TRANSPORT PROCESSES IN A MULTICOMPONENT ASSEMBLY ON THE BASIS OF RECONSIDERED GENERALIZED B-G-K COLLISION MODEL

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ABSTRACT

Bhatnagar, Gross and Krook developed a collision model for one component neutral assembly in order to overcome the inherent difficulties of the Boltzmann collision integral. This has been generalized to an N-component assembly of charged and neutral particles by Bhatnagar and Devanathan. However, for the simplicity of the model, the above mentioned authors have taken the collision cross section to be a constant. In fact the collision cross section is some unknown function of the relative velocity between the particles, as has already been pointed out by Krook and also by Koga. Koga gave some expressions for the cross section, which are linear and exponential function of the square of relative velocity, and calculated the coefficients involved in the model. In this paper, choosing the same types of cross section as suggested by Koga, we have obtained the transport equations by expanding the distribution functions in generalized Hermite polynomials following Grad. It is found that variable cross section enhances the relaxations times of all physical variables and thereby decreases the electrical conductivity and diffusion coefficients. Also such variable cross section introduces very high anisotropy even in the absence of magnetic field and the magnitudes of the viscosity coefficients in the principal directions are decreased. The bulk viscosity of the plasma is increased apart from introducing anisotropy. Further this model introduces anisotropy in the heat flux tensor also. All the transport properties have been obtained in an earlier paper by Devanathan, Uberoi and Bhatnagar. We have obtained the modified expressions for viscosity, electrical conductivity, heat conductivity and diffusion coefficients.

1. INTRODUCTION

Bhatnagar, Gross and Krook[1] gave a simple meaningful model for the binary collisions in an assembly. This was later generalized by Bhatnagar[2], Bhatnagar and Devanathan[3] for two component and multicomponent assemblies respectively including the electromagnetic effects. One of the basic assumptions in the above quoted investigations is that they considered the collision cross section between two interacting particles of two specified species to be constant. Devanathan, Uberoi and Bhatnagar[4] have used the generalized B-G-K model for evaluating the transport properties using the generalized Hermite polynomial. It is very well known that the collision cross section of interacting particles depend on the magnitude of their relative velocity. Consequently, in a recent paper Koga[5] has reconsidered the B-G-K model taking into account the functional dependance of cross section on the relative velocity of the interacting particles. He has considered the following two cases: (1) Collision cross section as linear function of square of relative velocity and (2) Collision cross section as exponential function of the square of relative velocity.

In the present paper we have taken the cross section to be of the form

$$\frac{1}{\sigma_{\delta\alpha}} = \frac{1}{\sigma^{(0)}_{\delta\alpha}} \cdot \left[A + B\, C^2_{\delta\alpha} \right], \qquad [1.1]$$

where $C_{\delta\alpha}$ is the relative velocity between the interacting particles. Using the above cross section, [1.1] feeding it into the collision integral of the Maxwell-Boltzmann equation, and integrating over the entire velocity range of the scatterer denoted by suffix δ we get the right hand side of the B-G-K model as a function of the first and the second power of the molecular velocity of the scattered particle (α).

After reconsidering the B-G-K model, in section 2 we establish the consistant set of transport equations following the procedure of Grad[6] and in section 3 we calculate all the transport coefficients like viscosity, heat and electrical conductivities, by interpreting the transport equations in an appropriate manner.

2. Derivation of Transport Equations

In order to derive the transport equations governed by the kinetic equations, we introduce the non-dimensional velocity $\mathbf{v}_\alpha$ and the non-dimensional distribution function g_α given by

$$\mathbf{v}_\alpha = \left(\frac{m_\alpha}{k\, T_{\alpha\alpha}} \right)^{1/2} \vec{\xi}_\alpha = \vec{\xi}_\alpha / c_\alpha \qquad [2.1]$$

where

$$C_\alpha = \left(\frac{k\, T_{\alpha\alpha}}{m_\alpha} \right)^{1/2} \qquad [2.2]$$

and

$$g_\alpha = \frac{C^3_\alpha}{N_\alpha} \cdot f_\alpha, \qquad [2.3]$$

where N_α is the number density, $T_{\alpha\alpha}$ the mean temperature of the particles of type α with molecular velocity $\vec{\xi}_\alpha$ and mass m_α and k is the Boltzmann constant. The non-dimensional distribution functions satisfy the kinetic equation.

