AN ANALYSIS OF THE EFFECT OF FAULT IMPEDANCE ON THE OPERATION OF DISTANCE RELAYS

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The exact solution of the equations for the behaviour of transmission lines leads to terms involving hyperbolic functions of the quantity $(ZY)^{\frac{1}{2}}$. It is customary to call this argument the complex or hyperbolic angle of the line. In the analysis of relay operations, it has, however, been usual to assume linear variation of the line constants, without taking note of the hyperbolic angle of the line. In this paper the hyperbolic relation for transmission lines has been used to discuss the effect of fault impedance on the impedance seen by distance relays and on their operation. The effect of different types of variation in the fault impedance on the relay operation is also discussed. The final relations developed are applied to the case of a 235 miles long transmission line and the effect on relay operation indicated.

Introduction

The development of the stepped characteristic scheme of protection by means of a distance relay was mainly to ensure relay operation free from unnecessary time delay introduced by the distance of the fault from the relaying point. It is a definite improvement on the sloping characteristic necessitated by the extremely high speed of operation imposed upon the relay by such factors as limited power swing and safety to apparatus.

The sloping characteristic shown, in dotted line, in Fig. 1, had inherently an amount of time delay for faults away from the relaying end. Comparing this with the stepped characteristic given by the sections OB, CD, EF, it is obvious that the latter is advantageous in that it eliminates the unnecessary time delays shown by the slope of the curve.

The stepped characteristic has also another advantage in that its operation is less affected by variations in the measured impedance values due to impedance in the faults. Impedance in faults are the more usual in occurrence and this affects the measured impedance of the relay in a number of ways. A certain amount of immunity is imparted to the protective scheme by the stepped characteristic. Thus, the relay operating for section OB, in Fig. 1, will trip the circuit breaker in the same time for all faults within that section irrespective of its magnitude and location. But a certain amount

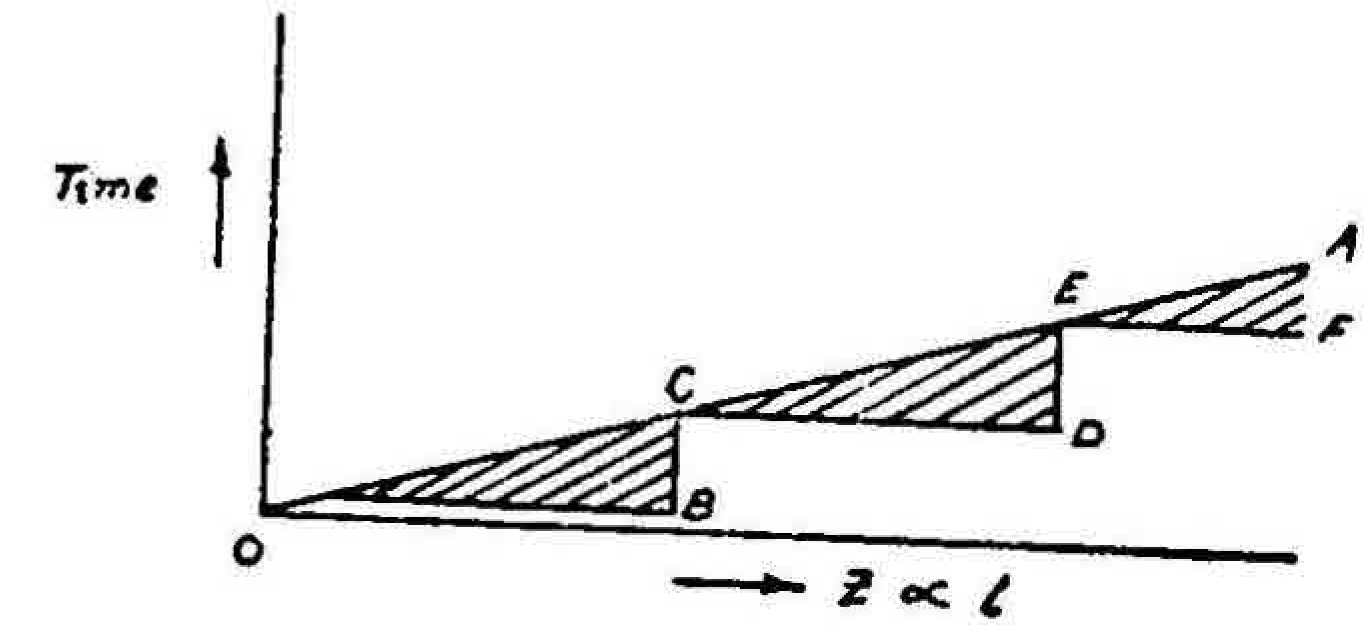


FIG. 1

of uncertainty is introduced when the fault is close to the point B, on either side of or at the point B itself. Under such cases, the magnitude of the total impedance to fault, which is affected by the fault impedance, may be such that the relay may misjudge its zone and cause an unnecessary time delay or operation in its clearance. It is, therefore, of interest to analyse and see how the fault impedance affects the measured impedance and, hence, the operation of the relay.

THE INPUT IMPEDANCE TERM

The terminal equations of a transmission line of length 1 is given by

$$E_S = E_L \cosh \gamma l + I_L Z_o \sinh \gamma l$$

$$I_S = I_L \cosh \gamma l + (E_L/Z_o) \sinh \gamma l \qquad (1)$$

where, E_S and I_S are sending end values and E_L and I_L are load-end values.

