sinh^2\alpha_n + \left\{\alpha_{4,n}\left(\frac{21}{16\alpha_n} + \frac{\alpha_n}{32}\right) - \alpha_{1,n}\left(\frac{9}{16} + \frac{5\alpha_n^2}{32}\right)\right\}\sinh 2\alpha_n\Bigg], \qquad [2.40a]$$

$$K_{2,n}=\frac{p_x^{(0)}}{2\alpha_n-\sinh 2\alpha_n}\left[\left\{\frac{\beta_{2,n}}{4}+\beta_{3,n}\left(\frac{11}{48}-\frac{1}{8\alpha_n}\right)\right\}\right.$$

$$\times\cosh^2\alpha_n-\left\{\frac{\beta_{2,n}}{4}+\beta_{3,n}\left(\frac{11}{48}-\frac{5}{8\alpha_n}\right)\right\}\sinh^2\alpha_n$$

$$\left.+\left\{\frac{\beta_{2,n}}{8\alpha_n}-\beta_{3,n}\left(\frac{5}{16\alpha_n^2}+\frac{1}{32}\right)\right\}\sinh 2\alpha_n\right]$$

$$+\frac{\lambda_1'^2 p_x^{(0)2}}{2\alpha_n-\sinh 2\alpha_n}\left[\left\{\alpha_{2,n}\left(\frac{19\alpha_n}{24}+\frac{13\alpha_n^3}{48}\right)-\alpha_{3,n}\right.\right.$$

$$\left.\times\left(\frac{13}{24}-\frac{91\alpha_n^2}{240}\right)\right\}\cosh^2\alpha_n-\left\{\alpha_{2,n}\left(\frac{9\alpha_n}{8}+\frac{13\alpha_n^3}{48}\right)\right.$$

$$\left.-\alpha_{3,n}\left(\frac{21}{8}+\frac{\alpha_n^2}{48}\right)\right\}\sinh^2\alpha_n+\left\{\alpha_{2,n}\left(\frac{9}{16}+\frac{5\alpha_n^2}{32}\right)\right.$$

$$\left.\left.-\alpha_{3,n}\left(\frac{21}{16\alpha_n}+\frac{\alpha_n}{32}\right)\right\}\sinh 2\alpha_n\right], \qquad [2.40b]$$

$$K_{3,n}=\frac{p_x^{(0)}}{2\alpha_n-\sinh 2\alpha_n}\left[\frac{\beta_{3,n}}{4}\cosh^2\alpha_n+\left\{\beta_{3,n}\left(\frac{5}{4\alpha_n^2}-\frac{5}{12}\right)-\frac{\beta_{2,n}}{2\alpha_n}\right\}\sinh^2\alpha_n\right.$$

$$\left.-\frac{3\beta_{3,n}}{8\alpha_n}\sinh 2\alpha_n\right]+\frac{\lambda_1'^2 p_x^{(0)2}}{2\alpha_n-\sinh 2\alpha_n}\left[\left\{\alpha_{3,n}\left(\frac{25\alpha_n}{24}+\frac{9\alpha_n^3}{40}\right)-\frac{\alpha_n^2\alpha_{2,n}}{6}\right\}\right.$$

$$\times\cosh^2\alpha_n-\left\{\alpha_{2,n}\left(\frac{9}{4}+\frac{5\alpha_n^2}{12}\right)+\alpha_{3,n}\left(\frac{9\alpha_n^3}{40}+\frac{23\alpha_n}{24}-\frac{21}{4\alpha_n}\right)\right\}\sinh^2\alpha_n$$

$$\left.+\left\{\frac{\alpha_n\alpha_{2,n}}{4}-\alpha_{3,n}\left(\frac{25}{16}+\frac{5\alpha_n^2}{16}\right)\right\}\sinh 2\alpha_n\right], \qquad [2.40c]$$

and

$$K_{4,n}=\frac{p_x^{(0)}}{2\alpha_n+\sinh 2\alpha_n}\left[\left\{\frac{\beta_{1,n}}{2\alpha_n}+\beta_{4,n}\left(\frac{5}{12}-\frac{5}{4\alpha_n^2}\right)\right\}\cosh^2\alpha_n\right.$$

$$\left.-\frac{\beta_{4,n}}{4}\sinh^2\alpha_n+\frac{3\beta_{4,n}}{8\alpha_n}\times\sinh 2\alpha_n\right]$$

$$+\frac{\lambda_1'^2 p_x^{(0)2}}{2\alpha_n+\sinh 2\alpha_n}\left[\left\{\alpha_{1,n}\left(\frac{9}{4}+\frac{5\alpha_n^2}{12}\right)\right.\right.$$

$$+ \alpha_{4,n}\left(\frac{9\alpha_n^3}{40} + \frac{23\alpha_n}{24} - \frac{21}{4\alpha_n}\right)\Big\} \cosh^2 \alpha_n$$

$$+ \left\{\frac{\alpha_n^2 \alpha_{1,n}}{6} - \alpha_{4,n}\left(\frac{25\alpha_n}{24} + \frac{9\alpha_n^3}{40}\right)\right\} \sinh^2 \alpha_n$$

$$+ \left\{-\frac{\alpha_n \alpha_{1,n}}{4} + \alpha_{4,n}\left(\frac{25}{16} + \frac{5\alpha_n^2}{16}\right)\right\} \sinh 2\alpha_n \Bigg] \qquad [2.40d]$$

The expression for $B_{2,n}$ and the corresponding dashed arbitrary constants $K'_{1,n}$, $K'_{2,n}$, ... etc., can be obtained from [2.39] and [2.40] by replacing $K_{1,n}$, $K_{2,n}$, $K_{3,n}$, $K_{4,n}$ by $K'_{1,n}$, $K'_{2,n}$, $K'_{3,n}$, $K'_{4,n}$; β'_s by α'_s with sign changed in the coefficient of $p_x^{(0)}$ and $\alpha's$ by $\beta's$ in the coefficient of $\lambda_1'^2 p_x^{(0)2}$.

Once again the expressions for $C_{2,n}$, $D_{2,n}$, $E_{2,n}$, $F_{2,n}$, $G_{2,n}$ and $H_{2,n}$ can be obtained from [2.30] on using [2.33], [2.34], [2.36], [2.39] and [2.40]. Integrating [1.5] and [1.6], we have

$$p + p' = xp_x^{(0)} + \sum_{n=1}^{\infty} \Bigg[\Bigg\{ C_n - \frac{1}{\alpha_n} F'_n + R^2 p_x^{(0)2} \left[\tfrac{1}{2}(y^2-1) A'_n - y A_n\right] + G_n$$

$$+ \alpha_n \int F_n \, dy + \frac{R^2 p_x^{(0)2} \alpha_n^2}{2} \int (y^2-1) A_n \, dy \Bigg\} \cos \alpha_n x$$

$$+ \Bigg\{ D_n + \frac{1}{\alpha_n} E'_n + R^2 p_x^{(0)2} \left[\tfrac{1}{2}(y^2-1) B'_n - y B_n\right] + H_n - \alpha_n \int E_n \, dy$$

$$+ \frac{R^2 p_x^{(0)2} \alpha_n^2}{2} \int (y^2-1) B_n \, dy \Bigg\} \sin \alpha_n x \Bigg], \qquad [2.41]$$

where p' is a constant pressure.

We note that the velocity field is affected only by the relaxation time while the stresses are affected by both relaxation and retardation times. This result is similar to that obtained by us in reference [3] where the stresses were affected both by visco-elasticity and cross-viscosity of the fluid while the velocity was affected by visco-elasticity only.

This completes the solution of the general flow problem.

Particular case:

Let the boundaries have sinusoidal deformation defined as

(i) $a_n = a'_n = 0$ for all n,

(ii) $b_n = b'_n = 0$, for $n > 1$ and $b_1 = b'_1 = 1$

In such a scheme, we have

$$\alpha_{1,1}=\alpha_{2,1}=\alpha_{3,1}=\alpha_{4,1}=\beta_{1,1}=\beta_{4,1}=0$$

$$\beta_{2,1}=\frac{2\cosh\alpha_1}{2\alpha_1-\sinh 2\alpha_1},$$

$$\beta_{3,1}=-\frac{2\sinh\alpha_1}{2\alpha_1-\sinh 2\alpha_1},$$

$$\alpha_1=\frac{1}{h}, \qquad [2.42]$$

so that

$$\left.\begin{aligned}&A_{0,1}=D_{0,1}=E_{0,1}=H_{0,1}\equiv 0,\\ &B_{0,1}=\beta_{3,1}\,y\cosh\alpha_1 y+\beta_{2,1}\sinh\alpha_1 y,\\ &F_{0,1}=-G_{0,1}=C_{0,1}=-2\,\alpha_1 p_x^{(0)}\left[(\alpha_1\beta_{2,1}+\beta_{3,1})\cosh\alpha_1 y\right.\\ &\qquad\left.+\alpha_1\beta_{3,1}\,y\sinh\alpha_1 y\right],\end{aligned}\right\} \qquad [2.43]$$

$$A_{1,1}=B_{1,1}=C_{1,1}=F_{1,1}=G_{1,1}\equiv 0, \qquad [2.44a]$$

$$\begin{aligned}D_{1,1}=\lambda_1' p_x^{(0)2}\alpha_1\big[&\{\alpha_1(\alpha_1\beta_{2,1}+\beta_{3,1})-(5\beta_{3,1}\\ &+\alpha_1\beta_{2,1})\alpha_1 y^2\}\cosh\alpha_1 y+\{(\alpha_1^2\beta_{3,1}-4\alpha_1\beta_{2,1}-4\beta_{3,1})y\\ &-\alpha_1^2\beta_{3,1}y^3\}\sinh\alpha_1 y\big]\end{aligned} \qquad [2.44b]$$

$$\begin{aligned}E_{1,1}=-\lambda_1'\alpha_1 p_x^{(0)2}\big[&\{(\alpha_1^2\beta_{3,1}-2\alpha_1\beta_{2,1}-\beta_{3,1})y-\alpha_1^2\beta_{3,1}y^3\}\cosh\alpha_1 y\\ &+\{(\beta_{2,1}+\alpha_1\beta_{3,1}+\alpha_1^2\beta_{2,1})-(3\beta_{3,1}+\alpha_1\beta_{2,1})\alpha_1 y^2\}\sinh\alpha_1 y\big],\end{aligned} \qquad [2.44c]$$

$$\begin{aligned}H_{1,1}=-\lambda_1'\alpha_1^2 p_x^{(0)2}\big[&\{(\beta_{3,1}+\alpha_1\beta_{2,1})+(\beta_{3,1}-\alpha_1\beta_{2,1})y^2\}\cosh\alpha_1 y\\ &+\{(2\beta_{2,1}+\alpha_1\beta_{3,1})y-\alpha_1\beta_{3,1}y^3\}\sinh\alpha_1 y\big];\end{aligned} \qquad [2.44d]$$

$$\begin{aligned}A_{2,1}=K_{3,1}\,y\cosh\alpha_1 y+K_{2,1}\sinh\alpha_1 y+p_x^{(0)}\Bigg[&-\frac{\beta_{3,1}}{8\alpha_1}y^3\cosh\alpha_1 y\\ &+\left\{\frac{\beta_{3,1}}{96}y^4+\left(\frac{5\beta_{3,1}}{16\alpha_1^2}-\frac{\beta_{2,1}}{8\alpha_1}-\frac{\beta_{3,1}}{8}\right)y^2\right\}\sinh\alpha_1 y\Bigg],\end{aligned} \qquad [2.45a]$$

$$B_{2,1} = K'_{3,1}\, y \cosh \alpha_1 y + K'_{2,1} \sinh \alpha_1 y + \lambda_1'^2 p_x^{(0)2} \Bigg[\Bigg\{ \frac{\alpha_1^2 \beta_{3,1}}{80} y^5$$

$$+ \left(\frac{\alpha_1 \beta_{2,1}}{12} - \frac{25 \beta_{3,1}}{48} - \frac{\alpha_1^2 \beta_{3,1}}{8} \right) y^3 \Bigg\} \cosh \alpha_1 y + \Bigg\{ \left(\frac{\alpha_1 \beta_{3,1}}{96} - \frac{\alpha_1^2 \beta_{2,1}}{96} \right) y^4$$

$$+ \left(\frac{21 \beta_{3,1}}{16 \alpha_1} - \frac{9 \beta_{2,1}}{16} - \frac{\alpha_1^2 \beta_{2,1}}{8} \right) y^2 \Bigg\} \sinh \alpha_1 y \Bigg], \qquad [2.45b]$$

$$C_{2,1} = \lambda_1'^2 \alpha_1 p_x^{(0)3} \Bigg[\Bigg\{ \left(\frac{209\, \alpha_1^2 \beta_{3,1}}{48} + \frac{25\, \alpha_1^3 \beta_{2,1}}{48} \right) y^4$$

$$+ \left(\frac{5 \alpha_1 \beta_{2,1}}{8} - \frac{11 \beta_{3,1}}{2} - \frac{3 \alpha_1^3 \beta_{2,1}}{4} - \frac{17 \alpha_1^2 \beta_{2,1}}{4} \right) y^2$$

$$+ \tfrac{1}{2} (\alpha_1^2 \beta_{3,1} + \alpha_1^3 \beta_{2,1}) \Bigg\} \cosh \alpha_1 y + \Bigg\{ \frac{19\, \alpha_1^3 \beta_{3,1}}{40} y^5$$

$$+ \left(\frac{119 \alpha_1 \beta_{3,1}}{24} + \frac{47 \alpha_1^2 \beta_{2,1}}{12} - \frac{3 \alpha_1^3 \beta_{3,1}}{4} \right) y^3 + \Bigg(\frac{\alpha_1^3 \beta_{3,1}}{2} - \frac{21 \beta_{3,1}}{4 \alpha_1}$$

$$- \frac{7 \alpha_1^2 \beta_{21}}{2} - 4 \alpha_1 \beta_{3,1} - \frac{15 \beta_{2,1}}{4} \Bigg) y \Bigg\} \sinh \alpha_1 y \Bigg]$$

$$- 2 \alpha_1 p_x^{(0)} [(\alpha_1 K'_{2,1} + K'_{3,1}) \cosh \alpha_1 y + \alpha_1 K'_{3,1}\, y \sinh \alpha_1 y], \qquad [2.45c]$$

$$D_{2,1} = \lambda_2' \alpha_1 p_x^{(0)2} [\{\alpha_1 (\alpha_1 \beta_{2,1} + \beta_{3,1}) y^2 - \alpha_1 (\alpha_1 \beta_{2,1} + \beta_{3,1})\} \cosh \alpha_1 y$$

$$+ \{\alpha_1^2 \beta_{3,1} y^3 + 6 \alpha_1 \beta_{3,1} y^2 + (8 \beta_{3,1} + 6 \alpha_1 \beta_{2,1} - \alpha_1^2 \beta_{2,1}) y\} \sinh \alpha_1 y]$$

$$+ 2 \alpha_1 p_x^{(0)} \Big\langle \{(\alpha_1 K_{2,1} + K_{3,1}) \cosh \alpha_1 y + \alpha_1 K_{3,1}\, y \sinh \alpha_1 y\}$$

$$+ p_x^{(0)} \Bigg\{ \left[\frac{\alpha_1 \beta_{3,1}}{96} y^4 - \left(\frac{\beta_{3,1}}{16 \alpha_1} + \frac{\beta_{2,1}}{8} + \frac{\alpha_1 \beta_{3,1}}{8} \right) y^2 \right] \cosh \alpha_1 y$$

$$+ \left[- \frac{\beta_{3,1}}{12} y^3 + \left(\frac{5 \beta_{3,1}}{8 \alpha_1^2} - \frac{\beta_{2,1}}{4 \alpha_1} - \frac{\beta_{3,1}}{8} \right) y \right] \sinh \alpha_1 y \Bigg\} \Big\rangle, \qquad [2.45d]$$

$$E_{2,1} = \lambda_2' \alpha_1 p_x^{(0)2} [\{(\alpha_1^2 \beta_{3,1} - \beta_{3,1} - 2\alpha_1 \beta_{2,1}) y \quad \alpha_1^2 \beta_{3,1} y^3\} \cosh \alpha_1 y$$

$$+ \{(\alpha_1 \beta_{3,1} + \alpha_1^2 \beta_{2,1} + \beta_{2,1}) - (3\alpha_1 \beta_{3,1} + \alpha_1^2 \beta_{2,1}) y^2\} \sinh \alpha_1 y]$$

$$- p_x^{(0)} \Big\langle 2\{\alpha_1^2 K_{3,1} \times y \cosh \alpha_1 y + (\alpha_1^2 K_{2,1} + \alpha_1 K_{3,1}) \sinh \alpha_1 y\}$$

$$+ p_x^{(0)} \left\{ \left[-\frac{\alpha_1 \beta_{3,1}}{6} y^3 + \left(-\frac{\beta_{2,1}}{2} - \frac{3\alpha_1 \beta_{3,1}}{8} + \frac{\beta_{3,1}}{2\alpha_1} \right) y \right] \cosh \alpha_1 y \right.$$

$$+ \left[\frac{\alpha_1^2 \beta_{3,1}}{48} y^4 - \left(\frac{\alpha_1^2 \beta_{3,1}}{4} + \frac{\alpha_1 \beta_{2,1}}{4} \right) y^2 + \left(\frac{5\beta_{3,1}}{8\alpha_1^2} - \frac{\beta_{2,1}}{4\alpha_1} \right. \right.$$

