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BOOK REVIEWS

Theodore S. Motzkin : Selected papers edited by D. Cantor, B. Gordon and B. Rothschild. Birkhauser-Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1983, pp. 530, S.Fr. 158.

Theodore Samuel Motzkin (1908–1970) belongs to the exceptional class of mathematicians who have made significant basic and beautiful contributions to several areas of mathematics. The admiration for his work is much more, considering the fact that this class is almost extinct in this era of specialisation.

The present volume partially fulfils a long-standing need for the collected works of Motzkin. As the editors remark, Motzkin was a mathematician of great erudition, versatility and ingenuity. Exceptionally broad, the range of his work included beautiful contributions to the theory of linear inequalities and programming, approximation theory, convexity, combinatorics, algebraic geometry, number theory, algebra function theory and numerical analysis. The very name Motzkin has become familiar as the descriptive title of entities, concepts and theorems belonging to several different parts of mathematics. The many areas in which he worked were, however, unified by the thread of his own characteristic approach and style.

The editors have carefully selected some of the papers of Motzkin and classified them in seven broad areas, Linear inequalities and Linear programming, Convexity, Algebra, Combinatorics and Graph theory, Power series, Approxmation theory and Miscellaneous papers.

Among the papers in linear inequalities and programming included is a translation of Motzkin's Ph.D. thesis which influenced immensely the development of these areas.

Apart from presenting a first coherent synthesis of all the work previously done in this area, and proving the most general form of transposition duality theorem and several other important new results, this thesis provided the basic ideas for the double description method in linear programming, developed later by Motzkin (with Raifa, Thompson and Thrall) in 1953. Among some other papers in this field, included in the volume is an important paper (with I. J. Schoenberg, 1954) on the relation method for solving linear inequalities, which also has interesting connections with pattern recognition.

Motzkin's work in combinatorics and graph theory had a great influence in the rapid development of these fields. In fact, there are certain methods and styles in these areas which one could really identify with Motzkin or his associates. An example is his beautiful short paper on convex polytopes (1964) in which he develops an ingenious method to prove a well-known conjecture of Eberhard (1891) on the evenness of the number of edges of a convex trivalent polyhedron, in which every face has edge number a multiple of 3. Using some simple properties of the group $SL_2(F_3)$, which at first seems completely unrelated to the problem, he proves the conjecture in just few lines. The method used here should be useful for studying many other combinatorial objects and has still not been exploited completely. Among the other papers included in this are the well-known generalisation of Sylvester Gallai theorem (1949), application of Ramsey's theorem to the convex sets (with P. E. O'Neil, 1967), the paper with E. G. Strauss (1965) giving a new proof of Turan's theorem, the paper on assignment problem (included in the linear programming section), ballot problem (with A. Dvoretzky, 1947) and the well-known paper with B. Grunbaum (1962) on the falsity of the conjecture about Hamiltonianness of polyhedral graphs.

Motzkin's characteristic style is quite visible in his well-known paper on Euclidean algorithm (1949), where he gives a new formulation of Euclidean algorithm and then uses it to give a first proof of the fact that there are PID's which are not Euclidean with respect to any norm (for example $Z\,[(1+\sqrt{-19})/2]$. Among some other papers in algebra included in the volume are his papers with O. Tausky-Todd (1952–55) on pairs of matrices with property *L*, which have several applications in statistics and physics and an abstract from *Bulletin of the American Mathematical Society*, **61** (1955) (included in Miscellaneous Section) on an efficient algorithm on evaluation of polynomials. This algorithm was originally used in the development of subroutines for digital computers and was also influential in the origination of an important field of mathematics, the computational complexity.

Motzkin's remarkable proof of Hilbert Nullstellestaz (1955) and the paper on questions related to Hilbert's 17th problem (1967) are also included. The last one gives the celebrated Motzkin polynomial $f(x_1, x_2) = 1 + x_1^4 x_2^2 + x_1^2 x_2^4 - 3x_1^2 x_2^2$ which is always $\geqslant 0$ but cannot be expressed as a sum of squares of real polynomials.

Power series and approximation theory are some of the other areas in which Motzkin made significant contributions. Very characteristic of Motzkin are his papers (some jointly with E. G. Strauss) on co-positive functions (1965–69), *i.e.*, functions >0 when independent variables are >0. In joint papers with J. L. Walsh, one of his many collaborators in the approximation theory, he produced a series of articles concerning approximation to specified values on certain point sets by special classes of functions. In the process they defined a number of new measures of closeness for approximations. They also studied analogues of best approximations by Chebycheff polynomials. Included in the volume are also the important papers with A. Sharma and E. G. Strauss on use of various averages as measure of interpolation (published in 1973) and the paper with I. Schoenberg (1952) on entire functions in variables which can be approximated by products of linear polynomials.

The reviewer only wishes that a collected work of all the papers of Motzkin is published in the near future. However, it can be remarked that the selection and presentation of the papers by the editors of the present volume in their limitation of space is excellent.

Tata Institute of Fundamental Research
Bombay 400 005.

N. M. SINGHI

Leonhard Euler (1707–1783). Beitrag zu Leben und Werk. Birkhauser Verlag, Basel, 1983, pp. 555, S.Fr. 58.

This is a memorial volume for Euler brought out by Basel-City Canton. The editors point out that this differs in character from the usual run of jubilee volumes in that care has been taken to organize the contents into a unity which the latter often lack; what is attempted is a unified presentation of Euler's life and work. The various facts of his scientific activity in a synoptic version, the subsequent working out of his ideas and the impact of his work on later science. The bulk of the work is in German but for five articles in English and five in French. The book is beautifully produced and the numerous photographs and facsimiles add to its attractiveness.

The contents fall into well-marked divisions. The book opens with an account by Fellmann on Euler's life and a very readable and systematic analysis of his work in mathematics (number theory, algebra, infinite series, analysis and function theory, calculus of variations, geometry) and other fields like mechanics, optics, astronomy, physics, philosophy and music. The next four articles pertain to number theory ; these include a survey article by Andre Weil on Euler's arithmetical work, one by Gelfond on Euler's notions of the infinite in analysis, one on generating functions in the theory of partitions, and one on a manuscript supplement to number theory. Items 6–9 are on analysis and deal with a hitherto unpublished manuscript on differential calculus, the fundamentals of analysis a la Euler–d' Alembert, ideas on non-standard analysis in Euler and Spline interpolation. Next, there is a group of seven articles pertaining to applied mathematics–optics, electricity and magnetism, Euler's contribution to mathematical hydraulics, theory of ships, angular momentum (by Van der Waerden), Euler's place in the development of optics, potential theory. The next two relate to astronomy: Euler's work on the three-body problem and Euler as astronomer. These are followed by two papers on Euler and the Petersburgh Academy and Euler's physics in Russia. There is one on Euler's role in the development of French science followed by four at a more personal level (like Euler and Lagrange, Euler and Cramer). Next we have a paper on Euler's philosophical ideas. Finally, there is an informative account of the history of the publication of Euler's collected works, and an index of Euler's correspondence. In such a varied collection, not all contributions naturally are of equal interest and judgement is bound to be subjective, depending to some extent on the reader's tastes and interests. However, together they succeed eminently in presenting an overall picture of Euler and his work. Considering that a large part of Euler's professional life was spent in Russia, there are several contributions by Russian authors.

Euler's domain was 'natural philosophy', to use an outdated but still appropriate and expressive phrase. Great mathematicians—Archimedes, Newton, Gauss or Riemann and Poincare—do not seem to have accepted the dichotomy of pure and applied mathematics. Their work was often what in current jargon would be called inter-disciplinary in nature, but this was in response to inner compulsions (in the subject and in the person) and not due to external pressures. And they succeeded to an amazing extent because, for one thing, the urge was genuine and for another, because besides being masters in their chosen fields, their command of other disciplines was little short of the expert level. Social relevance was hardly a fad with them ; by sheer scientific insight they isolated significant scientific problems and an unerring instinct directed them to sensible questions, and while they had no built-in bias against the genesis of a problem their primary interest in it was scientific. Gauss willingly spent years computing the orbits of asteroids and devoted time and energy to carrying out geodetic surveys, but he makes no secret of the fact that for him fulfilment came from the theorema Egregium rather than from the political importance of his work for the state of Hanover. And it is not incongruous to find in the collected works of Riemann a memoir on prime numbers rubbing shoulders with one on the mechanics of the ear.

Euler's impact on mathematics and mathematicians was universal. His successors usually started at the points where he had left of, and often it took decades to weld his rich formal work into a theory. Gauss in his writings reserves the adjective 'illustrious' for Euler–Newton was 'supreme'. Probably Euler's impact on mathematical pedagogy was even more pervasive. How many of us realize that the pattern for a first course in calculus was set for all time by Euler and that the way such a course is organized today is exactly as it was done in Euler's infinitesimal calculus—the choice of topics, the arrangement and order of presentation and the essential ingredients of the proofs ? It is revealing to see that sometimes even exercises in a text-book on differential equations (like the old one by Murray) have been bodily taken from Euler's writings. College books like the one on algebra by Chrystal or Smith (on which past generations of Indian students were reared) were in parts often resumes of Euler's treatises—even to the extent of treating convergence which it would be sacrilege to do today in a book on algebra. One must really be grateful to these authors for the good sense they exhibited in going straight to the masters. The generations which immediately followed Euler had the good fortune to learn their mathematics directly from him. The easy availability of a multiplicity of text-books without personaliti, written 'in accordance with the syllabus prescribed' is not an unmixed advantage. How many today who want to learn integration theory would think of reading Lebesgue ?

