STUDIES FOR A NEW HOT AIR ENGINE

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FOREWORD

The thermodynamic analysis of a new Hot Air Engine forms the first account of work initiated and done in the Department of Internal Combustion Engineering under the auspices of the Council of Scientific and Industrial Research (C.S.I.R.) within the " Research Scheme on Hot Air Engines and Their Development," an investigation devoted to fundamental data. It is written in two parts. An account of further work with sub-title "Heat Transfer to Pulsating Air Flowing in a Tube" will be published subsequently as Part III in this Journal.

PART I :

A Principal Thermodynamic Analysis

SUMMARY

This paper deals with the thermodynamic aspects of a new hot air engine cycle which provides for compression of, heat addition to and expansion of, clean air which is used to sustain combustion, with the combustion gases heating externally the compressed air in a heat exchanger. In the first part of this report, the cycle is considered particularly in its simple form without the complications of multistage processes occurring in any of the cylinders. The topics studied include the expansion ratio, cylinder configurations, the concept of necessary and useful expansion, the process of heat addition, and the indices of compression and expansion. The necessary equations are derived and numerical tables are given. In the second part, the effects of introducing multistage units, the air consumption and part-load operation will be examined.

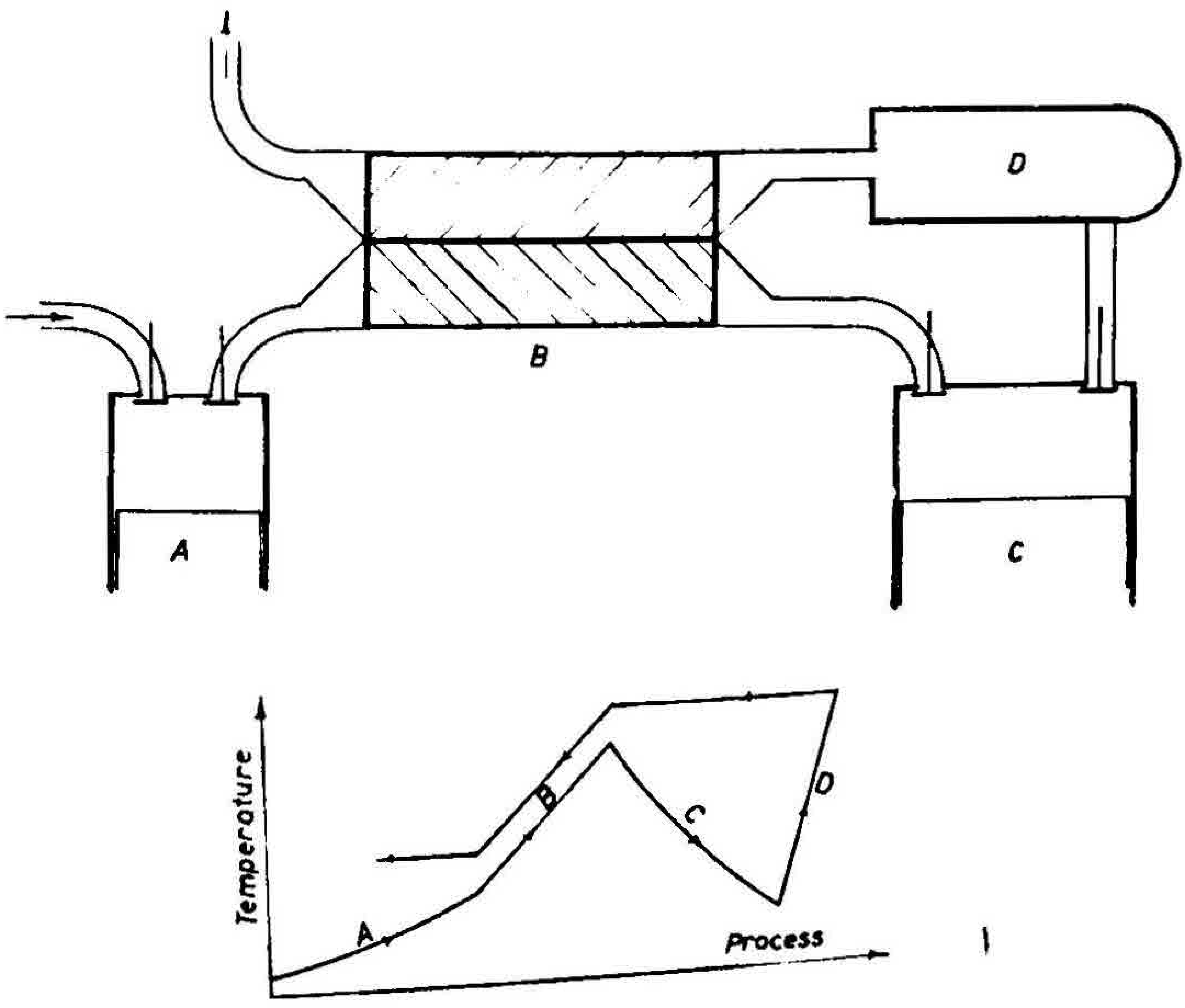
CONTENTS

1. Introduction

The development of a hot air engine, some fundamental aspects of which are examined in this Report, is important where cheap, indigenous, low-grade fuels for power production should be used instead of costly fuels as accepted by the conventional types of internal combustion engines. This is made possible by the introduction of a cycle in which the products of combustion do not come into contact with any of the surfaces of, and between, moving parts of the power unit.

Briefly, the cycle, which by applying terms of gas turbine technology, may be called an exhaust-heated open cycle, can be described as follows.[1]*

Atmospheric air is inducted into a compressor A (Fig. 1) and after being compressed to a suitable pressure, is made to pass through the



A: Compressor; B: Heat exchanger; C: expansion motor
D: Combustion chamber

Fig. 1. The process diagram.

