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Two-temperature noble gas plasmas
Part 1 : Thermodynamics and transport coefficients
Part 2 : Transport properties of argon plasma

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Abstract

From thermodynamic principles, species, kinetic temperature and state equations are defined. The different equilibrium models to compute a two-temperature plasma composition are presented. The effect of lowering of ionization potential on plasma composition is found to be very small. Expressions for two-temperature plasma transport coefficients are presented. Using the expressions for electron and heavy particle transport coefficients, the properties of non-equilibrium argon plasma are computed and the computed values in the limit $\theta = 1$ agree satisfactorily with the exact theory.

Key words : Local thermodynamic equilibrium, two-temperature plasma, ambipolar and reactive heat diffusion coefficients, electrons, heavy-particles transport properties.

1. Introduction

In recent years, a great deal of attention has been paid to studies of high temperature non-equilibrium plasma situations such as glow discharges, constricted arcs, plasma jets, shock-heated plasmas and magneto-gasdynamic energy conversion devices[1-6]. In the above cases, the non-equilibrium effects observed are ionization non-equilibrium, non-equipartition of translational energy between electrons and heavy particles and chemical non-equilibrium depending on the physical situation. For example, in a constricted arc, the processes which tend to promote these non-equilibrium effects are external electric field, thermal conduction, ambipolar diffusion and elastic/inelastic collisions. An effective utilization and exploitation of the plasma under such situations requires a knowledge of the thermodynamics of a plasma state first and subse-

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quently the study of transport processes based on a proper kinetic theory yields transport coefficients.

Only those references related to the study of thermodynamics and transport phenomena of a two-temperature plasma are cited in the present work since the literature available on this subject is wide and a vast growing one. The method of approach of the study of thermodynamics of two-temperature plasma follows that of Drawin[6], Pfender[13] and Bose[7]. Devoto[8-10] has presented expressions for transport coefficients based on a rigorous kinetic theory for an argon plasma under equilibrium conditions. Kannappan and Bose[11,12] have reviewed several methods for calculation of transport properties of a plasma under equilibrium and non-equilibrium conditions. Two-temperature plasma composition and transport propeities were presented by them for argon and helium plasmas for electron to heavy particle temperature ratio of three.

The highlights of the present paper are as follows : 1. Two different equilibrium composition models based on the Saha equation are presented. The partial derivatives of electron mole fraction with respect to electron and heavies temperatures are discussed for the two models. 2. The concept of ambipolar diffusion is extended to two-temperature plasma situations. 3. Equations are presented to compute heat flux due to conduction and diffusion-reaction.

Part 1 deals with the definitions of local thermodynamic equilibrium and kinetic temperatures of species. Multiple temperature concept is introduced in context with non-equipartition of energy. Thermodynamic state equations and the Saha equation are described/discussed in computing the plasma composition. Also shown in this section is the derivation of transport coefficients for a two-temperature plasma. Simplified expressions for the transport properties of electron and heavies subgases constituting an ionized gas mixture obtained from a rigorous kinetic theory are presented in Part 2. The computed transport properties for argon at $p = 1$ bar are presented followed by a brief discussion on the variation of species' transport properties with temperature.

1.1. *Local thermodynamic equilibrium*

In order to allow a laboratory plasma system to have thermal and chemical variations and still be characterized by a unique temperature and pressure at a point, the concept local thermodynamic equilibrium originated. Under the restrictions of LTE, the state of a system at a point is defined uniquely by a definite set of thermodynamic functions in conjunction with the equilibrium law of mass action. However, the state parameters are allowed to vary spatially.

Depending upon the purpose of study, the plasma system conditions and on the definition of temperature, numerous definitions of LTE are possible. For more details the reader is referred to the works of Pfender[13] and Drawin[6]. It is seen that the most appropriate interpretation depends upon which modes of energy storage are dominant for a plasma under given conditions. In the present work of collision-dominated three-component plasma, the dominant modes of energy storage are the species, thermal energy and the ionization energy. Hence LTE exists when $T_e = T_i = T_n = T_I$ at each point in the plasma where each T is defined *via* the appropriate equilibrium equation.

1.2. *Non-equipartition of energy and multiple temperature concepts*

In a constricted arc, Incropera and Viegas[2] made a time-scale study in which a representative relaxation time was associated with each of the physical mechanisms identified to promote non-equilibrium. Non-equipartition can be studied with reference to elastic collisions in a gas mixture containing species of disparate mass. McDaniel[18] has shown that, on the average, the fraction of the initial kinetic energy possessed by species with mass, m_i, and transferred to a less energetic species with mass, m_j, during a single elastic collision. It is given by :

$$\Delta E = \frac{2m_i m_j}{(m_i + m_j)^2} \tag{1}$$

if $m_i = m_j$, $\Delta E = 1/2$.

For an argon atom $m_i = m_n = 6{\cdot}635 \times 10^{-26}$ kg and for an electron, $m_e = 9{\cdot}1 \times 10^{-31}$ kg, the energy exchange is $\Delta E = 3 \times 10^{-5}$.

Thus it is apparent that energy transfer during an elastic collision between species of equal mass is highly efficient whereas it is highly inefficient between species of disparate mass.

Consider now the perturbation due to applied electric field, on a partially-ionized plasma at a certain instant in space. The electrons will very quickly equilibrate among themselves and approach an equilibrium (Maxwellian) thermal energy distribution, then heavier particles equilibrate among themselves and approach Maxwellian distribution. Under this condition of non-equipartition of energy, two distinct equilibrium distributions of thermal energies corresponding to lighter (electrons) and heavy particles are established. Hence the mixture of species of disparate mass is described by two different kinetic temperatures T_e and T_h.

From an energy balance at steady-state condition[7], the departure of the electron temperature from the gas temperature is given by the following expression

$$T_e = T_h + \frac{2m_h}{3k_B \delta}\left(\frac{j}{en_e}\right)^2 \tag{2}$$

1.3. *Thermodynamic state equations and composition of a two-temperature plasma*

The following are assumptions, approximations, definitions and derived relations :

(i) *Assumptions*

1. The argon plasma is a three-component mixture of free electrons, ground state singly charged ions and atoms or neutrals.
2. All species thermal velocity distributions are Maxwellian with appropriate kinetic temperatures. Atoms and ions have the same velocity or atom and ion temperatures are equal (T_h).
3. The plasma is quasi-neutral with equal electron and ion number densities.
4. All particle collisions are elastic binary collisions.
5. All radiation is optically thin, hence the influence of radiation term is not included in the present analysis.
6. Energy storage in the form of internal electronic excitation is negligible.

(ii) *Thermodynamic state equations of a two-temperature plasma*

Equation of state

$$p = n_e k_B T_e + k_B T_h \sum_{j \in H} n_j \tag{3}$$

$$= n k_B T_e [x_e(\theta - 1) + 1]/\theta, \quad \text{where} \quad \theta = T_e/T_h$$

$j \in H$ means that species, j corresponds to a set of heavy particles, H.

