A NOTE ON PARTIALLY FIXED BEAM COLUMNS

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INTRODUCTION

In Airplane Structures, we often come across members acted on both by axial and transverse loads. In several of these cases, their ends will be partially fixed and so the design of these members assuming ideal fixity at their ends is not accurate. A more accurate design can be obtained by deriving the equations for these members in terms of partial end fixity coefficients, as in the case of partially fixed beams.[1] The end fixity coefficient is defined as the ratio of the moment at one end to the moment at the same end when both the ends are ideally fixed.

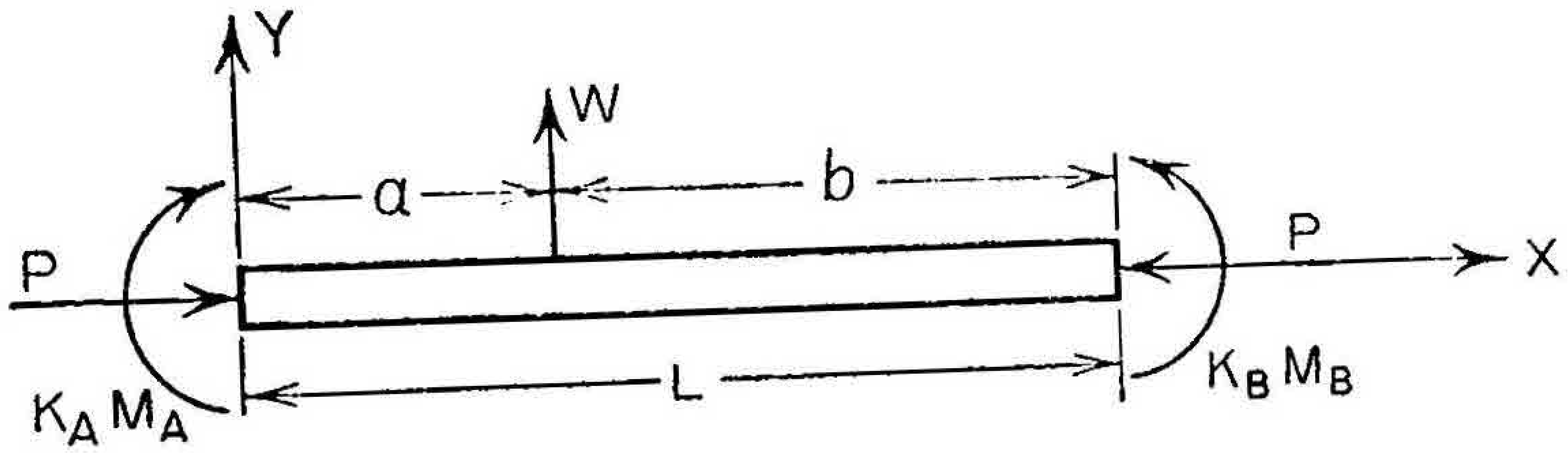

FIG. 1

For the beam column shown above, the equations for bending moment M_x and deflection y_x at any section distant x from left end can be obtained as

$x < a$

$$M_x = K_A M_A \frac{\sin \frac{L-x}{j}}{\sin \frac{L}{j}} + K_B M_B \frac{\sin \frac{x}{j}}{\sin \frac{L}{j}} - Wj \frac{\sin \frac{b}{j}}{\sin \frac{L}{j}} \sin \frac{x}{j} \tag{1}$$

$$y_x = \frac{1}{P}\left[K_A M_A \left(1 - \frac{x}{L} - \frac{\sin \frac{L-x}{j}}{\sin \frac{L}{j}} \right) + K_B M_B \left(\frac{x}{L} - \frac{\sin \frac{x}{j}}{\sin \frac{L}{j}} \right) \right.$$
$$\left. + W \left(j \frac{\sin \frac{b}{j}}{\sin \frac{L}{j}} \sin \frac{x}{j} - \frac{b}{L} x \right) \right] \tag{2}$$