Hence, the impedance, as viewed from the sending end, is given by

$$Z_S = \frac{E_S}{I_S} = \frac{E_L \cosh \gamma l + I_L Z_o \sinh \gamma l}{I_L \cosh \gamma l + (E_L/Z_o) \sinh \gamma l}. \qquad (2)$$

Since, the voltage across the load, $E_L = I_L Z_L$,

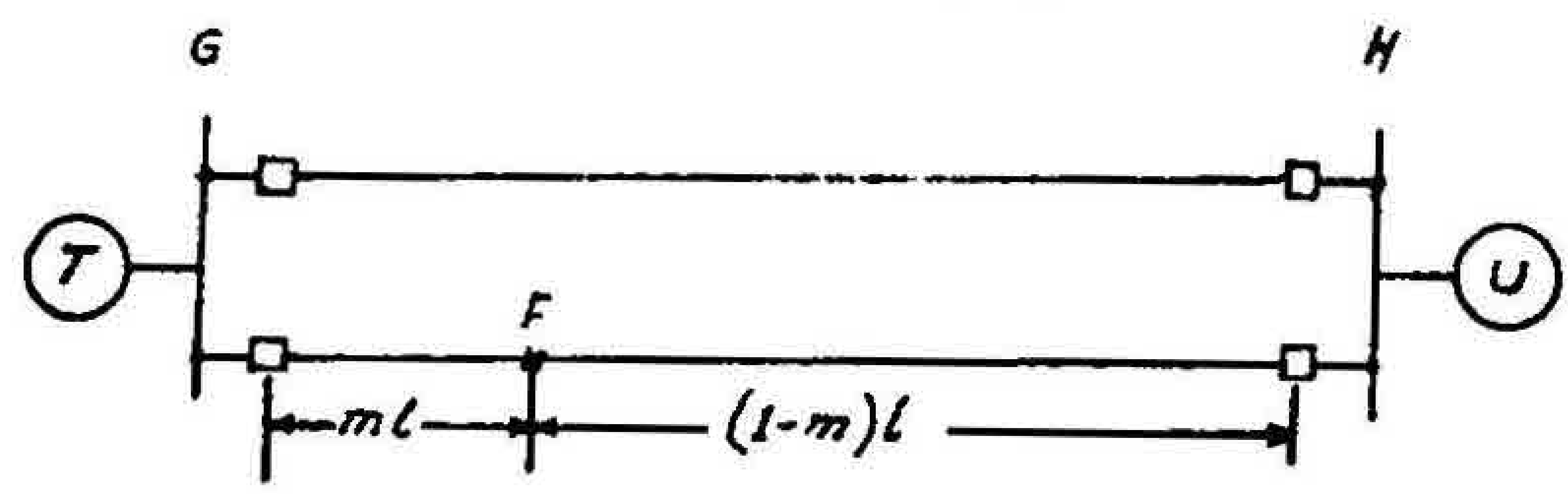
$$Z_S = Z_o \cdot \frac{Z_L + Z_o \tanh \gamma l}{Z_L \tanh \gamma l + Z_o}. \qquad (3)$$

If Z_L is considered as the impedance in a fault that has occurred at a point on the line at a distance l from the sending end, instead of the load impedance, the impedance given by the last equation above will serve as the 'Input Impedance' for a fault on the line. In the case of a three-phase system with unbalanced fault the quantities in this input impedance term must be replaced by the positive-, negative- and zero-sequence quantities for the respective sequence networks of the system.

THE TWO-LINE TWO-MACHINE SYSTEM

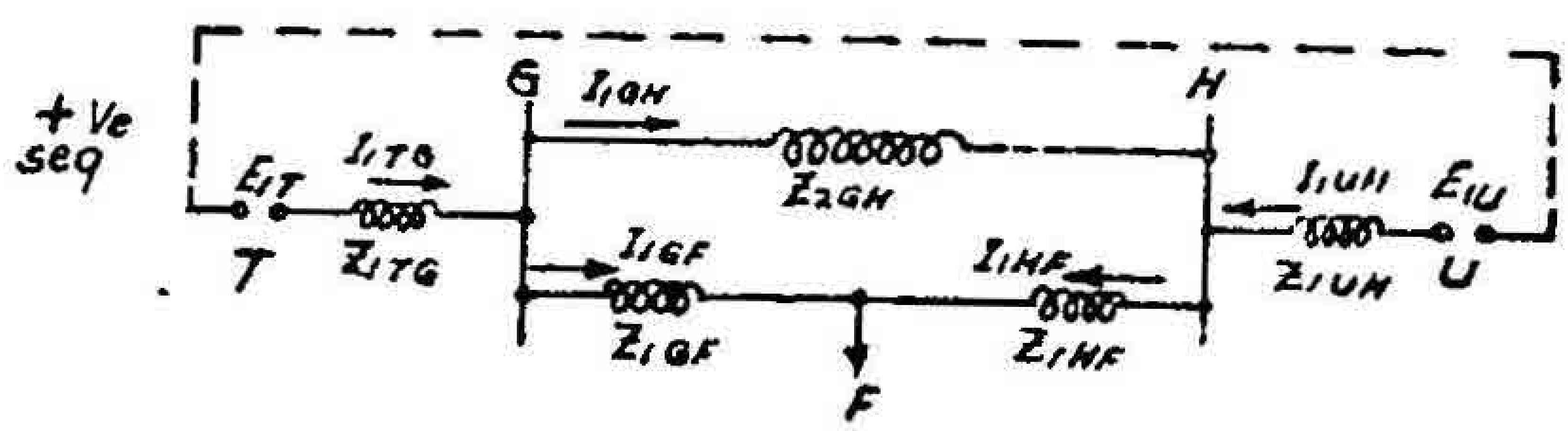
In the present analysis, the usual case of the two-line, two-machine system is taken for consideration. The expression for the impedance measured by the relay, in terms of the input and fault impedance terms analytically developed are as follows:—