$$\left. \left. \left. - \frac{\beta_{3,1}}{8} \right) \right] \sinh \alpha_1 y \right\} \Big\rangle, \qquad [2.45e]$$

$$F_{2,1} = -\lambda_2' \alpha_1 p_x^{(0)3} \left[\left\{ -\frac{19\alpha_1^3 \beta_{3,1}}{40} y^5 + \left(\frac{43\alpha_1 \beta_{3,1}}{24} + \frac{3\alpha_1^3 \beta_{3,1}}{4} - \frac{23\alpha_1^2 \beta_{2,1}}{12} \right) y^3 \right. \right.$$

$$\left. + \left(\frac{9\alpha_1 \beta_{3,1}}{4} + \frac{3\alpha_1^2 \beta_{2,1}}{2} - \frac{\alpha_1^3 \beta_{3,1}}{4} + \frac{25\beta_{3,1}}{8\alpha_1} - \frac{7\beta_{2,1}}{4} \right) y \right\} \cosh \alpha_1 y$$

$$+ \left\{ \left(\frac{3\alpha_1^2 \beta_{3,1}}{4} + \frac{23\alpha_1^3 \beta_{2,1}}{4} \right) y^4 + \left(\frac{\beta_{3,1}}{8} + \alpha_1^3 \beta_{2,1} + 3\alpha_1^2 \beta_{3,1} \right. \right.$$

$$\left. + \frac{15\alpha_1 \beta_{2,1}}{4} \right) y^2 + \left(\frac{21\beta_{3,1}}{8\alpha_1^2} - \frac{3\alpha_1 \beta_{2,1}}{4} - \frac{\alpha_1^2 \beta_{3,1}}{4} - \frac{\alpha_1^3 \beta_{2,1}}{2} \right.$$

$$\left. \left. \left. - \frac{9\beta_{2,1}}{8\alpha_1} \right) \right\} \sinh \alpha_1 y \right] - p_x^{(0)} [2\{\alpha_1^2 K_{3,1}' y \cosh \alpha_1 y$$

$$+ (\alpha_1^2 K_{2,1}' + \alpha_1 K_{3,1}') \sinh \alpha_1 y\}] \qquad [2.45f]$$

$$G_{2,1} = -\lambda_1'^2 \alpha_1 p_x^{(0)3} \left[\left\{ \left(\frac{31\alpha_1^2 \beta_{3,1}}{48} - \frac{25\alpha_1^3 \beta_{2,1}}{48} \right) y^4 + \left(-\frac{3\alpha_1^2 \beta_{3,1}}{4} + \frac{3\alpha_1^3 \beta_{2,1}}{4} \right. \right. \right.$$

$$\left. \left. - \frac{5\alpha_1 \beta_{2,1}}{8} - \frac{\beta_{3,1}}{2} \right) y^2 - \left(\frac{\alpha_1^2 \beta_{3,1}}{2} + \frac{\alpha_1^3 \beta_{2,1}}{2} \right) \right\} \cosh \alpha_1 y$$

$$+ \left\{ -\frac{19\alpha_1^3 \beta_{3,1}}{40} y^5 + \left(\frac{3\alpha_1^3 \beta_{3,1}}{4} + \frac{13\alpha_1^2 \beta_{2,1}}{12} - \frac{23\alpha_1 \beta_{3,1}}{24} \right) y^3 \right.$$

$$+\left(-\frac{9\,\beta_{2,1}}{4}+\frac{21\,\beta_{3,1}}{4\,\alpha_1}-\frac{\alpha_1^3\,\beta_{3,1}}{2}-\frac{3\,\alpha_1^2\,\beta_{2,1}}{2}\right)y\Big\}\sinh\alpha_1 y\Big]$$

$$+\,p_x^{(0)}\left[2\left\{\alpha_1^2 K'_{3,1}\,y\cosh\alpha_1 y+(\alpha_1^2 K'_{2,1}+\alpha_1 K'_{3,1})\sinh\alpha_1 y\right\}\right], \quad [2.45g]$$

$$H_{2,1}=\lambda_2' p_x^{(0)2}\,\alpha_1^2\left[\left\{(\alpha_1\,\beta_{2,1}+\beta_{3,1})+(\beta_{3,1}-\alpha_1\,\beta_{2,1})\,y^2\right\}\cosh\alpha_1 y\right.$$

$$\left.+\left\{(2\,\beta_{2,1}+\alpha_1\,\beta_{3,1})\,y-\alpha_1\,\beta_{3,1}\,y^3\right\}\sinh\alpha_1 y\right]$$

$$-\,p_x^{(0)}\Big\langle 2\left\{\alpha_1^2 K_{3,1}\,y\cosh\alpha_1 y+(\alpha_1^2 K_{2,1}+\alpha_1 K_{3,1})\sinh\alpha_1 y\right\}$$

$$+\,p_x^{(0)}\left\{\left[\frac{\alpha_1^2\,\beta_{3,1}}{48}\,y^4-\left(\frac{\beta_{3,1}}{8}+\frac{\alpha_1\,\beta_{2,1}}{4}+\frac{\alpha_1^2\,\beta_{3,1}}{4}\right)y^2\right]\cosh\alpha_1 y\right.$$

$$\left.+\left[-\frac{\alpha_1\,\beta_{3,1}}{6}\,y^3+\left(\frac{5\,\beta_{3,1}}{4\,\alpha_1}-\frac{\beta_{2,1}}{2}-\frac{\alpha_1\,\beta_{3,1}}{4}\right)y\right]\sinh\alpha_1 y\right\}\Big\rangle \quad [2.45h]$$

where

$$K_{2,1}=\frac{p_x^{(0)}}{2\,\alpha_1-\sinh 2\,\alpha_1}\left[\left\{\frac{\beta_{2,1}}{4}+\beta_{3,1}\times\left(\frac{11}{48}-\frac{1}{84\alpha_1}\right)\right\}\cosh^2\alpha_1+\left\{\left(\frac{5}{8\,\alpha_1}-\frac{11}{48}\right)\beta_{3,1}\right.\right.$$

$$\left.\left.-\frac{\beta_{2,1}}{4}\right\}\times\sinh^2\alpha_1+\left\{\frac{\beta_{2,1}}{8\alpha_1}-\beta_{3,1}\left(\frac{5}{16\,\alpha_1^2}+\frac{1}{32}\right)\right\}\sinh 2\,\alpha_1\right] \quad [2.46a]$$

$$K_{3,1}=\frac{p_x^{(0)}}{2\,\alpha_1-\sinh 2\,\alpha_1}\left[\frac{\beta_{3,1}}{4}\cosh^2\alpha_1+\left\{\beta_{3,1}\left(\frac{5}{4\,\alpha_1^2}-\frac{5}{12}\right)-\frac{\beta_{2,1}}{2\,\alpha_1}\right\}\sinh^2\alpha_1\right.$$

$$\left.-\frac{3\,\beta_{3,1}}{8\,\alpha_1}\sinh 2\,\alpha_1\right], \quad [2.46b]$$

$$K'_{2,1}=\frac{\lambda_1'^2\,p_x^{(0)2}}{2\,\alpha_1-\sinh 2\,\alpha_1}\left[\left\{\beta_{3,1}\left(\frac{25\,\alpha_1}{24}+\frac{9\,\alpha_1^3}{40}\right)-\frac{\alpha_1^2\,\beta_{2,1}}{6}\right\}\cosh^2\alpha_1\right.$$

$$-\left\{\beta_{2,1}\left(\frac{9}{4}+\frac{5\,\alpha_1^2}{12}\right)+\beta_{3,1}\left(\frac{9\,\alpha_1^3}{40}+\frac{23\,\alpha_1}{24}-\frac{21}{4\,\alpha_1}\right)\right\}\sinh^2\alpha_1$$

$$\left.+\left\{\frac{\alpha_1\,\beta_{2,1}}{4}-\beta_{3,1}\left(\frac{25}{16}+\frac{5\,\alpha_1^2}{16}\right)\right\}\sinh 2\,\alpha_1\right], \quad [2.46c]$$

$$K'_{3,1}=\frac{\lambda_1'^2\,p_x^{(0)2}}{2\,\alpha_1-\sinh 2\alpha_1}\left[\left\{\beta_{2,1}\left(\frac{19\,\alpha_1}{24}+\frac{13\,\alpha_1^3}{48}\right)-\beta_{3,1}\left(\frac{13}{24}-\frac{91\,\alpha_1^2}{240}\right)\right\}\cosh^2\alpha_1\right.$$

$$- \left\{ \beta_{2,1} \left(\frac{9\alpha_1}{8} + \frac{13\alpha_1^3}{48} \right) - \beta_{3,1} \left(\frac{21}{8} + \frac{\alpha_1^2}{48} \right) \right\} \sinh^2 \alpha_1$$

$$+ \left\{ \beta_{2,1} \left(\frac{9}{16} + \frac{5\alpha_1^2}{32} \right) - \beta_{3,1} \left(\frac{21}{16\alpha_1} + \frac{\alpha_1}{32} \right] \right\} \sinh 2\alpha_1 \Bigg] \qquad [2.46d]$$

Equation for pressure:

The pressure is given by

$$p + p' = x\, p_x^{(0)} + \epsilon \Big\langle \{[C_{0,1} - (1/\alpha_1) F'_{0,1} + G_{0,1} + \alpha_1 \int F'_{0,1}\, dy] + R^2 [C_{2,1}$$
$$- (1/\alpha_1) F'_{2,1} + G_{2,1} + \alpha_1 \int F_{2,1}\, dy] + R^4 p_x^{(0)2} [\tfrac{1}{2}(y^2-1) A'_{2,1} - y A_{2,1}$$
$$+ (\alpha_1^2/2) \int (y^2-1) A_{2,1}\, dy]\} \times \cos \alpha_1 x + \{[D_{1,1} + (1/\alpha_1) E'_{1,1} + H_{1,1}$$
$$- \alpha_1 \int E_{1,1}\, dy] + R^2 [D_{2,1} + (1/\alpha_1) E'_{2,1} + p_x^{(0)2} \{\tfrac{1}{2}(y^2-1) B'_{0,1} - y B_{0,1}$$
$$+ (\alpha_1^2/2) \int (y^2-1) B_{0,1}\, dy\}] + R^4 p_x^{(0)2} [\tfrac{1}{2}(y^2-1) B'_{2,1} - y B_{2,1}$$
$$+ (\alpha_1^2/2) \int (y^2-1) B_{2,1}\, dy]\} \sin \alpha_1 x \Big\rangle. \qquad [2.47]$$

In Fig. 1, we have drawn the stream lines for $R = 0.1$, $p_x^{(0)} = 1$, $\epsilon = 0.05$, $h = 1$, $\lambda_1' = 0.1$, We note that the stream lines near the boundaries run parallel to them, the deformity of the stream lines decreases as they approach the mid-plane where they are just straight as expected, in view of the symmetry about the mid-plane. This result is similar to that in reference [3]. The stream lines are slightly pisplaced due to the effect of relaxation time relative to the corresponding stream lines for Newtonian fluids.

3. Solution of enfrgy equation

Let us now consider the energy equation [1.7] to be solved under the thermal boundary conditions [1.9]

Setting $T = T^{(0)} + \epsilon T^{(1)}$, $T_x^{(0)} \equiv 0$, $T^{(0)}$ being a function of y alone. in [1.7] and [1.9], the zeroth and first order equations in terms of ψ are

$$T_{yy}^{(0)} + E\sigma R\, p_{xy}^{(0)} \cdot \psi_{yy}^{(0)} = 0, \qquad [3.1]$$

and

$$\psi_y^{(0)} T_x^{(1)} - \psi_x^{(1)} T_y^{(0)} = (1/R\sigma) [T_{xx}^{(1)} + T_{yy}^{(1)}]$$
$$+ E [(p_{xx}^{(0)} - p_{yy}^{(0)}) \psi_{xy}^{(1)} + p_{xy}^{(1)} \psi_{yy}^{(0)} + p_{xy}^{(0)} (\psi_{yy}^{(1)} - \psi_{xx}^{(1)})] \qquad [3.2]$$

The boundary conditions satisfied by zeroth and first order temperatures in said two cases reduce to:

Case (*a*):

$$T^{(0)}(\pm 1) = 1, \qquad [3.3]$$

$$\left.\begin{aligned} T^{(1)}(x,1) &= -T_y^{(0)}(1)\sum_{n=1}^{\infty}(a_n \cos\alpha_n x + b_n \sin\alpha_n x), \\ T^{(1)}(x,-1) &= T_y^{(0)}(-1)\sum_{n=1}^{\infty}(a'_n \cos\alpha_n x + b'_n \sin\alpha_n x). \end{aligned}\right\} \qquad [3.4]$$

Case (*b*):

$$T^{(0)}(1) = 1,\ T_y^{(0)}(-1) = 0, \qquad [3.5]$$

$$\left.\begin{aligned} T^{(1)}(x,1) &= -T_y^{(o)}(1)\sum_{n=1}^{\infty}(a_n \cos\alpha_n x + b_n \sin\alpha_n x), \\ T_y^{(1)}(x,-1) &= T_{yy}^{(o)}(-1)\sum_{n=1}^{\infty}(a'_n \cos\alpha_n x + b'_n \sin\alpha_n x). \end{aligned}\right\} \qquad [3.6]$$

We now take these cases in turn.

Case (*a*):

Solution of zeroth order equation;

The zero order flow is the same as the flow between two plane parallel walls for which, on substituting for $\psi^{(0)}$ and $p_{xy}^{(0)}$, $T^{(0)}(y)$ is given by

$$T^{(0)}(y) = 1 + \frac{E\sigma R^2 p_x^{(0)2}}{12}(1 - y^4). \qquad [3.7]$$

in view of [3.3].

We note that the zero order temperature field is not affected by relaxation and retardation times. This result is the same as obtained by us for Revlin-Ericksen fluids in reference [3] where visco-elasticity and cross-viscosity of the fluid have no effect on the temperature distribution in the absence of deformation in the boundaries.

Solution of first order equation:

Using [3.7] the boundary conditions [3.4] reduce to

$$\left.\begin{aligned} T^{(1)}(x, 1) &= \frac{E\,\sigma\,R^2\,p_x^{(0)2}}{3} \sum_{n=1}^{\infty} (a_n \cos \alpha_n x + b_n \sin \alpha_n x), \\ T^{(1)}(x, -1) &= \frac{E\,\sigma\,R^2\,p_x^{(0)2}}{3} \sum_{n=1}^{\infty} (a_n' \cos \alpha_n x + b_n' \sin \alpha_n x), \end{aligned}\right\} \quad [3.8]$$

so that we choose

$$T^{(1)}(x, y) = E\,\sigma\,R^2\,p_x^{(0)2} \sum_{n=1}^{\infty} [I_n(y) \cos \alpha_n x + J_n(y) \sin \alpha_n x]. \quad [3.9]$$

The comparison of [3.8] and [3.9] leads to

$$\left.\begin{aligned} I_n(1) &= \frac{a_n}{3}, \quad I_n(-1) = \frac{a_n'}{3}, \\ J_n(1) &= \frac{b_n}{3}, \quad J_n(-1) = \frac{b_n'}{3}. \end{aligned}\right\} \quad [3.10]$$

The equations determining I_n and J_n are:

$$\begin{aligned} (D^2 - \alpha_n^2)\, I_n = {} & [(A_n'' + \alpha_n^2 A_n) - (E_n/p_x^{(0)})]\, y + 2\,R\,p_x^{(0)}\,\alpha_n\,\lambda_1'\,y^2\,B_n' \\ & + R^2\,p_x^{(0)}\,\alpha_n\,[\sigma\,\{\tfrac{1}{2}(y^2 - 1)\,J_n - (y^3/3)\,B_n\} - 2\,\lambda_2'\,y^2\,B_n']. \quad [3.11] \end{aligned}$$

and

$$\begin{aligned} (D^2 - \alpha_n^2)\, J_n = {} & y\,[(B_n'' + \alpha_n^2 B_n) - (F_n/p_x^{(0)})] - 2\,R\,\alpha_n\,p_x^{(0)}\,\lambda_1'\,y^2\,A_n' \\ & - R^2\,\alpha_n\,p_x^{(0)}\,[\sigma\,\{\tfrac{1}{2}(y^2 - 1)\,I_n - (y^3/3)\,A_n\} - 2\,\lambda_2'\,y^2\,A_n']. \quad [3.12] \end{aligned}$$