$$\frac{\partial g_a}{\partial t} + C_a v_{ai} \frac{\partial g_a}{\partial x_i} + \left[\frac{1}{c_a}\left(\frac{e_a E_i}{m_a} + \frac{F_{ai}}{m_a}\right) + \frac{e_a \epsilon_{ijk} H_k v_{aj}}{c m_a}\right] \frac{\partial g_a}{\partial v_{ai}}$$

$$+ g_a \frac{\partial \log (N_a / c_a^3)}{\partial t} + c_a v_{ai} \frac{\partial (\log N_a / c_a^3)}{\partial x_i}$$

$$- \sum_\delta \left(\frac{T_{aa}}{2 \pi T_{\delta a}}\right)^{3/2} \frac{N_\delta}{\sigma_{\delta a}^{(0)}} \cdot \exp. \left[\frac{-T_{aa}}{2 T_{\delta a}} (\mathbf{v}_a - \mathbf{u}'_{\delta a})^2\right] \cdot [K_{\delta a} + L_{\delta ai} v_{ai} + M_{\delta a} \mathbf{v}_a^2]$$

$$- \sum_\delta \frac{N_\delta}{\sigma_{\delta a}^{(0)}} \cdot [K_{\delta a} + L_{\delta ai} v_{ai} + M_{\delta a} \mathbf{v}_a^2] g_a \qquad [2.4]$$

In writing the above we have denoted by $\mathbf{u}_{\delta a}$ the non-dimensional velocity and by $T_{\delta a}$ the temperature of the scattered a particles due to their encounter with δ type particles; $K_{\delta a}$, $L_{\delta a}$ and $M_{\delta a}$ are the constant coefficients of the reconsidered B-G-K model. We expand the non-dimensional distribution function g_a in the form

$$g_a (\mathbf{v}_a, \mathbf{r}, t) = \omega (\mathbf{v}_a) \sum_{n=0}^{\infty} a_a^{(n)} (\mathbf{r}, t) \cdot H^{(n)} (\mathbf{v}_a), \qquad [2.5]$$

where

$$\omega (\mathbf{v}_a) = \frac{1}{(2 \pi)^{3/2}} \cdot \exp.\left(-\frac{\mathbf{v}_a^2}{2}\right) \qquad [2.6]$$

It has already been pointed out by Grad that all the physical variables are given by only considering $n = 3$ terms in the above expansion of distribution function. Hence we will truncate the series for the non-dimensional distribution function only at $n = 3$. By the orthogonality property of the generalised Hermite polynomials $H^{(n)} (\mathbf{v}_a)$ we have

$$a_a^{(n)} (\mathbf{r}, t) = (1/X_a^{(n)}) \cdot \int H^{(n)} (\mathbf{v}_a) g_a \, d \mathbf{v}_a \qquad [2.7]$$

where

$$X_a^{(n)} = \int \omega (\mathbf{v}_a) \cdot [H^{(n)} (\mathbf{v}_a)]^2 \, d \mathbf{v}_a \qquad [2.8]$$

From the kinetic equation [2·4] the equations governing the coefficients $a_a^{(n)}$ can be obtained by multiplying throughout by $H^{(n)}(\mathbf{v}_a)$ and integrating over the entire velocity range of $\mathbf{v}_a$. In Cartesian tensor product notation, we have

$$X_a^{(n)} \frac{\partial a_a^{(n)}}{\partial t} + [(n+3) X_a^{(n)} a_a^{(n)} + 2 X_a^{(n-2)} \delta^{(2)} a_a^{(n-2)}] \frac{\partial \log c_a}{\partial t} + c_a \left[X_a^{(n+1)} \frac{\partial a_{ai}^{(n+1)}}{\partial x_i}\right.$$