Single-line diagram of system



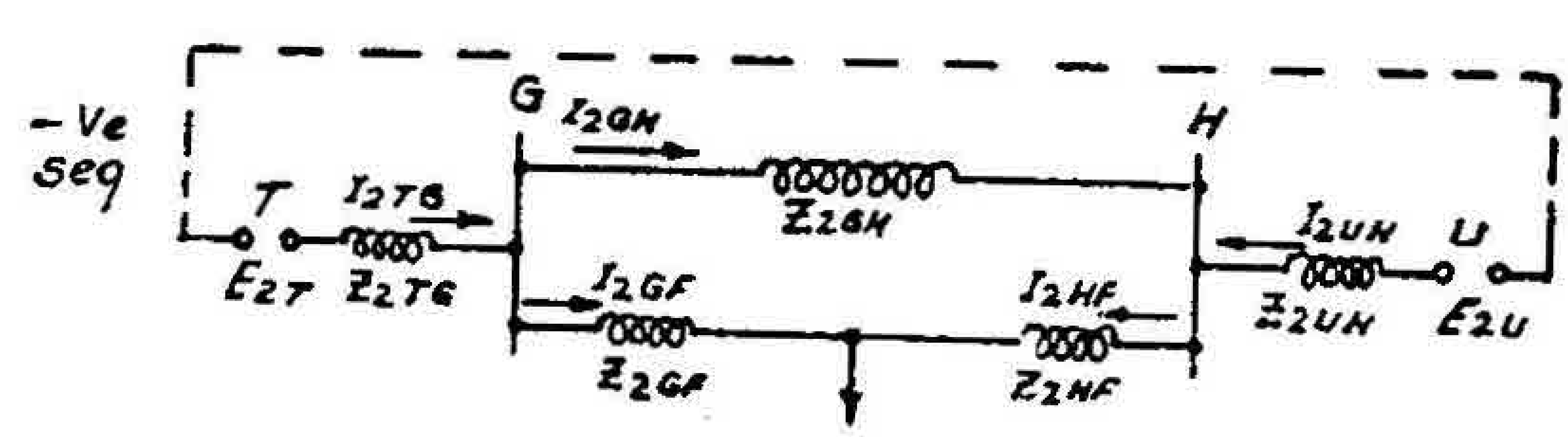
Positive-sequence network

FIG. 2 *a*



Negative-sequence network

FIG. 2 *b*



Zero-sequence network

FIG. 2 *c*

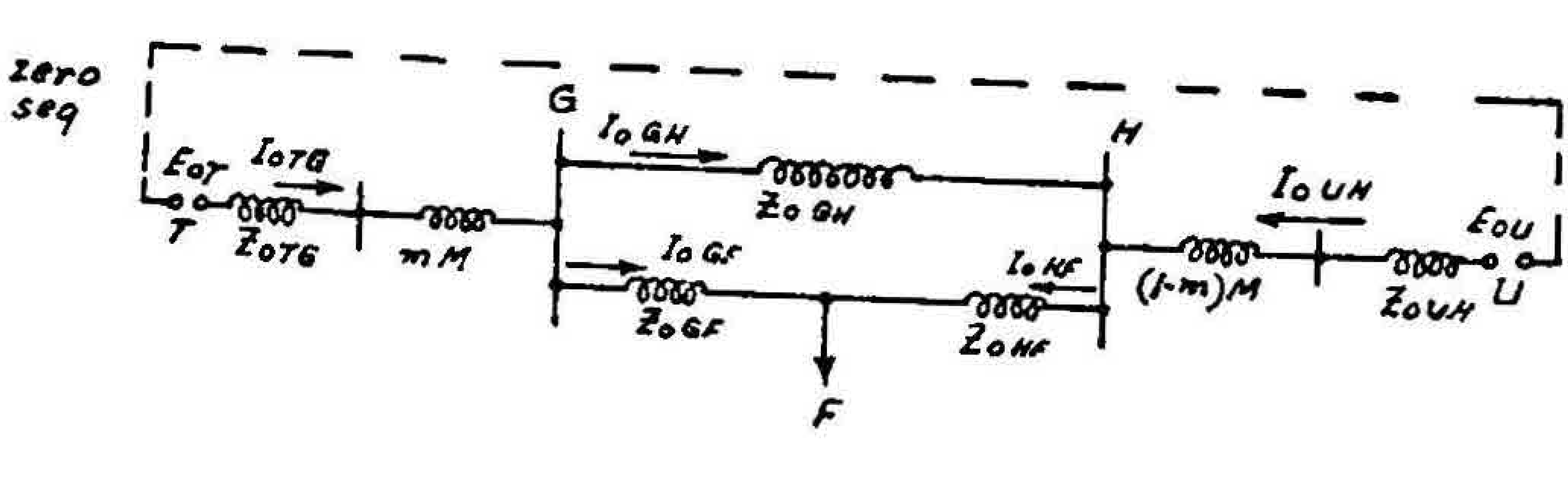


FIG. 2 *d*

The sequence input impedance terms are

$$Z_{1GF} = Z_{O1} \frac{Z_{O1} \tanh \gamma_1 (ml) + Z_g}{Z_{O1} + Z_g \cdot \tanh \gamma_1 (ml)} \tag{4}$$

$$Z_{2GF} = Z_{O2} \frac{Z_{O2} \tanh \gamma_2 (ml) + Z_g}{Z_{O2} + Z_g \cdot \tanh \gamma_2 (ml)} \tag{5}$$

$$Z_{OGF} = Z_{OO} \frac{Z_{OO} \tanh \gamma_0 (ml) + Z_g}{Z_{OO} + Z_g \cdot \tanh \gamma_0 (ml)} \tag{6}$$

where Z_g stands for fault impedance; and Z_O stands for line impedance.