* Numbers refer to the list of references given in Section 8, p. 240.

heat exchanger B where it is heated to a predetermined temperature. The hot and still clean air is now allowed to expand in the expansion motor C where its thermal energy is partly converted into useful mechanical energy. After expansion the air alone or in addition with other air is used as oxygen carrier to burn fuel in a combustion apparatus D, suitably designed for the particular fuel. The products of combustion, mixed with excess air, if necessary, are sent into the heat exchanger B where they give up their heat content as far as is possible, to the incoming compressed air. At the outlet end of the hot side of the heat exchanger, the hot gases would still have some energy left and this may be utilised in various ways depending on the application of the power unit—say for drying moist fuel —and on convenience, or alternatively they may be allowed to escape into the atmosphere.

It will be noticed immediately that parts of the heat exchanger only come into contact with the products of combustion and not any of the surfaces of the moving parts like the piston-cylinder assembly or a turbine wheel in the case of a rotary power plant. Thus, when low-grade, ash-forming fuels are used, the problem of precautions against corrosion and abrasion on moving parts, seizure and similar difficulties are completely eliminated and instead the problem now becomes one of designing for high temperature resisting materials in the heat exchanger where the highest cycle temperature occurs. It may also be pointed out that contamination of the working medium with lubricants will always be restricted to a very small percentage, with no dangerous accumulation being possible.

During the years immediately after the war, another power plant has been developed in Holland by Phillips.[2] This works on the principle of the closed cycle, wherein the same working medium—usually air—is continuously subjected to the thermodynamic processes of the cycle. This results, in the course of time, in the contamination of the working medium with increasing amounts of lubricating oil to the point when the oil vapour medium mixture reaches explosive limits. This difficulty is obviated in the open cycle, described above, as every cycle operates with a fresh portion of the working medium. The cycle under study also eliminates one of the components of the closed cycle, namely the cooler, and thus makes the power unit more suitable for arid zones.

The principle of the exhaust-heated open cycle is not new having been described by many others. It has been mentioned as a possible variation of the gas turbine cycle by Constant,[3] Lysholm,[4] Haverstick[5] and Kreitner and Nettel.[6] Mordell[7] has investigated theoretically the thermodynamics of the cycle and Johnson[8] has described how it could be profitably combined with steam and other turbine cycles to achieve higher efficiencies. Veits and Jacks[9] have investigated the energy characteristics of air turbines. Tyler and McPhail[10] have reported the use of the cycle for the development of a coal-burning gas turbine locomotive. All these researches have been made in connection with the development of high-output gas turbines. No one seems to have specially reported using the cycle for low-output (less than 20 H.P.) reciprocating units. Ricardo and Co. Ltd.,[11] considered the cycle for such a use but seem to be in favour of a small vertical steam engine designed on modern principles. In many backward countries and out-of-the way places, however, the hot air engine has the advantage over the steam engine that it requires no water for its operation. This alone justifies the expenditure of time and energy on the former in preference to the latter, its main problem being to harmonize the quest for good efficiency, *i.e.*, for a high cycle peak temperature with the temperature to which the material in the heat exchanger can safely be subjected.

This part, Part I of the Report, contains a first theoretical analysis of the thermodynamic and related principles of the exhaust-heated open cycle hot air engine as applied to low output reciprocating units.

2. The Simple Cycle

The phrase, " Simple Cycle ", is here used to denote the cycle as described at the beginning of the introduction, with single-stage compression and single-stage expansion occurring in two different cylinders and without the complication of multistage processes occurring.

Thermodynamically, the cycle consists of polytropic compression, followed by heat addition substantially at constant pressure and polytropic (in the ideal case, adiabatic) heat rejection.

The ideal efficiency of the cycle may be calculated by assuming that the working fluid is a perfect gas and both the machine components are 100% efficient. Under such circumstances the useful work of the cycle is given by the expression†:

$$L_u = L_e - L_c. \tag{1}$$

The heat supplied to the ideal plant (with a heat exchanger efficiency equal to unity is:

$$Q_i = C_p (T_3 - T_2)\cdot \tag{2}$$

The ideal cycle efficiency is therefore

$$\eta_a = \frac{L_u}{JQ_i} = \frac{L_e - L_c}{JC_p (T_3 - T_2)}. \tag{3}$$

Assuming the compression and expansion processes to be isentropic the respective changes in enthalpy will be:

$$L_e = JC_p (T_3 - T_4) \tag{4}$$

and

$$L_c = JC_p (T_2 - T_1). \tag{5}$$

$$\therefore \eta_a = \frac{JC_p (T_3 - T_4) - JC_p (T_2 - T_1)}{JC_p (T_3 - T_2)}. \tag{6}$$

† A list of symbols and units is given in Section 9, p. 241.

Since the specific heat C_p is constant for a perfect gas, equation (6) may be simplified into:

$$\eta_a = 1 - \frac{T_4 - T_1}{T_3 - T_2}. \tag{7}$$

For an ideal process where compression ratio = expansion ratio = E,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = E^{n-1}. \tag{8}$$

Hence, the ideal efficiency η_a is given by the expression

$$\eta_a = 1 - \frac{1}{E^{n-1}}. \tag{9}$$

This is the same expression as that for the constant pressure Otto cycle. The values of the ideal efficiency at different values of E between 2 and 14 are given in Table I.

The ideal efficiency can be used to obtain the value of the relative efficiency but for purposes of design as well as for comparison with other cycles, the actual or overall thermal efficiency is a more useful concept. This takes into account all the losses which are necessarily involved in the physical design of the power plant.