Of late, several titles of the type of the book under review have appeared and the tendency fortunately seems to be on the increase. It marks a healthy trend ; it is stimulating occasionally to return to the sources.

Department of Statistics
University of Kerala
Kariavattom 695 581.
Kerala State, India.

K. BALAGANGADHARAN

Collected mathematical papers of Alexander Ostrowski (Vol.2) Multivariate algebra and formal algebra, Birkhauser Verlag, Basel, Switzerland, 1983, pp. 652, S. Fr. 93.

Professor Alexander Ostrowski is one of the greatest mathematicians of this century who made fundamental contributions to virtually all branches of mathematics—algebra, number theory, function theory, analysis and numerical analysis. It is therefore of inestimable importance to collect all his fundamental papers which are no longer readily available—and publish these as comprehensive volumes for the benefit of contemporary researchers and the coming generation of mathematicians. Mathematicians should therefore welcome the commendable efforts made by Birkhauser Verlag to publish these papers as a series of six volumes arranged under sixteen topic headings.

Prof. Ostrowski was born in Kiev in 1893 and was a very distinguished member of the famous 'Gottingen School'; later he earned his reputation as a leading mathematician while he was a Professor at the University of Basel, for over 30 years, until his retirement in 1958. His mathematical investigations gave impetus to several branches of mathematics and computer science.

The present volume consists of fourteen papers in multivariate algebra and eleven papers from formal algebra. Fifteen of these papers are the original articles in German, one in Russian and the remaining ones are in English.

The significance of all these papers arises from the fact that these are seminal and gave birth to radically new branches of contemporary mathematics and computer science—complexity theory, symbolic integration and algebraic computation. Of particular interest to computer scientists and numerical analysts are those classic papers on multiplication and factorization of polynomials, abstract algebra connected with Horner's rule, and iterative solution of functional equations.

For mathematicians interested in multivariate algebra and formal algebra this is a rare collection (in one volume) of papers which are not so readily accessible.

It is always extremely exciting, brilliantly illuminating and intellectually rewarding when one reads the original works of a maestro who has such a remarkable power of exposition and pedagogical clarity of expression.

Ostrowski's collected works are appearing in six volumes and it is needless to say that every mathematical library should possess all of them.

Applied Mathematics Department
Indian Institute of Science
Bangalore 560 012.

E. V. KRISHNAMURTHY

Girolamo Cardano edited by Markus Fierz, translated by Helga Niman. Birkhauser Verlag AG, P.O. Box 133, CH-4010, Basel, Switzerland, 1983, pp. xxii + 202, S. Fr. 30.

Girolamo Cardano (1501-1576) is a familiar name in mathematics. The Cardano's solution of a cubic equation are given in any under-graduate algebra text-book. His book 'Arc Magna' printed in 1545 in Nuremberg and translated and edited by Richard Witmer as 'The Great Art or The Rules of Algebra' has always been highly praised and is considered as a milestone in the history of mathematics. Cardano's place in the history of science as a mathematician is therefore assured. Markus Fierz in his study of Girolamo Cardano's life focuses the reader's interest on the other relatively less known aspects of Cardano's work in the field of medicine, natural philosophy, astrology and psychology. The book originally written in the German is translated very well in English by Helga Niman. The English edition includes a list of references quoted in the notes given at the end of the book and a record—as complete as possible—of the original editions of Cardano's writings. This makes this edition a very important contribution in the field of history of science and the history of ideas. Besides making a very interesting reading for a general reader it is of great value to research workers in the history of science.

Cardano wrote about forty-seven books on natural philosophy, theology, mathematics, medicine, astrology and interpretation of dreams. These writings were collected by Cardano in the ten volumes of 'Opera Omnia' published in 1663. Markus Fierz has studied Cardano's life and work on the basis of the writings in the Opera. Many excerpts and translations, taken directly from these work, are given in the book.

The intellectual life of the High Renaissance period and the influence it had on Cardano's views on science, medicine and philosophy is described very vividly in the first chapter on Cardano's life and writings. In the following chapter Markus Fierz discusses Cardano as a physician. Cardano was a practising physician as well as a Professor of Medicine. Cardano owed his worldwide reputation not due to his theoretical views though, but in his abilities as a medical practitioner. The author impresses upon the reader the sensible human attitude that was characteristic of Cardano's practice of medicine.

The third and fourth chapters are devoted to the study of Girolamo Cardano as a natural philosopher and theologist. Markus has successfully attempted to present Cardano's thoughts in natural philosophy in a systematic manner. The complete fourth chapter discusses the very important writings of Cardano on natural philosophy published as *De Subtilitate* and *De Rerum Varietate*. The latter contains the material which Cardano gathered in preparation of his former book. *De Subtilitate* enjoyed enormous success in Cardano's time and continued to be frequently reprinted throughout the seventeenth century, long after Cardano's death. With this book Cardano exerted a marked influence on the intellectual life of the barouque period. Unfortunately, this influence is rarely acknowledged !

Astrology was a respectable science in the sixteenth century and physicians educated at the universities frequently were also astrologers. The fifth chapter tells us about Cardano's views and writings on astrology. Cardano, who was highly esteemed as an astrologer, seems to have compiled one hundred horoscopes, mostly of notable personalities, emperors and kings. The author has interestingly given three examples of the horoscopes written by Cardano.

Cardano, as an interpreter of dreams, showed remarkable originality. In the sixth chapter—Cardano's work on dreams—types of dreams, principles and rules for interpretation of dreams—is discussed.

The last chapter in the book entitled ' on the art of living with oneself ' relates the sadness and disturbances in the last years of Cardano's life, which he was forced to spend in Rome. To cope with his troubled mind Cardano in his seventy-fourth year wrote a strange dialogue between himself and his father who had died long ago. Markus Fierz ends his book by giving the literal translation of this dialogue, which appears to be more of a testimonial on coming to terms with one's own existence. A sad ending of a rich life and a beautiful ending of a good book.

Applied Mathematics Department
Indian Institute of Science
Bangalore 560 012.

CHANCHAL UBEROI

Numerical treatment of inverse problems in differential and integral equations edited by P. Deuflhard and E. Hairer. Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1983, pp. 357, S.Fr. 70. Indian orders to Allied Publishers Pvt. Ltd., New Delhi 110 002.

The book, Vol. 2 in the series ' Progress in Scientific Computing ', is the collection of articles from the proceedings of an international workshop, Heidelberg, Federal Republic of Germany (August 30–September 3, 1982). Beyond doubt, it is of immense value to research workers in mathematical modelling and identification/parameter estimation of dynamic systems governed by ordinary differential (OD), partial differential (PD) or integral (I) equations (E). The applications considered are diverse and quite impressive : chemical engineering, molecular biology, physics, geophysics, astronomy, reservoir simulation, electrocardiology, computer topography, and, interestingly enough, control system design. As far as the last one is concerned, the results are not striking in comparison with the rest. It should, however, be mentioned that most of the terminology has originated in the control system literature of the 60's and early 70's.

The subject matter of the book is the computational treatment of inverse problems originating in practical situations. The inverse problem, in contrast with the 'direct' problem of mere computational simulation of a physical process based on known physical laws, deals with the estimation of unknown parameters in the assumed ODE, PDE or IE model of the process under study, using the measurement data.

The book is divided into four parts, each containing 6 chapters on the average. The first two parts deal with the ODE models, and the remaining two with the PDE and IE models. On the whole, it is a balanced collection of theoretical results and practical applications. Almost all the chapters are replete with examples and comprehensive references to literature. Each chapter could as well be the subject of a separate review. However, in what follows, a concise summary of the chapters is presented in an attempt to highlight the major contributions.

Chapter 1 : Preliminary experiments with Runge-Kutta like codes are presented, and the influence of parameter changes studied. Computation of values at the mesh points of an automatic integrator is shown. These values are smooth functions of the parameters.

Chapters 2, 3 *and* 4 : The common theme is the inverse problem of chemical kinetics in the modelling of chemical reaction systems. In Chapter 2, the simulation package (of the direct problem) is combined with a nonlinear least squares fit routine, leading to a Gauss–Newton iteration for the l_2-minimization discrete problem. This procedure seems to be new in the field of chemical kinetics. Chapter 3 deals with the ODE model of Noyes, Field and Thomson of a large-scale biochemical system. In Chapter 4 the problem is solved not by analyzing the concentration matrix, but by using a new graphical matrix anlaysis technique on the absorbance matrix.

Chapter 5 : The problem is one of determining the several small perturbing functions in a system represented by ODE. The method employed is deterministic, and is based on the assumption that the solutions of the DE can be expressed piece-wise in short intervals by a Taylor convergent expansion and that the unknown perturbations can be approximated by elementary functions This is in contrast with the assumption of random vector function for the perturbations, as found in the literature.