Caloric equations of state of species

$$h_j = \frac{5}{2} \frac{k_B T_j}{m_j} \tag{4}$$

Equation (4) defines actually species translation enthalpy. However, in the calculation of heavy species enthalpy, the energy residing in the plasma in the form of electronic excitation, though small, is taken into account. Accordingly, the species total enthalpy is

$$h_j = \frac{5}{2} R^* T_h + h_{j,\text{exec}} + h_{j,0,\, j \in H} \tag{5}$$

where

$$h_{j,\text{exec}} = R^* T_e^2 \frac{\partial \ln Z_{j,\text{exec}}}{\partial T_e}$$

$$Z_{j,\text{exec}} = \sum_{j \in H} g_{jk} \, exp \, [-E_{jk}/k_B T_e]$$

$$h_{j,0} = \text{heat of formation at } 0^\circ K$$

Enthalpy of mixture is

$$h = \frac{5}{2} R^* [x_e T_e + (1 - x_e) T_h] + \sum_{j \in H} x_j h_{j0}$$
$$+ R^* \sum_{j \in H} x_j \frac{(g_{jk} E_{jk}/k_B) \, exp \, [-E_{j,k}/k_B T_e]}{g_{jk} \, exp \, (-E_{j,k}/k_B T_e)} \tag{6}$$

(iii) *Approximations and definitions*

$$(m_e/m_i) \ll 1, \quad m_i \simeq m_n, \quad n_e \simeq n_i,$$

$$\alpha = n_e/(n_e + n_n), \quad R^* = k_B N_A, \quad m_i = \frac{R^*}{k_B} M_i,$$

$$x_j = n_j/n, \quad \rho = \sum n_j M_j = \frac{k_B}{R^*} n \left[\frac{x_e}{(m_e/m_n)} + (x_e + x_a)\, m_h\right]. \tag{7}$$

(iv) *Plasma composition and discussion of models A and b*

Numerical computation of transport properties of coefficients requires an accurate evaluation of plasma composition for which two different equilibrium composition models are used.

Following the principle of minimum chemical potential, Monti and Napolitano[19] and Veis[19,20] has given the following expression for a three-component plasma in a state of chemical, not thermal, equilibrium, which we refer to as model *A*.

$$S = n x_e \left(\frac{x_{i\,1}}{x_i}\right)^{1/\theta} \tag{8}$$

From the basics of kinetics and use of the product of partition function evaluated at electron temperature T_e Kerrebrock[21] presented the following expression-model *B*.

$$S = n\left(\frac{x_e x_{i\,1}}{x_i}\right) \tag{9}$$

where the subscript $i = 1$ for neutral particles and $i = 2$ for singly charged ions.

The Saha function, S_i is given by

$$S_i = 2\,\frac{Z_{i\,1}}{Z_i}\left[\frac{2\pi m_e k_B T_e}{\bar{h}^2}\right]^{3/2} exp\ - [(I_i - \triangle I_i)/k_B T_e)] \tag{10}$$

In eqn. (9), I_i is the ionization potential and $\triangle I_i$ takes into account the lowering of the ionization potential due to fields of the charged particles. The expression for $\triangle I_i$ for the present case is as follows :

$$\Delta I_i \sim i^2 \sqrt{[n_e(\theta + 1)/T_e]} \tag{11}$$

It is found that ΔI_i is very small even if T_e is small, for example, near a cold wall. Consideration of $\triangle I_i$ affects the mole fraction only very marginally[22]. The partition functions $Z_{i\,1}$, Z_i are evaluated using the Unsöld criterion[23] for cutting off the series.

From the following definitions

$$x_e + \sum_{i \in H} x_i = 1 \tag{12}$$

$$x_e = (i - 1)\, x_i + i x_{i+1}' \tag{13}$$

a generalized form of the Saha equation for model A in the present case is

$$x_{i+1}^{\theta+1} + i x_{i+1}^{\theta} + (i + 2)\, S_i^{\theta}\, x_{i+1} - S_i^{\theta} = 0 \tag{14}$$

Figures 1 to 4 present representative results of calculation of argon species mole fractions at one atmosphere of pressure and for θ values up to 10 for both models A and

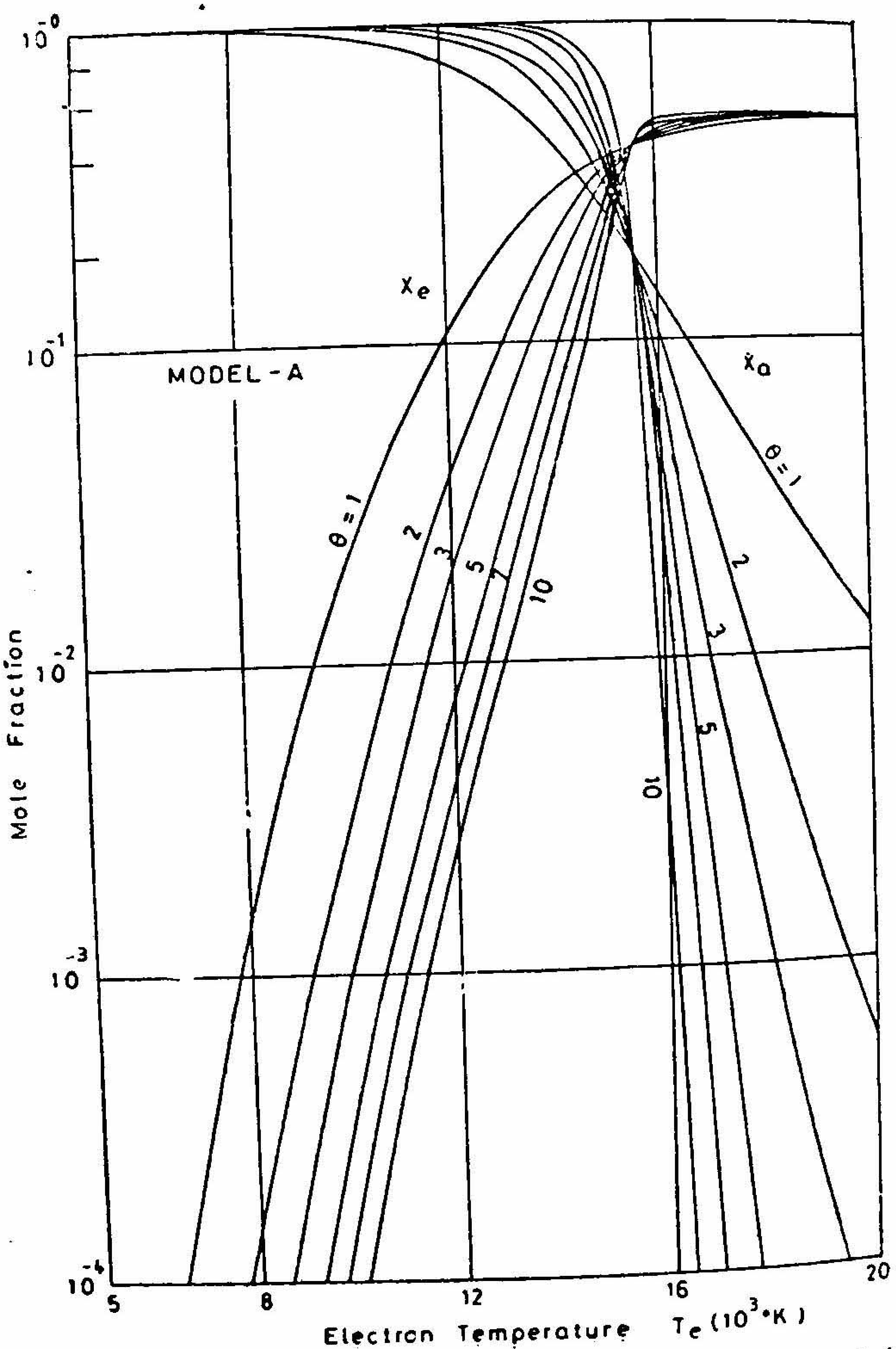

FIG. 1. Composition of two-temperature argon plasma at $p = 1$ bar for Model A.