In a real engine, the actual compression work is given by L_c/η_c and actual expansion work by $\eta_e L_e$. Therefore, the net output per kg of air, reduced by the mechanical losses represented by η_M and the parasite losses represented by Δh is given by the expression

$$L_n = \eta_M \left(\eta_e L_e - \frac{L_c}{\eta_c}\right) - J\Delta h. \tag{10}$$

If the temperature of the working medium at the end of heat addition (or before the commencement of expansion) is T_3 and the temperature before the beginning of heat addition is T_2, then the heat supplied to the power plant per kg of air is given by

$$L_i = \frac{1}{\eta_B \eta_R} \bar{C}_p J (T_3 - T_2), \tag{11}$$

where η_B and η_R are the combustion and heat transfer efficiencies respectively. The overall thermal efficiency can therefore be expressed as:

$$\eta = \frac{L_n}{L_i} = \frac{\eta_M \left(\eta_\epsilon L_e - \frac{L_c}{\eta_c}\right) - J\Delta h}{\frac{1}{\eta_B \eta_R} \cdot \bar{C}_p J (T_3 - T_2)}. \tag{12}$$

In order to simplify this expression for working purposes, the parasite losses, instead of being expressed as Δh may be approximated as a fixed percentage η_L of the net output. Further substituting the following well-known expressions for L_c and L_e namely

$$L_c = \frac{n}{n-1} RT_1 (1 - E^{n-1}) \tag{13}$$

and

$$L_e = \frac{n}{n-1} RT_3 \left(1 - \frac{1}{E^{n-1}}\right), \tag{14}$$

equation (12) can be reduced to the form

$$\eta = \eta_L \eta_M \eta_B \eta_R \left(\frac{n}{n-1}\right) \left(\frac{R}{C_p J}\right) \left(\frac{E^{n-1} - 1}{E^{n-1}}\right) \frac{T_3 - \dfrac{T_1 E^{n-1}}{\eta_e \eta_c}}{T_3 - T_1 E^{n-1}}. \tag{15}$$

This as well as all the following equations have been evaluated with typical values for the various quantities. Some have been assumed to be constant and some were varied within reasonable ranges. The effect of varying some of the quantities originally assumed to be constant was also ascertained.

The constant values used were: $\eta_L = 0 \cdot 9$, $\eta_M = 0 \cdot 85$, $\eta_B = 0 \cdot 98$ (assuming an oil-fired, gas turbine type combustion chamber), $\eta_R = 0 \cdot 8$, $n = 1 \cdot 3$, $R = 29 \cdot 27$ kg-m/kg-° C, $J = 427$ kg-m/kcal, $\eta_e = 0 \cdot 85$, $\eta_c = 0 \cdot 85$ and $T_1 = 303°$ K (30° C). With substitution of these values and simplification, equation (15) is reduced to

$$\eta = \frac{0 \cdot 1513}{C_p} \frac{E^{0.3} - 1}{E^{0.3}} \frac{T_3 - 419\, E^{0.3}}{T_3 - 303\, E^{0.3}}. \tag{16}$$

The specific heat of air was calculated to the fifth decimal in the relevant temperature ranges.

The change in η with values of E varying from 2 to 14 and T_3 from 325 to 925° C is shown in Table II and plotted in Fig. 2. It can be seen that upto about 600° C the thermal efficiency is a maximum at a compression ratio increasing from 4 to 7 while above that temperature there is no well-defined maximum but the increase in thermal efficiency is negligible for even a considerable increase in compression ratio. As the thermal efficiency is more sensitive to temperature at lower temperatures than at high temperatures, due to the variation of specific heat, the inadvisability of designing for very high temperatures may be deduced. It may also be remembered here that too high a temperature would increase the initial cost of the power plant inordinately due to the necessity of providing high quality steels in the heat exchanger.

Apart from the overall thermal efficiency which represents the effectiveness of utilisation of the fuel, the other important thermodynamic factor to be considered is the specific output which represents the effectiveness of utilisation of the air used in the power plant. The specific output obtained by expressing the net output per kg of air in H.P. per sec is,

$$L_s = \frac{\eta_L \eta_M \eta_C}{76 \cdot 04} \frac{n R}{n-1} \frac{E^{n-1} - 1}{E^{n-1}} \left(T_3 - \frac{T_1 E^{n-1}}{\eta_e \eta_c}\right). \tag{17}$$

