A NOTE ON THE ARMSTRONG SYSTEM OF FREQUENCY MODULATION

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Abstract

A relation between the degree of amplitude modulation produced in the Armstrong FM system as a function of the modulating frequency and modulating voltage has been derived from the vector diagram drawn by utilising the 'zero frequency carrier' concept (Wheeler, 1941). A graphical relation between the degree of amplitude modulation and the total harmonic distortion produced in the Armstrong system has been established.

Introduction

The vector diagram of the Armstrong FM system shows that the frequency modulation produced by this system involves invariably some amplitude modulation. It is the object of this paper to calculate from the vector diagram the degree of amplitude modulation that appears as a function of modulating frequency and modulating voltage when frequency modulation is achieved by the Armstrong system. The paper also presents a graphical relation between the degree of modulation and the total harmonic distortion produced in the Armstrong system.

Degree of Amplitude Modulation

The Armstrong system of frequency modulation is represented in Fig. 1.

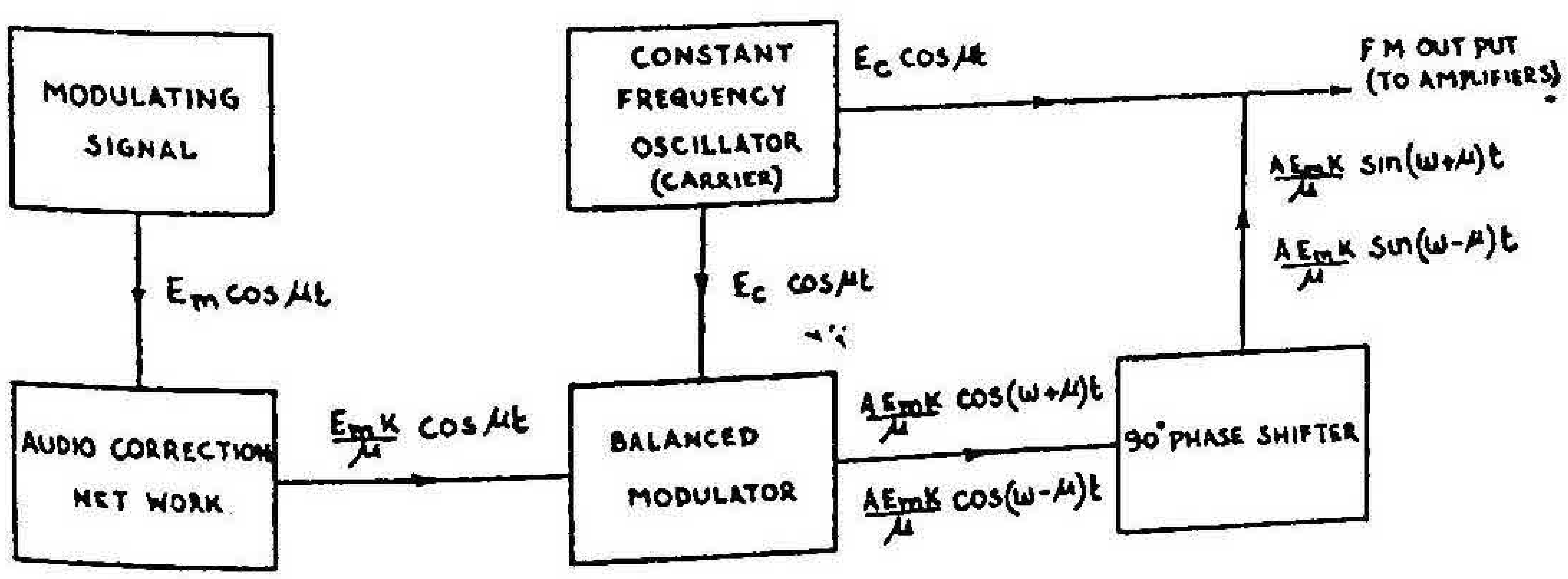

Fig. 1. Armstrong System of Frequency Modulation

The sidebands at the output of the balanced modulator are

$$\frac{AE_m k}{\mu} \sin (\omega + \mu)\, t \text{ and } \frac{AE_m k}{\mu} \sin (\omega - \mu)\, t,$$

where k is a constant and A is the gain of the balanced modulator. ω and μ represent the carrier and the modulating angular frequencies respectively. E_m represents the amplitude of the modulating voltage.