$$+ X_a^{(n-1)} \delta_i^{(2)} \frac{\partial a_a^{(n-1)}}{\partial x_i} + (n+4) \{X_a^{(n-1)} a_{ai}^{(n+1)} + X_a^{(n-1)} \delta_i^{(2)} a_a^{(n-1)}\} \frac{\partial \log c_a}{\partial x_i}$$

$$+2\{X_\alpha^{(n-1)}\,\delta^{(2)}\,a_{\alpha i}^{(n-1)}+X_\alpha^{(n-3)}\,\delta_i^{(2)}\,\delta^{(2)}\,a_\alpha^{(n-3)}\}\frac{\partial \log c_\alpha}{\partial x_i}\Bigg]-\frac{1}{c_\alpha}\left(\frac{e_\alpha E_i}{m_\alpha}\right.$$

$$\left.+\frac{F_{\alpha i}}{m_\alpha}\right)X_\alpha^{(n-1)}\,\delta_i^{(2)}\,a_\alpha^{(n-1)}-\frac{e_\alpha\,\epsilon_{ijk}\,H_k}{c\,m_\alpha}\{X_\alpha^{(n)}\,\delta_i^{(2)}\,a_{\alpha j}^{(n)}+X_\alpha^{(n-2)}\,\delta_i^{(2)}\,\delta_j^{(2)}\,a_\alpha^{(n-2)}\}$$

$$+X_\alpha^{(n)}\,a_\alpha^{(n)}\frac{\partial(\log N_\alpha/c_\alpha^3)}{\partial t}+c_\alpha\{X_\alpha^{(n+1)}\,a_{\alpha i}^{(n+1)}+X_\alpha^{(n-1)}\,\delta_i^{(2)}\,a_\alpha^{(n-1)}\}\frac{\partial(\log N_\alpha/c_\alpha^3)}{\partial x_i}$$

$$-\sum_\delta\left(\frac{T_{\alpha\alpha}}{T_{\delta\alpha}}\right)^{3/2}\cdot\frac{N_\delta}{\sigma_{\delta\alpha}^{(0)}}\cdot A_{\delta\alpha}^{(n)}-\sum_\delta\frac{N_\delta}{\sigma_{\delta\alpha}^{(0)}}\cdot\Big[K_{\delta\alpha}\,X_\alpha^{(n)}\,a_\alpha^{(n)}+L_{\delta\alpha i}\{X_\alpha^{(n+1)}\,a_{\alpha i}^{(n+1)}$$

$$+X_\alpha^{(n-1)}\,\delta_i^{(2)}\,a_\alpha^{(n-1)}\}+M_{\delta\alpha}\{X_\alpha^{(n+2)}\,a_{\alpha ii}^{(n+2)}+2\,X_\alpha^{(n)}\,\delta_i^{(2)}\,a_{\alpha i}^{(n)}$$

$$+X_\alpha^{(n-2)}\,\delta_i^{(2)}\,\delta_i^{(2)}\,a_\alpha^{(n-2)}\}\Big] \qquad [2.9]$$

where

$$A_{\delta\alpha}^{(n)}=[1/(2\pi)^{3/2}]\cdot\int[K_{\delta\alpha}+L_{\delta\alpha i}\,v_{\alpha i}$$

$$+v_{\alpha i}\,v_{\alpha i}\,M_{\delta\alpha}]\,H^{(n)}(\mathbf{v}_\alpha)\exp[-(T_{\alpha\alpha}/2\,T_{\delta\alpha})(\mathbf{v}_\alpha-\mathbf{u}'_{\delta\alpha})^2]\,d\,\mathbf{v}_\alpha. \qquad [2.10]$$