Setting, as before

$$\left.\begin{aligned} I_n &= I_{0,n} + R\,I_{1,n} + R^2\,I_{2,n}, \\ J_n &= J_{0,n} + R\,J_{1,n} + R^2\,J_{2,n}, \end{aligned}\right\} \quad [3.13]$$

the equations determining $I_{0,n}$, $J_{0,n}$; $I_{1,n}$, $J_{1,n}$; $I_{2,n}$ and $J_{2,n}$ are :

$$(D^2 - \alpha_n^2)\, I_{0,n} = y\,[(A''_{0,n} + \alpha_n^2 A_{0,n}) - E_{0,n}/p_x^{(0)}], \qquad [3.14]$$

$$(D^2 - \alpha_n^2)\, J_{0,n} = y\,[(B''_{0,n} + \alpha_n^2 B_{0,n}) - F_{0,n}/p_x^{(0)}], \qquad [3.15]$$

$$(D^2 - \alpha_n^2)\, I_{1,n} = 2\,\alpha_n\, p_x^{(0)}\, \lambda'_1\, y^2\, B'_{0,n} - (y\, E_{1,n}/p_x^{(0)}), \qquad [3.16]$$

$$(D^2 - \alpha_n^2)\, J_{1,n} = -2\,\alpha_n\, p_x^{(0)}\, \lambda'_1\, y^2\, A'_{0,n} - (y\, F_{1,n}/p_x^{(0)}), \qquad [3.17]$$

$$(D^2 - \alpha_n^2)\, I_{2,n} = y\,[(A''_{2,n} + \alpha_n^2 A_{2,n}) - E_{2,n}/p_x^{(0)}] + \alpha_n\, p_x^{(0)}\, [\sigma\, \{\tfrac{1}{2}(y^2 - 1)\, J_{0,n} - (y^3/3)\, B_{0,n}\} - 2\,\lambda'_2\, y^2\, B'_{0,n}] \qquad [3.18]$$

and

$$(D^2 - \alpha_n^2)\, J_{2,n} = y\,[(B''_{2,n} + \alpha_n^2 B_{2,n}) - F_{2,n}/p_x^{(0)}] - \alpha_n\, p_x^{(0)}\, [\sigma\, \{\tfrac{1}{2}(y^2 - 1)\, I_{0,n} - (y^3/3)\, A_{0,n}\} - 2\,\lambda'_2\, y^2\, A'_{0,n}], \qquad [3.19]$$

which have to be solved under the boundary conditions :

$$\left.\begin{aligned} I_{0,n}(1) = a_n/3, \quad I_{0,n}(-1) = a'_n/3, \\ J_{0,n}(1) = b_n/3, \quad J_{0,n}(-1) = b'_n/3, \end{aligned}\right\} \qquad [3.20]$$

$$I_{1,n}(\pm 1) = J_{1,n}(\pm 1) = 0, \qquad [3.21]$$

$$I_{2,n}(\pm 1) = J_{2,n}(\pm 1) = 0. \qquad [3.22]$$

The equations [3.14], [3.15] and the conditions [3.20] give

$$I_{0,n} = [L_{1,n} - \alpha_{1,n}\, y + \alpha_n\, \alpha_{2,n}\, y^2 + (2/3)\, \alpha_n\, \alpha_{4,n}\, y^3] \cosh \alpha_n y + [L_{2,n} - \alpha_{2,n}\, y + \alpha_n\, \alpha_{1,n}\, y^2 + \tfrac{2}{3}\, \alpha_n\, \alpha_{3,n}\, y^3] \sinh \alpha_n y, \qquad [3.23]$$

and

$$J_{0,n} = [M_{1,n} - \beta_{1,n}\, y + \alpha_n\, \beta_{2,n}\, y^2 + \tfrac{2}{3}\, \alpha_n\, \beta_{4,n}\, y^3] \cosh \alpha_n y + [M_{2,n} - \beta_{2,n}\, y + \alpha_n\, \beta_{1,n}\, y^2 + \tfrac{2}{3}\, \alpha_n\, \beta_{3,n}\, y^3] \sinh \alpha_n y, \qquad [3.24]$$

where

$$L_{1,n} = \frac{a_n + a'_n}{6 \cosh \alpha_n} - \alpha_n\, \alpha_{2,n} + (\alpha_{2,n} - \tfrac{2}{3}\, \alpha_n\, \alpha_{3,n}) \times \tanh \alpha_n, \qquad [3.25a]$$

$$L_{2,n} = \frac{a_n - a'_n}{6 \sinh \alpha_n} - \alpha_n\, \alpha_{1,n} + (\alpha_{1,n} - \tfrac{2}{3}\, \alpha_n\, \alpha_{4,n}) \coth \alpha_n. \qquad [3.25b]$$

$M_{1,n}$ and $M_{2,n}$ can be obtained from [3.25] by replacing $a's$ by $b's$ and $\alpha's$ by $\beta's$.

The equations [3.16], and [3.17] and the conditions [3.21] give

$$I_{1,n} = P_{1,n} \cosh \alpha_n y + P_{2,n} \sinh \alpha_n y + \lambda_1' p_x^{(0)} \times \Bigg[\Bigg\{ \left(\frac{\beta_{1,n}}{8\alpha_n} - \frac{\alpha_n \beta_{1,n}}{4} - \frac{\beta_{4,n}}{8\alpha_n^2} \right) y + \left(\frac{\alpha_n^2 \beta_{2,n}}{4} - \frac{\beta_{2,n}}{8} + \frac{\beta_{3,n}}{8\alpha_n} \right) y^2 + \left(\frac{\alpha_n \beta_{1,n}}{4} + \frac{\alpha_n^2 \beta_{4,n}}{6} - \frac{\beta_{4,n}}{12} \right) y^3 + \left(\frac{\alpha_n \beta_{3,n}}{8} - \frac{\alpha_n^3 \beta_{2,n}}{8} \right) y^4 - \frac{\alpha_n^2 \beta_{4,n}}{10} y^5 \Bigg\} \cosh \alpha_n y$$
$$+ \Bigg\{ \left(\frac{\beta_{2,n}}{8\alpha_n} - \frac{\alpha_n \beta_{2,n}}{4} - \frac{\beta_{3,n}}{8\alpha_n^2} \right) y + \left(\frac{\alpha_n^2 \beta_{1,n}}{4} - \frac{\beta_{1,n}}{8} + \frac{\beta_{4,n}}{8\alpha_n} \right) y^2 + \left(\frac{\alpha_n \beta_{2,n}}{4} + \frac{\alpha_n^2 \beta_{3,n}}{6} - \frac{\beta_{3,n}}{12} \right) y^3 + \left(\frac{\alpha_n \beta_{4,n}}{8} - \frac{\alpha_n^2 \beta_{1,n}}{8} \right) y^4 - \frac{\alpha_n^2 \beta_{3,n}}{10} y^5 \Bigg\} \sinh \alpha_n y \Bigg], \qquad [3.26]$$

where

$$P_{1,n} = \lambda_1' p_x^{(0)} \Bigg[\left\{ \beta_{2,n} \left(\frac{1}{8} - \frac{\alpha_n^2}{8} \right) - \beta_{3,n} \left(\frac{\alpha_n}{8} + \frac{1}{8\alpha_n} \right) \right\} - \left\{ \frac{\beta_{2,n}}{8\alpha_n} + \beta_{3,n} \left(\frac{\alpha_n^2}{15} - \frac{1}{12} - \frac{1}{8\alpha_n^2} \right) \right\} \times \tanh \alpha_n \Bigg], \qquad [3.27a]$$

and

$$P_{2,n} = \lambda_1' p_x^{(0)} \Bigg[- \left\{ \frac{\beta_{1,n}}{8\alpha_n} + \beta_{4,n} \left(\frac{\alpha_n^2}{15} - \frac{1}{12} - \frac{1}{8\alpha_n^2} \right) \right\} \coth \alpha_n + \left\{ \beta_{1,n} \left(\frac{1}{8} - \frac{\alpha_n^2}{8} \right) - \beta_{4,n} \left(\frac{\alpha_n}{8} + \frac{1}{8\alpha_n} \right) \right\} \Bigg] \qquad [3.27b]$$

The expression for $J_{1,n}$ and the corresponding constants obtained by replacing $P_{1,n}$, $P_{2,n}$ by $Q_{1,n}$ and $Q_{2,n}$ can be obtained from [3.26] and [3.27] by replacing $\beta's$ by $\alpha's$ with sign changed.

The equations [3.18], [3.19] and the conditions [3.22] give

$$I_{2,n} = R_{1,n} \cosh \alpha_n y + R_{2,n} \sinh \alpha_n y + [(\tfrac{2}{3}\alpha_n K_{4,n} y^3 + \alpha_n K_{1,3} y^2$$

$$- K_{2,n} y) \cosh \alpha_n y + (\tfrac{2}{3}\alpha_n K_{3,n} y^3 + \alpha_n K_{2,n} y^2 - K_{1,n} y) \sinh \alpha_n y]$$

$$+ p_x^{(0)} \left[\left\{ \frac{\alpha_n \beta_{3,n}}{288} y^6 - \frac{11 \beta_{4,n}}{480} y^5 + \left(\frac{7 \beta_{3,n}}{64 \alpha_n} - \frac{\alpha_n \beta_{3,n}}{16} - \frac{\beta_{2,n}}{16} \right) y^4 \right.\right.$$

$$- \left(\frac{\beta_{1,n}}{24\alpha_n} + \frac{5 \beta_{4,n}}{96 \alpha_n^2} \right) y^3 + \left(\frac{\beta_{2,n}}{2 \alpha_n} + \frac{25 \beta_{3,n}}{64 \alpha_n^3} - \frac{\beta_{2,n}}{16 \alpha_n^2} \right) y^2 + \left(\frac{\beta_{1,n}}{16 \alpha_n^3} + \frac{\beta_{4,n}}{16 \alpha_n^2} \right.$$

$$\left.\left. - \frac{25 \beta_{4,n}}{64 \alpha_n^4} \right) y \right\} \cosh \alpha_n y + \left\{ \frac{\alpha_n \beta_{4,n}}{288} y^6 - \frac{11 \beta_{3,n}}{480} y^5 + \left(\frac{7 \beta_{4,n}}{64 \alpha_n} \right.\right.$$

$$\left. - \frac{\alpha_n \beta_{4,n}}{16} - \frac{\beta_{1,n}}{16} \right) y^4 - \left(\frac{\beta_{2,n}}{24 \alpha_n} + \frac{5 \beta_{3,n}}{96 \alpha_n^2} \right) y^3 + \left(\frac{\beta_{4,n}}{2 \alpha_n} + \frac{25 \beta_{4,n}}{64 \alpha_n^3} \right.$$

$$\left.\left.\left. - \frac{\beta_{1,n}}{16 \alpha_n^2} \right) y^2 + \left(\frac{\beta_{2,n}}{16 \alpha_n^3} + \frac{\beta_{3,n}}{16 \alpha_n^2} - \frac{25 \beta_{3,n}}{64 \alpha_n^4} \right) y \right\} \sinh \alpha_n y \right]$$

$$+ \lambda_1'^2 p_x^{(0)2} \left[\left\{ -\frac{9 \alpha_n^3 \alpha_{4,n}}{280} y^7 - \left(\frac{103 \alpha_n^2 \alpha_{3,n}}{1440} + \frac{13 \alpha_n^3 \alpha_{2,n}}{288} \right) y^6 \right.\right.$$

$$+ \left(\frac{31 \alpha_n \alpha_{4,n}}{96} + \frac{\alpha_n^3 \alpha_{4,n}}{20} - \frac{23 \alpha_n^2 \alpha_{1,n}}{480} \right) y^5 + \left(\frac{95 \alpha_n \alpha_{2,n}}{192} + \frac{\alpha_n^3 \alpha_{2,n}}{8} \right.$$

$$\left. + \frac{\alpha_n^2 \alpha_{3,n}}{4} - \frac{149 \alpha_{3,n}}{192} \right) y^4 + \left(\frac{83 \alpha_{4,n}}{32 \alpha_n} - \frac{151 \alpha_{1,n}}{96} - \frac{\alpha_n^2 \alpha_{1,n}}{4} \right) y^3$$

$$+ \left(\frac{115 \alpha_{2,n}}{64 \alpha_n} - \frac{\alpha_n \alpha_{2,n}}{8} - \frac{\alpha_n^3 \alpha_{2,n}}{8} - \frac{165 \alpha_{3,n}}{64 \alpha_n^2} \right) y^2 + \left(\frac{\alpha_{1,n}}{8} + \frac{\alpha_n^2 \alpha_{1,n}}{8} \right.$$

$$\left.\left. - \frac{115 \alpha_{1,n}}{64} + \frac{165 \alpha_{4,n}}{64 \alpha_n^3} + \frac{\alpha_{4,n}}{8} - \frac{\alpha_n \alpha_{4,n}}{8} + \frac{9 \alpha_{4,n}}{8 \alpha_n} \right) y \right\} \cosh \alpha_n v$$

$$+ \left\{ -\frac{9 \alpha_n^3 \alpha_{3,n}}{280} y^7 - \left(\frac{103 \alpha_n^2 \alpha_{4,n}}{1440} + \frac{13 \alpha_n^3 \alpha_{1,n}}{288} \right) y^6 + \left(\frac{31 \alpha_n \alpha_{3,n}}{96} \right.\right.$$

$$\left. + \frac{\alpha_n^3 \alpha_{2,n}}{20} - \frac{23 \alpha_n^2 \alpha_{2,n}}{480} \right) y^5 + \left(\frac{95 \alpha_n \alpha_{1,n}}{192} + \frac{\alpha_n^3 \alpha_{1,n}}{8} + \frac{\alpha_n^2 \alpha_{4,n}}{4} \right.$$

$$- \frac{149\, \alpha_{4,n}}{192}\Big) y^4 + \left(\frac{83\, \alpha_{3,n}}{32\, \alpha_n} - \frac{151\, \alpha_{2,n}}{96} - \frac{\alpha_n^2 \alpha_{2,n}}{4}\right) y^3 + \Big(\frac{115\, \alpha_{1,n}}{64\, \alpha_n}$$

$$- \frac{\alpha_n \alpha_{1,n}}{8} - \frac{\alpha_n^3 \alpha_{1,n}}{8} - \frac{165\, \alpha_{4,n}}{64\, \alpha_n^2}\Big) y^2 + \Big(\frac{\alpha_{2,n}}{8} + \frac{\alpha_n^2 \alpha_{2,n}}{8} - \frac{115\, \alpha_{2,n}}{64\, \alpha_n^2}$$

$$+ \frac{165\, \alpha_{3,n}}{64\, \alpha_n^3} + \frac{\alpha_{3,n}}{8} - \frac{\alpha_n \alpha_{3,n}}{8} + \frac{9\, \alpha_{3,n}}{8}\Big) y\Big\} \sinh \alpha_n y\Big]$$

$$+ \lambda_2' p_x^{(0)} \Big[\Big\{\frac{\alpha_n^2 \beta_{4,n}}{10} y^5 + \left(\frac{\alpha_n^2 \beta_{2,n}}{8} - \frac{\alpha_n \beta_{3,n}}{8}\right) y^4 + \Big(\frac{\beta_{4,n}}{12} - \frac{\alpha_n^2 \beta_{4,n}}{6}$$

$$- \frac{\alpha_n \beta_{1,n}}{4}\Big) y^3 + \left(\frac{\beta_{2,n}}{8} - \frac{\alpha_n^2 \beta_{2,n}}{4} - \frac{\beta_{3,n}}{8\, \alpha_n}\right) y^2 + \Big(\frac{\alpha_n \beta_{1,n}}{4} - \frac{\beta_{1,n}}{8\, \alpha_n}$$

$$+ \frac{\beta_{4,n}}{8\, \alpha_n^2}\Big) y\Big\} \cosh \alpha_n y + \Big\{\frac{\alpha_n^2 \beta_{3,n}}{10} y^5 + \left(\frac{\alpha_n^2 \beta_{1,n}}{8} - \frac{\alpha_n \beta_{4,n}}{8}\right) y^4$$

$$+ \left(\frac{\beta_{3,n}}{12} - \frac{\alpha_n^2 \beta_{3,n}}{6} - \frac{\alpha_n \beta_{2,n}}{4}\right) y^3 + \left(\frac{\beta_{1,n}}{8} - \frac{\alpha_n^2 \beta_{1,n}}{4} - \frac{\beta_{4,n}}{8\, \alpha_n}\right) y^2$$

$$+ \left(\frac{\alpha_n \beta_{2,n}}{4} - \frac{\beta_{2,n}}{8\, \alpha_n} + \frac{\beta_{3,n}}{8\, \alpha_n^2}\right) y\Big\} \sinh \alpha_n y\Big] + p_x^{(0)} \sigma \Big[\Big\{\frac{\alpha_n \beta_{3,n}}{36} y^6$$

$$+ \left(\frac{\alpha_n \beta_{1,n}}{20} - \frac{7\, \beta_{4,n}}{60}\right) y^5 + \left(\frac{7\, \beta_{3,n}}{24\, \alpha_n} - \frac{\alpha_n \beta_{3,n}}{24} - \frac{11\, \beta_{2,n}}{48}\right) y^4$$

$$+ \left(\frac{M_{2,n}}{12} + \frac{\alpha_n \beta_{1,n}}{12} + \frac{11\, \beta_{1,n}}{24\, \alpha_n} + \frac{\beta_{4,n}}{12} - \frac{7\, \beta_{4,n}}{12\, \alpha_n^2}\right) y^3 + \Big(\frac{7\, \beta_{3,n}}{8\, \alpha_n^3} - \frac{\beta_{3,n}}{8\, \alpha_n}$$

$$- \frac{7\, \beta_{2,n}}{12\, \alpha_n^2} - \frac{M_{1,n}}{8\, \alpha_n}\Big) y^2 + \Big(\frac{M_{2,n}}{8\, \alpha_n^2} + \frac{11\, \beta_{1,n}}{16\, \alpha_n^3} + \frac{\beta_{4,n}}{8\, \alpha_n^2} - \frac{7\, \beta_{4,n}}{8\, \alpha_n^4}$$

$$- \frac{M_{2,n}}{4}\Big) y\Big\} \cosh \alpha_n y + \Big\{\frac{\alpha_n \beta_{4,n}}{36} y^6 + \left(\frac{\alpha_n \beta_{2,n}}{20} - \frac{7\, \beta_{3,n}}{60}\right) y^5$$