The vector diagram (Fig. 2) is drawn by assuming (Wheeler, *loc. cit.*) a stationary carrier and the sidebands rotating in opposite directions not with their absolute velocities but with velocities relative to that of the carrier. It is further assumed that the amplitude of the carrier is unity. The amplitude of the upper and lower sidebands AS_1 and AS_2 are k'/E_c, where $k' = AE_m k/\mu$. The resultant of the two sidebands is $AB = p \cos \mu t$, where $p = 2\ AE_m k/\mu$. It is evident that the resultant AB varies in magnitude but remains stationary as the two sidebands rotate in the opposite directions with velocities corresponding to μ. The modulated carrier will therefore change in amplitude and its instantaneous value is given by $[1 + p^2 \cos^2 \mu t]^{\frac{1}{2}}$.

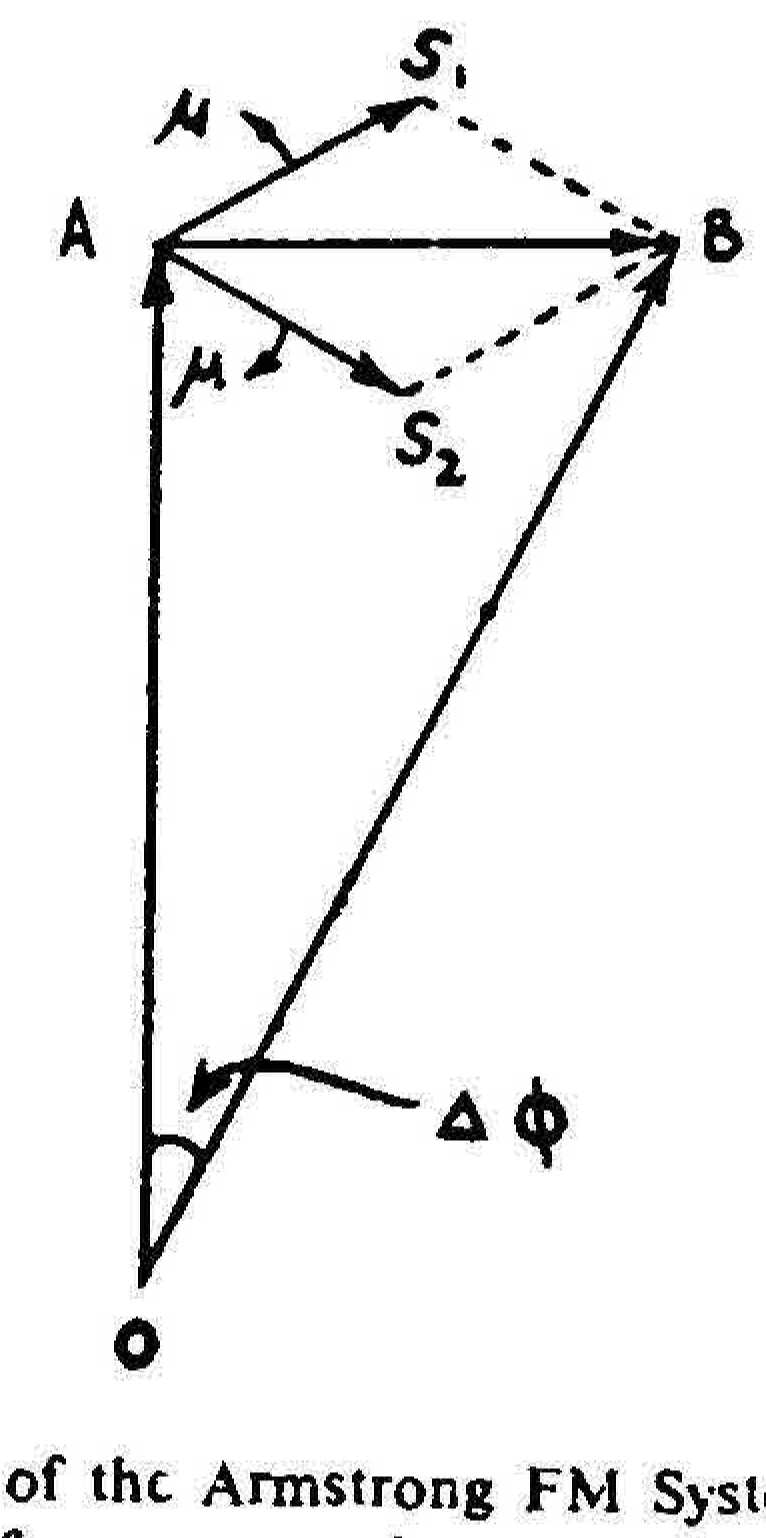

FIG. 2. Victor Diagram of the Armstrong FM System based on the zero frequency carrier concept