We note the following points about the equations [2·9]. The variable cross section introduces coupling with lower and higher order moments. Following Grad, we truncate the expansion at $n=3$ With the help of the above assumption, we get the consistent set of equations. Since $a_\alpha^{(n)}$ are nothing but linear combinations of the physical moments, converting back we obtain the following transport equations:

$$\frac{\partial N_\alpha}{\partial t}+\frac{\partial(N_\alpha\,u_{\alpha\alpha i})}{\partial x_i}=0 \qquad [2.11]$$

$$\frac{1}{N_\alpha}\cdot\frac{\partial(N_\alpha\,u_{\alpha\alpha p})}{\partial t}+\frac{1}{N_\alpha}\cdot\frac{\partial\cdot(N_\alpha\,P_{\alpha ip})}{\partial x_i}-\left(\frac{e_\alpha\,E_p}{m_\alpha}+\frac{F_{\alpha p}}{m_\alpha}\right)-\frac{e_\alpha\,\epsilon_{pik}\,H_k}{cm_\alpha}\,u_{\alpha\alpha j}$$

$$-c_\alpha\sum_\delta\left(\frac{T_{\alpha\alpha}}{T_{\delta\alpha}}\right)^{3/2}\cdot\frac{N_\delta}{\sigma_{\delta\alpha}^{(0)}}\cdot A_{\delta\alpha p}^{(1)'}-\sum_\delta{}'\frac{N_\delta}{\sigma_{\delta\alpha}^{(0)}}\cdot\Bigg[K_{\delta\alpha}\,u_{\alpha\alpha p}+\frac{L_{\delta\alpha i}\,P_{\alpha ip}}{c_\alpha}$$

$$+\frac{M_{\delta\alpha}\,S_{\alpha iip}}{c_\alpha^2}\Bigg] \qquad [2.12]$$

$$\frac{1}{N_\alpha}\cdot\frac{\partial (N_\alpha P_{\alpha pp})}{\partial t}+\frac{1}{N_\alpha}\cdot\frac{\partial (N_\alpha S_{\alpha ipp})}{\partial x_i}-2\left(\frac{e_\alpha E_p}{m_\alpha}+\frac{F_{\alpha p}}{m_\alpha}\right)u_{\alpha\alpha p}$$

$$-\frac{2\,e_\alpha\,\epsilon_{pjk}\,H_k}{c\,m_\alpha}\cdot P_{\alpha jp}=c_\alpha^2\sum_\delta\left(\frac{T_{\alpha\alpha}}{T_{\delta\alpha}}\right)^{3/2}\frac{N_\delta}{\sigma^{(0)}_{\delta\alpha}}\cdot\left(2\,A^{(2)}_{\delta\alpha pp}+A^{(0)}_{\delta\alpha}\right)$$

$$-\sum_\delta\frac{N_\delta}{\sigma^{(0)}_{\delta\alpha}}\cdot\left[K_{\delta\alpha}\cdot P_{\alpha pp}+\frac{L_{\delta\alpha i}\,S_{\alpha ipp}}{c_\alpha}+M_{\delta\alpha}\{P_{\alpha ii}+\delta_{ip}(7\,P_{\alpha ii}-5\,c_\alpha^2)\}\right] \quad [2.13]$$

$$\frac{1}{N_\alpha}\cdot\frac{\partial (N_\alpha P_{\alpha pq})}{\partial t}+\frac{1}{N_\alpha}\cdot\frac{\partial (N_\alpha S_{\alpha ipq})}{\partial x_i}-\left(\frac{e_\alpha E_p}{m_\alpha}+\frac{F_{\alpha p}}{m_\alpha}\right)u_{\alpha\alpha q}$$

$$-\left(\frac{e_\alpha E_q}{m_\alpha}+\frac{F_{\alpha q}}{m_\alpha}\right)u_{\alpha\alpha p}-\frac{e_\alpha H_k}{c m_\alpha}\cdot\left[\epsilon_{pjk}\,P_{\alpha jq}+\epsilon_{qjk}+P_{\alpha jp}\right]$$