Since, in the case of transmission lines, the positive- and negative-sequence quantities are equal, $Z_{1GF} = Z_{2GF}$. Also in deriving the above networks, it has been assumed that the variation of the mutual inductance for zero-sequence is linear with respect to the length of the line. Then the measured impedance, for a relay at the G-end, for the various types of faults, are:—

(1) *Three-phase fault*

$$Z_{O1} \cdot \frac{Z_{O1} \tanh \gamma_1 (ml) + \frac{1}{2} Z_g}{Z_{O1} + \frac{1}{2} Z_g \tanh \gamma_1 (ml)} \left\{2 + \frac{I_{aGF}}{I_{bGF}}\right\} + \frac{1}{2} Z_g \left\{\frac{2\, I_{bHF}}{I_{bGF}} + \frac{I_{aHF}}{I_{bGF}}\right\} \tag{7}$$

(2) *Line-to-Line-fault*

$$Z_{O1} \cdot \frac{Z_{O1} \tanh \gamma_1 (ml) + \frac{1}{2} Z_g}{Z_{O1} + \frac{1}{2} Z_g \tanh \gamma_1 (ml)} \left\{2 + \frac{I_{aGF}}{I_{bGF}}\right\} + \frac{1}{2} Z_g \left\{\frac{2\, I_{bHF}}{I_{bGF}} + \frac{I_{aHF}}{I_{bGF}}\right\} \tag{8}$$

(3) *Line-to-Line-ground fault*

$$Z_{O1} \cdot \frac{Z_{O1} \tanh \gamma_1 (ml) + \frac{1}{2} Z_g}{Z_{O1} + \frac{1}{2} Z_g \tanh \gamma_1 (ml)} \left\{2 - \frac{3\, I_{OGF}}{I_{bGF}} + \frac{I_{aGF}}{I_{bGF}}\right\}$$

$$+ \frac{1}{2} Z_g \left\{\frac{I_{aHF}}{I_{bGF}} + \frac{2\, I_{bHF}}{I_{bGF}} - \frac{3\, I_{OHF}}{I_{bGF}}\right\} \tag{9}$$

(4) *Single-Line-to-ground fault*

$$Z_{O1} \cdot \frac{Z_{O1} \tanh \gamma_1 (ml) + Z_g}{Z_{O1} + Z_g \tanh \gamma_1 (ml)} + Z_g \frac{I_{aHF}}{I_{aGF}} + \frac{I_{OGH}}{I_{aGF}} . m\, M_{OGH}$$

$$+ \frac{I_{OGF}}{I_{aGF}} \left\{Z_{OO} \cdot \frac{Z_{OO} \tanh \gamma_0 (ml) + Z_g}{Z_{OO} + Z_g \tanh \gamma_0 (ml)} - \right.$$

$$\left. Z_{O1} \cdot \frac{Z_{O1} \tanh \gamma_1 (ml) + Z_g}{Z_{O1} + Z_g \tanh \gamma_1 (ml)}\right\} . \tag{10}$$

The above equations serve to show the various ways in which the fault impedance Z_g enters into the expression for the measured impedance of the relay.

REACTANCE DUE TO PHASE DIFFERENCE

The effect of the coefficients of the fault impedance term in these equations can now be discussed. It will be found that the coefficients are of the form I_{HF}/I_{GF}, the ratio of the currents flowing to the fault from either end. These coefficients alter the nature of the impedance seen by the relay as discussed below.

Since I_{HF} and I_{GF} are both vector quantities, the ratio may be expressed in the form $K \angle \theta_{HF} - \theta_{GF}$. Even if the impedance term that this constant multiplies is purely resistive, it is clear that the impedance added to the measured impedance is not only resistive but has also a quadrature component. It becomes purely resistive only when $\theta_{HF} = \theta_{GF}$.

Since impedance in the fault is primarily only resistance reactance relays were developed to overcome the uncertainties introduced by the variation in fault impedances, the response of the relay being limited only to the reactance component of the measured impedance. In the case where the fault is fed from more than one source, the equations developed above show that the reactance component in the measured impedance is not merely proportional to the length of the line to fault, but includes a certain amount of fault impedance which distorts the proportionality. Thus, the reactance relay, although considerably free from the effect of fault resistances in comparison with the impedance relay, is nevertheless affected by it.

When the circuit breaker at the H end of the system is opened out, all the currents having the HF subscript are omitted. This eliminates the term in $K \angle \theta_{HF} - \theta_{GF}$ and so reduces the reactance component added to the measured impedance term. Then, the measured impedance is mostly dependent upon the input impedance term, whose variation is considered below.

VARIATION OF THE INPUT IMPEDANCE TERM

The nature of the input impedance term can now be examined for the variation in the nature of the fault, its position, magnitude and power factor. The input impedance is represented as a locus on a *r-x* axes system, for variations in the fault positions. Assuming a constant magnitude of the fault impedance, this locus can be derived in two steps:—(A) for various power factors, and (B) for various positions.

For convenience in this analytic study, the input impedance term can be put in the general form

$$Z_S = P \cdot \frac{P \tanh \gamma l + Q}{P + Q \tanh \gamma l}, \tag{11}$$

where P represents the surge impedance of the line

Q represents the fault impedance

$\gamma l = (\alpha + j\beta)\, l$, where β represents the position of the fault.

Assuming that the line has no attenuation

$$Z_S = P \cdot \frac{jP \tan \beta l + Q}{P + jQ \tan \beta l}. \tag{12}$$

Assuming Q to be of the form $K \angle \theta$, the locus of Z_S can now be determined for constant K and various values of θ, the values of βl remaining unchanged.

Equation (12) can be put in the form

$$\frac{Z_S}{P} = \frac{j \tan \beta l + (Q/P)}{1 + j(Q/P) \tan \beta l}. \tag{13}$$

Z_S/P, Q/P are 'normalised' values of impedances with respect to P. Let these be denoted by Z_S' and Q'. Then

$$Z_S' = \frac{Q' + j \tan \beta l}{1 + jQ' \tan \beta l} \tag{14}$$

Putting $Z_S' = a + jb$, $Q' = K' \angle \theta$, and $\tan \beta l = c$, then

$$a + jb = \frac{K' \angle \theta + jc}{1 + jK' \angle \theta \cdot c} \tag{15}$$

Solving for $K' \angle \theta$ and equating magnitudes,

$$K'^2 = \frac{a^2 + (b - c)^2}{(1 + cb)^2 + c^2 a^2} \tag{16}$$

which can be rewritten as

$$a^2 + \left\{ b - \frac{c(K'^2 + 1)}{1 - c^2 K'^2} \right\}^2 = \left[\frac{(c^2 + 1)^2}{1 - c^2 K'^2} \right] K'^2 \tag{17}$$

This is the equation of a circle, whose centre is at $\left[0, \frac{c(K'^2+1)}{1-c^2 K'^2}\right]$ and radius $\left(\frac{c^2+1}{1-c^2 K'^2}\right) K'$. The locus of Z' for various values of θ is shown in Fig. 3.