The average amplitude of the modulated carrier is

$$E_{av} = \frac{1}{2\pi} \int_0^{2\pi} [1 + p^2 \cos^2 \mu t]^{\frac{1}{2}}\, d(\mu t)$$

which can be written as

$$E_{av} = \frac{a}{2\pi} \int_0^{2\pi} [1 - b^2 \sin^2 \mu t]^{\frac{1}{2}}\, d(\mu t) \qquad (1)$$

where $$a^2 = 1 + p^2 \text{ and } b^2 = p^2/(1 + p^2) \tag{1 a}$$

As the instantaneous amplitude repeats every quarter cycle, eq. (1) can be reduced to

$$E_{av} = \frac{2a}{\pi} \int_0^{\frac{\pi}{2}} [1 - b^2 \sin^2 \mu t]^{\frac{1}{2}} \, d(\mu t) \tag{2}$$

which is an elliptic integral of the second kind. The percentage of amplitude modulation is given by the following relation

$$m\% = \frac{E_{av} - 1}{E_{av}} \times 100 \tag{3}$$

The relation between E_{av} and b can be found by evaluating (Spenceley, 1947); Adams and Hippisley, 1922) the integral in eq. (2) and is given in the following table:

$\sin^{-1} b$	5°	8°	11°	14°	18°	22°	26°	30°	34°	38°	42°	45°
E_{av}	1·002	1·005	1·009	1·015	1·026	1·040	1·057	1·078	1·106	1·139	1·180	1·215

The variation of $m\%$ with p has been calculated with the help of the eqs. (1 a), (3) and the above table and is represented in Fig. 3.