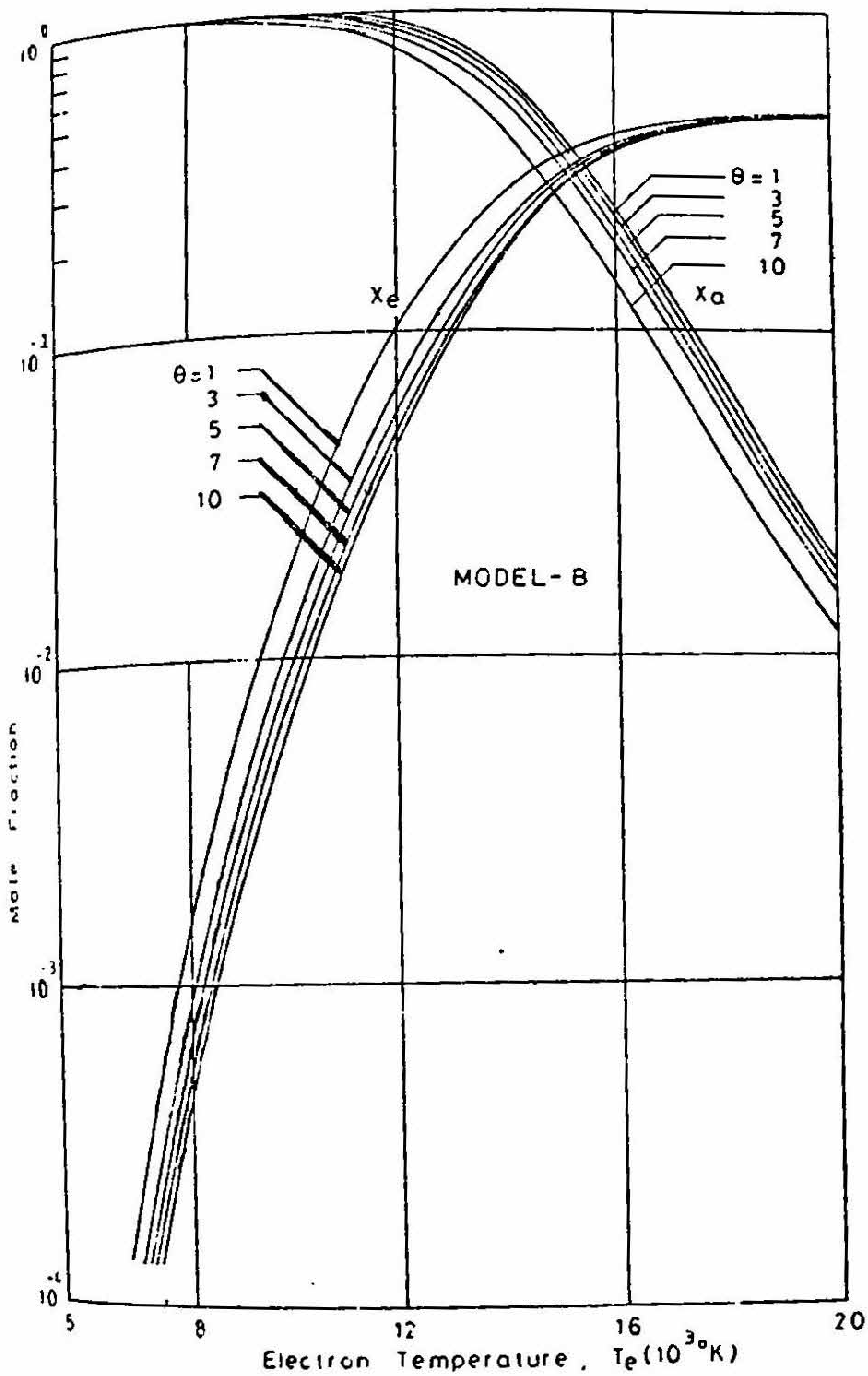

FIG. 2. Composition of two-temperature argon plasma at $p = 1$ bar for Model B.

8. From figs. 1 and 2 it is seen that the variation of x_e with T_e for $\theta > 1$ is generally much larger in model A than in model B.

Table I shows the values of electron mole fraction for various values of T_e and θ. Around 15000° K and at one atmosphere the number densities of equilibrium species

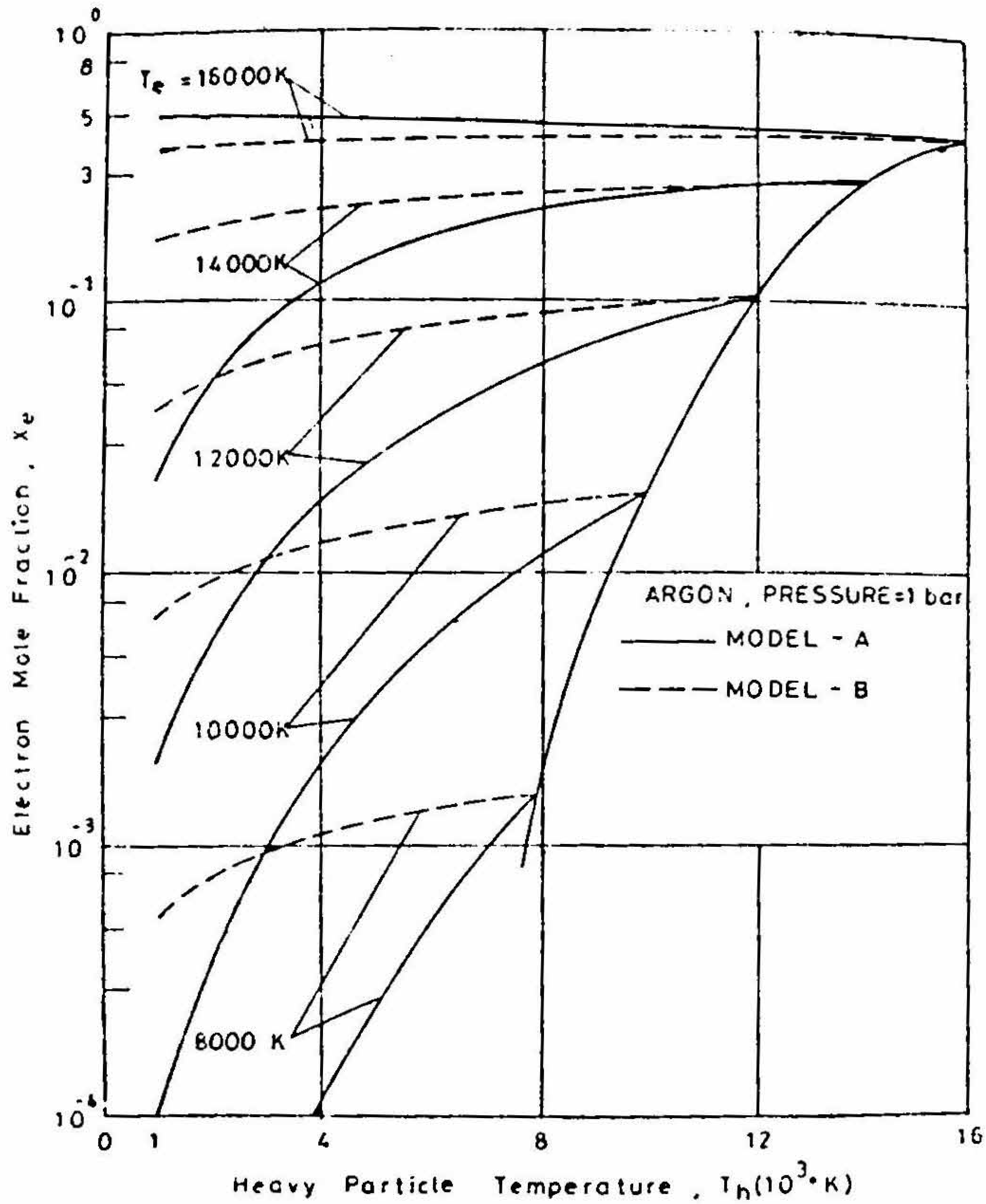

FIG. 3. Electron mole fraction *vs.* heavy particle temperature with electron temperature as parameter.

in an argon plasma are almost equal. However, the trend of variation of x_e in model A in the range 5000 to 15000° K is different from that above 15000° K.