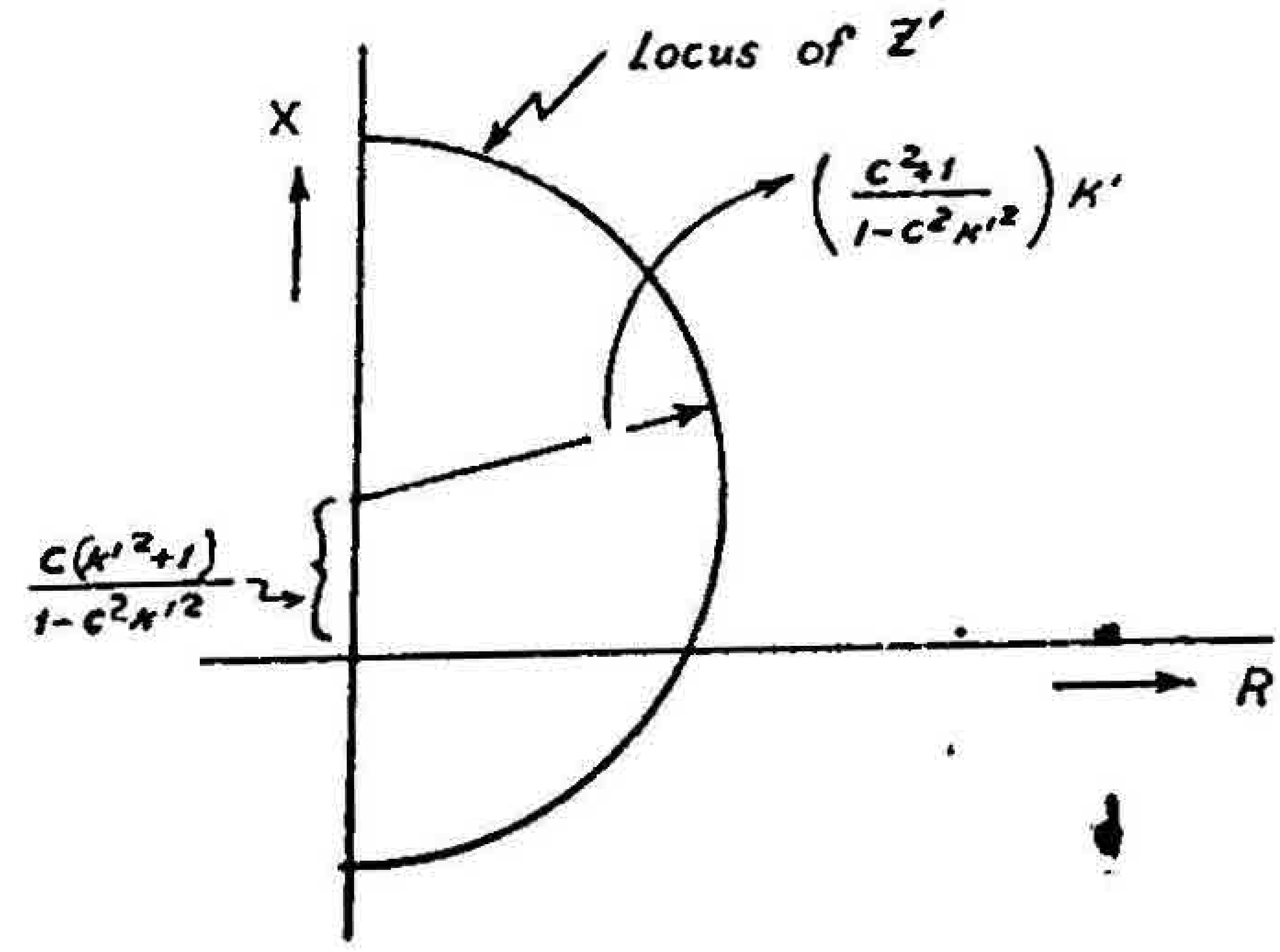


FIG. 3

The maximum values of Z' is given by

$$\frac{c(K'^2+1)}{1-c^2 K'^2} + \frac{(c^2+1) K'}{(1-c^2 K'^2)} = \frac{c+K'}{1-cK'} \tag{18}$$

LOCUS OF R' FOR VARIATION IN THE POSITION OF THE FAULT

This implies that Q' is constant and βl varies.

In equation (14), βl varies from 0 to s radians, where s radians determine the angle subtended by the full length of the line section protected, indicating that the fault with impedance Q moves from the receiving end to the sending end, throughout the length of the line. Due to the low frequency of transmission employed in the case of power lines, the length of the line in terms of angle is quite small.

Equation (14) under such conditions represent an arc of a circle of centre $(h, 0)$ and radius $\sqrt{h^2-1}$, where h is given by $\frac{1+|Q'|^2}{2q}$, q representing the real part of the normalised fault impedance Q'. The length of arc is from 0 to s radians, traced clockwise, as indicated in Fig. 4.