$$=c_\alpha^2\sum_\delta\left(\frac{T_{\alpha\alpha}}{T_{\delta\alpha}}\right)^{3/2}\frac{N_\delta}{\sigma^{(0)}_{\delta\alpha}}\cdot A^{(2)}_{\delta\alpha pq}-\sum_\delta\frac{N_\delta}{\sigma^{(0)}_{\delta\alpha}}\cdot\left[K_{\delta\alpha}\,P_{\alpha pq}+\frac{L_{\delta\alpha i}\,S_{\alpha ipq}}{c_\alpha}+7\,M_{\delta\alpha}\,P_{\alpha pq}\right] \quad [2.14]$$

$$\frac{1}{N_\alpha}\cdot\frac{\partial (N_\alpha S_{\alpha ppp})}{\partial t}-\frac{1}{N_\alpha}\cdot\frac{\partial (N_\alpha Q_{\alpha ippp})}{\partial x_i}-3\left(\frac{e_\alpha E_p}{m_\alpha}+\frac{F_{\alpha p}}{m_\alpha}\right)P_{\alpha pp}$$

$$-\frac{3\,e_\alpha\,\epsilon_{rjk}\,H_k}{cm_\alpha}\cdot S_{\alpha jpp}=3\,c_\alpha^3\sum_\delta\left(\frac{T_{\alpha\alpha}}{T_{\delta\alpha}}\right)^{3/2}\frac{N_\delta}{\sigma^{(0)}_{\delta\alpha}}\cdot\left(2\,A^{(3)}_{\delta\alpha ppp}+A^{(1)}_{\delta\alpha p}\right)$$

$$-\sum_\delta\frac{N_\delta}{\sigma^{(0)}_{\delta\alpha}}\cdot\left\{K_{\delta\alpha}\,S_{\alpha ppp}+\frac{L_{\delta\alpha i}\cdot Q_{\alpha ippp}}{c_\alpha}+M_{\delta\alpha}\left(3\,S_{\alpha iip}+9S_{\alpha ppp}-21\,u_{\alpha\alpha p}\,c_\alpha^2\right)\right\} \quad [2.15]$$

$$\frac{1}{N_\alpha}\cdot\frac{\partial (N_\alpha S_{\alpha ppq})}{\partial t}+\frac{1}{N_\alpha}\cdot\frac{\partial (N_\alpha Q_{\alpha ippq})}{\partial x_i}-2\left(\frac{e_\alpha E_p}{m_\alpha}+\frac{F_{\alpha p}}{m_\alpha}\right)P_{\alpha pq}$$

$$-\left(\frac{e_\alpha E_q}{m_\alpha}+\frac{F_{\alpha q}}{m_\alpha}\right)P_{\alpha pp}-\frac{e_\alpha H_k}{c\,m_\alpha}\cdot\left(2\,\epsilon_{pjk}\,S_{\alpha jpq}+\epsilon_{qjk}\,S_{\alpha jpp}\right)$$

$$=c_\alpha^3\sum_\delta\left(\frac{T_{\alpha\alpha}}{T_{\delta\alpha}}\right)^{3/2}\frac{N_\delta}{\sigma^{(0)}_{\delta\alpha}}\left(2\,A^{(3)}_{\delta\alpha ppq}+A^{(1)}_{\delta\alpha q}\right)-\sum_\delta\frac{N_\delta}{\sigma^{(0)}_{\alpha\delta}}\left[K_{\delta\alpha}\,S_{\alpha ppq}\right.$$

$$\left.+\frac{L_{\delta\alpha i}\,Q_{\alpha ippk}}{c_\alpha}+M_{\delta\alpha}\left(9S_{\alpha ppq}+S_{\alpha iiq}-8u_{\alpha\alpha q}c_\alpha^2\right)\right] \quad [2.16]$$

$$\frac{1}{N_\alpha}\cdot\frac{\partial(N_\alpha S_{\alpha pqr})}{\partial t}+\frac{1}{N_\alpha}\cdot\frac{\partial(N_\alpha Q_{\alpha ipqr})}{\partial x_i}-\left(\frac{e_\alpha E_p}{m_\alpha}+\frac{F_{\alpha p}}{m_\alpha}\right)P_{\alpha qr}$$