Chapter 6 : Multiple shooting techniques for the numerical solution of standard nonlinear boundary value problem (BVP) for ODE (without explicit dependence on parameters) are considered. A detailed elementwise rounding error analysis is given of those BVP's for which the condensing algorithm is numerically stable.

Chapter 7 : The problem considered is one of parameter identification technique for the discretized nonlinear ODE model BVP on the basis of generalized Gauss–Newton method. For the efficiency, stability and applicability, features of the implementation are presented. Reference is made to a notorious test problem.

Chapter 8 : Here the multiple shooting method is applied to four parameter estimation problems (two from the field of fluid dynamics, and the others concerned with nuclear or plasma physics). Choice of step size is done adaptively.

Chapter 9 : The method of unrestricted harmonic balance is applied to a chemical reaction, diffusion system, characterized by parabolic differential equations with cyclic boundary conditions (but transformed to a system of ODE).

Chapter 10 : The inverse eigenvalue problem for the earth's mantle is reduced to the determination of the density of the earth's mantle from the overtone eigen-frequencies. Many open problems to account for inhomogeneities and *a priori* knowledge, and to improve algorithmic accuracy are briefly presented.

Chapter 11 : The inverse problem of quantal scattering defined by the partial wave Schrödinger equation amounts to the determination of the potential $V(r)$ from the scattering function $S_1(E)$ when E is fixed. The new technique is based on interpolation. Some practical applications are given.

Chapters 12 and 13 : The well known problem of pole assignment by feedback for multivariable control systems is considered and reduced to an inverse eigenvalue problem which requires the determination of a matrix having prescribed eigenvalues. Algorithms are given for computing solutions to the pole assignment problem which satisfy certain robustness criteria.

Chapter 14 : Direct and inverse problems of electrocardiology (in terms of cardiac sources and potential alone) are formulated involving partial differential equations with presciibed boundary conditions and their models.

Chapter 15 : A mathematical reservoir model with unknown parameter values such as porosity and permeability is examined using the technique of history matching which is distinct from standard parameter estimation procedures (in view of the large dimensionality of the system state and the number of unknown parameters).

Chapters 16, 17 & 18 : Identification of parameters of nonlinear parabolic systems is exlmplified by water flow through porous media. Computer simulation is used to study the problem of parameter sensitivity. In Chapter 17 similar problem is solved. An iterative method (due to Crank–Nicolson) is used to estimate the diffusion function suitably approximated. In Chapter 18 the same problem (in one dimension) is reformulated as a Volterra integral equation of the first kind and is solved numerically.

Chapter 19: Determination of the geometric and physical parameters of the non-linear model of the ground in geophysical prospecting is described. Measurements are of the natural and artificial fields at the earth's surface.

Chapter 20: The two-dimensional inversion problem involving the index of refraction or propagation speed of a medium is examined in some localized region up to small perturbations of order ϵ of the known index of refraction. These perturbations are to be estimated from the observations of the scattered field. The problem is reduced to an integral equation of the second kind (Fredholm).

Chapter 21: The reconstruction problem in computer topography with incomplete data and unknown operators is treated.

Chapter 22: Ill-posed linear, very large nonlinear and computationally complex problems are considered with reference to molecular biology. Also dealt with are the use of inequality constraints by 'regularization approach' and choice of minimum number of parameters in a model recommended to reduce the complexity of the problems.

Chapter 23: Fredholm integral equation of the first kind of convolution type is analysed with the help of Tikhonov regularization and Fourier extrapolation methods.

Chapter 24: Numerical solution of convolution equations for non-negative kernels is derived using discrete Fourier transforms.

Chapter 25: A common abstract framework (Hilbert space) is developed for studying the convergence and effect of data perturbations for models of systems of various types (integral equations of the first kind, two-point BVP and others) assuming finite measurements.

Each of the chapters is contributed by leading workers in the area of computational mathematics (pure and applied). A consquence of this diversity in content and style is that no coherence in mathematical techniques is transparent to the serious reader. Nevertheless, the book does contain a wealth of material of some value to research workers in the numerical analysis of physical systems modelled by ODE, PDE and IE.

Electrical Engineering Department
Indian Institute of Science
Bangalore 560 012.

Y. V. VENKATESH

Differential-difference equations—Applications and numerical problems edited by L. Collatz, G. Meinardus and W. Wetterling, International Series of Numerical Mathematics, Vol. 62, Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Swizerland, 1983, pp. 196, S.Fr. 42. Indian orders to Allied Publishers Pvt. Ltd., New Delhi 110 002.

This volume contains manuscripts of 15 lectures delivered at a conference organised by the editors from 6–12 June, 1982. The aim of the conference was to make different aspects of the subject (which appear to be still quite heterogeneous) clear and to work out common features in the course of the discussions. This explains a selection of almost disconnected topics covered by the different participants in their lectures, as for example,

1. The influence of interpolation on the global error in retarded differential equations by H. Arndt.

2. A numerical procedure to compute many solutions of diffusion-reaction systems by E. Bohl.

3. Behaviour of the solution of the difference equation $y(n) = y(n-1) - cy(an)$, $n = 2, 3, 4, \ldots, y(1) = 1$ by L. Collatz.

4. Estimating the global error of Runge-Kutta approximations for ordinary differential equations by K. Dekker and J. G. Verwer.

5. Characteristic roots of discretized functional differential equations by de Gee, M.

6. Automatic delay-control in two-phase Stefan problem by K. H. Hoffmann and J. Sprekels, etc.

The contributions are similar to research papers rather than in the form of review articles. The collection contains neither reporting of the disscusion by the participants following the lectures nor a summary indicating unifying theme by the editors. Due to the absence of the report on the discussions and summary by editors or some other expert, a reader gets the feeling of going through a research journal. The book certainly contains article of good quality, which are also important because, due to the availability of very fast computers, the discrete mathematical models are going to play an important role in the understanding of many systems where classical continuum models are either not valid or are not suitable.

Applied Mathematics Department
Indian Institute of Science
Bangalore 560 012.

PHOOLAN PRASAD

Uniqueness and non-uniqueness in the Cauchy problem (Progress in Mathematics—Vol. 33) by Claude Zuily. Birkhäuser Verlag, P.O. Box 133, CH-4010 Basel, Switzerland, 1983, pp. 168, S.Fr. 32. Indian orders to Allied Publishers Pvt. Ltd., New Delhi 110 002.

The uniqueness of solution in the non-characteristic Cauchy problem for operators with analytic coefficients follows from the classical theorem of Holmgren. The existence and uniqueness in the analytic set-up is given by the celebrated theorem of Cauchy–Kovalevskaia.

In the C^∞ frame work, the non-characteristic Cauchy problem is well posed, in general, for hyperbolic operators ; indeed, at least in the constant coefficient case, hyperbolicity is necessary for well posedness. On the other hand, uniqueness holds for a much wider class of operators, though not for all.

Uniqueness theorems depend on successful application of variants of the 'Carleman estimates' which are weighted integral inequalities in L^2 norm, first used by Carleman[1] in a special case. Non-uniqueness theorems involve constructions using delicate variants of the methods of geometrical optics and are very reminiscent of solving for the 'phase' and 'transport' equations.

Since there is a definite link between uniqueness in the Cauchy problem and local solvability of an 'associated' pseudo-differential equation, uniqueness theorems often involve conditions of local solvability, either that of Hörmander or those of Nirenberg-Treves[2]. Uniqueness also depends on multiplicities of the real and complex characteristics, regularity of the coefficients and indeed on all terms of the operator. If the initial hypersurface is allowed to be characteristic there is non-uniqueness in general and uniqueness depends on the geometrical conditions between the operator and the hyper surface, called 'pseudo-convexity conditions' introduced by Hörmander[3].

The first major result for uniqueness in the non-characteristic Cauchy problem assuming simple real and atmost double complex characteristics was given by Calderon[4] in 1957 who demonstrated the flexibility and power of the use of pseudo-differential operators as a tool.

The book by Zuily gives an excellent introduction to the uniqueness and non-uniqueness in the Cauchy problem and contains three carefully arranged chapters on the results available to date.

The first chapter deals with uniqueness and non-uniqueness results for first order operators verifying (or violating) the solvability condition P of Nirenberg–Treves.

The second chapter contains an extension of Calderon's uniqueness theorem, not assuming constant multiplicity of the characteristics. Uniqueness result of Watanabe in the case of atmost triple complex roots is mentioned without proof and a counter example is given in the case of higher multiplicity. Finally, some uniqueness results are proved in the case of arbitrary high but constant multiplicity of the roots involving conditions on the sub-principal symbol.

The third chapter makes use of modern tools such as the Weyl calculus of pseudo-differential operators and symplectic geometry of the contangent bundle. Hörmander's uniqueness theorem for strongly pseudo-convex surfaces is proved and a non-uniqueness result is given otherwise. Symplectic geometry is applied to discuss uniqueness and non-uniqueness results for a class of quasihomogeneous operators.