$$+ \left(\frac{7\, \beta_{4,n}}{24\, \alpha_n} - \frac{\alpha_n \beta_{4,n}}{24} - \frac{11\, \beta_{1,n}}{48}\right) y^4 + \Big(\frac{M_{1,n}}{12} + \frac{\alpha_n \beta_{2,n}}{12} + \frac{11\, \beta_{2,n}}{12}$$

$$+ \frac{\beta_{3,n}}{12} - \frac{7\, \beta_{3,n}}{12\, \alpha_n^2}\Big) y^3 + \left(\frac{7\, \beta_{4,n}}{8\, \alpha_n^3} - \frac{\beta_{4,n}}{8\, \alpha_n} - \frac{7\, \beta_{1,n}}{12\, \alpha_n^2} - \frac{M_{2,n}}{8\, \alpha_n}\right) y^2$$

$$+ \left(\frac{M_{1,n}}{8\, \alpha_n^2} - \frac{M_{1,n}}{4} + \frac{11\, \beta_{2,n}}{16\, \alpha_n^3} + \frac{\beta_{3,n}}{8\, \alpha_n^2} - \frac{7\, \beta_{3,n}}{8\, \alpha_n^4}\right) y\Big\} \sinh \alpha_n y\Big] \quad [3.28]$$

where

$$R_{1,n} = -\alpha_n K_{2,n} + (K_{2,n} - \tfrac{2}{3}\alpha_n K_{3,n}) \tanh \alpha_n$$

$$+ p_x^{(0)} \left[\left\{ \beta_{2,n} \left(\frac{1}{16} + \frac{1}{16\alpha_n^2} \right) + \beta_{3,n} \left(\frac{17\alpha_n}{288} - \frac{39}{64} - \frac{25}{64\alpha_n^3} \right) \right\} \right.$$

$$\left. + \left\{ \beta_{2,n} \left(\frac{1}{24\alpha_n} - \frac{1}{16\alpha_n^3} \right) + \beta_{3,n} \left(\frac{11}{480} - \frac{1}{96\alpha_n^2} + \frac{25}{64\alpha_n^4} \right) \right\} \tanh \alpha_n \right]$$

$$+ \lambda_1'^2 p_x^{(0)2} \left[\left\{ \alpha_{2,n} \left(\frac{13\alpha_n^3}{288} - \frac{71\alpha_n}{192} - \frac{115}{64\alpha_n} \right) + \alpha_{3,n} \left(\frac{165}{64\alpha_n^2} + \frac{149}{192} \right. \right. \right.$$

$$\left. \left. - \frac{257\alpha_n^2}{1440} \right) \right\} + \left\{ \alpha_{2,n} \left(\frac{83\alpha_n^2}{480} + \frac{139}{96} + \frac{115}{64\alpha_n^2} \right) - \alpha_{3,n} \left(\frac{\alpha_n^3}{56} + \frac{19\alpha_n}{96} \right. \right.$$

$$\left. \left. \left. + \frac{119}{32\alpha_n} + \frac{165}{64\alpha_n^3} + \frac{1}{8} \right) \right\} \tanh \alpha_n \right] + \lambda_2' p_x^{(0)} \left[\left\{ \beta_{2,n} \left(\frac{\alpha_n^2}{8} - \frac{1}{8} \right) \right. \right.$$

$$\left. \left. + \beta_{3,n} \left(\frac{\alpha_n}{8} + \frac{1}{8} \right) \right\} + \left\{ \frac{\beta_{2,n}}{8\alpha_n} + \beta_{3,n} \left(\frac{\alpha_n^2}{15} - \frac{1}{12} - \frac{1}{8\alpha_n^2} \right) \right\} \tanh \alpha_n \right]$$

$$+ p_x^{(0)} \sigma \left[\left\{ \frac{M_{1,n}}{8\alpha_n} + \beta_{2,n} \left(\frac{11}{48} + \frac{7}{12\alpha_n^2} \right) + \beta_{3,n} \left(\frac{\alpha_n}{72} - \frac{1}{6\alpha_n} - \frac{7}{8\alpha_n^3} \right) \right\} \right.$$

$$+ \left\{ M_{1,n} \left(\frac{1}{6} - \frac{1}{8\alpha_n^2} \right) + \beta_{3,n} \left(\frac{1}{30} + \frac{11}{24\alpha_n^2} + \frac{7}{8\alpha_n^4} \right) - \beta_{2,n} \left(\frac{2\alpha_n}{15} + \frac{11}{24\alpha_n} \right. \right.$$

$$\left. \left. \left. + \frac{11}{16\alpha_n^3} \right) \right\} \tanh \alpha_n \right], \qquad [3.29a]$$

and

$$R_{2,n} = (K_{1,n} - \tfrac{2}{3}\alpha_n K_{4,n}) \coth \alpha_n - \alpha_n K_{1,n}$$

$$+ p_x^{(0)} \left[\left\{ \beta_{1,n} \left(\frac{1}{24\alpha_n} - \frac{1}{16\alpha_n^3} \right) + \beta_{4,n} \left(\frac{11}{480} - \frac{1}{96\alpha_n} + \frac{25}{64\alpha_n^4} \right) \right\} \coth \alpha_n \right.$$

$$\left. + \left\{ \beta_{1,n} \left(\frac{1}{16} + \frac{1}{16\alpha_n^2} \right) + \beta_{4,n} \left(\frac{17\alpha_n}{288} - \frac{39}{64\alpha_n} - \frac{25}{64\alpha_n^3} \right) \right\} \right]$$

$$+ \lambda_1'^2 p_x^{(0)2} \Bigg[\Bigg\{ \alpha_{1,n} \left(\frac{83\,\alpha_n^2}{480} + \frac{139}{96} + \frac{115}{64\,\alpha_n^2} \right) - \alpha_{4,n} \left(\frac{\alpha_n^3}{56} + \frac{19\,\alpha_n}{96} + \frac{119}{32\,\alpha_n} + \frac{165}{64\,\alpha_n^3} + \frac{1}{8} \right) \Bigg\} \coth \alpha_n + \Bigg\{ \alpha_{1,n} \left(\frac{13\,\alpha_n^3}{288} - \frac{71\,\alpha_n}{192} - \frac{115}{64\,\alpha_n} \right)$$

$$+ \alpha_{4,n} \left(\frac{165}{64\,\alpha_n^2} + \frac{149}{192} - \frac{257\,\alpha_n^2}{1440} \right) \Bigg\} \Bigg] + \lambda_2' p_x^{(0)} \Bigg[\Bigg\{ \frac{\beta_{1,n}}{8\,\alpha_n} + \beta_{4,n} \left(\frac{\alpha_n^2}{15} - \frac{1}{12} - \frac{1}{8\,\alpha_n^2} \right) \Bigg\} \coth \alpha_n + \left\langle \beta_{1,n} \left(\frac{\alpha_n^2}{8} - \frac{1}{8} \right) + \beta_{4,n} \left(\frac{\alpha_n}{8} + \frac{1}{8\,\alpha_n} \right) \right\rangle \Bigg]$$

$$+ p_x^{(0)} \sigma \Bigg[\Bigg\langle M_{2,n} \left(\frac{1}{6} - \frac{1}{8\,\alpha_n^2} \right) - \beta_{1,n} \left(\frac{2\,\alpha_n}{15} + \frac{11}{24\,\alpha_n} + \frac{11}{16\,\alpha_n^3} \right) + \beta_{4,n} \left(\frac{1}{30} + \frac{11}{24\,\alpha_n^2} + \frac{7}{8\,\alpha_n^4} \right) \Bigg\rangle \coth \alpha_n + \Bigg\langle \frac{M_{2,n}}{8\,\alpha_n} + \beta_{1,n} \left(\frac{11}{48} + \frac{7}{12\,\alpha_n^2} \right)$$

$$+ \beta_{4,n} \left(\frac{\alpha_n}{72} - \frac{1}{6\,\alpha_n} - \frac{7}{8\,\alpha_n^3} \right) \Bigg\rangle \Bigg] . \qquad [3.29b]$$

The expressions for $J_{2,n}$ and the corresponding constants $S_{1,n}$ and $S_{2,n}$ can be obtained from [3 28] and [3.29] by replacing R's by S's; K's by dash K's; β's by α's with sign changed into the coefficient of $p_x^{(0)}$, α's by β's in the coefficient of $\lambda_1'^2 p_x^{(0)2}$; β's by α's with sign changed in the coefficient of $\lambda_2' p_x^{(0)}$; β's by α's with sign changed, M's by L's with sign changed in the coefficient of $p_x^{(0)} \sigma$.

Particular case:

In the particular case when the boundaries have sinusoidal deformation defined by [2.42] we have,

$$L_{1,1} = L_{2,1} = M_{2,1} = R_{2,1} = S_{2,1} = P_{2,1} = Q_{1,1} = Q_{2,1} = 0, \; I_{0,1} = J_{1,1} = 0,$$

$$M_{1,1} = \frac{1}{3 \cosh \alpha_1} - \alpha_1 \beta_{2,1} + (\beta_{2,1} - \tfrac{2}{3} \alpha_1 \beta_{3,1}) \tanh \alpha_1, \qquad [3.30a)$$

$$R_{1,1} = -\alpha_1 K_{2,1} + (K_{2,1} - \tfrac{2}{3} \alpha_1 K_{2,1}) \tanh \alpha_1$$

$$+ p_x^{(0)} \Bigg[\Bigg\langle \beta_{2,1} \left(\frac{1}{10} + \frac{1}{16\,\alpha_1^2} \right) + \beta_{3,1} \left(\frac{17\,\alpha_1}{288} - \frac{39}{64\,\alpha_1} - \frac{25}{64\,\alpha_1^3} \right) \Bigg\rangle$$

$$+\left\langle \beta_{2,1}\left(\frac{1}{24\,\alpha_1}-\frac{1}{16\,\alpha_1^3}\right)+\beta_{3,1}\left(\frac{11}{480}-\frac{1}{96\,\alpha_1^2}+\frac{25}{64\,\alpha_1^4}\right)\right\rangle \tanh\alpha_1\Bigg]$$

$$+\lambda_2' p_x^{(0)}\Bigg[\left\langle \beta_{2,1}\left(\frac{\alpha_1^2}{8}-\frac{1}{8}\right)+\beta_{3,1}\left(\frac{\alpha_1}{8}+\frac{1}{8\,\alpha_1}\right)\right\rangle+\left\langle \frac{\beta_{2,1}}{8\,\alpha_1}\right.$$

$$\left.+\beta_{3,1}\left(\frac{\alpha_1^2}{15}-\frac{1}{12}-\frac{1}{8\,\alpha_1^2}\right)\right\rangle \tanh\alpha_1\Bigg]+p_x^{(0)}\sigma\Bigg[\left\langle \frac{M_{1,1}}{8\,\alpha_1}+\beta_{2,1}\left(\frac{11}{48}\right.\right.$$

$$\left.\left.+\frac{7}{12\,\alpha_1^2}\right)+\beta_{3,1}\left(\frac{\alpha_1}{72}-\frac{1}{6\,\alpha_1}-\frac{7}{8\,\alpha_1^3}\right)\right\rangle+\left\langle M_{1,1}\left(\frac{1}{6}-\frac{1}{8\,\alpha_1^2}\right)\right.$$

$$\left.-\beta_{2,1}\left(\frac{2\,\alpha_1}{15}+\frac{11}{24\,\alpha_1}+\frac{11}{16\,\alpha_1^3}\right)+\beta_{3,1}\left(\frac{1}{30}+\frac{11}{24\,\alpha_1^2}+\frac{7}{8\,\alpha_1^4}\right)\right\rangle \tanh\alpha_1\Bigg], \qquad [3.30b]$$

$$P_{1,1}=\lambda_1' p_x^{(0)}\Bigg[\left\langle \beta_{2,1}\left(\frac{1}{8}-\frac{\alpha_1^2}{8}\right)-\beta_{3,1}\left(\frac{\alpha_1}{8}+\frac{1}{8\,\alpha_1}\right)\right\rangle-\left\langle \frac{\beta_{2,1}}{8\,\alpha_1}+\beta_{3,1}\left(\frac{\alpha_1^2}{15}\right.\right.$$

$$\left.\left.-\frac{1}{12}-\frac{1}{8\,\alpha_1^2}\right)\right\rangle \tanh\alpha_1\Bigg], \qquad [3.30c]$$

$$S_{1,1}=-\alpha_1 K_{2,1}'+(K_{2,1}'-\tfrac{2}{3}\alpha_1 K_{3,1}')\tanh\alpha_1$$

$$+\lambda_1'^2 p_x^{(0)2}\Bigg[\left\langle \beta_{2,1}\left(\frac{13\,\alpha_1^3}{288}-\frac{71\,\alpha_1}{192}-\frac{115}{64\,\alpha_1^2}\right)+\beta_{3,1}\left(\frac{\alpha_1^3}{56}+\frac{149}{192}+\frac{165}{64\,\alpha_1^2}\right.\right.$$

$$\left.\left.-\frac{257\,\alpha_1^2}{1440}\right)\right\rangle+\left\langle \beta_{2,1}\left(\frac{83\,\alpha_1^2}{480}+\frac{139}{96}+\frac{115}{64\,\alpha_1^2}\right)-\beta_{3,1}\left(\frac{\alpha_1^3}{56}+\frac{19\,\alpha_1}{96}\right.\right.$$

$$\left.\left.+\frac{119}{32\,\alpha_1}+\frac{165}{64\,\alpha_1^3}+\frac{1}{8}\right)\right\rangle \tanh\alpha_1\Bigg] \qquad [3.30d]$$

Also

$$J_{0,1}=[M_{1,1}+\alpha_1\beta_{3,1}y^2]\cosh\alpha_1 y+[-\beta_{2,1}y+\tfrac{2}{3}\alpha_1\beta_{3,1}y^3]\sinh\alpha_1 y, \qquad [3.31a]$$

$$I_{1,1}=P_{1,1}\cosh\alpha_1 y+\lambda_1' p_x^{(0)}\Bigg[\left\langle\left(\frac{\alpha_1^2\beta_{2,1}}{4}-\frac{\beta_{2,1}}{8}+\frac{\beta_{3,1}}{8\,\alpha_1}\right)y^2\right.$$

$$+\left(\frac{\alpha_1\beta_{3,1}}{8}-\frac{\alpha_1^2\beta_{2,1}}{8}\right)y^4\Big\rangle\cosh\alpha_1 y+\Big\langle\left(\left(\frac{\beta_{2,1}}{8\alpha_1}-\frac{\alpha_1\beta_{2,1}}{4}-\frac{\beta_{3,1}}{8\alpha_1^2}\right)y\right.$$

$$\left.+\left(\frac{\alpha_1\beta_{2,1}}{4}+\frac{\alpha_1^2\beta_{3,1}}{6}-\frac{\beta_{3,1}}{12}\right)y^3-\frac{\alpha_1^2\beta_{2,1}}{10}y^5\right\rangle\sinh\alpha_1 y\Big] \qquad [3.31b]$$

$$I_{2,1}=R_{1,1}\cosh\alpha_1 y+[\alpha_1 K_{2,1}y^2\cosh\alpha_1 y+(\tfrac{2}{3}\alpha_1 K_{3,1}y^3-K_{2,1}y)\sinh\alpha_1 y]$$

$$+p_x^{(0)}\Big[\Big\langle\frac{\alpha_1\beta_{3,1}}{288}y^6+\left(\frac{7\beta_{3,1}}{64\alpha_1}-\frac{\alpha_1\beta_{3,1}}{16}-\frac{\beta_{2,1}}{16}\right)y^4+\left(\frac{\beta_{3,1}}{2\alpha_1}-\frac{25\beta_{3,1}}{64\alpha_1^3}\right.$$

$$\left.-\frac{\beta_{2,1}}{16\alpha_1^2}\right)y^2\Big\rangle\cosh\alpha_1 y+\Big\langle-\frac{11\beta_{3,1}}{480}y^5-\left(\frac{\beta_{2,1}}{24\alpha_1}+\frac{5\beta_{3,1}}{96\alpha_1^2}\right)y^3$$

$$+\left(\frac{\beta_{2,1}}{16\alpha_1^3}+\frac{\beta_{3,1}}{16\alpha_1^2}-\frac{25\beta_{3,1}}{64\alpha_1^4}\right)y\Big\rangle\sinh\alpha_1 y\Big]+\lambda_2' p_x^{(0)}\Big[\Big\langle\left(\frac{\alpha_1^2\beta_{2,1}}{8}\right.$$

$$\left.-\frac{\alpha_1\beta_{3,1}}{8}\right)y^4+\left(\frac{\beta_{2,1}}{8}-\frac{\alpha_1^2\beta_{2,1}}{4}-\frac{\beta_{3,1}}{8\alpha_1}\right)y^2\Big\rangle\cosh\alpha_1 y+\Big\langle\frac{\alpha_1^2\beta_{3,1}}{10}y^5$$

$$+\left(\frac{\beta_{3,1}}{12}-\frac{\alpha_1^2\beta_{3,1}}{6}-\frac{\alpha_1\beta_{2,1}}{4}\right)y^3+\left(\frac{\alpha_1\beta_{2,1}}{4}-\frac{\beta_{2,1}}{8\alpha_1}+\frac{\beta_{3,1}}{8\alpha_1^2}\right)y\Big\rangle\sinh\alpha_1 y\Big]$$

$$+p_x^{(0)}\sigma\Big[\Big\langle\frac{\alpha_1\beta_{3,1}}{36}y^6+\left(\frac{7\beta_{3,1}}{24\alpha_1}-\frac{\alpha_1\beta_{3,1}}{24}-\frac{11\beta_{2,1}}{48}\right)y^4$$