$$-\left(\frac{e_\alpha E_q}{m_\alpha}+\frac{F_{\alpha q}}{m_\alpha}\right)P_{\alpha rp}-\left(\frac{e_\alpha E_r}{m_\alpha}+\frac{F_{\alpha r}}{m_\alpha}\right)P_{\alpha pq}-\frac{e_\alpha H_k}{c\,m_\alpha'}(\epsilon_{pjk}S_{\alpha jqr}$$

$$+\epsilon_{qjk}S_{\alpha jrp}+\epsilon_{rjk}S_{\alpha jpq})=c_\alpha^3\sum_\delta\left(\frac{T_{\alpha\alpha}}{T_{\delta\alpha}}\right)^{3/2}\cdot\frac{N_\delta}{\sigma^{(0)}_{\delta\alpha}}A^{(3)}_{\delta\alpha pqr}$$

$$-\sum_\delta\frac{N_\delta}{\sigma^{(0)}_{\delta\alpha}}\cdot\left[K_{\delta\alpha}S_{\alpha pqr}+\frac{L_{\delta\alpha i}Q_{\alpha ipqr}}{c_\alpha}+9M_{\delta\alpha}S_{\alpha pqr}\right] \qquad [2.17]$$

The expressions $A^{(0)}_{\delta\alpha}$, $A^{(1)}_{\delta\alpha p}$, $A^{(2)}_{\alpha\alpha pp}$, $A^{(2)}_{\delta\alpha pq}$, $A^{(3)}_{\delta\alpha ppp}$, $A^{(3)}_{\delta\alpha ppq}$ and $A^{(3)}_{\delta\alpha pqr}$ are very lengthy and are given by the equations [2·0] after giving various values to n. We note the following important points regarding the above transport equations. The momentum equations are directly coupled to the stresses as well as with heat-flux vector. The heat flux tensors are coupled directly with all the lower order moments. Consequently, even in the absence of magnetic field, we expect intense anisotropy in the plasma.

3. Stationary Non-equilibrium Processes

In this section, we shall consider some simple stationary, non-equilibrium processes and we shall deduce the expression for electrical conductivity, viscosity, heat conductivity and diffusion coefficients of the plasma.

In order to find the expression for electrical conductivity and diffusivity terms, we consider the Lorentzian gas. Expressing the higher order moments in terms of equivalent lower order moments, the momentum equation reduces to

$$\frac{1}{N_\alpha}\cdot\frac{\partial(N_\alpha c_\alpha^2)}{\partial x_p}-\left(\frac{e_\alpha E_p}{m_\alpha}+\frac{F_{\alpha p}}{m_\alpha}\right)-\frac{e_\alpha\epsilon_{pjk}}{c\,m_\alpha}H_k u_{\alpha\alpha j}$$

$$=\sum_\delta\frac{N_\delta}{\sigma^{(0)}_{\delta\alpha}}\left(K_{\delta\alpha}+5M_{\delta\alpha}\frac{T_{\delta\alpha}}{T_{\alpha\alpha}}\right)u_{\delta\alpha p}-\sum_\delta\frac{N_\delta}{\sigma^{(0)}_{\delta\alpha}}(K_{\delta\alpha}+5M_{\delta\alpha})u_{\alpha\alpha p}+B_{\delta\alpha p} \qquad [3.1]$$

where

$$B_{\delta\alpha p}=-c_\alpha\sum_\delta\frac{N_\delta}{\sigma^{(0)}_{\delta\alpha}}L_{\delta\alpha p}+c_\alpha\frac{T_{\delta\alpha}}{T_{\alpha\alpha}}\cdot L_{\delta\alpha p}+\sum_\delta\frac{N_\delta}{\sigma^{(0)}_{\delta\alpha}}\cdot\left\{\frac{L_{\delta\alpha l}u_{\delta\alpha l}}{c_\alpha}\right.$$

$$\left.+\frac{M_{\delta\alpha}u^2_{\delta\alpha}}{c_\alpha^2}\right\}u^2_{\delta\alpha p}. \qquad [3.2]$$