In the expressions for species, diffusion flux and heat flux shown in the next section, ∇x_e appears and it is also noted that x_e is a function of electron and heavy particle temperatures.

Accordingly, the expression of ∇x_e is

$$\nabla x_e = \frac{\partial x_e}{\partial T_h} \frac{dT_h}{dy} + \frac{\partial x_e}{\partial T_e} \frac{dT_e}{dy} \tag{15}$$

Closed form analytical expressions for partial derivatives of x_e with respect to T_e and T_h are given in Appendix I for both models A and B.

Table I

Comparison of electron mole fraction at various values of T_e and θ for argon at $p = 1$ bar for model A

T_e, K	$x_e (\theta=1)$	$x_e (\theta=5)$	$x_e (\theta=10)$
10000	0·0202	0·000402	0·000105
12000	0·1072	0·0078	0·00068
14000	0·2816	0·0850	0·03985
15000	0·3725	0·2711	0·1926
16000	0·4314	0·4865	0·4990
20000	0·4979	0·50	0·50

In view of the peculiarity of variation in x_e in model A, we attempt to study and interpret the partial derivatives with physical relevance to our study. With the aim of understanding the situation further, x_e is plotted in fig. 3 for both the models as a function of T_e with T_h as parameter and in fig. 4 as a function of T_h with T_e as parameter. It is seen from fig. 3 that $\partial x_e/\partial T_e$ is positive at all T_e and T_h for both the models while $\partial x_e/\partial T_h$ can be seen in fig. 4 to be slightly negative for model A at $T_e \sim 16000°$ K. These can also be verified numerically from the expressions provided in Appendix I.

It is also well known and is the basis of electron temperature measurement with the tip of electrostatic probes[24], that T_e does not change significantly near the cooled probe, whereas T_h changes from wall temperature to the freestream temperature. Therefore, at $T_e > 15{,}500°$ K, atleast theoretically for model A to be valid, there has to be a small electron flux and resultant heat flux by diffusion from lower to higher temperature. However, model A violates the second law of thermodynamics in this range of electron temperatures and also the total heat flux due to pure conduction and diffusion becomes lower than due to pure conduction alone. It is also found that the absolute magnitude of this reactive heat flux for both the models is much smaller in comparison with the pure conductivity values. In view of the above reasoning it is suggested to put $\partial x_e/\partial T_h = 0$ for model A at this temperature range when it is negative.

3.4. *Species diffusion velocities and current densities*

With the knowledge of the equilibrium chemical composition of argon plasma, we proceed to study the species diffusion velocities and current densities due to concentra-

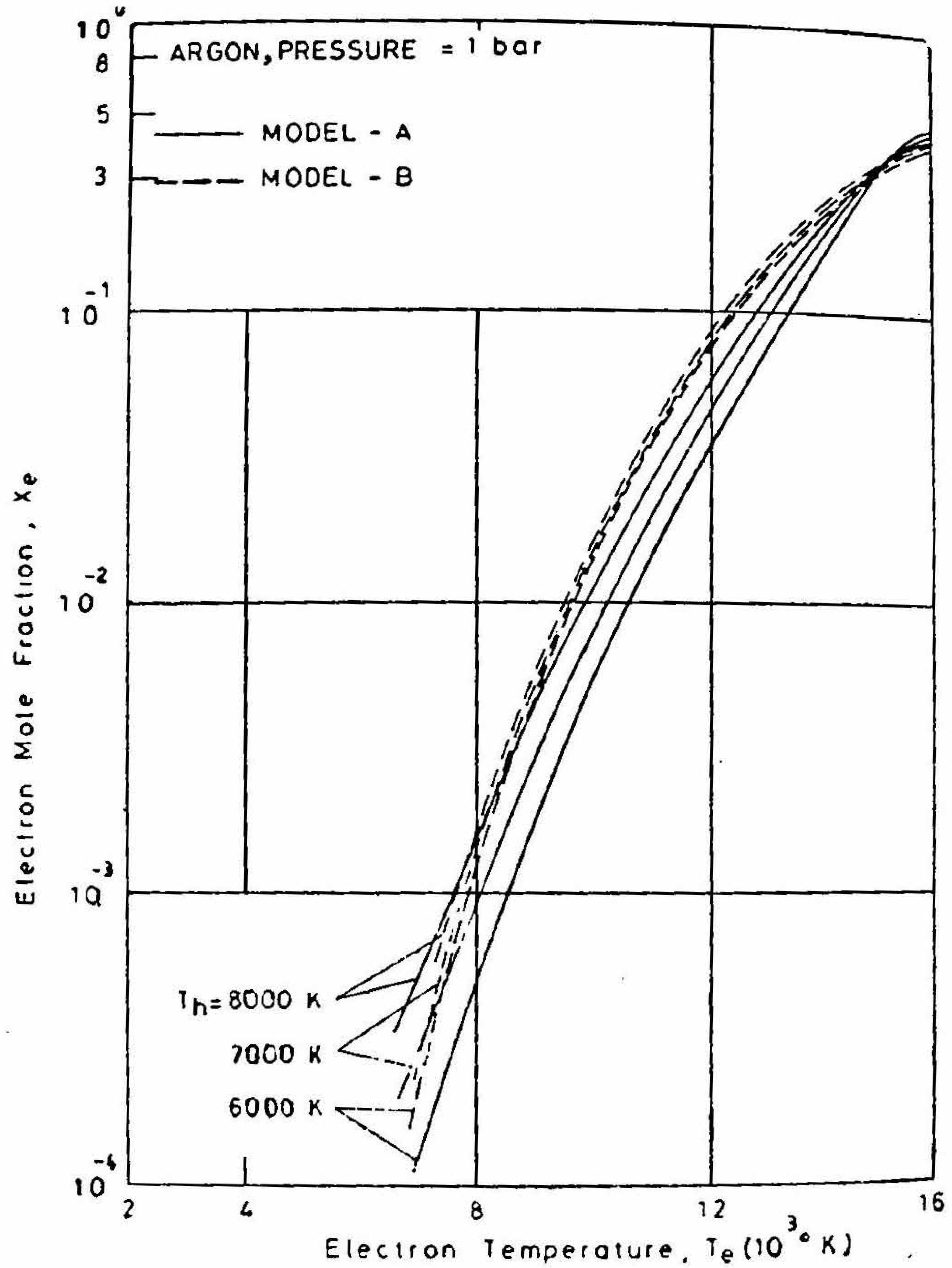

FIG. 4. Electron mole fraction *vs.* electron temperature with heavy particle temperature as parameter.

tion gradients and applied electric field under thermal non-equilibrium conditions. This follows from classical approach[24,25].

It is assumed that in any given state (p, T_e, T_h) only electrons, i and $(i+1)$th ions are present in the gas. From eqns. (12) and (13), x_i and x_{i+1} are expressed in terms of x_e as

$$x_i = i - (i+1)\,x_e \tag{16}$$

$$x_{i+1} = 1 + i\,(x_e - 1) \tag{17}$$

Upon operating with del operator, eqns. (16) and (17) are written as

$$\nabla x_i = -(1+i)\,\nabla x_e \tag{18}$$

$$\nabla x_{i+1} = i\,\nabla x_e \tag{19}$$

Current densities of species in a plasma with impressed electric field and concentration gradients are as follows :

$$J_e = -en_e V'_e = en_e b_e E + eD_e n \nabla x_e$$

$$J_i = (i-1)\,en_i V'_i = e(i-1)\,n_i b_i E - e(i-1)\,nD_i \nabla x_i$$

$$J_{i+1} = ien_{i+1} V'_{i+1} = ein_{i+1} b_{i+1} E - einD_{i+1} \nabla x_{i+1} \tag{19 a–c}$$

Substituting the gradients of mole fraction of individual species given in eqns. (18) and (19) into (19 a–c) current densities of species i and $(i+1)$ can be written in terms of ∇x_e. The expressions for the species diffusive velocities in the absence of applied field, E, obtained from eqns. (19 a–c), are expressed in terms of ∇x_e

$$V'_e = -(D_e/x_e)\,\nabla x_e$$

$$V'_i = -(1+i)\,(D_i/x_i)\,\nabla x_e$$

$$V'_{i+1} = +i\,(D_{i+1}/x_{i+1})\,\nabla x_e \tag{20 a–c}$$

It may be observed that eqns. (20 a–c) satisfy the condition, $\Sigma \rho_j V'_j = 0$.