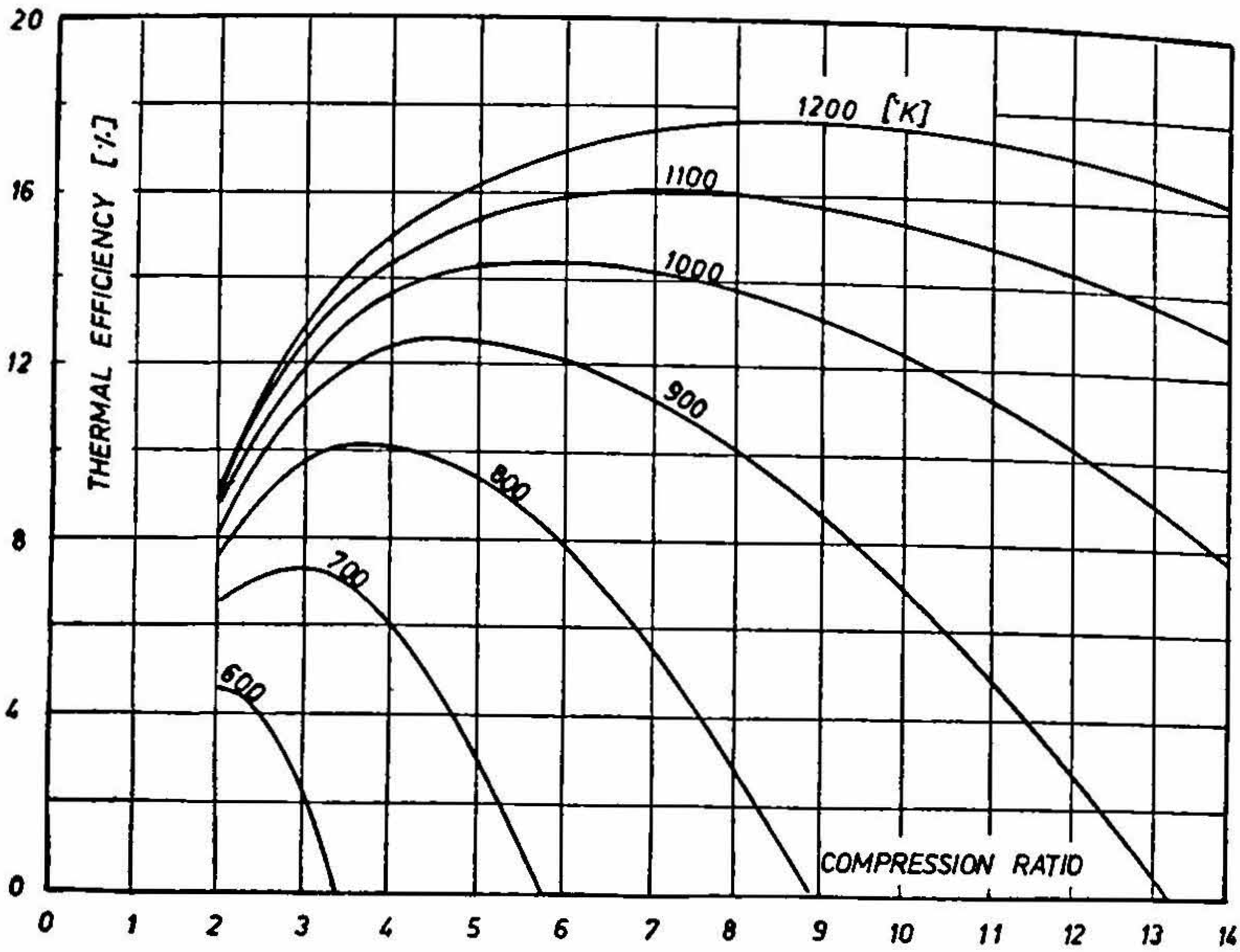


FIG. 2. Overall thermal efficiency.

When the numerical values mentioned above are substituted for the different factors in this equation, it is reduced to the form

$$L_s = 1 \cdot 085 \frac{E^{0.3} - 1}{E^{0.3}} (T_3 - 419 E^{0.3}). \qquad (18)$$

The values of L_s for different values of E and T_3 are given in Table III and plotted in Fig. 3. While the specific output is directly proportional to the temperature as may be expected, its maxima, as compared to those of the thermal efficiency, are spread over and shifted to, compression ratios between 5 and 11, increasing with increasing temperatures. Hence the optimum conditions with respect to *both* thermal efficiency and specific output for the simple cycle are given by (*a*) $5 < E < 7$ and (*b*) $T_3 < 800°$ C.

By equating either η or L_s (equations 15, 16, 17 or 18) to zero, $T_{3\,min.}$ is obtained, which is the temperature necessary to just overcome the losses in the engine

$$T_{3\,min} = \frac{T_1 E^{n-1}}{\eta_e \eta_c}, \qquad (19)$$

or with the numerical values assumed above

$$T_{3\,min} = 419 E^{n-1}. \qquad (20)$$

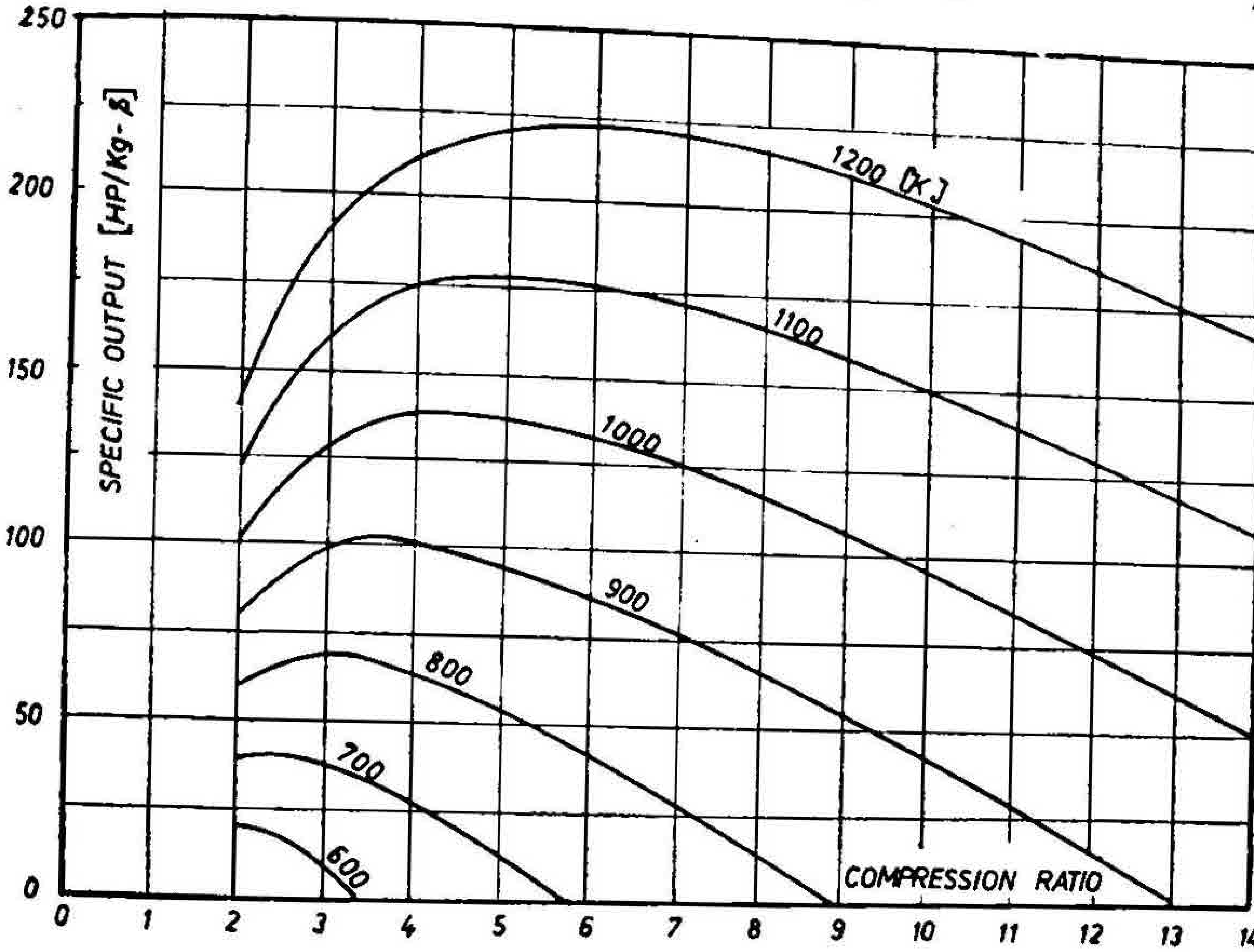


FIG. 3. Specific output.