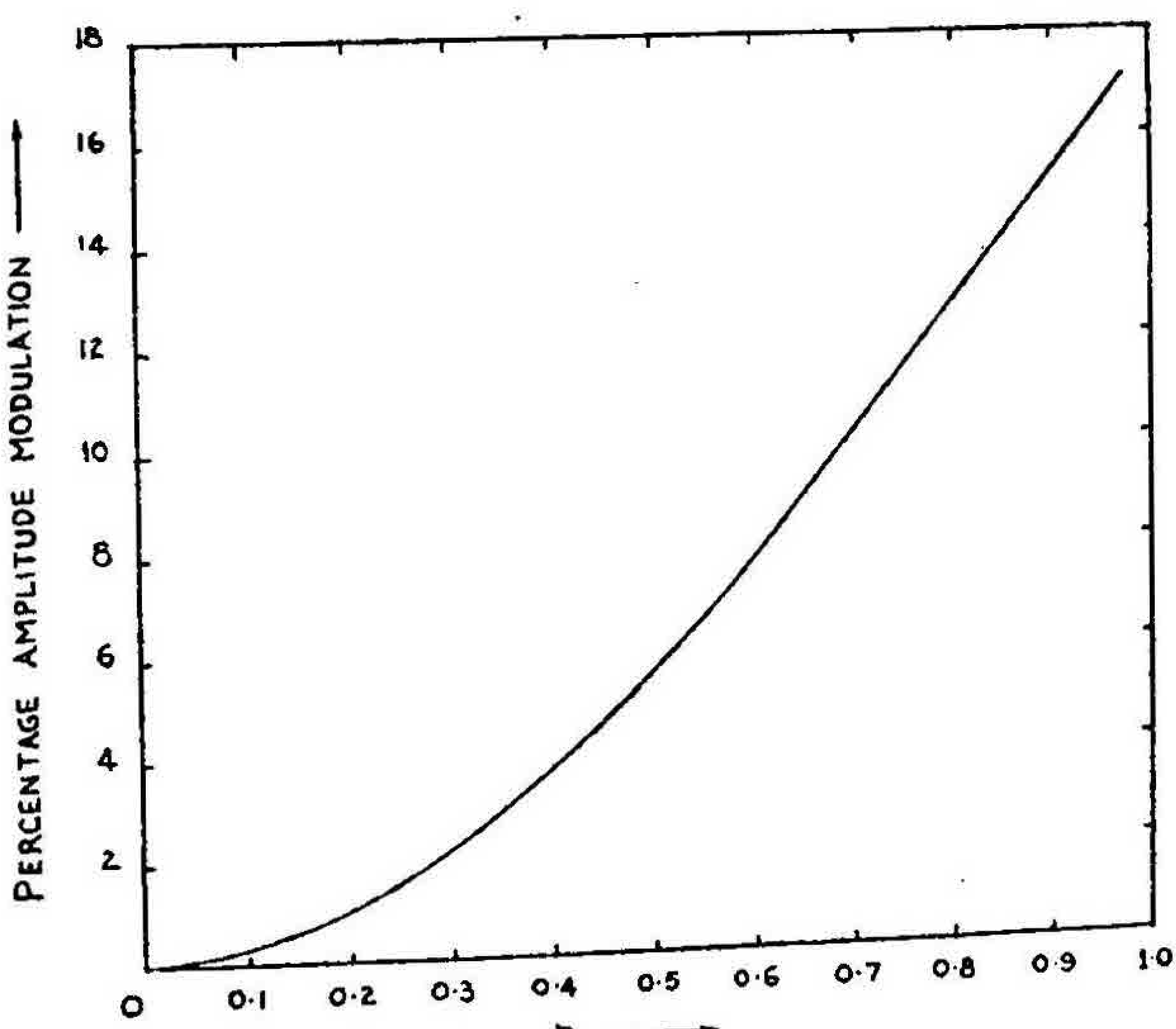

FIG. 3. Percentage of Amplitude Modulation *vs.* p.

MAXIMUM PHASE DEVIATION

It is evident from the vector diagram (Fig. 2) that the maximum phase deviation $\Delta\Phi_{max.}$ of the modulated carrier is given by $\tan^{-1} p$. Fig. 4 shows the relation between $m\%$ and $\Delta\Phi_{max.}$