15. *Ambipolar diffusion coefficient*

When diffusion is considered, one of the two possible approaches is used to calculate diffusion flux. One approach involves the use of individual species momentum equations which are solved to give separate species diffusion velocities as well as resultant electric field intensity due to space charge set up by charge separation. A second approach involves invoking the quasi-neutral assumption, thereby equal electron and ion diffusion velocities and an appropriate ambipolar diffusion model is used. In the present study, the later approach is employed by neglecting thermal diffusion.

From eqn. (19 a–c), we get

$$\frac{J_e}{e} = \frac{ix_e b_e D_{i+1} + x_{i+1} b_{i+1} D_e}{x_e b_e + x_{i+1} b_{i+1}} \nabla n_e$$

$$\simeq \frac{ix_e b_e D_{i+1} + x_{i+1} b_{i+1} D_e}{x_e b_e}\, n \nabla x_e$$

$$= n\left(iD_{i+1} + \frac{x_{i+1} b_{i+1}}{x_e b_e} D_e\right) \nabla x_e \tag{21}$$

Noting that

$$\frac{b_{i+1}}{D_{i+1}} = \frac{i_e}{k_B T_h}\ ; \quad \frac{b_e}{D_e} = \frac{e}{k_B T_e}$$

We get

$$\frac{J_c}{en} = iD_{i+1}\left(1 + \theta \frac{x_{i+1}}{x_i}\right) \nabla x_e \tag{22}$$

Hence, the ambipolar diffusion coefficient for a two-temperature plasma is given by the relation

$$D_{amb} = i\left(1 + \theta \frac{x_{i+1}}{x_i}\right) D_{i+1} \tag{23}$$

Equation (23) is a relation for an 'effective diffusion coefficient' and is a function of ratio of electron to heavy temperature, mole fraction and diffusion coefficient of species. This relation also holds good even in the absence of applied field, E. For $\theta = 1$ and $i = 1$, eqn. (23) reduces to the familiar expression for diffusion under equilibrium conditions. The numerical results and discussion on ambipolar diffusion are given in Part 2.

1.6. *Reactive heat conductivities and heat fluxes of species*

For a reacting and radiating gas mixture under certain circumstances, two components of energy transfer mechanisms which give rise to an effective increase in the value of the thermal conductivity coefficient, were identified. They are diffusion and radiation processes. A total thermal conductivity coefficient of j-th species in a mixture is defined as

$$k_j = k_{c_j} + k_{D_j} + k_R \tag{24}$$

If V_j' is the relative velocity due to diffusion of j-th species carrying an enthalpy, h_j, the total enthalpy flux due to diffusion carried on to the colder region in a two-temperature plasma, is given by

$$H = \Sigma\ \rho_j V_j' h_j, \quad j = e, i \text{ and } n \tag{25}$$

Since $\rho_j = (k_B/R^*)\ m_j n_j = m_i n_j/N_A$ and $H_j = m_j h_j$ we can arrive at the following expression for enthalpy flux and using eqns. (20) and (23)

$$H = -\frac{n\, D_{amb}}{N_A}(H_e + H_{i+1} - H_i)\, \Delta x_e \tag{26}$$

Upon realizing eqn. (15), eqn. (26) leads to two expressions for reactive conductivities of electrons and heavy particles

$$k_{re} = \left(\frac{nD_{amb}}{N_A}\right)[H_e + H_{i+1} - H_i]\,\frac{\partial x_e}{\partial T_e} \tag{27}$$

$$k_{rh} = \left(\frac{nD_{amb}}{N_A}\right)[H_e + H_{i+1} - H_i]\,\frac{\partial x_e}{\partial T_h} \tag{28}$$

The total heat flux due to conduction and diffusion in the absence of radiation can now be written in terms of gradients of temperature of electrons and heavy particles under LTE conditions as

$$q = -k_{eh} \nabla T_h - k_{ee} \nabla T_e + \sum_{j=e,H} p_j V_j' h_j$$
$$= -(k_{eh} + k_{rh}) \nabla T_h - (k_{ee} + k_{re}) \nabla T_e$$
$$= -k_h \nabla T_h - k_e \nabla T_e \tag{29}$$

In the most ideal case of θ = constant (similarity of electron and heavy particle temperature fields), eqn. (29) reduces to

$$q = -(k_h + \theta k_e) \nabla T_h = -k \nabla T_h \tag{30}$$

1.7. *Discussion of thermodynamics and transport coefficients*

Local thermal equilibrium, species' kinetic temperatures and state equations are defined/presented for a two-temperature plasma based on thermodynamic principles. Also derived are the transport coefficients for a multiple-ionized non-equilibrium plasma and expressions for species heat flux. Plasma composition is obtained for argon up to $\theta = 10$ at pressure one bar and electron heavy particle temperatures up to 20000° K. While Model *B* gives positive values of partial derivatives with respect to electron and heavy particle temperatures correctly, atleast the partial derivative becomes negative at certain temperatures for model *A*, which results into a small heat flux due to diffusion in a direction opposite to that due to the pure conduction. A suggestion is made to handle such a situation for Model *A*.

Part 2. Transport properties of a two-temperature argon plasma

In view of the difficulties in obtaining experimental values over a wide range of temperatures and pressures encountered in plasma situations, we find that a theoretical method for calculating the non-equilibrium transport properties is best suited for our study. With a small perturbation in distribution function and following the method of Chapman-Enskog, solutions[8,9] are reported for transport coefficients of heavies and electron subgases. We extensively use the results of Devoto[8] in conjunction with the expressions developed in Part 1 for numerical computation of two-temperature argon plasma transport properties. The average cross-sections required for the determination of transport properties of a singly ionized two-temperature argon plasma are as in Kannappan and Bose[11].

2.1. *Heavy particle transport coefficients*

We present here only the simplified expressions for heavies transport coefficients to the first approximation. These expressions are linked with the two-temperature plasma composition results in Part 1. The heavy particles (atoms and ions) are treated as a binary mixture.

Denoting the diffusivities of the pair *i-j* in a binary and a multi-component mixture by $\mathfrak{D}_{ij}$ and D_{ij} respectively, the diffusion coefficient is given by the expression

$$D_{ij} = \mathscr{D}_{ij}\left[1 + \frac{x_k (m_k/m_j)\, \mathscr{D}_{ik} - \mathscr{D}_{ij}}{x_i \mathscr{D}_{jk} + x_j \mathscr{D}_{jk} + x_k \mathscr{D}_{ij}}\right] \tag{31}$$

where

$$\mathscr{D}_{ij} = \frac{3}{16n}\left(\frac{2\pi k_B T_h}{m_{ij}}\right)\frac{1}{\bar{Q}_{i,j}^{(1,1)}}$$

The subscripts i and j take the following combinations in the expression for D_{ij}

$$(i,j) = (i+1,\ i),\ (i+1,\ e),\ (e,i) \text{ and } (e,\ i+1)$$

$$D_{i+1,9} = D_{i+1,i}\left[1 - \frac{m_e}{m_h}\frac{D_{i+1,e}}{D_{i+1,i}}\left(1 - \frac{D_{e,i}}{D_{e,i+1}}\right)\right] \tag{32}$$

and D_{amb} for a two-temperature plasma is given by eqn. (23) in Part 1.