3. Expansion Ratio

In the analysis it has so far been assumed that the compression and expansion ratios are equal. Matching the two ratios correctly is essential for maintaining the efficiency at a high level. The expansion ratio cannot usually be considerably greater than the compression ratio, as otherwise the pressure at the end of expansion will be too low to overcome the pressure drop across combustion chamber and heat exchanger. This gives the upper limit of the expansion ratio

$$E_{e\,max} \leqslant E. \tag{21}$$

The lower limit is given by the fact that with low expansion, the output would be less than what is necessary for overcoming the losses in the engine. If, in equations (13) and (14), the value of E is not the same but is E and E_e for compression and expansion respectively, and if these expressions are substituted in equation (12) and the resultant equation is simplified,

$$\eta = \eta_L \eta_M \eta_B \eta_K \frac{nR}{n-1} \frac{\left[\eta_e T_3 \left(1 - E_e^{\frac{1}{n-1}}\right) - \frac{T_1}{\eta_c}\left(E^{n-1} - 1\right)\right]}{C_p J (T_3 - T_2)}. \tag{22}$$

The values of η for different values of expansion ratio are given in Table IV. When this is equated to zero and solved for E_e, the minimum value of E_e is obtained. Hence,

$$E_{e\ min} > \left[\frac{1}{1 - \frac{1}{\eta_0 \eta_c} \frac{T_1}{T_3}(E^{n-1} - 1)}\right]^{\frac{1}{n-1}}. \tag{23}$$

$E_{c\ min}$ is therefore lowered by increasing T_3 at any compression ratio or by decreasing the compression ratio at any value of T_3.

Within the limits indicated by the expressions (21) and (23) the higher the expansion ratio, the higher will be the thermal efficiency at any given temperature T_3. It is more useful to design a lower compression engine with close values of E and E_e than a higher compression engine with too great a difference between the two. For instance, if $E = 7$ and $E_e = 5$, then $\eta = 6{\cdot}46\%$ at $T_3 = 750°$ C, but if $E = E_e = 5$, $\eta = 13{\cdot}58\%$ at the same temperature.

4. Cylinder Configurations

The calculations so far have been made under the assumption that compression occurs in one cylinder and that expansion occurs in another cylinder of a suitable size. However, some useful deductions follow if the case is considered where the size of the expansion cylinder is arbitrarily chosen. The analysis may be made as follows:—

If p_1, V_{1c} and T_1 are the initial conditions in the compressor and p_2, V_{2c} and T_2 are the final conditions, then

$$G = \frac{p_1 V_{1c}}{RT_1} = \frac{p_2 V_{2c}}{RT_2}. \tag{24}$$

If the pressure at the end of heat exchange is p_3, this may be expressed by

$$p_3 = \pi_{23} p_2 = \pi_{23} p_1 E^n, \tag{25}$$

where π_{23} is a constant which defines the ratio between the pressures at the end and the beginning of the heat exchanger at the moment when the expansion motor inlet valve opens. In the theoretical case, when heat exchange is at constant pressure, $\pi_{23} = 1$; but in the real case, there will be a slight pressure difference between the two due to two reasons: (*a*) the pressure drop across the heat exchanger tubes and (*b*) the effect of constant volume heating which occurs during a short period in each cycle when both the compressor outlet valve and the expansion motor inlet valve remain closed simultaneously. This effect is examined in greater detail in Section 7.

From the value p_3, the pressure at the hot end of the heat exchanger falls when the expansion motor inlet valve opens to the value p_a (say), when the valve closes. p_a is defined by the equation

$$p_a = \pi_{2a} \cdot p_2 = \pi_{2a} \cdot p_1 E^n, \tag{26}$$

where π_{2a} is a constant.

The drop in pressure from p_3 to p_a may be calculated from

$$\frac{p_3 V_{HR}}{R T_3} = \frac{p_a (V_{HR} + V_a)}{RT_a}. \tag{27}$$

At the end of expansion, if the pressure in the working medium is p_4, it may be defined by

$$p_4 = \frac{p_1}{\pi_{41}}. \tag{28}$$

During the expansion process, if p_a, V_a and T_a are the initial conditions and p_4, V_{1e}'' and T_4 are the final conditions, then it follows that

$$mG = \frac{p_a V_a}{RT_a} = \frac{p_4 V_{1e}''}{RT_4}, \tag{29}$$

where m represents the fraction of the air that is used for expansion. V_a may be defined by the expression

$$V_a = \frac{V_{1e}''}{E_e} = V_{1e}'' C, \tag{30}$$

where C is the cut-off ratio which is here defined as the ratio of the actual volume of hot air admitted into the expansion cylinder to the total volume of the cylinder and is, in effect, the reciprocal of the expansion ratio.