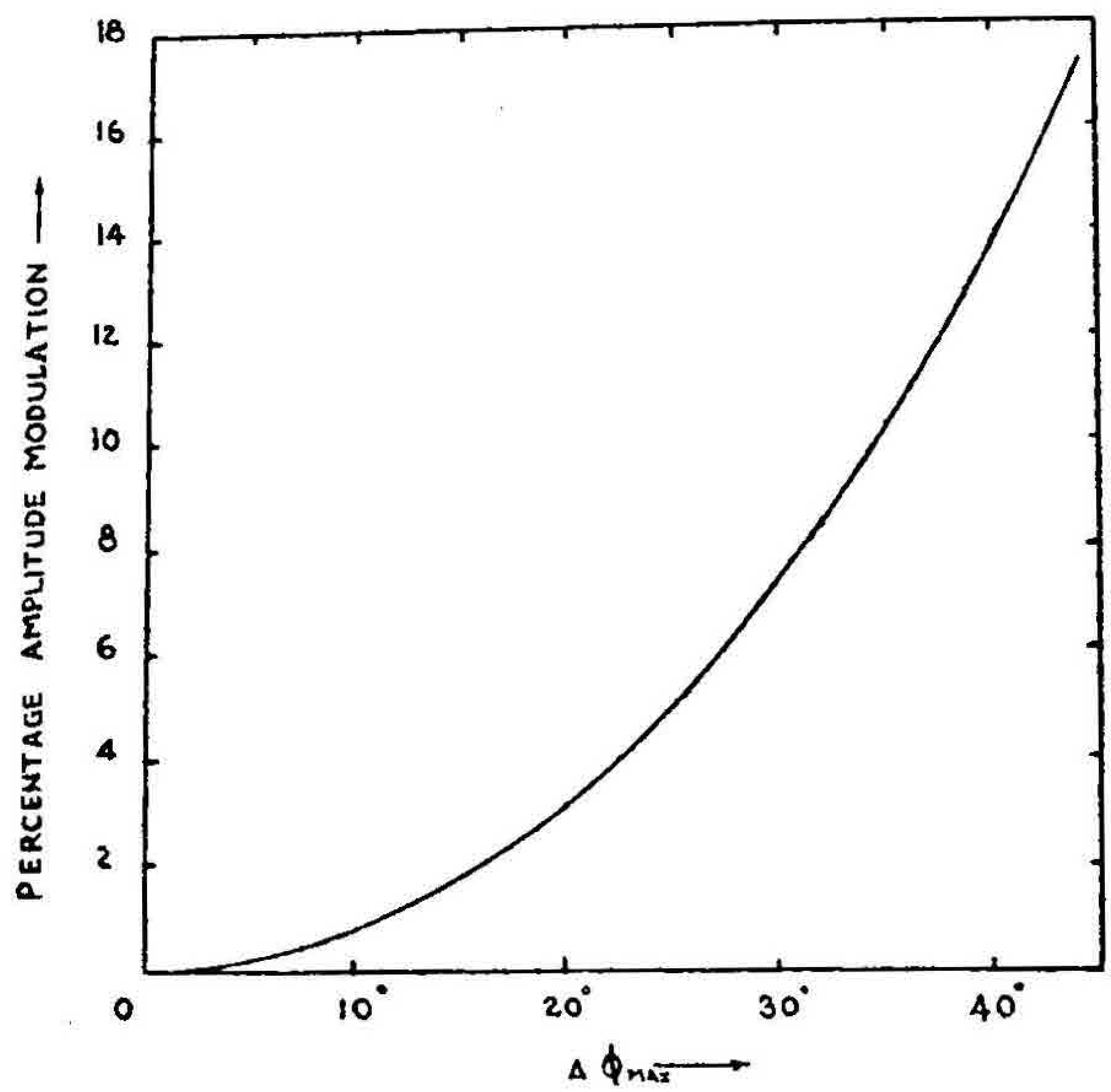

FIG. 4. Percentage of Amplitude Modulation *vs.* Maximum Phase Deviation

TOTAL HARMONIC DISTORTION

When the phase of the sidebands of an amplitude modulated signal is shifted by 90° and then recombined with the carrier, the resulting FM wave contains harmonic components of only odd order n whose amplitude is given (Jaffe, 1938) by the following relation

$$A_n = \frac{2}{p^{n+1}} [(1 + p^2)^{\frac{1}{2}} - 1]^n, \tag{4}$$

where A_n represents the magnitude of the nth harmonic. As both m and A_n involve p, it is evident that the total harmonic distortion is related to the degree of amplitude modulation present in this system. A graphical relation (Fig. 5) between $m\%$ and the percentage of the third and the fifth harmonic distortions has been found from eqs. (1 a), (3), (4) and the table. The ordinate in Fig. 5 represents the amount of the third and the fifth harmonic with respect to the fundamental in percentage. Fig. 5 shows that the amount of the fifth and hence of higher odd harmonics is much less than that of the third harmonic. Hence the total harmonic distortion present in the system can be considered as due to the third harmonic only without introducing appreciable error.