COMPARISON WITH THE USUAL METHODS

The variation of the input impedance term with the position of the fault along the line has been indicated above. By assuming a linear variation

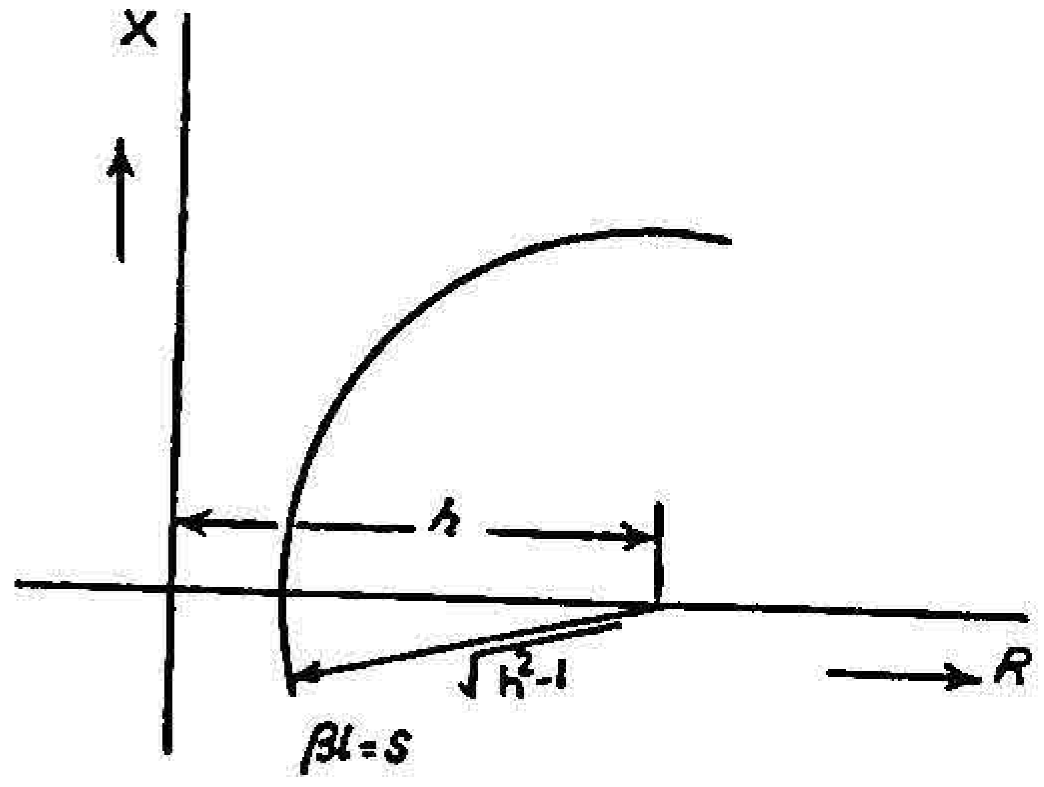


FIG. 4

of the line constants, the locus will be a straight line, and usually the fault resistance is added directly to this locus to obtain the locus of the measured impedance, as shown by the straight line BC in Fig. 5.

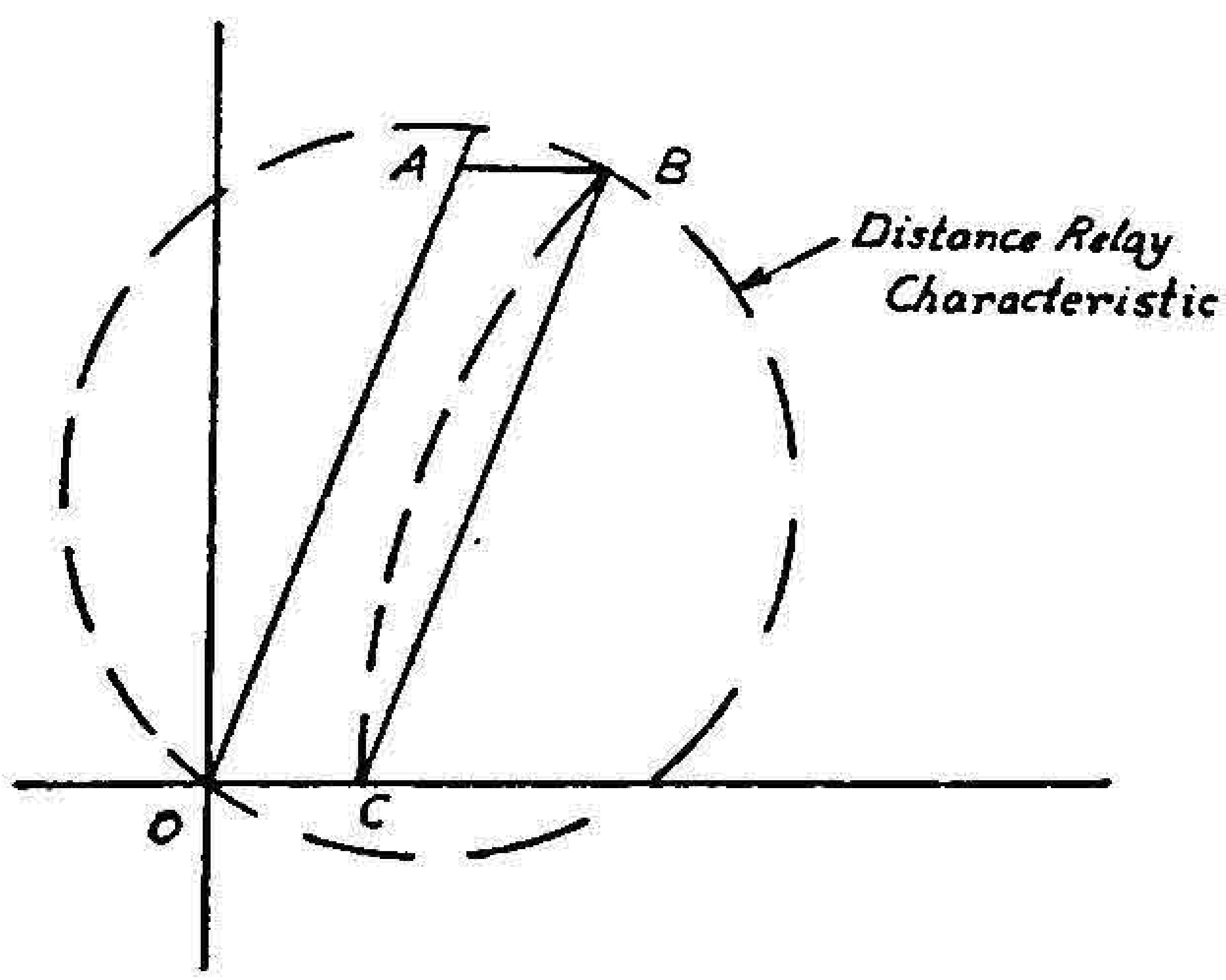


FIG. 5

But actually this should be replaced by the arc of the locus as proved above and as indicated by the arc BC. This same arc BC must be considered in setting the radius of the distance relay characteristic or in calculating the minimum length of the line for which the relay is set to protect for a certain value of fault impedance.