At the end, one feels that the problem of classification is still in the large, much as it is in the case of operators with multiple characteristics. The book fails to mention the extensive works of Ivrii and Petkoff[5] on the Cauchy problem for the case of variable multiplicity of the roots where useful concepts such as 'the index of well posedness' have been introduced.

The discerning reader will no doubt benefit from a wealth of ideas and techniques presented systematically in the book together with a comprehensive bibliography.

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TIFR Centre
Bangalore 560 012.

ADIMURTHI
B. R. NAGARAJ

Functional integration and semi-classical expansions edited by F. Langouche, D. Roecakerts and E. Tirapegui, D. Reidel Publishing Company, Dordrecht, 1982, pp. 328, $ 39.50.

In this book, functional integrals are defined as limits of discretized expressions. It is observed that several discretized expressions give the same limit and lead to formally distinct functional integral representations. Here, instead of considering a unique representation in each class, each such representation is considered separately.

The notion of discretization and the relation of the multiplicity of representation to the ordering problem of operators and the stochastic properties of the paths are also considered. Non-linear point transformations are studied. Covariant definitions of functional integrals are also dealt with. These techniques are then applied to Fokler-Planck dynamics and to the generalization of the Onsayer-Mechlup theory to general diffusion processes. Multiplicity problem is also considered from the point of view of stochastic differential equations, thus illustrating the stochastic origin of the different representations. Product integrals and their relation to path integrals are also treated and semi-classical expansion is also studied.

An exact calculation of the propagator on the manifold of SU(2) is presented and this serves as a good illustration of discretization technique.

The functional integrals are made use of in various branches of modern physics and as such, it is no doubt impossible to give a full account of all its applications in a single book. So, as to be expected, the authors do not deal with all the practical ramifications of functional methods. However, they give an account of the theory of functional integrals from a general point of view.

Finally, a remark regarding the typography used in the book. The size is so small that one has to unduly strain to read it.

Ramanujan Institute
University of Madras
Madras 600 005.

P. S. REMA

Geometric quantization in action (Mathematics and its applications/8) by Norman E. Hurt. D. Reidel Publishing Co., P.O. Box 17, 3300 AA Dordrecht, Holland, 1983, pp. 336, $ 49.50.

The procedure by which one passes from the classical description of a physical system to the 'corresponding' quantum mechanical description has always been a heuristic one. In fact, whenever one tries to compare some aspects of the quantum description with analogous aspects in the classical description, one usually has to make some conventions or rules as the basis for the comparison. There is, in the sense in which the term is used in mathematics, no natural correspondence or both ways unambiguous relationship between the two. This is seen, for instance, when one wishes to compare physical observables of a quantum system with those of the analogue classical system, and one tries to set up some sort of correspondence between the two sets of observables:

several correspondences are available, each being appropriate in a particular context but not in others. This ambiguity in the process of 'quantizing' a given classical system is quite familiar to physicists, whether they follow the methods of canonical quantization or the Feynman path integral approach.

In recent times the increasing use of differential geometric methods in this context has raised the hope that the issues involved may be clarified a great deal if not resolved to one's complete satisfaction. At the classical level, such methods have been successfully used to express the basic principles and intrinsic structure of classical dynamics in as concise and direct a manner as possible. One of the purposes of the relatively recent subject of geometric quantization has been to build on such a description of classical theory and then to see what additional geometric objects and assumptions are needed to pass to a quantum theory.

The reviewer expected from its title that the present book would explain to a physicist what geometric quantization was all about, show how it clarified the transition from classical to quantum theory, and then apply the new insights and methods to situations of physical interest. In actual fact, it turns out that the aims of this book are really very different from this. One can say right away that this is *not* a book that a reasonably well-trained physicist could expect to *learn* from without *enormous* effort. In particular, one would not turn to this book, nor recommend it to a student, to get to know what geometric quantization is or what classical dynamics looks like in intrinsic term, if one did not already know these things. The aim of this book is to cover briefly an extraordinarily large number of topics of current mathematical research, and roughly indicate their connections to one another and to questions of physics.

The author covers the main concept of group representation theory in a few pages, and reviews the basic ideas of calculus on manifolds, contact and symplectic manifolds and their use in formalizing the structure of classical dynamics in equally brief style. A chapter deals with the interesting Dirac problem of mapping classical observables onto quantum observables so as to preserve as many algebraic properties as possible. Then there are brief chapters devoted to the geometry of polarizations, orbits, C and R spaces, and de Sitter spaces. The discussion of geometric quantization itself takes up some 15 pages in all, in a book of about 300 pages. Throughout, as is apparently common practice in mathematical writing today, the style is extremely dry. Typically a page contains four definitions and four theorems. The author's conviction is that all the areas of mathematics touched upon are important to the physicist and should be assimilated and used by him. In a sense, this is a very rich book full of insights to the reader who has learnt the mathematics in a more leisurely fashion from some other sources. Probably more than this cannot be expected from a book in which the chapter on Quantum Statistical Mechanics is 22 pages long, and that on Quantum Field Theory is all of 10 pages.

The physicist certainly needs to work extremely hard to appreciate all the points that the author wants to make, and then to see what all this mathematics has to say concerning the kinds of problems mentioned at the beginning of this review.

Centre for Theoretical Studies
Indian Institute of Science
Bangalore 560 012.

N. MUKUNDA

Differential geometry edited by R. Brooks, A. Gray and B. L. Reinhart, (*Progress in Mathematics*, Volume No. 32, Proceedings, Special year in Geometry, Maryland 1981–82), Birkhäuser Verlag, Basel, 1983, pp. 254, S.Fr. 42.

There are in all 13 research articles on topics in Differential Geometry in this volume. They are individually reviewed here.

1. *Combinatorial Laplacians and Sullivan-Whitney forms* : *G. A. Baker, pp.* 1–33

Let M be N-dimensional compact oriented Riemannian manifold and K be a smooth triangulation of M. Let $A^*(K)$ denote the deRham Complex of smooth forms of K. Then the Sullivan Complex $S^*(K)$ of polynomial forms on K is a subcomplex of $A^*(K)$. The author considers finite dimensional subcomplexes $P_r^*(K)\,(r \geqslant N)$ of $A^*(K)$ by restricting the coefficients' degree of the polynomial forms to be r and defines the combinatorial Laplacian Δ_q^r and cohomology groups $H^{q,r}$ of $P_r^*(K)$ and the deRham-Hodge decomposition theorem is proved (Proposition 2.16). Further it was proved that this decomposition theorem is an asymptotically convergent approximation to the analytic Hodge decomposition (Proposition 4.1) and the eigenvalues (spaces) of Δ_q^r converge to those of analytic Laplacian Δ (Proposition 4.6).

2. *Critical points and curvature for embedded polyhedra II* : *T. Banchoff, pp.* 34–55

The critical set theorem : $\Sigma\, \chi(G)\, u(G,\pi) = \chi(S^{k-m-1})\, \chi(M^k)$ is stated for a smooth map $\pi : M^k \to E^m$, where G runs over components of singular set $S(\pi)$ of π, u denotes index of G and χ denotes the Euler characteristic (For $m = 1$ case, see *J. Diff. Geom.* 1967, 1, 257–268). This is verified by several examples and proved for the convex cell complex $M \subset E^n$. For such M Grassmann curvatures $K_m(\tau)$ are defined and Gauss-Bonnet and theorema Egregium are proved (Theorems 2 and 3, § 5). Finally higher curvatures are computed for (*a*) cubical complexes. (*b*) simplicial manifolds, and (*c*) the complex projective plane with 9-point triangulation.

3. *Some Riemannian and dynamical invariants of foliations* : *R. Brooks, pp.* 56–72

For a Riemannian L, the usual invariants (1) $\mu(L)$: the exponential growth (2) $h(L)$: the isoperimetric constant (3) $\lambda_0(L)$: the first eigenvalue (4) Sullivan's openness at infinity are introduced and inequalities and implications for them are given (Theorems 1.2, of Buser and of Gromov). Then for a foliation F over a compact manifold M with flow boxes $\{U_i\}$, transversal set T and pseudogroup structure Γ, the local invariants h_Γ, Følnerness of a Leaf L of F are defined and relations between them are given (Theorem 2.3 and Corollary 2.4). Finally invariant measures for foliation F are defined. The non-existence of such measures on F implies h (L) is bounded below by a positive constant h, for all leaves L of F (Corollary 3.2). Moreover, necessary and sufficient condition for the existence of such h is given (Theorem 3.4). Further, amenability of a foliation F(which is a global property of F) is also discussed.

4. *Conformal geometry* : *L. Ehrenpreis, pp.* 73–88

Conformal maps on $\mathbf{R}^n$ $(n \geqslant 2)$ are studied for the indefinite metric

$$ds^2 = \sum_1^p dx_i^2 - \sum_{p+1}^n dx_i^2$$

and Liouville's theorem is properly interpreted for this metric.