$x > a$

$$M_x = K_A M_A \frac{\sin \frac{L-x}{j}}{\sin \frac{L}{j}} + K_B M_B \frac{\sin \frac{x}{j}}{\sin \frac{L}{j}} - Wj \frac{\sin \frac{L-x}{j}}{\sin \frac{L}{j}} \sin \frac{a}{j} \qquad (3)$$

$$y_x = \frac{1}{P}\left[K_A M_A \left(1 - \frac{x}{L} - \frac{\sin \frac{L-x}{j}}{\sin \frac{L}{j}}\right) + K_B M_B \left(\frac{x}{L} - \frac{\sin \frac{x}{j}}{\sin \frac{L}{j}}\right) + W\left(\frac{ax}{L} - a + j \frac{\sin \frac{L-x}{j} \sin \frac{a}{j}}{\sin \frac{L}{j}}\right)\right] \qquad (4)$$

where $\frac{1}{j} = \sqrt{\frac{P}{EI}}$, E and I being the modulus of elasticity of the material of the beam and moment of inertia of the cross-section, assumed to be uniform, respectively; K_A and K_B are end fixity coefficients at left and right ends respectively; M_A and M_B are the moments at the ends of the beam column when they are rigidly fixed, and are given by

$$M_A = WL \frac{\frac{j}{L}\left(\sin \frac{L}{j} - \sin \frac{a}{j} - \sin \frac{b}{j}\right) - \frac{a}{L} - \frac{b}{L} \cos \frac{L}{j} + \cos \frac{b}{j}}{2 \cos \frac{L}{j} + \frac{L}{j} \sin \frac{L}{j} - 2} \qquad (5)$$

$$M_B = WL \frac{\frac{j}{L}\left(\sin \frac{L}{j} - \sin \frac{a}{j} - \sin \frac{b}{j}\right) - \frac{b}{L} - \frac{a}{L} \cos \frac{L}{j} + \cos \frac{a}{j}}{2 \cos \frac{L}{j} + \frac{L}{j} \sin \frac{L}{j} - 2} \qquad (6)$$

From the above two equations (5) and (6), it is evident that M_A and M_B are functions of $\frac{a}{L}$, $\frac{b}{L}$, WL and $\frac{L}{j}$ and they can conveniently be represented for a particular value of $\frac{L}{j}$ by

$$M_A = WL.\left(\frac{a}{L}\right)\left(\frac{b}{L}\right)^2 C_A \tag{7}$$

$$M_B = WL.\left(\frac{a}{L}\right)^2\left(\frac{b}{L}\right) C_B \tag{8}$$

where C_A and C_B are dimensionless coefficients.[2]

For the point just under the load the expressions (1) and (2) reduce to

$$M_w = K_A M_A \frac{\sin\frac{b}{j}}{\sin\frac{L}{j}} + K_B M_B \frac{\sin\frac{a}{j}}{\sin\frac{L}{j}} - Wj\frac{\sin\frac{a}{j}\sin\frac{b}{j}}{\sin\frac{L}{j}} \tag{9}$$

$$y_w = \frac{1}{P}\left[K_A M_A\left(\frac{b}{L} - \frac{\sin\frac{b}{j}}{\sin\frac{L}{j}}\right) + K_B M_B\left(\frac{a}{L} - \frac{\sin\frac{a}{j}}{\sin\frac{L}{j}}\right) + W\left(j\frac{\sin\frac{a}{j}\sin\frac{b}{j}}{\sin\frac{L}{j}} - \frac{ab}{L}\right)\right] \tag{10}$$

Now to find out the end fixity coefficients K_A and K_B of any beam column, it is sufficient as in the case of the beam to obtain two simultaneous equations in K_A and K_B. Then the two equations can be solved for K_A and K_B. To achieve this, either two bending stress measurements or two deflection measurements or one bending stress measurement and another deflection measurement are sufficient. The two appropriate equations can be used to get the two simultaneous equations required. But while computing the bending moment at any section from the value of the bending stress at that section, care should be taken to take into account the compressive stress due to the axial compressive load P. The two simultaneous equations can also be solved by means of nomograms but the labour is not worthwhile as the circle diagrams[3] can be utilised to determine K_A and K_B as shown below.

In this case it is better to have one stress measurement directly under the load, and the other at any other point. Since $\frac{a}{L}$ is known we can compute $\frac{M_A}{WL}$ and $\frac{M_B}{WL}$ from equations 5, 6 (or 7, 8) with a known value of $\frac{L}{j}$ and they can be marked on OC and OD respectively making an angle of $\frac{L}{j}$ radians,