$$+\left(\frac{7\beta_{3,1}}{8\alpha_1^3}-\frac{\beta_{3,1}}{8\alpha_1}-\frac{7\beta_{2,1}}{12\alpha_1^2}-\frac{M_{1,1}}{8\alpha_1}\right)y^2\Big\rangle\cosh\alpha_1 y$$

$$+\Big\langle\left(\frac{\alpha_1\beta_{2,1}}{20}-\frac{7\beta_{3,1}}{60}\right)y^5+\left(\frac{M_{1,1}}{12}+\frac{\alpha_1\beta_{2,1}}{12}+\frac{11\beta_{2,1}}{24}+\frac{\beta_{3,1}}{12}\right.$$

$$\left.-\frac{7\beta_{3,1}}{12\alpha_1^2}\right)y^3+\left(\frac{M_{1,1}}{8\alpha_1^2}-\frac{M_{1,1}}{4}+\frac{11\beta_{2,1}}{16\alpha_1^3}+\frac{\beta_{3,1}}{8\alpha_1^2}\right.$$

$$\left.-\frac{7\beta_{3,1}}{8\alpha_1^4}\right)y\Big\rangle\sinh\alpha_1 y\Big]. \qquad [3.31c]$$

$$J_{2,1} = S_{1,1}\cosh\alpha_1 y + [\alpha_1 K'_{2,1} y^2 \cosh\alpha_1 y + (\tfrac{2}{3}\alpha_1 K'_{3,1} y^3 - K'_{2,1} y)\sinh\alpha_1 y]$$

$$+ \lambda_1'^2 p_x^{(0)2}\left[\left\langle -\left(\frac{103\,\alpha_1^2}{1440}\beta_{3,1} + \frac{13\,\alpha_1^3}{288}\beta_{2,1}\right)y^6 + \left(\frac{95\,\alpha_1\beta_{2,1}}{192}\right.\right.\right.$$

$$\left. + \frac{\alpha_1^3\beta_{2,1}}{8} + \frac{\alpha_1^2\beta_{3,1}}{4} - \frac{149}{192}\beta_{3,1}\right)y^4 + \left(\frac{115\,\beta_{2,1}}{64\,\alpha_1} - \frac{\alpha_1\beta_{2,1}}{8} - \frac{\alpha_1^3\beta_{2,1}}{8}\right.$$

$$\left.\left. - \frac{165}{64\,\alpha_1^2}\beta_{3,1} y^2\right)\right\rangle \cosh\alpha_1 y + \left\langle -\frac{9\,\alpha_1^3\beta_{3,1}}{280}y^7 + \left(\frac{31\,\alpha_1\beta_{3,1}}{96}\right.\right.$$

$$\left. + \frac{\alpha_1^3\beta_{3,1}}{20} - \frac{23\,\alpha_1^2\beta_{2,1}}{480}\right)y^5 + \left(\frac{83\,\beta_{5,1}}{32\,\alpha_1} - \frac{151\,\beta_{2,1}}{96} - \frac{\alpha_1^2\beta_{2,1}}{4}\right)y^3$$

$$+ \left(\frac{\beta_{2,1}}{8} + \frac{\alpha_1^2\beta_{2,1}}{8} - \frac{115\,\beta_{2,1}}{64\,\alpha_1^2} + \frac{165\,\beta_{3,1}}{64\,\alpha_1^3} + \frac{\beta_{3,1}}{8} - \frac{\alpha_1\beta_{3,1}}{8}\right.$$

$$\left.\left.\left. + \frac{9\,\beta_{3,1}}{8\,\alpha_1}\right)y\right\rangle \sinh\alpha_1 y\right]. \qquad [3.31d]$$

Fig. 2 depicts the isotherms for $\epsilon = 0.05$, $h = 1$, $E = 0.5$, $R = 0.1$, $\sigma = 1$, $p_x^{(0)} = 1$, $\lambda_1' = 0.1$; $\lambda_2' = 0.011$. The isotherms near the boundaries proceed more or less parallel to them. In this case we do not get any straight isotherm as was the case with the Rivlin-Ericksen fluid in reference[3]. The isotherms near the mid-plane form a very complicated pattern so much so that between $x = \pi/2$ and $x = 2\pi$ they form closeed loops. This pattern is similar to that obtained in reference[3] but with a slight shift of the closed loops towards the origin in comparison to those for the Rivlin-Ericksen fluid.

Case (*b*):

Taking $T_x^{(0)} \equiv 0$, as in case (*a*), [3.1] and [3.5] give

$$T^{(0)}(y) = E\sigma R^2 p_x^{(0)2}[5/12 - y/3 - y^4/12], \qquad [3.32]$$

so that the boundary conditions [3.6] reduce to:

$$\left.\begin{aligned} T^{(1)}(x, 1) &= \frac{2E\sigma R^2 p_x^{(0)2}}{3}\sum_{n=1}^{\infty}(a_n\cos\alpha_n x + b_n\sin\alpha_n x),\\ T_y^{(1)}(x, -1) &= -E\sigma R^2 p_x^{(0)2}\sum_{n=1}^{\infty}(a_n'\cos\alpha_n x + b_n'\sin\alpha_n x).\end{aligned}\right\} \qquad [3.33]$$

Choosing $T^{(1)}(x, y)$ again as in case (*a*) and using [3.13], we find that $I_{0,n}$, $J_{0,n}$, $I_{1,n}$ and $J_{1,n}$ are determined by the same equations as in case (*a*) but satisfy the following boundary conditions

$$\left.\begin{aligned} I_{0,n}(1) &= (2a_n/3),\ I'_{0,n}(-1) = -a'_n, \\ J_{0,n}(1) &= (2b_n/3),\ J'_{0,n}(-1) = -b'_n, \\ I_{1,n}(1) &= I'_{1,n}(-1) = J_{1,n}(1) = J'_{1,n}(-1) = 0 \end{aligned}\right\} \qquad [3.34]$$

The solutions for $I_{0,n}$, $J_{0,n}$, $I_{1,n}$ and $J_{1,n}$ are also the same as in case (*a*) but with the new values of the constants $L_{1,n}$, $L_{2,n}$, $M_{1,n}$, $M_{2,n}$, $P_{1,n}$, $P_{2,n}$, $Q_{1,n}$ and $Q_{2,n}$ given as follows:

$$\begin{aligned} L_{1,n} = {} & [(1/\alpha_n \cosh 2\alpha_n)]\,[\tfrac{2}{3}\alpha_n a_n \cosh\alpha_n + a'_n \sinh\alpha_n - (\tfrac{2}{5}\alpha_n^2\alpha_{3,n} - \alpha_n\alpha_{4,n} \\ & + \tfrac{1}{2}\alpha_{1,n})\sinh 2\alpha_n + (2\alpha_n\alpha_{1,n} + \alpha_{2,n} - 2\alpha_n^2\alpha_{2,n} - 2\alpha_n\alpha_{3,n})\sinh^2\alpha_n \\ & + (\alpha_n\alpha_{1,n} - \alpha_n^2\alpha_{2,n} - \tfrac{2}{3}\alpha_n^2\alpha_{4,n})] \end{aligned} \qquad [3.35a]$$

$$\begin{aligned} L_{2,n} = {} & [(1/\alpha_n \cosh 2\alpha_n)]\,[\tfrac{2}{3}\alpha_n a_n \sinh\alpha_n - a'_n \cosh\alpha_n - (\tfrac{2}{3}\alpha_n^2\alpha_{4,n} - \alpha_n\alpha_{3,n} \\ & + \tfrac{1}{2}\alpha_{2,n})\sinh 2\alpha_n + (2\alpha_n\alpha_{2,n} + \alpha_{1,n} - 2\alpha_n^2\alpha_{1,n} - 2\alpha_n\alpha_{4,n}) \\ & \times \cosh^2\alpha_n + (\tfrac{2}{3}\alpha_n^2\alpha_{3,n} + \alpha_n^2\alpha_{1,n} - \alpha_n\alpha_{2,n})]. \end{aligned} \qquad [3.35b]$$

$M_{1,n}$ and $M_{2,n}$ can be obtained from [3.35] by replacing *a*'s by *b*'s and α's by β's.

$$\begin{aligned} P_{1,n} = {} & \frac{\lambda'_1 p_x^{(0)}}{\alpha_n \cosh 2\alpha_n}\Bigg[\Bigg\{-\frac{\beta_{1,n}}{8} + \beta_{2,n}\left(\frac{\alpha_n}{8} - \frac{\alpha_n^3}{8}\right) - \beta_{3,n}\left(\frac{\alpha_n^2}{8} + \frac{1}{8}\right) \\ & - \beta_{4,n}\left(\frac{\alpha_n^2}{15} - \frac{\alpha_n}{12} - \frac{1}{8\alpha_n}\right)\Bigg\}\cosh^2\alpha_n + \Bigg\{-\beta_{2,n}\left(\frac{1}{8\alpha_n} + \frac{3\alpha_n}{8} + \frac{\alpha_n^3}{8}\right) \\ & - \frac{\beta_{1,n}}{8} + \beta_{4,n}\left(\frac{1}{8\alpha_n} + \frac{5\alpha_n}{12} + \frac{\alpha_n^3}{15}\right) + \beta_{3,n}\left(\frac{1}{8} - \frac{\alpha_n^2}{8} + \frac{1}{8\alpha_n^2}\right)\Bigg\}\sinh^2\alpha_n \\ & + \Bigg\{\beta_{1,n}\left(\frac{\alpha_n}{4} + \frac{1}{16\alpha_n}\right) - \beta_{3,n}\left(\frac{\alpha_n^3}{15} + \frac{\alpha_n}{6}\right) \\ & - \beta_{4,n}\left(\frac{1}{8} + \frac{1}{16\alpha_n^2}\right)\Bigg\}\sinh 2\alpha_n\Bigg], \end{aligned} \qquad [3.35c]$$

$$P_{2,n} = \frac{\lambda_1' p_x^{(0)}}{\alpha_n \cosh 2\alpha_n}\Bigg[\Bigg\{-\frac{\beta_{2,n}}{8} + \beta_{1,n}\left(\frac{\alpha_n}{8} - \frac{\alpha_n^3}{8}\right) - \beta_{4,n}\left(\frac{\alpha_n^2}{8} + \frac{1}{8}\right)$$

$$- \beta_{3,n}\left(\frac{\alpha_n^3}{15} - \frac{\alpha_n}{12} - \frac{1}{8\alpha_n}\right)\Bigg\}\sinh^2\alpha_n + \Bigg\{-\beta_{1,n}\left(\frac{1}{8\alpha_n} + \frac{3\alpha_n}{8} + \frac{\alpha_n^3}{8}\right)$$

$$-\frac{\beta_{2,n}}{8} + \beta_{3,n}\left(\frac{1}{8\alpha_n} + \frac{5\alpha_n}{12} + \frac{\alpha_n^3}{15}\right) + \beta_{4,n}\left(\frac{1}{8} + \frac{1}{8\alpha_n^2} - \frac{\alpha_n^2}{8}\right)\Bigg\}\cosh^2\alpha_n$$

$$+ \Bigg\{\beta_{2,n}\left(\frac{\alpha_n}{4} + \frac{1}{16\alpha_n}\right) - \beta_{3,n}\left(\frac{1}{8} + \frac{1}{16\alpha_n^2}\right)$$

$$- \beta_{4,n}\left(\frac{\alpha_n^3}{15} + \frac{\alpha_n}{6}\right)\Bigg\}\sinh 2\alpha_n\Bigg]. \qquad [3.35d]$$

The expressions for $Q_{1,n}$ and $Q_{2,n}$ can be obtained from [3.35] by replacing β's by α's.

The equation determining $I_{2,n}$ and $J_{2,n}$ are :

$$(D^2 - \alpha_n^2) I_{2,n} = y\,[(A''_{2,n} + \alpha_n^2 A_{2,n}) - (E_{2,n}/p_x^{(0)})] + \alpha_n p_x^{(0)}\,[\sigma\{\tfrac{1}{2}(y^2-1) J_{0,n}$$
$$- \tfrac{1}{3}(y^3+1) B_{0,n}\} - 2\lambda_2' y^2 B'_{0,n}], \qquad [3.36]$$

$$(D^2 - \alpha_n^2) J_{2,n} = y\,[(B''_{2,n} + \alpha_n^2 B_{2,n}) - F_{2,n}/p_x^{(0)}] + \alpha_n p_x^{(0)}\,[\sigma\{\tfrac{1}{2}(y^2-1) I_{0,n}$$
$$- \tfrac{1}{3}(y^3+1) A_{0,n}\} - 2\lambda_2' y^2 A'_{0,n}], \qquad [3.37]$$

which have to be solved under the boundary conditions

$$\left.\begin{aligned} I_{2,n}(1) &= I'_{2,n}(-1) = 0,\\ J_{2,n}(1) &= J'_{2,n}(-1) = 0. \end{aligned}\right\} \qquad [3.38]$$

Equations [3.36], [3.37] and [3.38] give

$$I_{2,n} = R_{1,n}\cosh\alpha_n y + R_{2,n}\sinh\alpha_n y + [(\{2\alpha_n/3\}K_{4,n}y^3 + \alpha_n K_{2,n}y^2$$
$$- K_{1,n}y)\cosh\alpha_n y + (\{2\alpha_n/3\}K_{3,n}y^3 + \alpha_n K_{1,n}y^2 - K_{2,n}y)\sinh\alpha_n y]$$
$$+ p_x^{(0)}\Bigg[\Bigg\langle\frac{\alpha_n\beta_{3,n}}{288}y^6 - \frac{11\beta_{4,n}}{480}y^5 + \left(\frac{7\beta_{3,n}}{64\alpha_n} - \frac{\alpha_n\beta_{2,n}}{16} - \frac{\beta_{2,n}}{16}\right)y^4$$

$$-\left(\frac{\beta_{1,n}}{24\,\alpha_n}+\frac{5\,\beta_{4,n}}{96\,\alpha_n^2}\right)y^3+\left(\frac{\beta_{3,n}}{2\,\alpha_n}+\frac{25\,\beta_{3,n}}{64\,\alpha_n^3}-\frac{\beta_{2,n}}{16\,\alpha_n^2}\right)y^2+\left(\frac{\beta_{1,n}}{16\,\alpha_n^3}+\frac{\beta_{4,n}}{16\,\alpha_n^2}\right.$$

$$\left.-\frac{25\,\beta_{4,n}}{64\,\alpha_n^4}\right)y\Big\rangle\cosh\alpha_n y+\Big\langle\frac{\alpha_n\,\beta_{4,n}}{288}y^6-\frac{11\,\beta_{3,n}}{480}y^5+\left(\frac{7\,\beta_{4,n}}{64\,\alpha_n}\right.$$

$$\left.-\frac{\alpha_n\,\beta_{1,n}}{16}-\frac{\beta_{1,n}}{16}\right)y^4-\left(\frac{\beta_{2,n}}{24\,\alpha_n}+\frac{5\,\beta_{3,n}}{96\,\alpha_n^2}\right)y^3+\left(\frac{\beta_{4,n}}{2\,\alpha_n}+\frac{25\,\beta_{4,n}}{64\,\alpha_n^3}\right.$$

$$\left.-\frac{\beta_{1,n}}{16\,\alpha_n^2}\right)y^2+\left(\frac{\beta_{2,n}}{16\,\alpha_n^3}+\frac{\beta_{3,n}}{16\,\alpha_n^2}-\frac{25\,\beta_{3,n}}{64\,\alpha_n^4}\right)y\Big\rangle\sinh\alpha_n y\Big]$$

$$+\lambda_1'^2\,p_x^{(0)2}\Big[\Big\langle-\frac{9\,\alpha_n^3\,\alpha_{4,n}}{280}y^7-\left(\frac{103\,\alpha_n^2\,\alpha_{3,n}}{1440}+\frac{13\,\alpha_n^3\,\alpha_{2,n}}{288}\right)y^6$$

$$+\left(\frac{31\,\alpha_n\,\alpha_{4,n}}{96}+\frac{\alpha_n^3\,\alpha_{4,n}}{20}-\frac{23\,\alpha_n^2\,\alpha_{1,n}}{480}\right)y^5+\left(\frac{95\,\alpha_n\,\alpha_{2,n}}{192}+\frac{\alpha_n^3\,\alpha_{2,n}}{8}\right.$$

$$\left.+\frac{\alpha_n^2\,\alpha_{3,n}}{4}-\frac{149\,\alpha_{3,n}}{192}\right)y^4+\left(\frac{83\,\alpha_{4,n}}{32\,\alpha_n}-\frac{151\,\alpha_{1,n}}{96}-\frac{\alpha_n^2\,\alpha_{1,n}}{4}\right)y^3$$

$$+\left(\frac{115\,\alpha_{2,n}}{64\,\alpha_n}-\frac{\alpha_n\,\alpha_{2,n}}{8}-\frac{\alpha_n^3\,\alpha_{2,n}}{8}-\frac{165\,\alpha_{3,n}}{64\,\alpha_n^2}\right)y^2+\left(\frac{\alpha_{1,n}}{8}+\frac{\alpha_n^2\,\alpha_{1,n}}{8}\right.$$

$$\left.-\frac{115\,\alpha_{1,n}}{64\,\alpha_n^2}+\frac{165}{64\,\alpha_n^3}+\frac{\alpha_{4,n}}{8}-\frac{\alpha_n\,\alpha_{4,n}}{8}+\frac{9\,\alpha_{4,n}}{8\,\alpha_n}\right)y\Big\rangle\cosh\alpha_n y$$