The notion of conformal compactification $\bar{M}$ of a smooth manifold M for a Laplacian Δ is given and several such $\bar{M}$ are found for known Ms. For instance, $\bar{M}$ = the projectivization of the space of light rays when $M = \mathbf{R}^4$. Further it is shown that every harmonic distribution on $\mathbf{R}^n$ with $p \neq 0$, n extends to a hyperfunction harmonic on $\bar{\mathbf{R}}^n$ (Theorem 1). Finally $\bar{\mathbf{R}}^n$ is interpreted as G^c/P where G^c is a noncompact real semi-simple Lie group and P is a maximal parabolic subgroup and the Light Cone as a real algebraic variety defined by quadratic equations.

5. *Vector bundles with harmonic connections over spheres* : *P. B. Gilkey, pp.* 89–102

Let W be a vector bundle with fiber metric and a unitary connection ∇_W over a Riemannian manifold M. The connection ∇_W (respectively its curvature Ω_W) is called covariant constant or harmonic if $\nabla \Omega_W = 0$, of M. Such harmonic connections occur in Yang-Mills theory. In this paper certain vector bundles W over spheres S^n arising from spinor representations are constructed admitting harmonic connections. A typical result is : Let τ be an irreducible unitary representation of $PIN(n+1)$. Form bundles $\pi_\pm$ with connection $\nabla_\pm$ and curvature $\Omega_\pm$. Then $\nabla \Omega_\pm = 0$ (Theorem 4.3). A special case of this for n is even studied in detail. This theorem (4.3) is generalized for bundles over a hyperbolic space (Theorem 7.1). Further, it is shown that certain bundles $\pi_\pm(P_\tau)$ over the sphere bundles $S(T^*M)$ will not admit harmonic connections in general where M is a Riemannian manifold with spin structure and τ is a representation of $PIN(m)$.

6. *Foliations and metrics* : *F. W. Kamber and P. Tondeur, pp.* 103–152

For a foliation F over a R-manifold M given by $O \to L \to TM \xrightarrow{\pi} Q \to 0$, its second fundamental form α of F is given by $\alpha = -\nabla\pi$. F is called *totally geodesic* if $\alpha \equiv 0$ on all leaves, F is called *harmonic* if its mean curvature 1-form K or its tension field τ vanishes. F is called *Taut* if F is harmonic for some g_M on M. Various characterizations and equivalences of these concepts of F are given (Theorems 1.18, 1.19). Then codimension 1 foliations are discussed (Theorem 1.27).

Using the spectral sequence $E(F)$ associated to a foliation F on M *via* the filtration of the deRham complex $\Omega^*(M)$, a characterization for a foliation F of codimension q (respectively for a R-foliation F), to be tense (taut), is given (Theorems 2.17, 4.23 and 4.25). Also compact foliations are studied using Theorem 4.25 and Remmler's theorem (4.26). Also deRham duality theorem is proved for a foliation F (Theorem 3.12). Then some examples of (*a*) foliations with a transversal symplectic structure, (*b*) codimension one foliations are studied. Finally, harmonicity of an R-foliation F is characterized as an extremal of the energy functional (Theorem 5.4) and the second variational formula is proved for them, and also the stability of foliations over sphere S^n is discussed.

7. *The growth of harmonic functions and maps* : *L, Karp, pp.* 153–161

In this paper the theorems proved are

Theorem A' : Let M^n be a complete R-manifold of non-negative Ricci curvature and u is a real function on M^n with $\Delta u = f(u)$ for f in C^1. If $u\,f(u) \geqslant 0$ and $f'(u) \geqslant 0$ then either ∇u is parallel on M^n or $\limsup_{r \to \infty} \frac{1}{r^n F(r)} \int_{B(r)} u^2 \, d\,\text{vol.} = +\infty$ (*).

Moreover, if the Ricci curvature is positive or if $f'(u) > 0$ at some point of M then either $u =$ constant or u satisfies (*).

Theorem B' : Similar result holds for an harmonic map $f : M^n \to N^k$, N manifold with a pole at y_0.

8. *Surface symmetrics, holomorphic maps and tessellations* : *R. S. Kulkarni. pp.* 162–176

Classification problem of holomorphic maps of R-surfaces (respectively of surface symmetrics) is reduced to classification of pairs (Γ, Φ) where Γ is a properly discontinuous group and Φ is a torsion-free subgroup (respectively normal subgroup) of finite index in Γ. The case studied here is for Γ finitely generated. Necessary and sufficient conditions in order that Φ be a subgroup of finite index are given (Theorem 2.2). These are found to be sufficient for cocompact Fuchsian groups Γ n case of genus Γ/H is positive. For the case of genus 0 (*i.e.*, $\Gamma/H = S^2$) some partial results are given (Theorems 3.4 and 3.5).

In Part II conditions for Φ to be normal in Γ are discussed and no good N and S conditions are obtained in general ; however, in the case of orientation-preserving finite group actions on 2-torus this is done, *via* ζ-functions $\zeta(\Gamma, S)$. Then density function $\delta(\Gamma)$ is defined and $N \times S$ conditions for $\delta(\Gamma) > 0$ are given.

9. *Three-dimensional Lorentz space forms and Seifert fiber spaces*: *R. S. Kulkarni and F. Raymond*, pp. 177–188

The Lorentz space forms are studied for the special quadric $S^{1,2}$ in $\mathbf{R}^4$. A close connection between (*a*) 3-dim. Lorentz space forms of type (1, 2) with constant curvature 1, (*b*) Seifert fiber spaces with non-zero Euler number, (*c*) extensions of $\mathbf{Z}$ by Fuchsian groups with non-zero Euler number are given. Roughly, a Seifert fiber space looks like solid tori M fibered by circles and this gives rise to an exact sequence of groups : $1 \to \mathbf{Z} \to \Gamma \to \Phi \to 1$ where $\Gamma (= \pi_1(M))$ is a central extension of $\mathbf{Z}$ by Φ. Typical results are : A standard compact Lorentz orbi fold is an orientable Seifert fiber space with non-zero Euler number and conversely (Theorems 4.1 and 4.2).

10. *Brownian motion and Riemannian geometry* : *M. A. Pinsky, pp.* 189–202

A Brownian motion is a Markov process with continuous path functions. Here Feller-Markov processes are defined and examples on manifolds like Wiener process, geodesic flows, isotropic transport process and horizontal (Riemannian) Brownian motions are discussed. These can be approximated by geodesics and are studied on manifolds with negative curvature. Two limit theorems for these processes are proved (Theorems 5.1 and 5.2). Also the spectrum of the Laplacian is discussed (Theorem 6.1). Finally mean value theorems for small geodesic balls are derived *via* stochastic and non-stochastic approaches (§ 7, 8, 9).

11. *Invariant hyperbolic systems on symmetric spaces*: *M. M. Shahshahani, pp.* 203–233

In this paper the initial value problem for the system (*) $p\left(\frac{\cdot}{\partial t}\right) \mu = L_p' \mu$ for each p in A subject to the Cauchy data $\mu_j(0, x) = f_j(x)$ on the symmetric space $X = G/K$, where $\mu : A \times X \to C^W$ is a function (A = a max. abelian subalgebra) and $f_j \in C(X)$: Schwartz space of X and L_p' is the transpose of the matrix of the endomorphism defined by $L_p(q) = pq$. Then (*) is a well-posed problem and for its solutions the law of conservation of energy holds. Let H' denote the space of initial data $F = (f_1, f_w)$ from $C(X)$. Then the energy form $E(F, H)$ is a symmetric bilinear form on H'. The main result proved is the following spectral representation theorem : The linear map $E : H' \to L^2(A^* \times K/M.,)$ defined by $E(F) = E(F, \mu)$ where μ is certain solution of system (*), is injective norm-preserving with image a dense subspace of L^2 (the difficult part of proof is in showing the denseness of image of E). As applications, Huygens' principle and the Lax-Philips axioms hold good on $X = G/K$ provided all the Cartan subalgebras of $\underline{G}$ are conjugate.

12. *Homogeneous structures* : *F. Tricerri and L. Vanhecke, pp.* 234–246

A homogeneous structure on an (M, g) is a tensor field T of type $(1, 2)$ satisfying certain identities. There are eight classes of homogeneous structures on (M, g). A non-zero T on (M, g) belongs to class one if and only if (M, g) is the hyperbolic space up to isometry (Theorem 3.2). T on (M, g) is a class three if and only if (M, g) is a naturally reductive homogeneous space (Theorem 4.2). In dimension 3, such manifolds are classified (Theorem 4.4). Among others, the following problems : (i) Do there exist homogeneous manifolds which are not naturally reductive, but in which all geodesics are orbits of 1-parameter subgroups? (ii) Is a homogeneous manifold with volume preserving geodesic symmetries necessarily naturally reductive? are considered. By studying the generalized Heisenberg groups N which have volume preserving geodesic symmetries (Theorem 6.2), the Cayley groups answer (ii) negatively and the 6-dimensional group of type H (of Kaplan) is not reductive and all its geodesics are orbits of 1-parameter subgroups of isometries (Theorem 6.3) and thus answers (i) also negatively.

13. *The number of ends of the universal leaf of a Riemannian foliation* : *H. E. Winkelnkemper, pp.* 247–254

Let F be a K-dimensional smooth R-foliation on R-manifold M. Let $\tilde{L}$ denote the universal leaf of F. The following two results are proved :

(i) If M is closed and simply connected then $e(\tilde{L}) = 0$, 1 or 2 where $e(\tilde{L})$ = number of ends of $\tilde{L}$.