From equations (25), (26), (27) and (30), it is possible to deduce that

$$T_a = \frac{\pi_{2u}}{\pi_{23}} \left(1 + C \frac{V_{1e}''}{V_{HR}}\right) (\Delta T + T_1 E^{n-1}), \tag{31}$$

where

$$\Delta T = T_3 - T_2 = T_3 - T_1 \cdot E^{n-1}. \tag{32}$$

From equations (24) and (29) one obtains

$$\frac{p_a V_a}{T_a} = m \frac{p_1 V_{1e}}{RT_1}. \tag{33}$$

From equations (26), (30), (31) and (33), the expression for the cut-off ratio is obtained as

$$C = \cfrac{1}{\cfrac{\pi_{23} E^n}{m \left(\cfrac{\Delta T}{T_1} + E^{n-1}\right) V_{1e}} - \cfrac{V_{1e}''}{V_{HR}}}. \tag{34}$$

Again from equations (24) and (29),

$$\frac{p_4 V_{1e}''}{RT_4} = m \frac{p_1 V_{1e}}{RT_1}, \tag{35}$$

where T_4 is given by the expression

$$T_4 = \frac{T_a}{E_e^{n-1}} = \frac{\pi_{2a}}{\pi_{23}} \left(1 + C \frac{V_{1e}''}{V_{HE}}\right) (\Delta T - T_1 E^{n-1}) C^{n-1}. \tag{36}$$

From equations (28), (35) and (36), one obtains

$$\pi_{23} = m \frac{V_{1c}}{V_{1e}''} \pi_{41} \pi_{2a} \left(1 + C \frac{V_{1e}''}{V_{HE}}\right) \left(\frac{\Delta T}{T_1} + E^{n-1}\right) C^{n-1}. \tag{37}$$

If the entire mass of air that is compressed is used for expansion in a single cylinder, then '*m*' will be equal to unity. Therefore, equations (34) and (37) may be combined and simplified into the form

$$\frac{V_{1c}}{V_{1e}''} \frac{1 + C \frac{V_{1e}''}{V_{HE}}}{C E^n} \left(\frac{\Delta T}{T_1} + E^{n-1}\right) = \pi_{23}. \tag{38}$$

If, further, π_{23} is equated to unity as an approximation, then

$$\frac{V_{1c}}{V_{1e}''} \frac{1 + C \frac{V_{1e}''}{V_{HE}}}{C E^n} \left(\frac{\Delta T}{T_1} + E^{n-1}\right) = 1. \tag{39}$$

This expression gives the relationship between cut-off ratio and compression ratio for different ratios of compression cylinder volume to expansion cylinder volume. The numerical values are given in Table V. The values shown when the latter ratio is two may be used if production considerations lead to the use of three cylinders for the entire power plant, the first one for compression and the other two for expansion.

On the other hand, compression and expansion might be made to occur in the same cylinder in what might be called the "four-stroke" version of the hot air engine. In such a case the condition is

$$V_{1c} = V_{1e}'', \tag{40}$$

and equation (39) is simplified into the form

$$\frac{1 + C \frac{V_{1e}''}{V_{HE}}}{C E^n} \left(\frac{\Delta T}{T_1} + E^{n-1}\right) = 1. \tag{41}$$

The numerical values can be seen in Table V in the column under $C_{1c}/V_{1e}'' = 1$. The value of the cut-off ratio may be obtained by combining equations (34) and (37) to

$$C = \frac{1}{(\pi_{41} \pi_{2a})^{1/n} E}. \tag{42}$$

These values are listed in Table VI and show that high values of T_3 would lead to high values of the cut-off ratio and consequently to a loss due to high discharge pressure. This is also shown in Fig. 4. If the expansion were to be reasonably complete, the temperature rise ΔT is limited to less than 300° C, giving a value