$$+\Big\langle-\frac{9\,\alpha_n^3\,\alpha_{3,n}}{280}y^7-\left(\frac{103\,\alpha_n^2\,\alpha_{4,n}}{1440}+\frac{1\quad\alpha_n^3\,\alpha_{1,n}}{288}\right)y^6+\left(\frac{31\,\alpha_n\,\alpha_{3,n}}{96}\right.$$

$$\left.+\frac{\alpha_n^3\,\alpha_{3,n}}{20}-\frac{23\,\alpha_n^2\,\alpha_{2,n}}{480}\right)y^5+\left(\frac{83\,\alpha_{3,n}}{32\,\alpha_n}-\frac{151\,\alpha_{2,n}}{96}-\frac{\alpha_n^2\,\alpha_{2,n}}{4}\right)y^3$$

$$+\left(\frac{95\,\alpha_n\,\alpha_{1,n}}{192}+\frac{\alpha_n^3\,\alpha_{1,n}}{8}+\frac{\alpha_n^2\,\alpha_{4,n}}{4}-\frac{149\,\alpha_{4,n}}{192}\right)y^4+\left(\frac{115\,\alpha_{1,n}}{64\,\alpha_n}-\frac{\alpha_n\,\alpha_{1,n}}{8}\right.$$

$$\left.-\frac{\alpha_n^3\alpha_{1,n}}{8}-\frac{165\,\alpha_{3,n}}{64\,\alpha_n^2}\right)y^2+\left(\frac{\alpha_{2,n}}{8}+\frac{\alpha_n^2\,\alpha_{2,n}}{8}-\frac{115\,\alpha_{2,n}}{64\,\alpha_n^2}+\frac{165\,\alpha_{3,n}}{64\,\alpha_n^3}+\frac{\alpha_{3,n}}{8}\right.$$

$$\left.-\frac{\alpha_n\,\alpha_{3,n}}{8}+\frac{9\,\alpha_{3,n}}{8\,\alpha_n}\right)y\Big\rangle\sinh\alpha_n y\Big]+\lambda_2'\,p_x^{(0)}\Big[\Big\langle\frac{\alpha_n^2\,\beta_{4,n}}{10}y^5+\left(\frac{\alpha_n^2\,\beta_{2,n}}{8}\right.$$

$$- \frac{\alpha_n \beta_{3,n}}{8}\Big) y^4 + \left(\frac{\beta_{4,n}}{12} - \frac{\alpha_n^2 \beta_{4,n}}{6} - \frac{\alpha_n \beta_{1,n}}{4}\right) y^3 + \left(\frac{\beta_{2,n}}{8} - \frac{\alpha_n^2 \beta_{2,n}}{4}\right.$$

$$\left. - \frac{\beta_{3,n}}{8\alpha_n}\right) y^2 + \left(\frac{\alpha_n \beta_{1,n}}{4} - \frac{\beta_{1,n}}{8\alpha_n} + \frac{\beta_{4,n}}{8\alpha_n^2}\right) y \Big\rangle \cosh \alpha_n y + \Big\langle \frac{\alpha_n^2 \beta_{3,n}}{10} y^5$$

$$+ \left(\frac{\alpha_n^2 \beta_{1,n}}{8} - \frac{\alpha_n \beta_{4,n}}{8}\right) y^4 + \left(\frac{\beta_{3,n}}{12} - \frac{\alpha_n^2 \beta_{3,n}}{6} - \frac{\alpha_n \beta_{2,n}}{4}\right) y^3$$

$$+ \left(\frac{\beta_{1,n}}{8} - \frac{\alpha_n^2 \beta_{1,n}}{4} - \frac{\beta_{4,n}}{8\alpha_n}\right) y^2 + \left(\frac{\alpha_n \beta_{2,n}}{4} - \frac{\beta_{2,n}}{8\alpha_n} + \frac{\beta_{3,n}}{8\alpha_n^2}\right) y \Big\rangle \sinh \alpha_n y \Big]$$

$$+ p_x^{(0)} \sigma \Big[\Big\langle \frac{\alpha_n \beta_{3,n}}{36} y^6 + \left(\frac{\alpha_n \beta_{1,n}}{20} - \frac{7\beta_{4,n}}{60}\right) y^5 + \left(\frac{7\beta_{3,n}}{24\alpha_n} - \frac{\alpha_n \beta_{3,n}}{24}\right.$$

$$\left. - \frac{11\beta_{2,n}}{48}\right) y^4 + \left(\frac{M_{2,n}}{12} + \frac{\alpha_n \beta_{1,n}}{12} + \frac{11\beta_{1,n}}{24\alpha_n} + \frac{\beta_{4,n}}{12} - \frac{7\beta_{4,n}}{12\alpha_n^2}\right) y^3$$

$$+ \left(\frac{7\beta_{3,n}}{8\alpha_n^3} - \frac{\beta_{3,n}}{8\alpha_n} - \frac{M_{1,n}}{8\alpha_n} - \frac{\beta_{4,n}}{12}\right) y^2 + \left(\frac{M_{2,n}}{8\alpha_n} - \frac{M_{2,n}}{4} + \frac{11\beta_{1,n}}{8\alpha_n^2}\right.$$

$$\left. - \frac{7\beta_{4,n}}{8\alpha_n^4} - \frac{\beta_{2,n}}{6\alpha_n} + \frac{\beta_{3,n}}{12\alpha_n^2}\right)\Big\rangle \cosh \alpha_n y + \Big\langle \frac{\alpha_n \beta_{4,n}}{36} y^6 + \left(\frac{\alpha_n \beta_{2,n}}{20}\right.$$

$$\left. - \frac{7\beta_{3,n}}{60}\right) y^5 + \left(\frac{7\beta_{4,n}}{24\alpha_n} - \frac{\alpha_n \beta_{4,n}}{24} - \frac{11\beta_{1,n}}{8}\right) y^4 + \left(\frac{M_{1,n}}{12} + \frac{\alpha_n \beta_{2,n}}{12}\right.$$

$$\left. + \frac{11\beta_{2,n}}{24\alpha_n} + \frac{\beta_{3,n}}{12} - \frac{7\beta_{3,n}}{12\alpha_n^2}\right) y^3 + \left(\frac{7\beta_{4,n}}{8\alpha_n^3} - \frac{\beta_{4,n}}{8\alpha_n} - \frac{7\beta_{1,n}}{12\alpha_n^2} - \frac{M_{2,n}}{8\alpha_n}\right.$$

$$\left. - \frac{\beta_{3,n}}{12\alpha_n}\right) y^2 + \left(\frac{M_{1,n}}{8\alpha_n^2} - \frac{M_{1,n}}{4} + \frac{11\beta_{2,n}}{16\alpha_n^3} + \frac{\beta_{3,n}}{8\alpha_n^2} - \frac{7\beta_{3,n}}{8\alpha_n^4}\right.$$

$$\left. - \frac{\beta_{1,n}}{6\alpha_n} + \frac{\beta_{4,n}}{12\alpha_n^2}\right) y \Big\rangle \sinh \alpha_n y \Big], \qquad [3.39]$$

where

$$R_{1,n} = [1/(\alpha_n \cosh 2\alpha_n)] [\{(\alpha_n K_{1,n} - \alpha_n^2 K_{2,n} - \tfrac{2}{3}\alpha_n^2 K_{4,n}) \cosh^2 \alpha_n + (\tfrac{2}{3}\alpha_n^2 K_{4,n}$$

$$- \alpha_n^2 K_{2,n} - 2\alpha_n K_{3,n} + \alpha_n K_{1,n} + K_{2,n}) \sinh^2 \alpha_n + (\alpha_n K_{4,n}$$

$$-\tfrac{2}{3}\alpha_n^2 K_{3,n} - \tfrac{1}{2} K_{1,n}) \sinh 2\alpha_n\} + p_x^{(0)}\{[\beta_{1,n}(1/24 - 1/16\alpha_n^2)$$

$$+\beta_{2,n}\left(\frac{\alpha_n}{16}+\frac{1}{16\alpha_n}\right)+\beta_{3,n}\left(\frac{17\alpha_n^2}{288}-\frac{39}{24}-\frac{25}{64\alpha_n^2}\right)+\beta_{4,n}\left(\frac{11\alpha_n}{480}\right.$$

$$\left.\left.-\frac{1}{96\alpha_n}+\frac{25}{64\alpha_n^3}\right)\right]\cosh^2\alpha_n+\left[\beta_{2,n}\left(\frac{\alpha_n}{16}+\frac{3}{16\alpha_n}-\frac{1}{16\alpha_n^3}\right)\right.$$

$$-\beta_{1,n}\left(\frac{7}{24}+\frac{1}{16\alpha_n^2}\right)+\beta_{4,n}\left(\frac{25}{64\alpha_n^3}+\frac{139}{96\alpha_n}-\frac{47\alpha_n}{160}\right)+\beta_{3,n}\left(\frac{17\alpha_n^2}{288}\right.$$

$$\left.\left.+\frac{25}{64\alpha_n^4}-\frac{19}{64\alpha_n^2}-\frac{95}{192}\right)\right]\sinh^2\alpha_n+\left[\beta_{1,n}\left(\frac{1}{32\alpha_n^3}-\frac{1}{16\alpha_n}\right)\right.$$

$$+\frac{\beta_{2,n}}{6}+\beta_{3,n}\left(\frac{19\alpha_n}{120}-\frac{35}{48\alpha_n}\right)-\beta_{4,n}\left(\frac{11}{192}+\frac{3}{64\alpha_n^2}\right.$$

$$\left.\left.+\frac{25}{128\alpha_n^4}\right)\right]\sinh 2\alpha_n\Big\}+\lambda_1'^2 p_x^{(0)\,2}\left\{\left[\alpha_{1,n}\left(\frac{83\alpha_n^3}{480}+\frac{139\alpha_n}{96}+\frac{115}{64\alpha_n}\right)\right.\right.$$

$$+\alpha_{2,n}\left(\frac{13\alpha_n^4}{288}-\frac{71\alpha_n^2}{192}-\frac{115}{64}\right)+\alpha_{3,n}\left(\frac{165}{64\alpha_n}+\frac{149\alpha_n}{192}-\frac{257\alpha_n^3}{1440}\right)$$

$$\left.-\alpha_{4,n}\left(\frac{\alpha_n^4}{56}+\frac{19\alpha_n^2}{96}+\frac{119}{32}+\frac{165}{64\alpha_n^2}+\frac{\alpha_n}{8}\right)\right]\cosh^2\alpha_n$$

$$+\left[\alpha_{2,n}\left(\frac{13\alpha_n^4}{288}+\frac{95\alpha_n^2}{192}+\frac{179}{64}+\frac{115}{64\alpha_n^2}\right)+\left(-\frac{31\alpha_n^2}{160}+\frac{9\alpha_n}{32}+\frac{115}{64\alpha_n}\right)\alpha_{1,n}\right.$$

$$+\alpha_{4,n}\left(\frac{\alpha_n^4}{56}+\frac{123\alpha_n^2}{160}-\frac{49}{96}+\frac{\alpha_n}{8}-\frac{201}{32\alpha_n^2}\right)+\alpha_{3,n}\left(-\frac{293\alpha_n^3}{1440}-\frac{137\alpha_n}{192}\right.$$

$$\left.\left.+\frac{1}{8}-\frac{405}{64\alpha_n}+\frac{165}{64\alpha_n^3}\right)\right]\sinh^9\alpha_n+\left[-\alpha_{1,n}\left(\frac{83\alpha_n^2}{192}+\frac{147}{64}+\frac{115}{128\alpha_n^2}\right)\right.$$

$$+\alpha_{2,n}\left(\frac{83\alpha_n^3}{960}+\frac{31\alpha_n^2}{320}+\frac{7\alpha_n}{12}\right)-\alpha_{3,n}\left(\frac{\alpha_n^4}{56}+\frac{29\alpha_n^2}{60}+\frac{77}{48}-\frac{165}{128\alpha_n^2}\right.$$

$$\left.\left.+\frac{\alpha_n}{8}-\frac{9}{16\alpha_n}\right)+\alpha_{4,n}\left(\frac{\alpha_n^3}{80}+\frac{143\alpha_n}{192}-\frac{1}{16}-\frac{165}{128\alpha_n^3}+\frac{285}{64\alpha_n}\right)\right]\sinh 2\alpha_n\Big\}$$

$$+ \lambda_2' p_x^{(0)} \left\{ \left[\frac{\beta_{1,n}}{8} + \beta_{2,n} \left(\frac{\alpha_n^3}{8} - \frac{1}{8} \right) + \beta_{3,n} \left(\frac{\alpha_n^2}{8} + \frac{1}{8} \right) \right. \right.$$

$$+ \beta_{4,n} \left(\frac{\alpha_n^3}{15} - \frac{\alpha_n}{12} - \frac{1}{8\alpha_n} \right] \cosh^2 \alpha_n + \left[\beta_{2,n} \left(\frac{\alpha_n^3}{8} + \frac{3\alpha_n}{8} + \frac{1}{8\alpha_n} \right) + \frac{\beta_{1,n}}{8} \right.$$

$$- \beta_{4,n} \left(\frac{\alpha_n^3}{15} + \frac{5\alpha_n}{12} + \frac{1}{8\alpha_n} \right) + \beta_{3,n} \left(\frac{\alpha_n^2}{8} - \frac{1}{8} - \frac{1}{8\alpha_n^2} \right) \Bigg] \sinh^2 \alpha_n$$

$$+ \left[- \beta_{1,n} \left(\frac{\alpha_n}{4} + \frac{1}{16\alpha_n} \right) + \beta_{3,n} \left(\frac{\alpha_n^3}{15} + \frac{\alpha_n}{6} \right) \right.$$

$$\left. \left. + \beta_{4,n} \left(\frac{1}{8} + \frac{1}{16\alpha_n^2} \right) \right] \sinh 2\alpha_n \right\} + p_x^{(0)} \sigma \left\{ \left[\frac{M_{1,n}}{8} + M_{2,n} \left(\frac{\alpha_n}{6} - \frac{1}{8\alpha_n} \right) \right. \right.$$

$$- \beta_{1,n} \left(\frac{2\alpha_n^2}{15} + \frac{11}{24} + \frac{11}{16\alpha_n^2} \right) + \beta_{2,n} \left(\frac{11\alpha_n}{48} + \frac{1}{6} + \frac{7}{12\alpha_n} \right) + \beta_{3,n} \left(\frac{\alpha_n^2}{72} \right.$$

$$\left. - \frac{1}{6} - \frac{1}{12\alpha_n} - \frac{7}{8\alpha_n^2} \right) + \beta_{4,n} \left(\frac{\alpha_n}{30} + \frac{11}{24\alpha_n} + \frac{7}{8\alpha_n^3} + \frac{1}{12} \right) \Bigg] \cosh^2 \alpha_n$$

$$+ \left[- M_{2,n} \left(\frac{\alpha_n}{6} + \frac{1}{8\alpha_n} \right) + M_{1,n} \left(\frac{1}{8} - \frac{1}{8\alpha_n^2} \right) - \beta_{2,n} \left(\frac{1}{6} + \frac{13\alpha_n}{48} \right. \right.$$

$$\left. + \frac{19}{24\alpha_n} + \frac{11}{16\alpha_n^3} \right) + \beta_{1,n} \left(\frac{2\alpha_n^2}{15} - \frac{11}{24} + \frac{1}{4\alpha_n^2} + \frac{1}{6\alpha_n} \right)$$

$$+ \beta_{4,n} \left(\frac{1}{12} + \frac{7}{8\alpha_n} + \frac{7}{8\alpha_n^3} - \frac{1}{12\alpha_n^2} - \frac{\alpha_n}{30} \right) + \beta_{3,n} \left(\frac{\alpha_n^2}{72} + \frac{1}{6} + \frac{3}{4\alpha_n^2} - \frac{1}{12\alpha_n} \right.$$

$$\left. \left. + \frac{7}{8\alpha_n^4} \right) \right] \sinh^2 \alpha_n + \left[\frac{\alpha_n M_{1,n}}{6} + \frac{M_{2,n}}{16\alpha_n^2} + \beta_{1,n} \left(\frac{1}{6} + \frac{\alpha_n}{4} + \frac{11}{16\alpha_n} + \frac{11}{32\alpha_n^3} \right) \right.$$

$$- \beta_{2,n} \left(\frac{2\alpha_n^2}{15} + \frac{15}{32\alpha_n^2} + \frac{1}{12\alpha_n} \right) + \beta_{3,n} \left(\frac{\alpha_n}{30} - \frac{5}{24\alpha_n} + \frac{1}{24\alpha_n^2} \right)$$

$$\left. \left. - \beta_{4,n} \left(\frac{1}{6} + \frac{13}{16\alpha_n^2} + \frac{7}{16\alpha_n^4} \right) \right] \sinh 2\alpha_n \right\} \Bigg], \qquad [3.40a]$$

$$R_{2,n}=\frac{1}{\alpha_n\cosh 2\alpha_n}\Big\langle\ \{([2\,\alpha_n^2/3]\,K_{3,n}-\alpha_n^2 K_{1,n}-2\,\alpha_n K_{4,n}+\alpha_n K_{2,n}$$

$$+K_{1,n})\cosh^2\alpha_n+(\alpha_n K_{2,n}-\alpha_n^2 K_{1,n}-\tfrac{2}{3}\alpha_n^2 K_{3,n})\sinh^2\alpha_n+(\alpha_n K_{3,n}$$