(ii) Let $M, F, \tilde{L}$ be above and let $e(L) \neq 0$. Then the involution $A_\xi : S(\xi) \to S(\xi)$ is cobordant to a trivial involution unless there exists non-trival elements of order 2 in holonomy where ξ is the tangent bundle of F and $S(\xi)$ is the sphere bundle of ξ and A_ξ is the involution on $S(\xi)$ defined by the antipodal map on each fiber.

There are some typing mistakes, though not of a serious nature. On the whole, the get-up and printing are good and the topics covered vary over a wide range in differential geometry and will be useful for research workers in this area.

School of Mathematics
University of Hyderabad
Hyderabad 500 001.

M. SITARAMAYYA

Seminar on stochastic processes 1981 edited by E. Cinclar, K. L. Chung and R. K. Getoor (Vol. 1 of Progress in Probability and Statistics edited by Peter Huber and Murray Rosenblatt), Birkhauser, Basel. 1981, pp. 242, S.Fr. 34. Indian orders to Allied Publishers Pvt. Ltd., New Delhi 110 002.

The book consists of a fragment of the work discussed at the Seminar on Stochastic Processes held at the Northwestern University in April 1981. Beyond doubt, the eight papers in it contain a masterly presentation of the latest results by eminent workers in the area of stochastic processes. The problems mostly refer to Markov processes and the techniques devised to resolve them are the probabilistic versions of classical analysis.

A glance through the book provides the observant reader with a rich panorama of solved and unsolved problems as stimulation for further work.

Each of the papers makes a brief reference to the enormous literature—most of it by the authors themselves—on stochastic processes, and exhibits the main contributions. Of course, it requires a somewhat formidable mathematical framework to present all the papers in their proper perspective. However, only the essential ideas are mentioned here.

The first paper (Feynman-Kac functional and the Schrödinger equation by K. L. Chung and K. M. Rao) treats (i) the development of potential theory using probabilistic methods and Harnack's inequality ; and (ii) the solution of the boundary value problem for the Schrödinger equation $(\Delta + 2q)\varphi = 0$ (elliptic partial differential equation) for which the Feynman-Kac formula supplies the natural Green's operator. The solution includes the classical solution of the Dirichlet problem obtained by probabilistic methods. Comparison with the classical methods would be of considerable interest. It is noted that the Schrödinger equation differs from the Laplacian in that a condition on the size of the domain is necessary to guarantee the uniqueness of the solution. The finiteness of this domain is necessary for probabilistic considerations. In effect, the results are extensions of the one-dimensional case analysed by Chung.

In the second paper (Two results on dual excursions by R. K. Getoor and M. J. Sharpe) a pair X, $\hat{X}$ of standard Markov processes is considered on a common state space E having a dual density relative to some σ-finite measure on E. The first result is concerned with the families of measures P^{xly} governing the distribution of excursions of X, straddling a stopping time T, from a given closed homogeneous optional set M, conditional on the excursion starting at x, ending at y and having length l. Pitman's technique is used for the case of the excursion straddling a general stopping time T. In the second result, it is shown that if b is a regular point for X, b is recurrent, and all excursions of X from b begin and end at b, then the path map Φ on Ω that reverses every excursion away from b maps P^b to $\hat{P}^b$.

The third paper (Characterization of the Levy measure of inverse local times of gap diffusion by F. B. Knight) deals with $X(t)$, a persistent non-singular diffusion on an interval Q containing O. Assuming a natural scale, X is characterized by its speed measure $m(dx)$ on Q, and finite endpoints are reflecting. The problem then is to characterize the class of all $n(dy)$ which can appear when Q and $m(dx)$ vary. The inverse spectral theorem of Krein is applied to the problem after employing a couple of probabilistic transformations.

The fourth paper (Levy systems and path decompositions by J. W. Pitman) explains, in terms of point processes (attached to a Markov process as introduced by Ito), exactly how a Levy system induces a path decomposition which is 'right'. (A path decomposition theorem is a result to the effect that some fragment of the trajectory X is conditionally independent of some other fragment given suitable conditioning variables, with one or more of the fragments being conditionally Markovian.)

Given a suitable point process π of excursions, the splitting time theorems amount to a decomposition of π at the first time $t = \tau_\mu$ that one of the points $\pi_{t\mu}$ hits a set $A_{t\mu}$, which may in general depend either optionally or predictably on information up to time t. The intuitive basis for this decomposition is the obvious splitting of the information that $\tau = t$ into past and present components

$$\{\tau = t\} = \{\pi_s \notin A_s,\ O < s < t\} \cap \{\pi_t \in A_t\}$$

The result of the application of Levy systems to prove path decompositions is an extension of the one due to Weil. Other results are obtained as special cases : for instance, the last exit decompositions of Pittenger and Shih and others.

The fifth paper (Regular birth and death times for Markov processes by A. O. Pittenger) considers another aspect of Markovian processes : Class of random times called optional times or stopping times, characterized by the strong Markov property, $\{T < t\} \in F_t$, where F_t denotes the usual completion of $\sigma(X_s . s \leq t)$, Methods used to obtain a generalization of the results due to Jacobsen and Pitman are described.

The paper, 'Some results on energy' by Z. R. Pop-Stojanovic and K. M. Rao, uses probabilistic tools such as sub-Markov resolvents, Revuz measures and additive functionals (as against the classical Dirichlet spaces techniques and the kernel theory) to cover separability, limits of potentials with bounded energy, excessive functions of finite energy and convergence in energy. In a general setting, the complete analogue of Cartan's result in classical potential theory (that the space of positive measures of finite energy is complete) is presented.

The next paper, 'Absolute continuity and the finite topology' by J. Walsh and W. Winkler, takes up a slightly different problem : can one express a measure-theoretic condition, like the hypothesis (L) of Meyer in purely topological terms ? It is shown that hypothesis (*L*) holds if and only if the fine topology satisfies the countable chain condition that every disjoint collection of finely open sets is countable.

The last paper, 'Representation of semi-martingale Markov processes in terms of Wiener Processes and Poisson measures' by E. Cinlar and J. Jacod, the longest in the book, is a review of results concerned with the representation of Markov processes taking values in R^m in terms of well-understood processes and operations (such as Wiener and Poisson processes). The major result is that every semi-martingale Hunt process is obtained by a random time change from a Markov process that satisfies a stochastic integral equation driven by a Wiener process and a Poisson random measure. The paper extends earlier investigations due to Dynkin, Feller and Skorokhod.

On the whole, this collection of research papers is of immense value to mathematicians interested in stochastic processes. If it is to be used as a text-book, considerable supplementary material is required.

Department of Electrical Engineering
Indian Institute of Science
Bangalore 560 012.

Y. V. VENKATESH

Nonlinear parametric optimization by B. Bank, J. Guddat, D. Klatte, B. Kummer and K. Tammer. Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1983, pp. 228, S.Fr. 58. Indian orders to Allied Publishers Pvt. Ltd., New Delhi 110002.

Study of optimization has now become a necessity for a wide class of engineers, scientists, managers and applied mathematicians. Courses on optimization form parts of standard curricula of engineering and management programmes. The general problem usually studied here is the minimization of a scalar-valued function $f(x)$ where x takes values in the n-dimensional Euclidean space and is subject to equality and/or inequality constraints. A large number of books advocating various approaches have appeared on such nonlinear programming problems.

The subject matter of the book under consideration is a much more enlarged one in the sense that it considers the dependence of the fnuction f on a set of parameters as well. The problem can be stated as one of minimizing $f(x, \lambda)$ where $x \in M(\lambda)$, the constraint set and λ is a parameter vector. Both x and λ are regarded as elements of metric spaces. Qualitatively, λ could represent initial data which is variable and problems with dependence on λ are referred to as parameter optimization problems.

Since there is one optimization problem for each value of parameter, the parametric problem represents an infinite class of optimization problems. A hopeful approach is to partition the problem into a finite number of subclasses and attempt to obtain the solution set in a closed form for each subclass. Much of the book is oriented towards this objective and is naturally led to an examination of several multivalued mappings which are referred to in the book as point-to-set mappings.

Stability of such point-to-set mappings is investigated in great detail under various assumptions on the function f as well as the constraint set. Here stability does not stand for any well-defined property but rather for certain types of semicontinuity. The depth of treatment in the book can be judged by the fact that two types of semicontinuity (both upper and lower) are defined, one due to Berge, the other due to Hausdorff and the conditions under which the properties result are stated with great care.

A vast expanse of problems are considered in the six chapters of the book starting from a general function f defined over metric spaces and then specializing to convexity conditions, quadratic optimization with linear inequalities and with or without integer restrictions. In each case the results are stated in the Theorem–Lemma–Proof format. It is refreshing to note that the reason for introducing certain conditions is brought out in several instances by short illustrative numerical examples.