In the case of power swings, it is to be noted that the locus of power swing passes orthogonally to this arc BC. It may be further noted that the locus is further distorted due to the effect of currents flowing from the H-end.

REACTIVE AND RESISTIVE FAULTS

Pure reactive faults are only of theoretical interest as they seldom occur in practice. On the other hand, pure resistive faults are quite common.

If the fault impedance is a pure reactance then in the input impedance equation (13)

$$Z_S = P \cdot \frac{j \tan \beta l + (Q/P)}{1 + j (Q/P) \tan \beta l}$$

since $Q = \pm jX$, $Q/P = \pm jX/P = j \tan \beta d$,

$$Z_S = P \cdot \frac{j \tan \beta l \pm j \tan \beta d}{1 \pm \tan \beta d \cdot \tan \beta l} \quad (19)$$

$$= j P \cdot \tan \beta (1 + d) \text{ for } + X\Omega \quad (19\,a)$$

or

$$= j P \cdot \tan \beta (1 - d) \text{ for } - X\Omega \quad (19\,b)$$

This shows that the effect of a pure reactance in the fault is to make the fault appear to be at a larger distance d, where $d = \frac{\tan^{-1} (X/P)}{\beta}$. If the reactance were capacitive the distance term would be $(1 - d)$ showing that the fault appears to be nearer.

EFFECT OF SMALL ATTENUATION

Power lines have a very small amount of attenuation; they are not lossless as has been assumed throughout the above analysis. The effect of small attenuation combined with low values of fault impedances can be analysed as follows:

Let the attenuation constant be α,

then,

$$Z_S = Z_o \frac{Z_o \tanh \gamma l + Z_L}{Z_o + Z_L \tanh \gamma l}$$

$$= Z_o \cdot \frac{Z_o \sinh \gamma l + Z_L \cosh \gamma l}{Z_o \cosh \gamma l + Z_L \sinh \gamma l}$$

$$= Z_o \cdot \frac{Z_o \sinh (\alpha + j\beta) l + Z_L \cosh (\alpha + j\beta) l}{Z_o \cosh (\alpha + j\beta) l + Z_L \sinh (\alpha + j\beta) l} \quad (20)$$

on expanding and simplifying

$$Z_S = Z_o \frac{\cos \beta l (Z_o \alpha l + Z_L) + j \sin \beta l (Z_o + Z_L \alpha l)}{(Z_o + Z_L \alpha l) \cos \beta l + j \sin \beta l (Z_L + Z_o \alpha l)} \quad (20\,a)$$

when Z_L and αl are small, $Z_L \alpha l$ can be neglected, and then

$$Z_S = Z_0 \cdot \frac{(Z_0 \alpha l + Z_L) \cos \beta l + j Z_0 \sin \beta l}{Z_0 \cos \beta l + j \sin \beta l \, (Z_L + Z_0 \alpha l)}$$

$$= Z_0 \cdot \frac{j Z_0 \tan \beta l + (Z_0 \alpha l + Z_L)}{Z_0 + j \tan \beta l \, (Z_0 \alpha l + Z_L)} . \quad (21)$$

Thus a line with a small attenuation constant, gives a value for the input impedance term Z_S, as would be given by a lossless line with a fault impedance $Z_0 \alpha l + Z_L$. The effect of attenuation is therefore to increase the fault impedance.

APPENDIX I

Notations Used

F	..	Physical point of fault along transmission line.
G	..	Substation where relaying is considered.
H	..	Substation at opposite end line section.
m	..	Fraction of distance from G to H at which fault is assumed to occur.
M_{OGH}	..	Zero-sequence mutual impedance between the two parallel lines from G to H.
I	..	Current symbol with subscript notation as follows:

First subscripts:

1, 2, 0	..	Symmetrical components of currents in line.
a, *b*, *c*	..	Line currents of corresponding phases.

Following subscripts:

GF, HF, GH	..	Currents flowing between these two points.
Z_1, Z_2, Z_0	..	Sequence input impedances.
Z_0	..	Characteristic line impedance.
Z_g	..	Fault impedance for line faults between line conductors, or fault impedance between any single line and ground.

APPENDIX II

Calculation for a 235 miles long transmission line. Power transmitted 30,000 KVa at 110 KV.

266,800 c.m. A.C.S.R. Conductor with 14′ horizontal spacing. Ground wire 5′ 8″ above conductor cross arm and at 13′ 5″ spacing.

Resistance of conductor 25° C.	= 0·278 Ω per mile
Inductive Reactance at 50 cycles	= 0·712 Ω per mile
Capacitive Susceptance at 50 cycles	= $3 \cdot 85 \times 10^{-6}$ Mho per mile
Line impedance per mile	= 0·7643 Ω
Total impedance	= 179·6 Ω
Characteristic line impedance Z_0	.. 445·5 Ω

Assuming a fault at a point 90% of the length of the line, let fault impedance presented to the instantaneous step of the relay be 8·661.

The normalised value = 0·001944 = K′ of eqn. (17)

Fault impedance presented to relay with 0·5 sec. delay = 30·65 Ω

The normalised value of this impedance = 0·06878

Value of β for the line	= 0·097 degrees per mile.
Hence βl for 90% of the line	= 21·38 degrees.
and tan βl	= 0·3906 = c of equn. (17)

(i) *Locii for the Instantaneous unit*

(*a*) Location of fault fixed, power factor of fault varies.