Concentrating on the first power of velocity components in [3.1] the expressions for the electrical conductivity σ and the generalized diffusion coefficients σ_α and σ_β are given by

$$\sigma = \frac{(e_\beta N_\beta - e_\alpha N_\alpha)}{4 N_\alpha N_\beta} \cdot \frac{D_2}{D_1} \qquad [3.3]$$

$$\sigma_\alpha = \frac{(e_\beta N_\beta - e_\alpha N_\alpha) D_{\gamma\alpha}}{2 D_1} \qquad [3.4]$$

$$\sigma_\beta = \frac{(e_\alpha N_\alpha - e_\beta N_\beta) D_{\beta\gamma}}{2 D_1} \qquad [3.5]$$

where

$$D_1 = N_\alpha D_{\beta\alpha} D_{\alpha\gamma} + N_\beta D_{\beta\alpha} D_{\beta\gamma} + N_\gamma D_{\beta\gamma} D_{\gamma\alpha} \qquad [3.6]$$

and

$$D_2 = N_\alpha D_{\alpha\gamma} + N_\beta D_{\beta\gamma} \qquad [3.7]$$

where

$$D_{\delta\alpha} = \frac{m_\alpha (K_{\delta\alpha} T_{\alpha\alpha} + 5 M_{\delta\alpha} T_{\delta\alpha})}{\sigma^{(0)}_{\delta\alpha} T_{\alpha\alpha}} a_{\delta\delta} \qquad [3.8]$$

From [3.3] we see that electrical conductivity decreases in the case of variable cross section. because D_1 has increased. We now consider the gradient dependance of the stresses. For simplicity, we have chosen the Z-axis along the direction of magnetic field and deduced the stress components and hence the expressions for visocity and bulk viscosity :

$$P_{\alpha 33} = -\mu^{(0)}_{\alpha 33} e_{\alpha 33} - \mu^{(1)}_{\alpha 33} (e_{\alpha 11} + e_{\alpha 22} + e_{\alpha 33}) \qquad [3.9]$$

where

$$\mu^{(0)}_{\alpha 33} = \frac{2 k^2 T^2_{\alpha\alpha}}{m^2_\alpha (\tau_\alpha + 7 I_\alpha)} \qquad [3.10]$$

and

$$\mu^{(1)}_{\alpha 33} = \frac{k^2 T^2_{\alpha\alpha} (\tau_\alpha + 5 I_\alpha)}{m^2_\alpha (\tau_\alpha + 10 I_\alpha)(\tau_\alpha + 7 I_\alpha)} \qquad [3.11]$$

$$\begin{Bmatrix} P_{\alpha 23} \\ P_{\alpha 31} \end{Bmatrix} = -\mu^{(0)}_{\alpha 3} \begin{Bmatrix} e_{\alpha 23} \\ e_{\alpha 31} \end{Bmatrix}, \qquad [3.12]$$

where $$\mu_{\alpha 3}^{(0)} = \frac{2 k^2 T_{\alpha\alpha}^2}{m_\alpha^2 [\omega_\alpha^2 + \tau_\alpha + 7 I_\alpha)^2]} \cdot \begin{bmatrix} \tau_\alpha + 7 I_\alpha & -\omega_\alpha \\ \omega_\alpha & \tau_\alpha + 7 I_\alpha \end{bmatrix}; \qquad [3.13]$$

$$P_{\alpha 12} = -\mu_{\alpha 12}^{(0)} \begin{Bmatrix} e_{\alpha 11} \\ e_{\alpha 12} \\ e_{\alpha 22} \end{Bmatrix} \qquad [3.14]$$

where $$\mu_{\alpha 12}^{(0)} = \frac{2 k^2 T_{\alpha\alpha}^2 [-\omega_\alpha, \tau_\alpha + 7 I_\alpha, \omega_\alpha]}{m_\alpha^2 [4 \omega_\alpha^2 + (\tau_\alpha + 7 I_\alpha)^2]}, \qquad [3.15]$$