The viewpoint of the authors is decidedly that of the mathematician : the aim appears to be to obtain as much generality as possible for the theorems, though an attempt is made in many places to stop at a point so that the proofs remain transparent. No reference is made to any application of the results to the physical world and it is clear that it is not part of the authors' intention. However, with its wealth of material the book could serve as a source for several books which are needed to bridge the gap between it and particular classes of problems and specific applications.

An understanding of the book requires a good mathematical training in function spaces and n-dimensional geometry. A shortcoming of the book experienced by a reader with an engineering background is the lack of motivation for passing from one type of result to another ; such a treatment may be perfectly acceptable to a mathematician on logical grounds.

The book concludes with an appendix containing neatly summarized results pertaining to optimization and an extensive bibliography which includes basic literature on mathematical programming as well. The authors make a thorough literature survey of parametric optimization in the introductory chapter and this should greatly help any one starting to work in the field to go to the source papers. To conclude, the book by Bank *et al* is among the first attempts in a difficult area and should be able to lead to the creation of more lucid books with a fundamental as well as applied orientation.

Electrical Engineering Department
Indian Institute of Science
Bangalore 560 012.

M. A. L. THATHACHAR

Percolation theory for mathematicians by Harry Kesten. Birkhauser Verlag,, Basel, Switzerland, 1982, pp. 423, S.Fr. 68.

This research monograph, comprising 12 chapters covering 423 pages, is devoted entirely to certain mathematical questions and proofs of theorems on percolation in random lattices and the related random resistor networks. It is addressed to mathematicians as advertised and emphasis is on mathematical rigour. The latter has imposed severe restrictions. Thus, only the Bernoulli percolation on undirected lattices with purely bond or purely site disorder *sans* correlation is considered. Also, except for Ch. 5, the treatment is limited to essentially an integer lattice or equivalently to an infinite planar periodic graph imbedded in the 2-dimensional space $\mathbf{R}^2$. Given this restriction, the book is very impressive in its content. Several conjectures and folk-lore truths arrived at by physicists and mathematicians on the basis of intuitive arguments have been confirmed rigorously. Among these is the equality $P_H = P_S = P_T$ for planar graphs imbedded in $\mathbf{R}^2$ and the result $P_H = 1/2$ for Z^2. These results follow from the two general theorems (3.1 and 3.2) proved in Ch. 3. The monograph is fairly self-contained and introduces the relevant ideas of periodic planar graph theory and the point-set topological results in the very beginning (Ch. 2). One may, however, need some facility with the probability theory, *i.e.*, probability triple, σ-field, cylinder sets, etc. Throughout the book the notion of matching pairs takes precedence over that of duals and quite rightly so. Among the results obtained rigorously, one should note the results on percolative regions in the multiparameter space (Ch. 3), the improved lower bounds for the cluster size distribution in the percolative region for R^d (Ch. 5) and bounds for resistances of random networks (Ch. 11). There is also a chapter (12) on unsolved problems. The book concludes with an appendix on some planar graphs. The book is highly recommended for specialists in the area. But, for physicists working on physical problems involving percolation ideas and following the 'broad highway of common sense', I am afraid there may not be much here, except for getting some flavour of mathematical reasoning at work.

Department of Physics
Indian Institute of Science
Bangalore 560 012.

N. KUMAR

Quantum theory of chemical reactions, Vol. III. *Chemisorption, catalysis, biochemical reactions* edited by Raymond Daudel, Albert Pullman, Lionel Salem and Alain Veillard, 1982, pp. viii + 180, \$ 32·50.

Intramolecular dynamics edited by Joshua Jortner and Bernard Pullman, 1982, pp. 564, \$ 74.

D. Reidel Publishing Company, Dordrecht, Holland/Boston, USA.

Two stimulating regions of enquiry that contemporary quantum chemistry has turned to are theories of chemical reactions and of time-dependent phenomena. The former is concerned with, among other things, the topics of heterogeneous catalysis, chemisorption, clusters, enzyme action and quantum pharmacology. The latter *viz.* time-dependent processes focus inter- and intramolecular relaxation processes related to excited-state dynamics of chemical and biophysical systems. The focussing of research in the above topics is closely related to the development of theoretical tools and experimental techniques over the past two or three decades in and outside quantum chemistry. In particular, we may refer to the growth of many-body techniques to handle co-operative phenomena; elucidation of photochemical (–physical) processes in atoms, molecules, clusters, biological macromolecules and in condensed phase; chemisorption, and catalysis; and varied applications of quantum and statistical mechanics to chemistry. Consequently, the range of theoretical chemistry, in general, has become wider, its emphasis even more interdisciplinary and its impact vast. Any one who entertains doubts on the above claims is only to refer to these two volumes.

Both the volumes are in the nature of a collection of contributions from experts. The former is the last in the series of these volumes on the same title. The latter carries the invited lectures at the fifteenth Jerusalem Symposium on Quantum Chemistry and Biochemistry held in early 1982.

Together, the two volumes—though unequal in scope, level and several other respects (including editing !)—convey the flavour of much of the quantum chemistry of today.

Quantum theory of chemical reactions (Vol. III) contains ten papers spanning 175 pages. The topics are widely varied—talking of quantum theory of metal clusters and action of schistosomicidal agents, in the same breath !

Two papers deal with the cluster-approaches. E. J. Baerandas and D. Post advocate an improvement based on the Hartree–Fock–Slater method using Slater's local ($\rho^{1/3}$) exchange correlation potential. Applications to clusters are illustrated through a discussion of the electronic structure of the bridging CO's and an investigation of the electronic factors that determine the chemisorption sites in the case of Fe_2 CO_9 and Cu_5–CO (Analysis of CO-metal cluster interaction energies by the HF-Slater method, p. 15). In another interesting paper, F. Cyrot Lackmann demonstrates how chemisorptive behaviour is linked to the electronic properties of transition metal clusters and surfaces. The main trends in variation of the binding energies are demonstrated using a simplified description of the local density of states employing its first few moments. The importance of spin-orbit coupling is stressed (Chemisorption properties of transition metal clusters, p. 35).

G. Bertholon provides a lucid introduction to the notion of reactivity of an organic solid. By comparing the variation in selectivity and in conversion ratio (as one goes through alkaline peroxides) and also through mechanistic studies of several gas-organic crystal reactions, the importance of crystalline state/structure and morphology is brought out (Gas, organic solid-state reactions and their applications, p. 55).

Quantum chemistry of biochemical reactions require calculations pertaining to possible pathways, active-site-molecular model systems and environmental effects. Interesting details on the pursuit of these in two different contexts are provided by P. Th. van Duijnen and B. T. Thole (Environmental effects on proton transfer, *ab initio* calculations on systems in a semi-classical polarisable environment, p. 85) and O. Tapia *et al* (Recent quantum/statistical mechanics on enzyme activity—Serine proteases and alcohol dehydrogenases, p. 97). The former deals with the reaction mechanism of paparin and thiosubtilisin and seeks to incorporate the environmental effects quantum mechanically *via* 'direct reaction field' method. The latter paper is based on ISCRF (inhomogeneous self-consistent reaction field) theory of protein core effects. P. Claverie treats environmental effect of a different sort (Intermolecular interacations and solvent effects—Simplified theoretical methods, p. 151).

The volume also contains two articles on quantum pharmacology (On the pharmacophore and mode of action of some schistosomicidal of agents—Conformational aspect, p. 135; Applications of quantum chemistry to pharmacology, p. 125). There is also a useful overview on the 'theoretical background of heterogeneous catalysis' (p. 1).

Intramolecular dynamics has thirty-seven papers over 550 pages.

Atom-molecule scattering computations reveal a resonance structure even though there is no static potential energy well on the PE surface. One may therefore visualise quasi-periodic bound orbits and classically call these 'vibrationally adiabatic molecules' (noting that the binding force is dynamic). In his report, E. Pollak analyses the intriguing question : is there a quantal analogy ? In other words, 'do vibrationally adiabatic molecules exist' ? (p. 1).

Several questions related to collision dynamics beg proper description. One of these pertains to 'the constants of motion' *vis-a-vis* stationary and time-evolving states—especially, their 'stability' which appears to be closely connected to the onset, nature and experimental manifestations of the *chaotic* regime for intermolecular dynamics. The objectives are to understand the classical and the quantum dichotomy, or to follow the transition from quasi-periodic to chaotic in various contexts (*e.g.* vibrational motion) or to develop appropriate mathematical approaches (*e.g.* Morse-like anharmonic oscillators). Quite a few papers refer to the above topic in the volume (Comparison between classical and quantum dynamical chaos, p. 107 ; An analytic approach to time averages and chaotic behaviour in quantum mechanics, p. 391 ; Classical aspects of wave packet dynamics, p. 97 ; The (Lie) 'Algebraic approach to molecular structure and dynamics, p. 17 ; Dynamical studies of excited states in triatomic molecules, p. 191).

In the context of *many-body dynamics*, one ends up invariably with a hierarchial way of visualising the apparent complexity in simpler, reduced format—each element usually imbedding the other. Closely following this, one wonders if there are such patterns in *few-body dynamics* as in electrons in atoms, atoms in small molecules and clusters. In his 'Are atoms and molecules almost the same ?' R. S. Berry analyses (p. 29) the relative importance of kinematics and dynamics in collective or independent particle motion and the transition between them. Existence of molecule-like quantization and collective motion in some states of doubly-excited helium and helium-like ions is indicated.