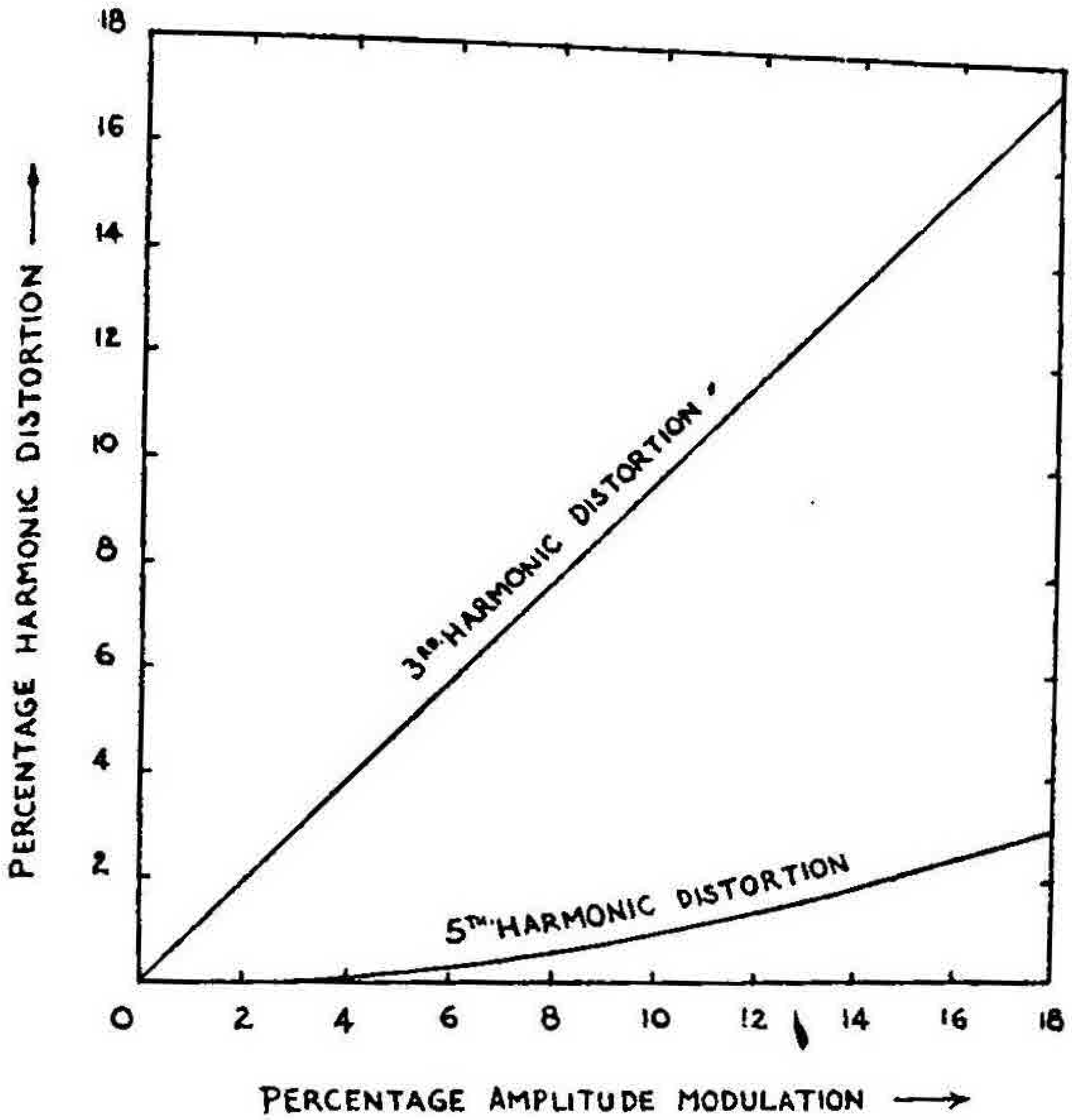

FIG. 5. Percentage of Harmonic Distoration *vs.* Percentage of Amplitude Modulation

CONCLUSION

1. Some degree of amplitude modulation is inherently associated with the Armstrong FM system.

2. The degree of amplitude modulation bears almost a parabolic relation with the parameter p. The degree of amplitude modulation can be reduced by reducing p which can be achieved by keeping the modulating voltage and the gain of the balanced modulator at a low level. This is possible in the Armstrong system as the modulation is inherently a low level one.

3. The degree of amplitude modulation is inversely proportional to the modulating frequency. From Fig. 5 and Jaffe's (*loc. cit.*) graphical relation between the total harmonic distortion and the modulating frequency at constant modulating voltage, it can be found that the percentage of amplitude modulation corresponding to the modulating frequency of 20 c.p.s.

is 5·2%. At modulating frequencies of 100 c.p.s. and above the total harmonic distortion is very small and hence the degree of amplitude modulation is much less than a per cent.

4. In the Armstrong system (Armstrong, 1936) a maximum phase deviation of 30° is used at a modulating frequency of 30 c.p.s. The harmonic distortion is 7·2% at 30 c.p.s. and 0·01% at 1,000 c.p.s. The corresponding percentage amplitude modulation is 7·4% and 0·01% at 30 c.p.s. and 1,000 c.p.s. respectively.

5. The total harmonic distortion bears a linear relation with the percentage amplitude modulation.

REFERENCES

Wheeler, H. A. .. *Proc. I. R. E.*, 1941, 29, 446.

Spenceley, G. W. and R. M. .. *Smithsonian Elliptic Functions Tables*, 1947. Fublished by the Smithsonian Institution.

Adams, E. P. and Hippisley, R. L. *Smithsonian Mathematical Formulae and Tables of Elliptic Functions*, 1922. Published by the Smithsonian Institution.

Jaffe, D. L. .. *Proc. I. R. E.*, 1938, 26, 475.

Armstrong, E. H. .. *Ibid.*, 1936, 24, 689.