$$P_{\alpha 11} = -\mu_{\alpha 11}^{(0)} \begin{Bmatrix} e_{\alpha 11} \\ e_{\alpha 12} \\ e_{\alpha 13} \end{Bmatrix} - \mu_{\alpha 11}^{(1)} (e_{\alpha 11} + e_{\alpha 22} + e_{\alpha 33}), \qquad [3.16]$$

where $$\mu_{\alpha 11}^{(0)} = \frac{2 k^2 T_{\alpha\alpha}^2 [\{2 \omega_\alpha^2 + (\tau_\alpha + 7 I_\alpha)^2\}, 2 \omega_\alpha (\tau_\alpha + 7 I_\alpha), 2 \omega_\alpha^2]}{m_\alpha^2 (\tau_\alpha + 7 I_\alpha) [4 \omega_\alpha^2 + (\tau_\alpha + 7 I_\alpha)^2]} \qquad [3.17]$$

and $$\mu_{\alpha 11}^{(1)} = \frac{k^2 T_{\alpha\alpha}^2 [\tau_\alpha + 5 I_\alpha]}{m_\alpha^2 (\tau_\alpha + 7 I_\alpha)(\tau_\alpha + 10 I_\alpha)} \qquad [3.18]$$

With similar expressions for $P_{\alpha 22}$, In all these expressions, we have

$$\tau_\alpha = \sum_\delta \frac{N_\delta}{\sigma_{\delta\alpha}^{(0)}} K_{\delta\alpha}, \quad I_\alpha = \sum_\delta \frac{N_\delta}{\sigma_{\delta\alpha}^{(0)}} M_{\delta\alpha} \qquad [3.19]$$

Due to the dependance of cross section on the relative velocity between two species of particles, a very high anisotropy is introduced even in the absence of magnetic field and the magnitudes of viscosity coefficients in the principal directions are decreased. Further the diagonal terms of the stress tensor depend also on the dilatation term $(e_{\alpha 11} + e_{\alpha 22} + e_{\alpha 33})$ apart from other couplings. Hence we can conclude that variable cross section not only introduces anisotropy, but the bulk viscosity of the plasma is also increased. The viscocity along the direction of magnetic field is decreased by a factor

$$= 1/(\tau_\alpha + 7 I_\alpha). \qquad [3.20]$$

If we examine the viscocity coefficients corresponding to stress components $P_{\alpha 12}$, $P_{\alpha 22}$, $P_{\alpha 31}$, $P_{\alpha 11}$ etc., the anisotropy is evident even in the absence of magnetic field.

Proceeding in a similar manner for the heat flux vector we obtain

$$S_{\alpha 3} = -K_{\alpha 3}^{(0)} (\partial T_{\alpha\alpha} / \partial x_3) \qquad [3.21]$$

where

$$K_{a3}^{(0)} = \frac{5k^2 T_{aa}(\tau_a + 9I_a)}{m_a^2 [\tau_a^2 + 23\tau_a I_a + 126 I_a^2]} \qquad [3.22]$$

$$\begin{bmatrix} S_{a1} \\ S_{a2} \end{bmatrix} = -K_a^{(0)} \begin{bmatrix} \partial T_{aa}/\partial x_1 \\ \partial T_{aa}/\partial x_2 \end{bmatrix} \qquad [3.23]$$

where

$$K_a^{(0)} = \frac{5k^2 T_{aa}}{m_a^2 [\omega_a^2 + (\tau_a + 14 I_a)^2]} \cdot \begin{bmatrix} \tau_a + 14 I_a & \omega_a \\ -\omega_a & \tau_a + 14 I_a \end{bmatrix}; \qquad [3.24]$$

As in the case of stress tensor both the magnetic field and variable cross section introduce anisotropy in the heat flux vector. The coefficient of heat conductivity along the magnetic field is given by

$$= \frac{5k^2 T_{aa}(\tau_a + 9I_a)}{m_a^2 [\tau_a^2 + 23\tau_a I_a + 126 I_a^2]} \qquad [3.25]$$

This expression for heat conductivity along the magnetic field shows that the ratio of heat conductivity and the coefficient of viscosity along the magnetic field is no longer a constant.

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