The notion of 'complex energy' is useful in intramolecular relaxation like 'complex frequency' in classical analysis. Resonance widths and hence the rates can be determined by employing the Siegart schemes as shown by Lefebure (p. 55). The extraction of information on intramolecular dynamics from the spectra is an important task (p. 89). And so are the studies of collision dynamics—especially differential cross sections and the glory oscillations in the integral cross section, life-times of vibrational excitations and the like—associated with van der Waals molecules (pp. 63 and 269).

Even physically identical bonds do not behave alike in different excitation states, giving rise to local mode phenomenon. Existence and characteristics of local mode behaviour in the stretching vibrations of small symmetrical molecules and quantum mechanical time evolution for an exicted local mode in a methyl group form the subject matter of two papers (pp. 115 and 129).

Next comes—in the hierarchy of analysis—the intramolecular (vibrational) energy flow and redistribution in polyatomic systems. Investigations span a wide range—from preparing initially a non-stationary wave packet and analysing its quantum mechanical evolution to the study of manifestation of intramolecular vibrational and rotational coupling in multiphoton excitation of polyatomic molecules by intense infrared laser. Besides, questions on modelling photochemical and unimolecular decay processes, fast collision-induced vibrational relaxation, etc., bring forth considerations of mechanistic schemes, calculation of stationary and dynamical properties of highly excited vibrational states. There are several reports dealing with many aspects of this area (pp. 139, 171, 205, 311, 325, 371 and 403).

Laser-induced fluorescence spectroscopy used to study dimer of *s*-tetrazine and several porphyrins in supersonic expansion provides information on the structure and dynamical processes (pp. 219, 227 and 241).

Dissociation can unravel the finer aspects of intramolecular dynamics. M. Shapiro considers four cases : *fast* or *slow* processes probed in *ground* or *excited* states (p. 299). Another aspect is the time-dependent quantum mechanical approach to laser-induced dissociation—based on optical potentials and R-matrix approach utilising Floquet amplitudes (p. 403).

Three reports dealing with model analysis—even if on different aspects—deserve mention here. One deals with the tunnelling of a coherent state wave packet across the barrier to levels at the same height and the importance of these in many chemical, photo chemical processes (p. 287). Another is concerned with the Morse potential as a bridge between molecular structure and dyamics (p. 341) and the third with the construction and study of a pressure-dependent effective Hamiltonian (p. 351).

The role of rotations in elecronic decay is not limited to its effects on the coupling element between a singlet and a triplet manifold (intersystem crossing) or between singlet-excited and gıound states (internal conversion). Its effect on the number of levels to which a state is coupled is also interesting. Strong rotational effect on the electronic decay in the case of pyrazine is analysed (p. 259).

Another field of research covered in the volume pertains to the microscopic description of chemical reactions and description processes, *albeit* in passing. Examples of applications are homogeneous reactions *especially* near critical points and heterogeneously catalysed reactions near magnetic phase transitions of the catalyst (pp. 429 and 447).

Polymers and biologically important (super) molecules are but a next step in the conceptual thoroughfare linking atoms to molecules, van der Waals complexes and clusters and condensed phases ! Though, of course, the *ab initio* attitude to the dynamics appears to fade a little, the pursuit has similarities nevertheless.

Discussion now covers a slightly different terrain—thermal motion of the chain ends of polymers relative to one another in a given molecule, serving as an index of the dynamic flexibility of the macromolecule (p. 459), probing conformation in peptides (p. 473), many-pronged ' NMR studies of protein dynamics ' (p. 481), fast non-radiative decay rates in metalloproteins (p. 497) , steroid conformations (p. 505) and studies on DNA (p. 537).

There is no doubt that the two volumes reviewed are useful additions to the existing literature. But in the absence of good overviews or perspective papers or a discussion section, it is not clear whether these will remain as reference volumes for long. An intriguing aspect of publishing such direct reproductions from the Mss is the issue of editorial responsibility (and professionalism) in such publications edited by working scientists. *Quantum theory of chemical reactions* leaves much to be desired in the field of editing. Typographical errors are carried over to the print from the Mss and several instances of lapses in grammar have been permitted to stay in the reproduced version. Index of subjects (in both volumes) is useful more as a collection of keywords and is neither exhaustive nor thorough.

Minor lapses apart, the two volumes serve as pointers to the trends in modern theoretical chemistry and carry authoritative reports on several topics of interest, These should be of interest to theoreticians and experimentalists, biochemists, physical chemists and pharmacologists.

Inorganic and Physical Chemistry Department
Indian Institute of Science
Bangalore 560 012.

S. K. RANGARAJAN

Organic chemistry (Third edition) by Morrison and Boyd. Allyn and Bacon inc., 1981, pp. ix + 1258, $ 19.95.

The purpose of a text-book should be (i) to provide an introductory course, *i.e.*,to describe the fundamental principles ; (ii) to give an opportunity to review and reinforce the understanding of many of the fundamentals; (iii) to help the student to learn to approach the solutions of problems in a systematic way and (iv) to consolidate his learning into a habit pattern of ready recognition of these principles when in need. The authors have done full justice to these requirements and have thoroughly revised and rewritten almost all chapters and also introduced seven new ones. The material presented is comprehensive without being exhaustive and concise without being superficial and is well organised. In this present form the subject matter really will permit rapid dissemination and acquisition of information. Also many of the problems provide a necessary first step forward for formal discussion which will result in increasing the interest of the reader.

Each chapter is provided with adequate information and also with succinct review of the most salient aspects of various fundamental features of functional groups ; and a few very selected problems to develop the ability to perceive the reactive capabilities of the fundamental groups and recognise the ways in which a compound will behave under a variety of experimental conditions.

Generalisation of facts, which appear to be isolated, gives a certain coherence, enables the student to read and study with quick comprehension, to retain vast information and the differences in intrinsic chemical behaviour allowing one to predict more intelligently. Or else, for a beginner, organic chemistry seems to consist of a variety of methods and reactions which appear isolated and never give a fillip to think. The authors fully justified this generalisation aspect in reorganising the subject matter. To mention a couple of generalisations, out of many :

(i) a separate chapter on carbanions, neighbouring group effects is a testimony;

(ii) the chapter Carbonions is well conceived and appropriately accommodated after carbonyl functions thus helping the student to realise the importance of the acidity of alpha hydrogens and also the resulting synthetic applications as a whole in a unified way;

(iii) generalisation of mechanisms, stereo and conformal aspects of the intra-molecular nucleophilic attack on electron-deficient sites like C,N,O, etc., (both classical and non-classical) makes the student feel that the majority of the rearrangements follow the same pattern in general.

Thus stress has been laid on the structure, its influence on reactivity and of the role of intermediates such as carbonium, carbonion, carbene, nitrene, etc. Even though the non-classical carbonium ions are discussed in a preliminary way still it gives an insight into the understanding the chemistry of natural products.

In addition to the vast information given about aliphatic nucleophilic substitution reactions, it would have been better, to mention the trend in substitution reactions. For example, if the leaving group is changed, the ease of substitution is effected on factors like inductive effect of L :, the basic nature of the non-bonding pair of electrons of L :, etc., (L : = Cl, Br, I, OH′, NH_2', SH′, H_2O^+, etc). This generalisation makes student to understand why OH is a bit difficult to remove than Cl, Br, etc., and why in acid medium the OH can more effectively be replaced.

Also it would have been helpful if the differences in nucleophilicity and basicity were introduced in a very simple qualitative way as the student would be in a position to understand why RS′ : or HS′ is more nucleophilic than RO′ and OH^- respectively, but how it differs in basicity in the same way as in Cl′, Br′, I′.

Aromaticity could have been introduced starting with butadiene and a little concept of anti-bonding and bonding orbitals, extending the same to benzene for a better appreciation of energy relation of the orbitals as aromaticity is a ground state property associated with the molecule.

The chapter on Orbital symmetry with stress on Woodward–Hoffman rules is a welcome addition as it almost becomes the basis to undeistand the organic photo-chemistry.

The unique speciality of this volume in comparison to various text-books for beginners is familiarising the student with :

- the stereo and conformational aspects of organic molecules and reactions with a clear three-dimensional representation ;
- the various aspects of NMR spectroscopy in a simple and organised way. After going through the 30 pages of text and problems the student will definitely feel confident to analyse the NMR spectra.

The authors should be complimented for carefully restructuring a large part of the text matter of the earlier edition into problems. By doing so, they did not detract the attention to the subject content, but provided a necessary first step forward giving the opportunity to review and reinforce the fundamental aspects. Most of the questions at the end of each chapter are largely in the form of problems and are considerably difficult. They provide a rapid revision of basic principles.

The resolutions of the peaks of the IR diagrams are more clearer in the previous edition than in the present one. In general, the new edition is a bible to the undergraduate if he believes that his religion is organic chemistry.

Department of Chemistry
Nanakram Bhagawandas Science College
Hyderabad 500 002.

K. N. SETTY
A. S. R. SAI