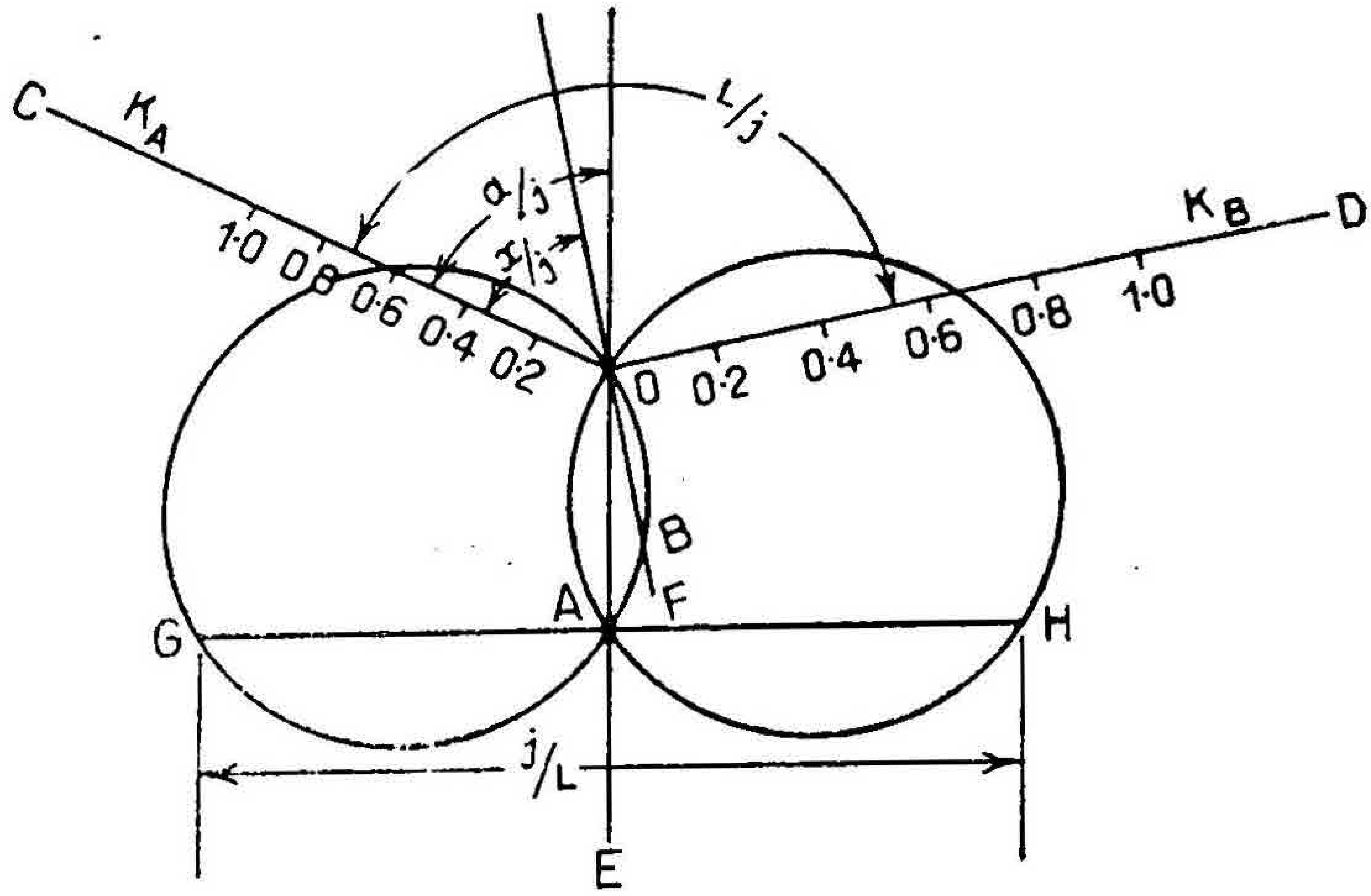

FIG. 2

The value of $\frac{M_A}{WL}$ is thought of as unity and is divided into equal parts. Similar procedure is followed with respect to $\frac{M_B}{WL}$. Now OE is drawn making an angle of $\frac{a}{j}$ radians with OC and OA is set out on it equal to the value of $\frac{M_w}{WL}$, M_w being the value of bending moment computed from stress measurement directly under load. Then OF is drawn making $\frac{x}{j}$ radians with OC and OB is set out on it equal to $\frac{M_x}{WL}$, M_x being the value of bending moment computed from stress measurement at distance x from left end of the beam column ($x < a$ for construction shown in Fig. 2). A circle through O, A and B is drawn to cut OC at the value of K_A. A perpendicular to OE at A is drawn to cut the circle at G. GA is produced to H such that GH is equal to $\frac{j}{L}$ and another circle passing through O, A and H is drawn to cut OD at the required value of K_B. The same procedure can be extended to any other type of loading.

REFERENCES

1. "Partially Fixed Beams" *Current Science*, Jan. 1953, pp. 11, 12.
2. *Airplane Structures* Niles and Newell, 2, p. 129 (3rd edition),
3. ——— .. *Ibid.*, 2, p. 144.