$$-\tfrac{1}{2}K_{2,n}-\tfrac{2}{3}\alpha_n^2 K_{4,n})\sinh 2\,\alpha_n\}+p_x^{(0)}\Big\{\Big[\beta_{1,n}\Big(\frac{\alpha_n}{16}+\frac{3}{16\,\alpha_n}-\frac{1}{16\,\alpha_n^3}\Big)$$

$$-\beta_{2,n}\Big(\frac{7}{24}+\frac{1}{16\,\alpha_n^2}\Big)+\beta_{3,n}\Big(\frac{25}{64\,\alpha_n^3}+\frac{139}{96\,\alpha_n}-\frac{47\,\alpha_n}{160}\Big)+\beta_{4,n}\Big(\frac{17\,\alpha_n^2}{288}$$

$$+\frac{25}{64\,\alpha_n^4}-\frac{19}{64\,\alpha_n^2}-\frac{95}{192}\Big)\Big]\cosh^2\alpha_n+\Big[\beta_{2,n}\Big(\frac{1}{24}-\frac{1}{16\,\alpha_n^2}\Big)$$

$$+\beta_{1,n}\Big(\frac{\alpha_n}{16}+\frac{1}{16\,\alpha_n}\Big)+\beta_{4,n}\Big(\frac{17\,\alpha_n^2}{288}-\frac{39}{64}-\frac{25}{64\,\alpha_n^2}\Big)+\beta_{3,n}\Big(\frac{11\,\alpha_n}{480}$$

$$-\frac{1}{96\,\alpha_n}+\frac{25}{64\,\alpha_n^3}\Big)\Big]\sinh^2\alpha_n+\Big[\frac{\beta_{1,n}}{6}+\beta_{2,n}\Big(\frac{1}{32\,\alpha_n^3}-\frac{1}{16\,\alpha_n}\Big)$$

$$-\beta_{3,n}\Big(\frac{11}{192}+\frac{3}{64\,\alpha_n^2}+\frac{25}{128\,\alpha_n^4}\Big)+\beta_{4,n}\Big(\frac{19\,\alpha_n}{120}$$

$$-\frac{35}{48\,\alpha_n}\Big)\Big]\sinh 2\,\alpha_n\Big\}+\lambda_1'^2\,p_x^{(0)2}\ \Big\{\Big[\alpha_{1,n}\Big(\frac{13\,\alpha_n^4}{288}+\frac{95\,\alpha_n^2}{192}+\frac{179}{64}$$

$$+\frac{115}{64\,\alpha_n^2}\Big)+\alpha_{2,n}\Big(-\frac{31\,\alpha_n^2}{160}+\frac{9\,\alpha_n}{32}+\frac{115}{64\,\alpha_n}\Big)+\alpha_{3,n}\Big(\frac{\alpha_n^4}{56}+\frac{123\,\alpha_n^2}{160}-\frac{49}{96}$$

$$+\frac{\alpha_n}{8}-\frac{165}{32\,\alpha_n^2}-\frac{9}{8\,\alpha_n}\Big)+\alpha_{4,n}\Big(-\frac{293\,\alpha_n^2}{1440}-\frac{137\,\alpha_n}{192}+\frac{1}{8}-\frac{405}{64\,\alpha_n}$$

$$+\frac{165}{64\,\alpha_n^3}\Big)\Big]\cosh^2\alpha_n+\Big[\alpha_{2,n}\Big(\frac{83\,\alpha_n^3}{480}+\frac{139\,\alpha_n}{96}+\frac{115}{64\,\alpha_n}\Big)$$

$$+\alpha_{1,n}\Big(\frac{13\,\alpha_n^4}{288}-\frac{71\,\alpha_n^2}{192}-\frac{115}{64}\Big)+\alpha_{4,n}\Big(\frac{165}{64\,\alpha_n}+\frac{149}{192}-\frac{257\,\alpha_n^3}{1440}\Big)$$

$$-\alpha_{3,n}\Big(\frac{\alpha_n^4}{56}+\frac{19\,\alpha_n^2}{96}+\frac{119}{32}+\frac{165}{64\,\alpha_n^2}+\frac{\alpha_n}{8}\Big)\Big]\sinh^2\alpha_n+\Big[\alpha_{1,n}\Big(\frac{83\,\alpha_n^3}{960}$$

$$+\frac{31\,\alpha_n^2}{320}+\frac{7\,\alpha_n}{12}\Big)-\alpha_{2,n}\left(\frac{83\,\alpha_n^2}{192}+\frac{147}{64}+\frac{115}{128\,\alpha_n^2}\right)+\alpha_{3,n}\Big(-\frac{\alpha_n^3}{80}+\frac{143\,\alpha_n}{192}$$

$$-\frac{1}{16}+\frac{285}{64\,\alpha_n}-\frac{165}{128\,\alpha_n^3}\Big)+\alpha_{4,n}\Big(-\frac{\alpha_n^4}{56}-\frac{29\,\alpha_n^2}{60}-\frac{\alpha_n}{8}-\frac{77}{48}+\frac{165}{128\,\alpha_n^2}$$

$$+\frac{9}{16\,\alpha_n}\Big)\Big]\sinh 2\,\alpha_n\Big\}+\lambda_2'\,p_x^{(0)}\Big\{\Big[\beta_{1,n}\left(\frac{\alpha_n^3}{8}+\frac{3\,\alpha_n}{8}+\frac{1}{8\,\alpha_n}\right)+\frac{\beta_{2,n}}{8}$$

$$-\beta_{3,n}\left(\frac{\alpha_n^3}{15}+\frac{5\,\alpha_n}{12}+\frac{1}{8\,\alpha_n}\right)+\beta_{4,n}\left(\frac{\alpha_n^2}{8}-\frac{1}{8}-\frac{1}{8\,\alpha_n^2}\right)\Big]\cosh^2\alpha_n$$

$$+\Big[-\beta_{2,n}\left(\frac{1}{16\,\alpha_n}+\frac{\alpha_n}{4}\right)+\beta_{3,n}\left(\frac{1}{8}+\frac{1}{16\,\alpha_n^2}\right)$$

$$+\beta_{4,n}\left(\frac{\alpha_n^3}{15}+\frac{\alpha_n}{6}\right)\Big]\sinh 2\,\alpha_n+\Big[\frac{\beta_{2,n}}{8}+\beta_{1,n}\left(\frac{\alpha_n^3}{8}-\frac{\alpha_n}{8}\right)$$

$$+\beta_{4,n}\left(\frac{\alpha_n^2}{8}+\frac{1}{8}\right)+\beta_{3,n}\left(\frac{\alpha_n^3}{15}-\frac{\alpha_n}{12}-\frac{1}{8\,\alpha_n}\right)\Big]\sinh^2\alpha_n\Big\}$$

$$+p_x^{(0)}\,\sigma\Big\{\Big[-M_{1,n}\left(\frac{\alpha_n}{6}+\frac{1}{8\,\alpha_n}\right)+M_{2,n}\left(\frac{1}{8}-\frac{1}{8\,\alpha_n^2}\right)+\beta_{1,n}\Big(\frac{1}{6}+\frac{13\,\alpha_n}{48}$$

$$+\frac{19}{24\,\alpha_n}+\frac{11}{16\,\alpha_n^3}\Big)+\beta_{2,n}\left(\frac{2\,\alpha_n^2}{15}-\frac{11}{24}+\frac{1}{4\,\alpha_n^2}+\frac{1}{6\,\alpha_n}\right)+\beta_{3,n}\Big(\frac{1}{12}+\frac{7}{8\,\alpha_n}$$

$$+\frac{7}{8\,\alpha_n^3}-\frac{1}{12\,\alpha_n^2}-\frac{\alpha_n}{30}\Big)+\beta_{4,n}\left(\frac{\alpha_n^2}{72}+\frac{1}{6}+\frac{3}{4\,\alpha_n^2}-\frac{1}{12\,\alpha_n}+\frac{7}{8\,\alpha_n^4}\right)\Big]\cosh^2\alpha_n$$

$$+\Big[\frac{M_{2,n}}{8}+M_{1,n}\left(\frac{\alpha_n}{6}-\frac{1}{8\,\alpha_n}\right)-\beta_{2,n}\left(\frac{2\,\alpha_n^2}{15}+\frac{11}{24}+\frac{11}{16\,\alpha_n^2}\right)$$

$$+\beta_{1,n}\left(\frac{11\,\alpha_n}{48}+\frac{1}{6}+\frac{7}{12\,\alpha_n}\right)+\beta_{4,n}\left(\frac{\alpha_n^2}{72}-\frac{1}{6}-\frac{1}{12\,\alpha_n}-\frac{7}{8\,\alpha_n^2}\right)$$

$$+\beta_{3,n}\left(\frac{\alpha_n}{30}+\frac{11}{24\,\alpha_n}+\frac{7}{8\,\alpha_n^3}+\frac{1}{12}\right)\Big]\sinh^2\alpha_n+\Big[\frac{M_{1,n}}{16\,\alpha_n^2}-\frac{M_{2,n}}{16\,\alpha_n}$$

$$-\beta_{1,n}\left(\frac{2\,\alpha_n^2}{15}+\frac{15}{32\,\alpha_n^2}+\frac{1}{12\,\alpha_n^{*}}\right)+\beta_{2,n}\left(\frac{1}{6}+\frac{\alpha_n}{4}+\frac{19}{48\,\alpha_n}+\frac{11}{32\,\alpha_n^3}\right)$$

$$- \beta_{3,n}\left(\frac{1}{6}+\frac{13}{16\,\alpha_n^2}+\frac{7}{12\,\alpha_n^4}\right)+\beta_{4,n}\left(\frac{\alpha_n}{30}-\frac{5}{24\,\alpha_n}\right.$$

$$\left.\left.\left.+\frac{1}{24\,\alpha_n^3}\right)\right]\sinh 2\,\alpha_n\right\}\Big\rangle. \qquad [3.40b]$$

The expressions for $J_{2,n}$ and the corresponding constants $S_{1,n}$ and $S_{2,n}$ for it can be obtained from [3.39] and [3.40] in the same way as in case (*a*).

Particular Case:

When the boundaries have sinusoidal deformation we have

$$L_{1,1}=L_{2,1}=Q_{1,1}=Q_{2,1}=',$$

$$I_{0,1}=J_{1,1}\equiv 0,$$

$$J_{0,1}=[M_{1,1}+\alpha_1\,\beta_{2,1}\,y^2]\cosh\alpha_1 y+[M_{2,1}-\beta_{2,1}\,y$$
$$+\tfrac{2}{3}\alpha_1\,\beta_{3,1}\,y^3]\sinh\alpha_1 y, \qquad [3.41]$$

$$I_{1,1}=P_{1,1}\cosh\alpha_1 y+P_{2,1}\sinh\alpha_1 y$$
$$+\lambda_1'\,p_x^{(0)}\left[\left\{\left(\frac{\alpha_1^2\beta_{2,1}}{4}-\frac{\beta_{2,1}}{8}+\frac{\beta_{3,1}}{8\,\alpha_1}\right)y^2+\left(\frac{\alpha_1\beta_{3,1}}{8}\right.\right.\right.$$
$$\left.\left.-\frac{\alpha_1^2\beta_{2,1}}{8}\right)y^4\right\}\cosh\alpha_1 y+\left\{\left(\frac{\beta_{2,1}}{8\alpha_1}-\frac{\alpha_1\beta_{2,1}}{4}-\frac{\beta_{3,1}}{8\alpha_1^2}\right)y+\left(\frac{\alpha_1\beta_{2,1}}{4}\right.\right.$$
$$\left.\left.\left.+\frac{\alpha_1^2\beta_{3,1}}{6}-\frac{\beta_{2,1}}{12}\right)y^3-\frac{\alpha_1^2\beta_{2,1}}{10}\,y^5\right\}\sinh\alpha_1 y\right], \qquad [3.42]$$

$$I_{2,1}=R_{1,1}\cosh\alpha_1 y+R_{2,1}\sinh\alpha_1 y+[\alpha_1 K_{2,1}\,y^2\cosh\alpha_1 y+(\tfrac{2}{3}\alpha_1/K_{3,1}\,y^3$$
$$-K_{2,1}\,y)\sinh\alpha_1 y]+p_x^{(0)}\left[\left\{\frac{\alpha_1\beta_{3,1}}{288}\,y^6+\left(\frac{7\beta_{3,1}}{64\,\alpha_1}-\frac{\alpha_1\beta_{3,1}}{16}-\frac{\beta_{2,1}}{16}\right)y^4+\left(\frac{\beta_{3,1}}{2\,\alpha_1}\right.\right.\right.$$
$$\left.\left.+\frac{25\,\beta_{3,1}}{64\,\alpha_1^3}-\frac{\beta_{2,1}}{16\,\alpha_1^2}\right)y^2\right\}\cosh\alpha_1 y+\left\{-\frac{11\,\beta_{3,1}}{480}\,y^5-\left(\frac{\beta_{2,1}}{24\alpha_1}+\frac{5\,\beta_{3,1}}{96\,\alpha_1^2}\right)y^3\right.$$

$$+\left(\frac{\beta_{2,1}}{16\alpha_1^3}+\frac{\beta_{3,1}}{16\alpha_1^2}-\frac{25\beta_{3,1}}{64\alpha_1^4}\right)y\Bigg\}\sinh\alpha_1 y\Bigg]+\lambda_2' p_x^{(0)}\Bigg[\Bigg\{\Bigg(\frac{\alpha_1^2\beta_{2,1}}{8}$$

$$-\frac{\alpha_1\beta_{3,1}}{8}\Bigg)y^4+\left(\frac{\beta_{2,1}}{8}-\frac{\alpha_1^2\beta_{2,1}}{4}-\frac{\beta_{3,1}}{8\alpha_1}\right)y^2\Bigg\}\cosh\alpha_1 y+\Bigg\{\frac{\alpha_1^2\beta_{3,1}}{10}y^5$$

$$+\left(\frac{\beta_{3,1}}{12}-\frac{\alpha_1^3\beta_{3,1}}{6}-\frac{\alpha_1\beta_{2,1}}{4}\right)y^3+\left(\frac{\alpha_1\beta_{2,1}}{4}-\frac{\beta_{2,1}}{8\alpha_1}+\frac{\beta_{3,1}}{8\alpha_1^2}\right)y\Bigg\}\sinh\alpha_1 y\Bigg]$$

$$+p_x^{(0)}\sigma\Bigg[\Bigg\{\frac{\alpha_1\beta_{3,1}}{36}y^6+\left(\frac{7\beta_{3,1}}{24}-\frac{\alpha_1\beta_{3,1}}{24}-\frac{11\beta_{2,1}}{48}\right)y^4+\frac{M_{2,1}}{12}y^3$$

$$+\left(\frac{7\beta_{3,1}}{8\alpha_1^3}-\frac{\beta_{3,1}}{8\alpha_1}-\frac{7\beta_{2,1}}{12\alpha_1^2}-\frac{M_{1,1}}{8\alpha_1}\right)y^2+\Bigg(\frac{M_{2,1}}{8\alpha_1^2}+\frac{\beta_{3,1}}{12\alpha_1^2}-\frac{\beta_{2,1}}{6\alpha_1}$$

$$-\frac{M_{2,1}}{4}\Bigg)y\Bigg\}\cosh\alpha_1 y+\Bigg\{\left(\frac{\alpha_1\beta_{2,1}}{20}-\frac{7\beta_{3,1}}{60}\right)y^5+\Bigg(\frac{M_{1,1}}{12}+\frac{\alpha_1\beta_{2,1}}{12}$$

$$+\frac{11\beta_{2,1}}{24\alpha_1}+\frac{\beta_{3,1}}{12}-\frac{7\beta_{3,1}}{12\alpha_1^2}\Bigg)y^3-\left(\frac{M_{2,1}}{8\alpha_1}+\frac{\beta_{3,1}}{12\alpha_1}\right)y^2+\Bigg(\frac{M_{1,1}}{8\alpha_1^2}+\frac{11\beta_{2,1}}{16\alpha_1^3}$$

$$+\frac{\beta_{3,1}}{8\alpha_1^2}-\frac{7\beta_{3,1}}{8\alpha_1^4}-\frac{M_{1,1}}{4}\Bigg)y\Bigg\}\sinh\alpha_1 y\Bigg], \qquad [3.43]$$

$$J_{2,1}=S_{1,1}\cosh\alpha_1 y+S_{2,1}\sinh\alpha_1 y+[\alpha_1 K_{2,1}' y^2\cosh\alpha_1 y$$

$$+(\tfrac{2}{3}\alpha_1, K_{3,1}' y^3-K_{2,1}' y)\sinh\alpha_1 y]$$

$$+\lambda_1'^2 p_x^{(0)2}\Bigg[\Bigg\langle-\left(\frac{103\alpha_1^2\beta_{3,1}}{1440}+\frac{13\alpha_1^3\beta_{3,1}}{288}\right)y^6+\Bigg(\frac{95\alpha_1\beta_{2,1}}{192}+\frac{\alpha_1^3\beta_{2,1}}{8}$$

$$+\frac{\alpha_1^2\beta_{3,1}}{4}-\frac{149}{192}\Bigg)y^4+\Bigg(\frac{115\beta_{2,1}}{64\alpha_1}-\frac{\alpha_1\beta_{2,1}}{8}-\frac{\alpha_1^3\beta_{2,1}}{8}$$