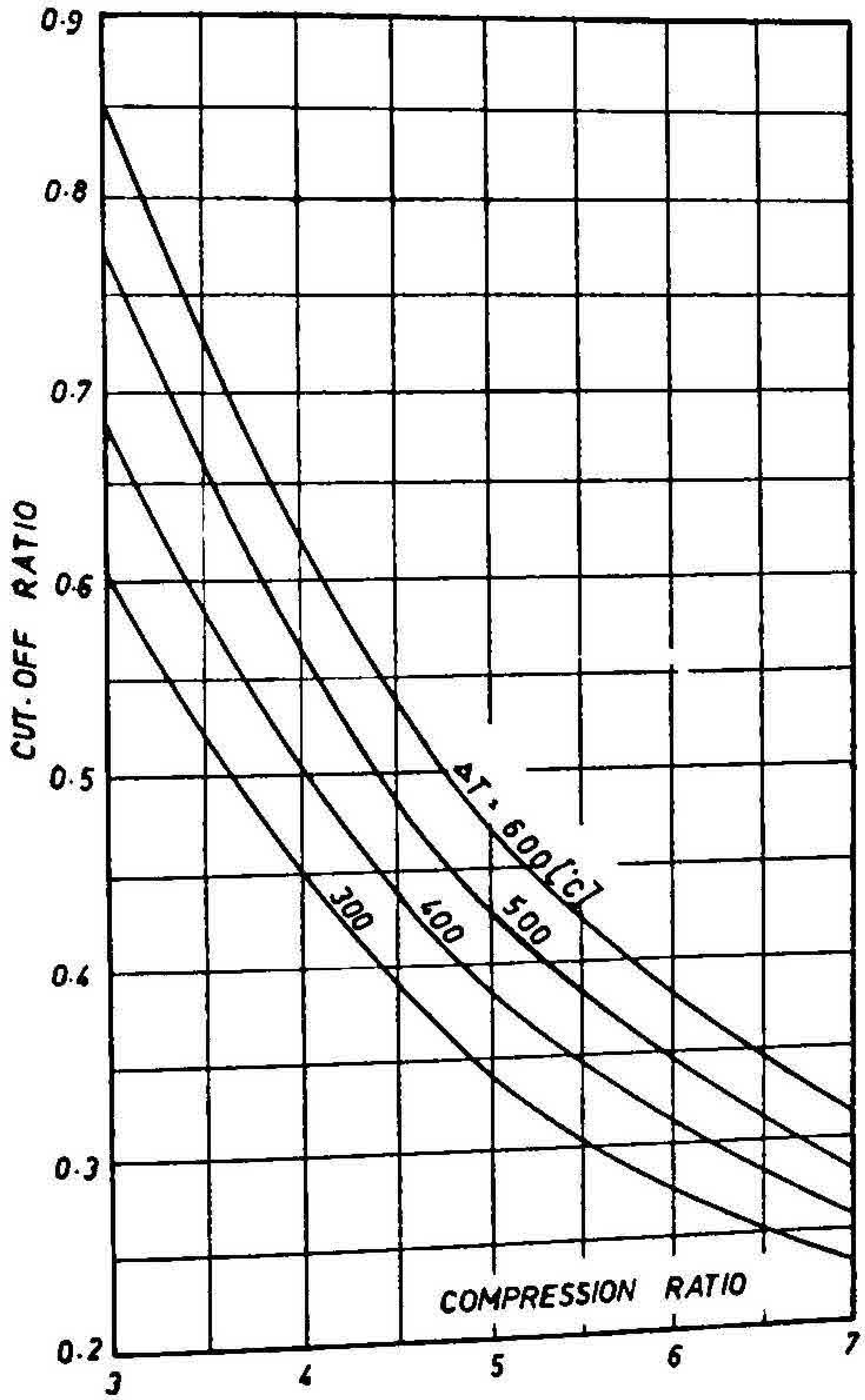


FIG. 4. Cut-off ratio.

of about 450 or 500° C for T_3. At this temperature the efficiency is a maximum at a compression ratio of about 5. At higher temperatures and compression ratios the efficiency drops to uneconomic levels.

The adoption of the single-cylinder unit, while reducing the number of cylinders by half, at the same time decreases—other quantities remaining equal—the mass flow through the power plant and hence the power output, by half.

5. THE CONCEPT OF NECESSARY AND USEFUL EXPANSION

In order to take advantage of higher temperatures and to attain greater flexibility of operation, expansion might be made to take place in two separate cylinders, one designated as the necessary expansion cylinder and the other as the useful expansion cylinder. The former produces just sufficient power for compression and to overcome the losses in the engine while the latter produces the useful power which may be drawn from the shaft and put to work.

Considering first " Necessary Expansion," the condition to be adhered to is

$$G.L_c = m' \, GL_{e\ \text{nec.}}, \tag{43}$$

where $m'G$ is the fraction of the air used for "Necessary Expansion". Substituting for L_c and L_e from equations (13) and (14) respectively and solving for m', gives

$$m' = \frac{E^{n-1} - 1}{\left(E^{n-1} + \frac{\Delta T}{T_1}\right)\left(1 - C^{n-1}\right)}. \tag{44}$$

These values are shown in Table VII. It is seen that the part of the air which has to be bled off for necessary expansion decreases with increasing temperature. By combining equations (37) and (44), the relationship between compression and cut-off ratios for necessary expansion is obtained, namely

$$\left(\frac{1 - E^{n-1}}{E^n}\right) \frac{1 + C \frac{V_{1e}''}{V_{HE}}}{C - C^n} = \pi_{23}. \tag{45}$$

Table VIII gives these values when $\pi_{23} = 1$.

The corresponding relationship for useful expansion is obtained by substituting the expression $(1 - m')$ from equation (44) into equation (37) and by simplifying:

$$\left[\left(E^{n-1} + \frac{\Delta T}{T_1}\right) - \left(\frac{E^{n-1} - 1}{1 - C^{n-1}}\right)\right]\left[\frac{1 + C \frac{V_{1e}''}{V_{HE}}}{E^n C}\right] = \pi_{23}. \tag{46}$$

These values are given in Table IX. According to equation (42) the temperatures are limited to about 700° C which is also the limit for easily available heat-resisting steels. At this temperature, m' is about 0·4. In other words, about 60% of the total mass flow produces useful work so that the size of the expansion cylinder may be equal to that of the compression cylinder with very little loss in efficiency but leading to ease of manufacture. In this case, the mass flow and hence power output can be increased by 100% by increasing the number of cylinders by only 50%.

6. The Process of Heat Addition

Heat addition should nominally occur at constant pressure, as otherwise, the compressor will not be able to discharge into the heat exchanger. Nevertheless, it actually takes place partly at constant pressure and partly at constant volume and this effect may be taken advantage of to give a slight increase in the thermal efficiency of the power plant. The effect occurs because the valve timing is usually such that in one part of each cycle, either both the compressor discharge valve and the expansion motor inlet valve remain open or at least one of them is open and heat addition therefore takes place at constant pressure; in the other part of the cycle, both the valves remain closed simultaneously whereby heat addition takes place at constant volume. When the two valves are open during immediately successive intervals (as is necessarily the case when compression and expansion occur in a single cylinder), one gets the maximum possible constant pressure period and the minimum possible constant volume period. The opposite effect is obtained when the overlap of the two opening periods is maximum. In general, constant volume heating may occupy about 60% of each cyclic period.

The problem here is to take advantage of the increased thermal efficiency due to constant volume heating while at the same time overcoming the difficulty of increased resistance to the compressor discharge. This problem can be solved by making the expansion motor inlet period precede the compressor discharge period. Then the complete process in one cycle would be as follows:—

At the moment when the compressor outlet valve opens, the expansion motor inlet valve would be closed and the mean pressure at the inlet of the heat exchanger would be p_2 while the pressure at the outlet of the heat exchanger would be $p_3 = p_2 - \Delta p$ where Δp is the pressure drop along the heat exchanger. When the compressor discharge process is completed, the expansion inlet valve would still be closed and the pressure p_2 would rise at the rate

$$\Delta\, p_c = p_2 \frac{\mathrm{G}}{\mathrm{W_{HX}}} \ \mathrm{kg/cm^2\ s}, \tag{47}$$

while the inertia of the air in the heat exchanger would continue the flow and at the same time tend to equalise the pressure. At this time, constant volume heating would commence, increasing the pressure at the rate

$$\Delta\, p_v = p_2 \frac{\mathrm{H}}{\mathrm{W_{HX}\, C_v}} \ \mathrm{kg/cm^2\ s}. \tag{48}$$

Then the expansion motor inlet valve opens and the pressure falls at the rate

$$\Delta\, p_e = p_3 \frac{\mathrm{G}}{\mathrm{W_{HX}}} \ \mathrm{kg/cm^2\ s}. \tag{49}$$