Thermal conductivity and viscosity with the first approximation are calculated from Devoto[8]. The viscosity of the mixture has been assumed to be entirely due to the heavy particles. The ion contribution of electrical conductivity is neglected because it is of the order of $\sqrt{(m_e/m_h)}$ times the electron contribution.

2.2. *Electron transport coefficients*

For the calculation of electron subgas transport properties, fourth approximation is used.

$$D_{ee} = \frac{3}{2} n_e \left(\frac{2\pi k_B T_e}{m_e}\right)^{1/2} \frac{1}{|q|} \begin{vmatrix} q^{11} & q^{12} & q^{13} \\ q^{21} & q^{22} & q^{23} \\ q^{31} & q^{32} & q^{33} \end{vmatrix},\quad m^2/s \tag{33}$$

The electrical conductivity due to electrons is

$$\sigma = e^2\, n_e D_{ee}/k_B T_e,\ A/Vm \tag{34}$$

The expression for thermal conductivity is (reactive heat conductivity is given in Part 1).

$$k_e = \frac{75 n_e^2\, k_B}{8}\left(\frac{2\pi k_B T_e}{m_e}\right)^{1/2} \frac{1}{|q|} \begin{vmatrix} q^{01} & q^{02} & q^{03} \\ q^{21} & q^{22} & q^{23} \\ q^{31} & q^{32} & q^{33} \end{vmatrix},\quad \frac{W}{mk} \tag{35}$$

The determinant, $|q|$ and the expressions for elements q's are from Devoto[8]. The contribution of electron to the mixture viscosity is neglected on the physical grounds that the viscosity of a species is proportional to the suqare root of its mass.

2.3. *Discussion of transport properties*

In figs. 5–8, only representative of the two-temperature transport properties of argon plasma is shown at $p = 1$ bar. Those curves for which $\theta = 1$ in all the figures correspond to the case of equilibrium conditions and agree with the results of Devoto[8].

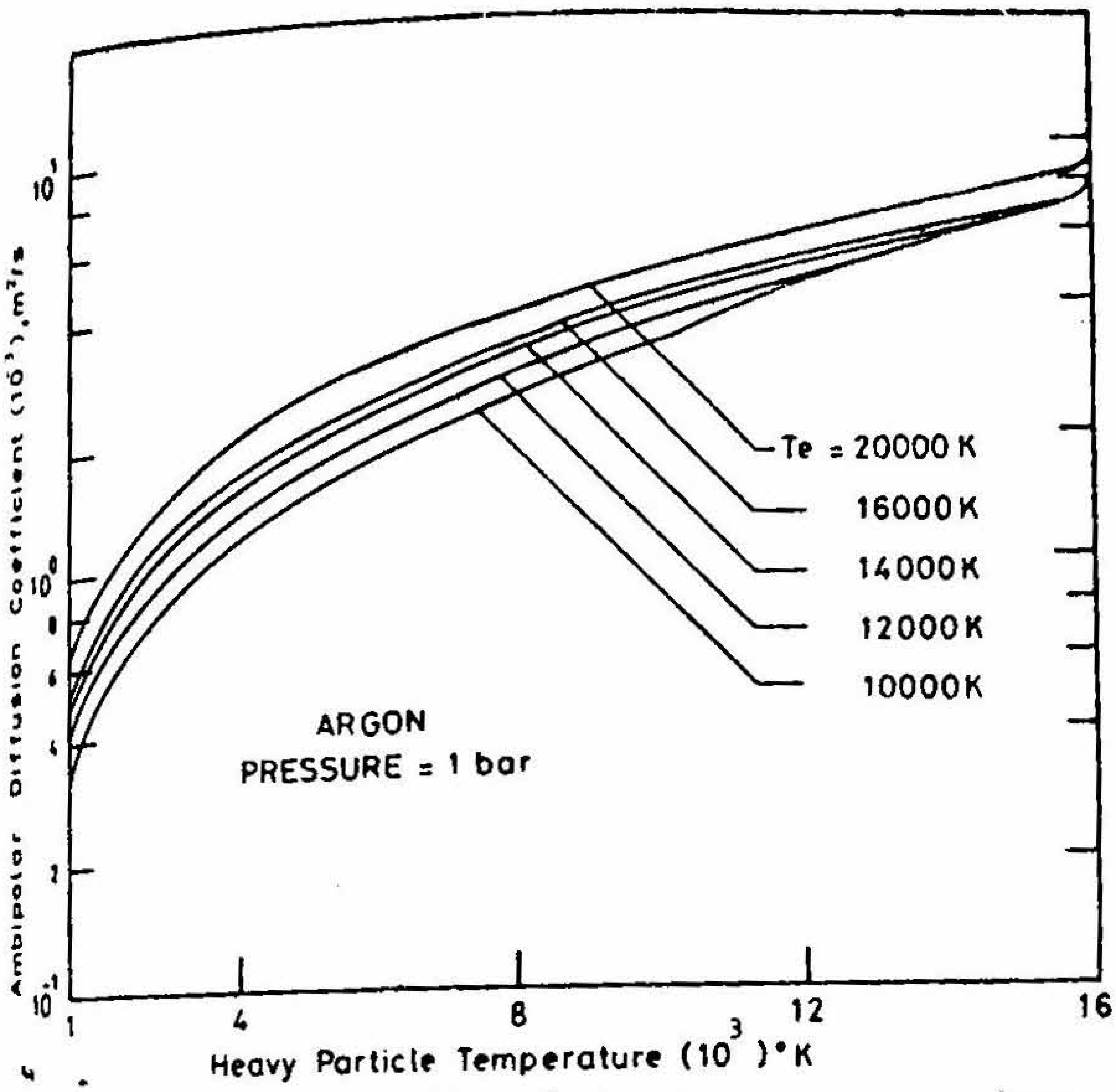

FIG. 5. Ambipolar diffusion coefficient of a two temperature argon plasma.

Figure 5 shows a uniform variation of D_{amb} with T_h, and figs. 6 and 7 the total electron and heavies thermal conductivity coefficients respectively. The total thermal conductivity variation of heavies is different from that of electrons as observed in figs. 6 and 7 respectively. The electron heat conductivity coefficient increases steeply up to $T_e \simeq 140000°$ K whereas the temperature above 14000° K, there is no appreciable increase. This is due to the fact that electrons up to $T_e \simeq 14000°$ K, acquire high mobility and also lesser fraction of neutrals present in the mixture (refer fig. 3 x_e vs T_h with parameter, T_e). The contribution of heavies to total mixture conductivity is appreciable at lower temperatures and is negligible at higher temperatures. The mixture total thermal conductivity at lower temperatures is due entirely to the atoms owing to the large neutral particle cross-sections. Figure 8 shows the variation of electrical conductivity with electron temperature and θ as parameter. The increase in electrical conductivity beyond a certain electron temperature shown in fig. 8 with increasing θ values is due to the domination of charged particle interaction at high electron temperatures. The decrease in σ with increasing θ values for a given T_e follows from the Ramsauer effect (*i.e.*, domination of e–A interaction at low values of T_h). The results shown in fig. 8 also demonstrate quantitatively the possibility of achieving high value of electrical conductivity at reasonably lower gas temperatures. The values of transport properties of argon shown in figs. 5–8 are limited to a maximum temperature of 20000° K because Ar^{++} ion should be considered in computing the composition and transport coefficients beyond this limit.