$$-\frac{1(5}{64\alpha_1^2}\Bigg)y\Bigg\rangle\cosh\alpha_1 y+\Bigg\langle-\frac{9\alpha_1^3\beta_{3,1}}{280}y^7+\Bigg(\frac{31\alpha_1\beta_{3,1}}{96}+\frac{\alpha_1^3\beta_{3,1}}{20}$$

$$-\frac{23\alpha_1^2\beta_{2,1}}{480}\Bigg)y^5+\left(\frac{83\beta_{3,1}}{32\alpha_1}-\frac{151\beta_{2,1}}{96}-\frac{\alpha_1^2\beta_{2,1}}{4}\right)y^3+\Bigg(\frac{\beta_{2,1}}{8}+\frac{\alpha_1^2\beta_{2,1}}{8}$$

$$-\frac{115\beta_{2,1}}{64\alpha_1^3}+\frac{\beta_{3,1}}{8\alpha_1}-\frac{\alpha_1\beta_{3,1}}{8}+\frac{9\beta_{3,1}}{8\alpha_1}\Bigg)y\Bigg\rangle\sinh\alpha_1 y\Bigg], \qquad [3.44]$$

where

$$M_{1,1} = [1/(\alpha_1 \cosh 2\alpha_1)]\,[\tfrac{2}{3}\alpha_1 \cosh\alpha_1 + \sinh\alpha_1 - \tfrac{2}{3}\alpha_1^2 \beta_{3,1} \sinh 2\alpha_1 + (\beta_{2,1} - 2\alpha_1^2 \beta_{2,1} - 2\alpha_1 \beta_{3,1}) \sinh^2\alpha_1 - \alpha_1^2 \beta_{2,1}], \quad [3.45a]$$

$$M_{2,1} = [1/(\alpha_1 \cosh 2\alpha_1)]\,[\tfrac{2}{3}\alpha_1 \sinh\alpha_1 - \cosh\alpha_1 + (\alpha_1\beta_{3,1} - \tfrac{1}{2}\beta_{2,1}) \sinh 2\alpha_1 + 2\alpha_1 \beta_{2,1} \cosh^2\alpha_1 + (\tfrac{2}{3}\alpha_1^2 \beta_{3,1} - \alpha_1 \beta_{2,1})], \quad [3.45b]$$

$$P_{1,1} = [\lambda_1' p_x^{(0)}/(\alpha_1 \cosh 2\alpha_1)]\,[\{\beta_{2,1}\,[(\alpha_1/8 - \alpha_1^3/8) - \beta_{3,1}(\alpha_1^2/8 + \tfrac{1}{8})\} \cosh^2\alpha_1 + \{-\beta_{2,1}(1/8\alpha_1 + 3\alpha_1/8 + \alpha_1^3/8) + \beta_{3,1}(\tfrac{1}{8} + 1/8\alpha_1^2 - \alpha_1^2/8)\} \sinh^2\alpha_1 - \beta_{3,1}(\alpha_1^3/15 + \alpha_1/6) \sinh 2\alpha_1], \quad [3.45c]$$

$$P_{2,1} = [(\lambda_1' p_x^{(0)}/(\alpha_1 \cosh 2\alpha_1)]\,[-\{\beta_{2,1}/8 + \beta_{3,1}(\alpha_1^3/15 - \alpha_1/12 - 1/8\alpha_1)\} \sinh^2\alpha_1 + \{-\beta_{2,1}/8 + \beta_{3,1}(1/8\alpha_1 + 5\alpha_1/12 + \alpha_1^3/15)\} \cosh^2\alpha_1 + \{\beta_{2,1}(\alpha_1/4 + 1/16\alpha_1) - \beta_{3,1}(\tfrac{1}{8} + 1/16\alpha_1^2)\} \sinh 2\alpha_1], \quad [3.45d]$$

$$\begin{aligned} R_{1,1} = {} & [1/(\alpha_1 \cosh 2\alpha_1)][\{-\alpha_1^2 K_{2,1} \cosh^2\alpha_1 + (K_{2,1} - \alpha_1^2 K_{2,1} - 2\alpha_1 K_{3,1}) \sinh^2\alpha_1 \\ & - \tfrac{2}{3}\alpha_1^2 K_{3,1} \sinh 2\alpha_1\} + p_x^{(0)} \{[\beta_{2,1}(\alpha_1/16 + 1/16\alpha_1) + \beta_{3,1}(17\alpha_1^2/288 \\ & - 39/64 - 25/64\alpha_1^2)] \cosh^2\alpha_1 + [\beta_{2,1}(\alpha_1/16 + 3/16\alpha_1 - 1/16\alpha_1^3) \\ & + \beta_{3,1}(17\alpha_1^2/288 + 25/64\alpha_1^4 - 19/64\alpha_1^2 - 95/192)] \sinh^2\alpha_1 + [\beta_{2,1}/6 \\ & + \beta_{3,1}(19\alpha_1/120 - 35/48\alpha_1)] \sinh 2\alpha_1\} + \lambda_2' p_x^{(0)} \{[\beta_{2,1}(\alpha_1^3/8 - \tfrac{1}{8}) \\ & + \beta_{3,1}(\alpha_1^2/8 + \tfrac{1}{8})] \cosh^2\alpha_1 [\beta_{2,1}(\alpha_1^3/8 + 3\alpha_1/8 + 1/8\alpha_1) + \beta_{3,1}(\alpha_1^2/8 \\ & - \tfrac{1}{8} - 1/8\alpha_1^2)] \sinh^2\alpha_1 + [\beta_{3,1}(\alpha_1^3/15 + \alpha_1/6)] \sinh 2\alpha_1\} + p_x^{(0)} \sigma \{[M_{1,1}/8 \\ & + M_{2,1}(\tfrac{1}{6} - 1/8\alpha_1^2) + \beta_{2,1}(11\alpha_1/48 + \tfrac{1}{6} + 7/12\alpha_1) + \beta_{3,1}(\alpha_1^2/72 - \tfrac{1}{6} \\ & - 1/12\alpha_1 - 7/8\alpha_1^2)] \cosh^2\alpha_1 + [-M_{2,1}(\alpha_1/6 + 1/8\alpha_1) + M_{1,1}(\tfrac{1}{8} \\ & - 1/8\alpha_1^2) - \beta_{2,1}(\tfrac{1}{6} + 13\alpha_1/48 + 19/24\alpha_1 + 11/16\alpha_1^3) + \beta_{3,1}(\alpha_1^2/72 + \tfrac{1}{6} \\ & + 3/4\alpha_1^2 - 1/12\alpha_1 + 7/8\alpha_1^4)] \sinh^2\alpha_1 + [-\alpha_1 M_{1,1}/6 + M_{2,1}/16\alpha_1^2 \\ & - \beta_{2,1}(2\alpha_1^2/15 + 15/32\alpha_1^2 + 1/12\alpha_1) + \beta_{3,1}(\alpha_1/30 - 5/24\alpha_1 \\ & + 1/24\alpha_1^2)] \sinh 2\alpha_1\}], \end{aligned} \quad [3.45e]$$

$$R_{2,1} = \frac{1}{\alpha_1 \cosh 2\alpha_1} \Big\langle \{(\tfrac{2}{3}\alpha_1^2 K_{3,1} + \alpha_1 K_{2,1}) \cosh^2 \alpha_1 + (\alpha_1 K_{2,1}$$

$$- \tfrac{2}{3}\alpha_1^2 K_{3,1}) \sinh^2 \alpha_1 + (\alpha_1 K_{3,1} - \tfrac{1}{2} K_{2,1}) \sinh 2\alpha_1\}$$

$$+ p_x^{(0)} \left\{\left[-\beta_{2,1}\left(\frac{7}{24} + \frac{1}{16\alpha_1^2}\right) + \beta_{3,1}\left(\frac{25}{64\alpha_1^3} + \frac{139}{96\alpha_1} - \frac{47\alpha_1}{160}\right)\right]\cosh^2\alpha_1\right.$$

$$+ \left[\beta_{2,1}\left(\frac{1}{24} - \frac{1}{16\alpha_1^2}\right) + \beta_{3,1}\left(\frac{11\alpha_1}{480} - \frac{1}{96\alpha_1} + \frac{25}{64\alpha_1^3}\right)\right]\sinh^2\alpha_1$$

$$\left. + \left[\beta_{2,1}\left(\frac{1}{32\alpha_1^3} - \frac{1}{16\alpha_1}\right) - \beta_{3,1}\left(\frac{11}{192} + \frac{3}{64\alpha_1^2} + \frac{25}{128\alpha_1^4}\right)\right)\sinh 2\alpha_1\right\}$$

$$+ \lambda_2' p_x^{(0)} \left\{\left[\frac{\beta_{2,1}}{8} - \beta_{3,1}\left(\frac{\alpha_1^3}{15} + \frac{5\alpha_1}{12} + \frac{1}{8\alpha_1}\right)\right]\cosh^2\alpha_1 + \left[\frac{\beta_{2,1}}{8}\right.\right.$$

$$\left. + \beta_{3,1}\left(\frac{\alpha_1^3}{15} - \frac{\alpha_1}{12} - \frac{1}{8\alpha_1}\right)\right]\sinh^2\alpha_1 + \left[-\beta_{2,1}\left(\frac{1}{16\alpha_1} + \frac{\alpha_1}{4}\right)\right.$$

$$\left.\left. + \beta_{3,1}\left(\frac{1}{8} + \frac{1}{16\alpha_1^2}\right)\right]\sinh 2\alpha_1\right\} + p_x^{(0)}\sigma \left\{\left[-M_{1,1}\left(\frac{\alpha_1}{6} + \frac{1}{8\alpha_1}\right)\right.\right.$$

$$+ M_{2,1}\left(\frac{1}{8} - \frac{1}{8\alpha_1^2}\right) + \beta_{2,1}\left(\frac{2\alpha_1^2}{15} - \frac{11}{24} + \frac{1}{4\alpha_1^2} + \frac{1}{6\alpha_1}\right) + \beta_{3,1}\left(\frac{1}{12} + \frac{7}{8\alpha_1}\right.$$

$$\left.\left. + \frac{7}{8\alpha_1^3} - \frac{1}{12\alpha_1^2} - \frac{\alpha_1}{30}\right)\right]\cosh^2\alpha_1 + \left[\frac{M_{2,1}}{8} + M_{1,1}\left(\frac{\alpha_1}{6} - \frac{1}{8\alpha_1}\right)\right.$$

$$\left. - \beta_{2,1}\left(\frac{2\alpha_1^2}{15} + \frac{11}{24} + \frac{11}{16\alpha_1^2}\right) + \beta_{3,1}\left(\frac{11}{24\alpha_1} + \frac{7}{8\alpha_1^3} + \frac{1}{12} + \frac{\alpha_1}{30}\right)\right]\sinh^2\alpha_1$$

$$+ \left[\frac{M_{1,1}}{16\alpha_1^2} - \frac{M_{2,1}}{16\alpha_1} + \beta_{2,1}\left(\frac{1}{6} + \frac{\alpha_1}{4} + \frac{19}{48\alpha_1} + \frac{11}{32\alpha_1^3}\right) - \beta_{3,1}\left(\frac{1}{6} + \frac{13}{16\alpha_1^2}\right.\right.$$

$$\left.\left.\left. + \frac{7}{16\alpha_1^4}\right)\right]\sinh 2\alpha_1\right\}\Big\rangle, \qquad [3.45f]$$

$$S_{1,1} = \frac{1}{\alpha_1 \cosh 2\alpha_1}\Big\langle\Big\{-\alpha_1^2 K_{2,1}' \cosh^2\alpha_1 + (\alpha_1 K_{2,1}' - \tfrac{2}{3}\alpha_1^2 K_{3,1}')\sinh^2\alpha_1$$

$$- \frac{2\alpha_1^2}{3} K_{3,1}' \sinh 2\alpha_1\Big\} + \lambda_2'^2 p_x^{(0)2}\left\{\left[\beta_{2,1}\left(\frac{13\alpha_1^4}{288} - \frac{71\alpha_1^2}{192} - \frac{115}{64}\right)\right.\right.$$

$$+ \beta_{3,1}\left(\frac{165}{64\,\alpha_1}+\frac{149\,\alpha_1}{192}-\frac{257\,\alpha_1^3}{1440}\right)\Bigg]\cosh^2\alpha_1 + \Bigg[\beta_{2,1}\left(\frac{13\,\alpha_1^4}{288}+\frac{95\,\alpha_1^2}{192}\right.$$

$$\left.+\frac{179}{64}+\frac{115}{64\,\alpha_1^2}\right) + \beta_{3,1}\left(-\frac{293\,\alpha_1^3}{1440}-\frac{137\,\alpha_1}{192}+\frac{1}{8}-\frac{405}{64\,\alpha_1}\right.$$

$$\left.\left.+\frac{165}{64\,\alpha_1^3}\right)\right]\sinh^2\alpha_1 + \left[\beta_{2,1}\left(\frac{83\,\alpha_1^3}{960}+\frac{31\,\alpha_1^2}{320}+\frac{7\,\alpha_1}{12}\right) - \beta_{3,1}\left(\frac{\alpha_1^4}{56}+\frac{29\,\alpha_1^2}{60}\right.\right.$$

$$\left.\left.+\frac{78}{48}-\frac{165}{128\,\alpha_1^2}+\frac{\alpha_1}{8}-\frac{9}{16\,\alpha_1}\right)\right]\sinh 2\,\alpha_1\Big\}\Big\rangle, \qquad [3.45g]$$

$$S_{2,1} = \frac{1}{\alpha_1\cosh 2\,\alpha_1}\Big\langle \{(\tfrac{2}{3}\alpha_1^2 K'_{3,1} + \alpha_1 K'_{2,1})\cosh^2\alpha_1 + (\alpha_1 K'_{2,1}$$

$$-\tfrac{2}{3}\,\alpha_1^2 K'_{3,1})\sinh^2\alpha_1 + (\alpha_1 K'_{3,1} - \tfrac{1}{2}K'_{2,1})\sinh 2\,\alpha_1\}$$

$$+\lambda_1'^2 p_x^{(0)2}\Bigg\{\Bigg[\beta_{2,1}\left(-\frac{31\,\alpha_1^2}{60}+\frac{9\,\alpha_1}{32}+\frac{115}{64\,\alpha_1}\right) + \beta_{3,1}\left(\frac{\alpha_1^4}{56}+\frac{123\,\alpha_1^2}{160}\right.$$

$$\left.-\frac{49}{96}+\frac{\alpha_1}{8}-\frac{165}{32\,\alpha_1^2}-\frac{9}{8\,\alpha_1}\right)\Bigg]\cosh^2\alpha_1 + \Bigg[\beta_{2,1}\left(\frac{83\,\alpha_1^3}{480}+\frac{139\,\alpha_1}{96}+\frac{115}{64\,\alpha_1}\right.$$

$$\left.-\beta_{3,1}\left(\frac{\alpha_1^4}{56}+\frac{19\,\alpha_1^2}{96}+\frac{119}{32}+\frac{165}{64\,\alpha_1^2}+\frac{\alpha_1}{8}\right)\right]\sinh^2\alpha_1$$

$$+\Bigg[-\beta_{2,1}\left(\frac{83\,\alpha_1^2}{192}+\frac{147}{64}+\frac{115}{128\,\alpha_1^2}\right) + \beta_{3,1}\left(-\frac{\alpha_1^3}{80}+\frac{143\,\alpha_1}{192}-\frac{1}{16}\right.$$

$$\left.+\frac{285}{64\,\alpha_1}-\frac{165}{128\,\alpha_1^3}\right)\Bigg]\sinh 2\,\alpha_1\Big\}\Big\rangle \qquad [3.45h]$$

Fig. III depicts the isotherms for the same set of parameters as in case (*a*). The upper wall being an isotherm, the isotherm in its neighbourhood proceed more or less parallel to it. In this case also we do not get any straight isotherm and this is a difference between the flow patterns for Rivlin-Ericksen and the present model. The temperature increases on the isotherms situated farther and farther away from the upper boundary. The lower wall is not an isotherm and the temperature on this boundary decreases from $x = 0$ to $x = \pi/2$, increases from $x = \pi/2$ to $x = 3\pi/2$ and again decreases from $x = 3\pi/2$ to $x = 2\pi$.

We also note that very near to the insulated wall, the isotherms starts on it and ends on it The maximum and minimum temperatures are at $x = 3\pi/2$ and $x = \pi/2$ on it.

4. CONCLUSIONS

In conclusion, we record some of the important points of our investigation.

(1) The velocity field is affected by the stress relaxation time only, while the stresses and the temperature field are affected by both the stress relaxation and the rate of strain retardation times. The stream lines and the isotherms show a slight shift in comparison to Rivlin Ericksen fluids.

(2) The stream lines near the boundaries run parallel to them and the deformity of the stream lines decreases as they approaeh the mid-plane where they become just straight in view of the symmetry about the mid-plane.

(3) When both the boundaries are maintained at the same constant temperature, the isothrems near the boundaries, which themselves are isotherms are more or less parallel to them. The deformity of the isotherms increases as we approach the mid-plane so much so that between $x = \pi/2$ and $x = 2\pi$ the isotherms form closed loops.

(4) When the upper wall is maintained at a constant temperature and the lower wall is heat insulated, the isotherms near the upper wall, which itself, is an isotherm, proceed more or less parallel to it. The temperature increases on the isotherms situated farther and farther away from this wall. The lower wall is not an isotherm and temperature on it decreases from $x = 0$ to $x = \pi/2$, increases from $x = \pi/2$ to $x = 3\pi/2$ and again decreases from $x = 3\pi/2$ to $x = 2\pi$.

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