The locus is a circle of which the centre is at

$$0, \frac{c(K'^2 + 1)}{1 - c^2K'^2}, \textit{ i.e., } \text{at } 0, 0 \cdot 39$$

and the radius is $\frac{c^2 + 1}{1 - c^2K'^2} K'$, *i.e.*, 0·002242

The impedance presented to the instantaneous unit is not much affected by the power factor of the fault.

(*b*) Location of fault varies, power factor of fault constant.

The locus is an arc of a circle of which the centre is at $\frac{1 + Q'^2}{2q}$, 0,

i.e., at 257·3, 0 assuming that the fault is a pure resistance fault.

The radius is $\sqrt{257 \cdot 3^2 - 1} = 257 \cdot 3$

Showing that the arc of the circle within the line angle is too flat and can be considered to be almost straight.

b5

(ii) Loci for unit with time delay of 0·5 sec.

(*a*) for different power factor of fault at a fixed location.

here $c = 0\cdot 3906$, and $K' = 0\cdot 06878$
the centre of the circle is at 0, 0·3938
Radius of the circle $= 0\cdot 07966$

Showing that the variation of the impedance measured with variation in power factor of the fault is negligibly small.

(*b*) for different location of faults at constant power factor assuming purely resistive fault,

Centre of the circle is at 7·609, 0
Radius of the circle $= 7\cdot 542$

This is shown in Fig. 6, as a locus of greater curvature than the locus of the previous case.

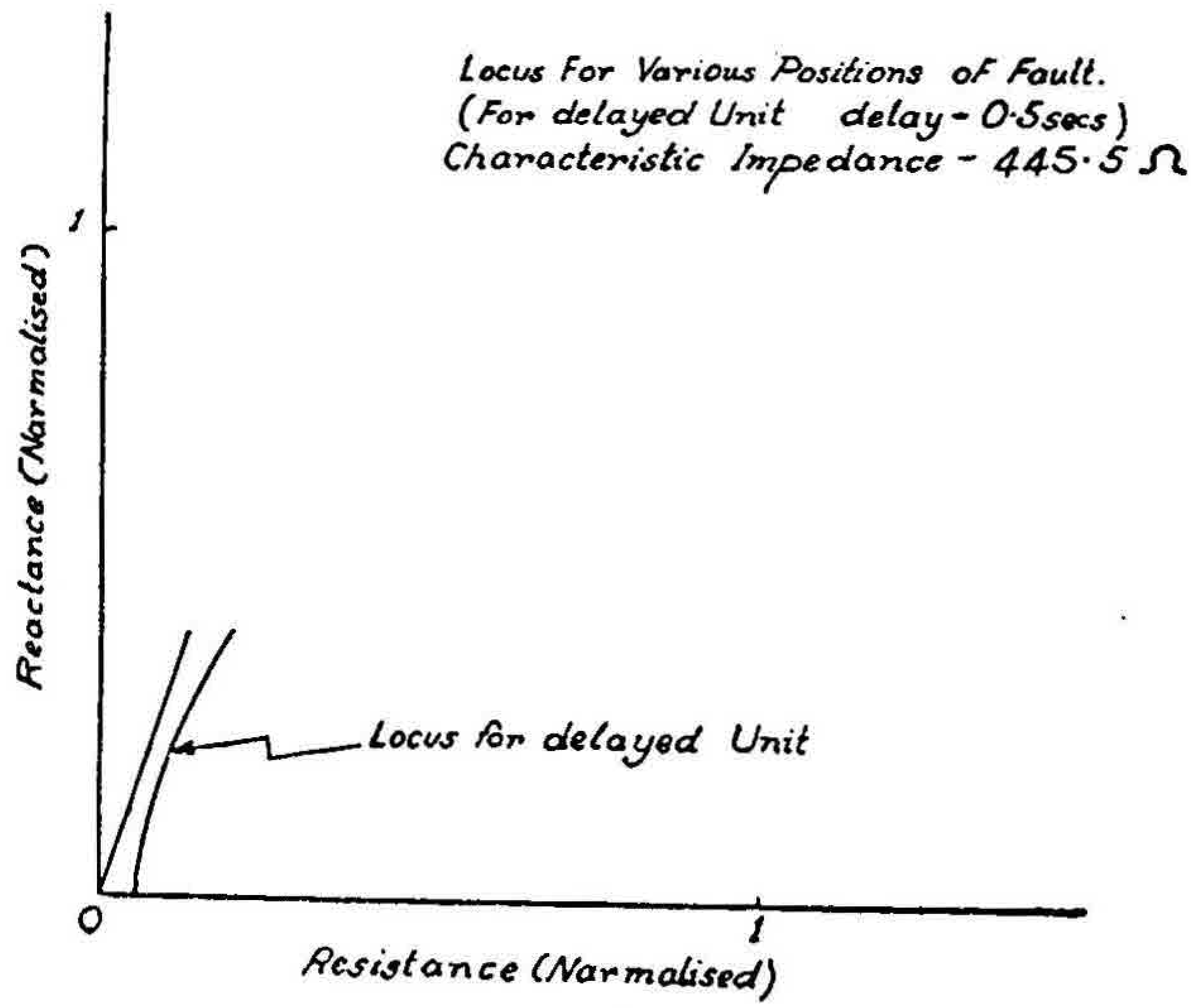


FIG. 6.

REFERENCES

W. A. Lewis and L. S. Tippett .. *Electrical Engineering*, 1931, **50**, 420.

——— .. *Trans. A.I.E.E.*, 1947, **66**, 694.

J. H. Neher .. *Ibid.*, 1937, **56**, 833.

Edith Clarke .. *Ibid.*, 1945, **64**, 372.

W. A. Wolfe .. *Ibid.*, 1946, **65**, 458.