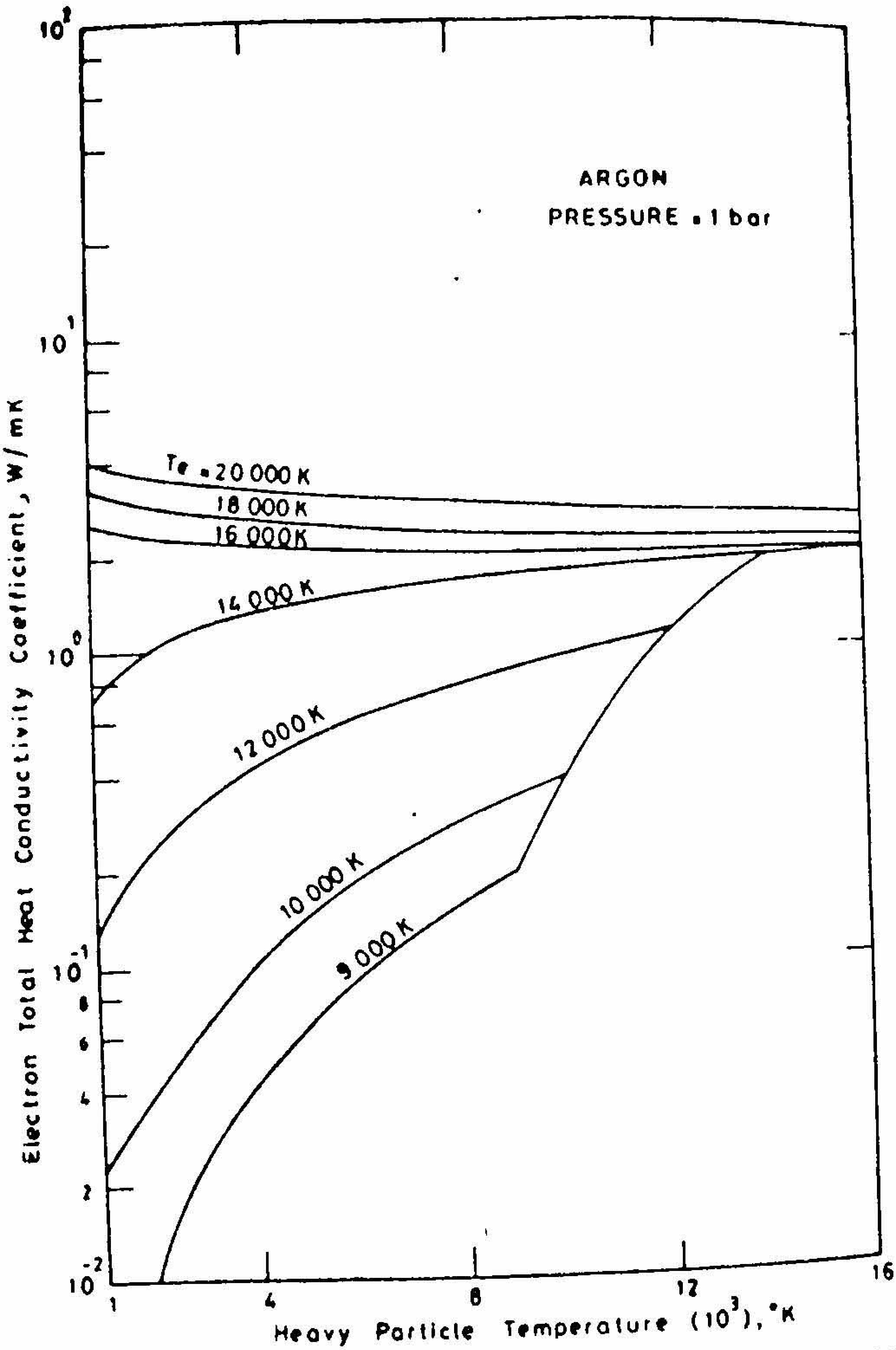

FIG. 6. Electron total heat conductivity coefficient of a two-temperature argon plasma.

3. *Summary and conclusion*

In part 1 we have discussed the LTE, the non-equipartition of energy and the multiple-temperature concept in a two-temperature plasma. Thermodynamic state equations and Saha equation are defined/discussed. Also shown are the derivations of expressions for transport coefficients of a two-temperature plasma. Variation of electron mol-

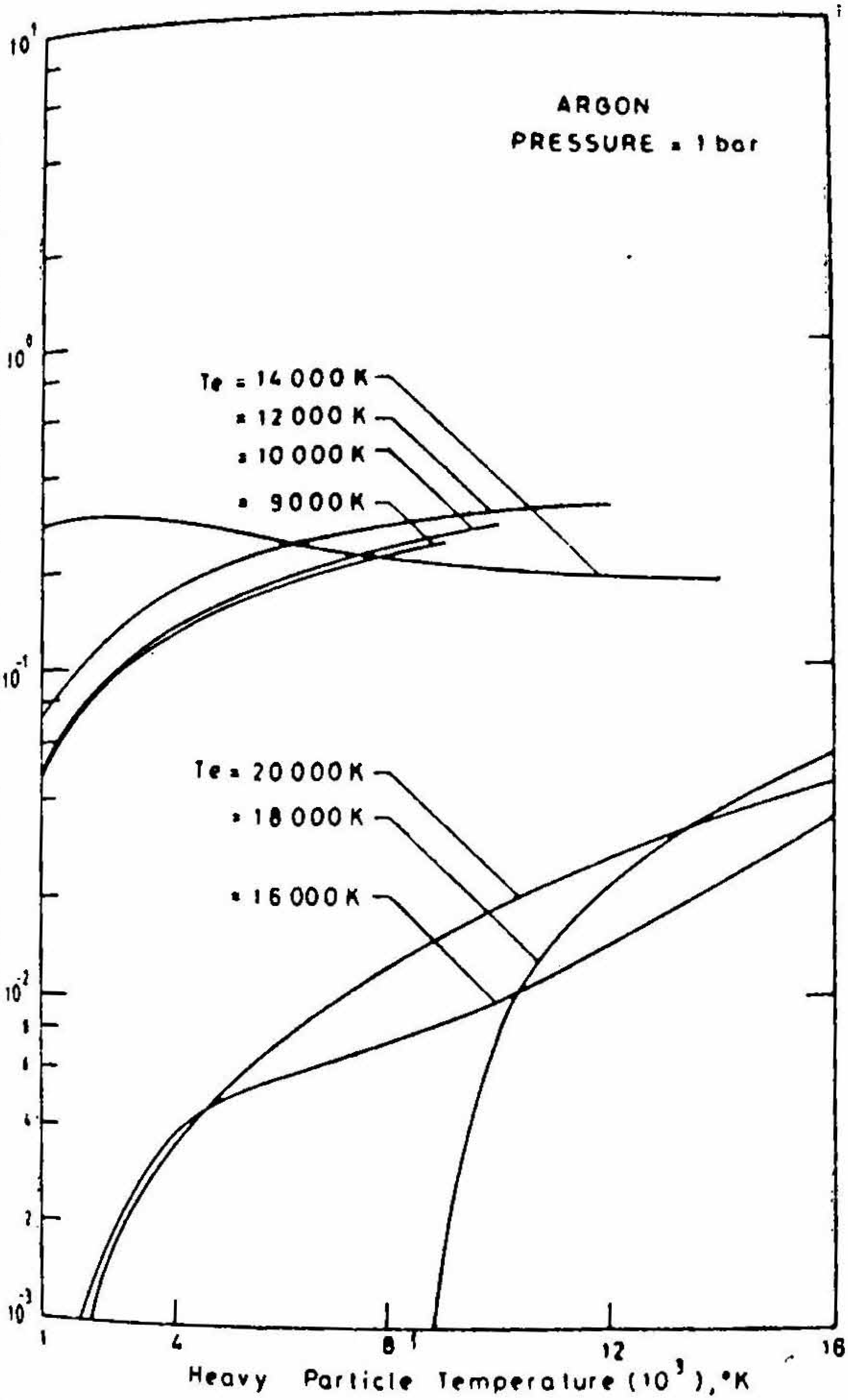

FIG. 7. Heavy particle total heat conductivity coefficient of a two-temperature argon plasma.

fraction as a function of electron and heavy particle temperatures is studied for both models with special reference to argon at $p = 1$ bar and temperatures up to 20000° K. The partial derivatives of x_e with respect to T_e and T_h are discussed with reference to the direction and heat flux diffusion. In Part 2 are shown the expressions of transport coefficients of electron and heavies subgases constituting a plasma mixture. The

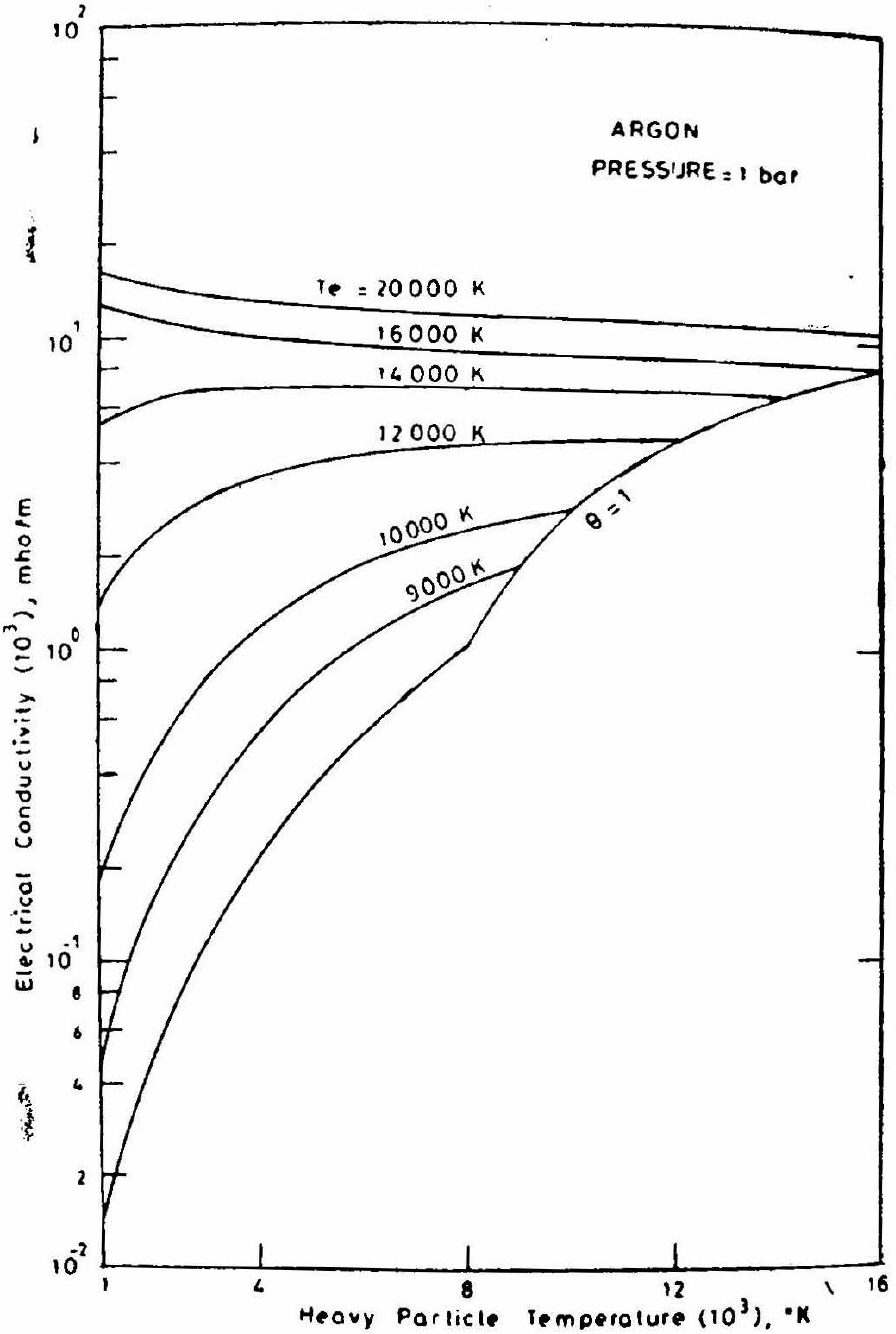

FIG. 8. Electrical conductivity of a two-temperature argon plasma.

computed results of transport properties of two-temperature argon plasma at $p = 1$ bar are shown and discussed. Comparison with earlier results for the special case of a plasma under thermal equilibrium shows a good agreement.

Nomenclature

a, b variables defined in Eqn. (*A.* 5)

b	mobility coefficient
D	diffusion coefficient
$\mathscr{D}_{ij}$	binary diffusion coefficient
E	electric field
e	elementary charge of electron
f, g	functions defined in Eqns. (A. 3, 4 and A. 8, 9)
H	molar enthalpy
h	specific enthalpy
h	Planck constant
I	ionization potential
i	ionization index, for neutrals $i = 1$
J	current density
k_B	Boltzmann constant
k_c	pure heat conduction coefficient
k_r	reactive heat conduction coefficient
M	mass of a particle
m	molar mass
N_A	Avagadro number
n	number density
p	pressure
q	heat flux vector
R°	universal gas constant
S	Saha function
T	temperature
V	diffusive velocity
x	mole fraction
Z	partition function
α	degree of ionization
δ	energy loss factor
ρ	density
θ	temperature ratio, T_e/T_h

Subscripts

c	conduction
e	electron
eqm	equilibrium
h	heavy particle, ion or atom
H	heavies (ions plus neutrals)
i	ionization index, for neutrals i = 1
j	species
r	reactive or diffusive part

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Appendix A

Two-temperature Saha equation and its partial derivative expressions

The following analytical expressions of partial derivatives for model A are used in the calculation of species coefficients, and fluxes as well as in the systems of equations formulated to sudy the energy transfer processes.

Model A

$$\frac{\partial x_e}{\partial T_e} = \frac{g\theta}{fT_h} \tag{A.1}$$

$$\frac{\partial x_e}{\partial T_h} = -\frac{g}{fT_h} + \frac{1}{f}\left[2\cdot5\frac{\theta}{T_e} + \frac{I_i}{(k_BT_e)^2}\right] \tag{A.2}$$

where

$$f = \frac{[\theta(x_e + a) + x_e)}{x_e(x_e + a)} - \frac{[\theta^2(1 - bx_e) - b(1 - x_e) - \theta]}{(1 - bx_e)\,[x_e(\theta - 1) + 1]} \tag{A.3}$$

$$g = \frac{(1 - x_e)}{[x_e(\theta - 1, + 1]} - \log_e(S/x_e) \tag{A.4}$$

$$a = (1 - i)/i,\ b = (i + 1)/i \tag{A.5}$$

The following are the expressions for model B.

$$\frac{\partial x_e}{\partial T_e} = \frac{g}{fT_h} \tag{A.6}$$

$$\frac{\partial x_e}{\partial T_h} = \frac{g}{fT_h} + \frac{1}{f}\left[\frac{2.5}{T_e} + \frac{I_i}{(k_BT_e)^2}\right] \tag{A.7}$$

where

$$f = \frac{2x_e + a}{x_e(x_e + a)} - \frac{[(\theta - 1)(1 - 2bx_e) - b}{(1 - bx_e)\,[x_e(\theta - 1) + 1]} \tag{A.8}$$

$$g = \frac{1 - x_e}{[x_e(\theta - 1) + 1]} \tag{A.9}$$

and b from equation (A.5).