The process would be in dynamic equilibrium when

$$c\,.\,\Delta\, p_c + v\,\Delta\, p_v = e\,\Delta\, p_e \tag{50}$$

where c, v and e are the compressor discharge, constant volume heating and expansion motor inlet periods respectively. c, v and e are related by the equation

$$c + v + e = \frac{60}{N} \cdot \tag{51}$$

If it is assumed that

$$\frac{v}{(e + c)} = \alpha \text{ (say)} \tag{52}$$

then, equations (50) and (51) give

$$c = \frac{1 - \alpha \dfrac{\Delta p_v}{\Delta p_e}}{1 + \dfrac{\Delta p_c}{\Delta p_e}} \frac{60}{N(\alpha + 1)}, \tag{53}$$

$$e = \frac{1 + \alpha \dfrac{\Delta p_v}{\Delta p_c}}{1 + \dfrac{\Delta p_e}{\Delta p_c}} \frac{60}{N(\alpha + 1)}, \tag{54}$$

and

$$v = \frac{60}{N} \cdot \frac{\alpha}{\alpha + 1} \cdot \tag{55}$$

It remains for further examination whether equation (50) is really valid in actual practice. According to the conventional principles of design of a compressor and an expansion motor (or compressed air motor), it may be stated that e is slightly greater than c while v is approximately equal to $2(e + c)$; on the other hand, Δp_e and Δp_c are of the same order of magnitude while Δp_v is comparatively negligible. Hence equation (50) is apparently satisfied but this analysis does not take into account the pressure waves which are generated whenever the valves are operated.

When the compressor discharge valve opens, a high pressure wave is generated and though partially damped by the heat exchanger headers, travels along the heat exchanger tubes at more than sound velocity at the prevailing temperature. This vave is reflected back and forth a number of times depending on the length of the tubes and the magnitude of the constant volume heating period which immediately follows the compressor diacharge period. When the expansion motor inlet valve opens, a low pressure wave is generated and superimposes itself on the high pressure wave. Just sufficient time may be allowed now for this low pressure wave to reach the compressor end of the heat exchanger and then the compressor discharge valve may be opened. A simple calculation shows this time to be of the order of 3 to 4° of crank rotation. In this way the compressor discharge process is improved. Thus this analysis only strengthens the validity of equation (50).

The pressure pulsations described here also have another and possibly beneficial effect on the performance of the heat exchanger. It is proved in a further part of this Report that pulsations increase the heat transfer coefficient in a heat exchanger tube by nearly 30%, if not more, under well-defined conditions of frequency, amplitude and Reynolds Number. Thus, by suitable design, the pulsations can be taken advantage of to decrease the volume of the heat exchanger for any particular power plant. The headers in the heat exchanger can be used to provide just the degree of damping that is necessary to optimise the rate of heat transfer.

7. The Indices of Compression and Expansion

The mean values of the indices of compression as well as expansion have been assumed to be 1·3 in the present calculations. In practice they change with various conditions but their theoretical limits are 1·0 and γ (= 1·4 for air).

During compression, the work input is a minimum with isothermal compression ($n = 1$) and a maximum with adiabatic compression ($n = 1 \cdot 4$); the difference between the two is greater at higher compression ratios. Ideal cooling would abstract all the heat produced by compression and bring the value of n nearer unity but the cooling work increases considerably. On the other hand, compression along the adiabatic line reduces the volumetric efficiency but at the same time, increases the temperature at the end of compression thus allowing for a slight decrease in the duty (or increase in efficiency) of the heat exchanger.

If however, the temperature at the end of compression rises above 200° C, lubrication and sealing difficulties may appear and cooling work increases forbiddingly above a compression ratio of about 10 or 11. The value of n also varies along the p-v curve, being greater at the beginning and smaller at the end than the mean value because of the different quantities of heat flow occurring at different temperature differences. It is further effected by the speed of the compressor, being lower at lower speeds. Similar considerations also apply for the process of expansion during which the work done is a maximum when $n = \gamma$. The value of 1·3 was assumed in the calculations to allow also for the loss due to radiation.

In practice, there is bound to be a variation in the indices and the effect of variation was calculated. By putting n_e and n_c for the indices of compression and expansion in equations (13) and (14) respectively and substituting in equation (12) the following equation is obtained

$$\eta = \eta_L \eta_M \eta_B \eta_K \eta_e \, R \, \frac{\left[\frac{n_e}{n_e - 1}\left(1 - \frac{1}{E^{n_e - 1}}\right) T_3 - \frac{1}{\eta_e \eta_c} \frac{n_e}{n_c - 1} T_1 (1 - E^{n_c - 1})\right]}{\bar{C}_p \, J \, (T_3 - T_2)} . \tag{56}$$

Introducing the previously mentioned numerical values and simplifying leads to

$$\eta = 3 \cdot 4946 \, \frac{\left[\frac{n_e}{n_e - 1}\left(\frac{E^{n_e - 1} - 1}{E^{n_e - 1}}\right) T_3 - 415 \cdot 2 \, \frac{n_c}{n_c - 1} (1 - E^{n_c - 1}) \, T_1\right]}{C_p \, J \, (T_3 - T_2)} . \tag{57}$$

Table X shows these numerical values. While variation in the expansion index is not so vital, the thermal efficiency is very sensitive to the compression index. This is presumably so due to the fact that the fall in temperature during expansion is more than twice the rise in temperature during compression.

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9. List of Symbols and Units

Symbol	Significance	Units
c	Compressor discharge period	s
C	Cut-off ratio	..
C_p	Specific heat	kcal/kg–° C
C_p	Mean specific heat at constant pressure	kcal/kg–° C
C_v	Mean specific heat at constant volume	kcal/kg–° C
e	Expansion motor inlet period	s
E	Compression ratio	..
E_e	Expansion ratio	..
G	Mass flow of process air	kg/cycle
H	Rate of heat transfer	kcal/s
J	Mechanical equivalent of heat	kg–m/kcal
L_c	Compressor power input	kg–m/s
L_e	Expansion motor power output	kg–m/s
L_i	Energy input	kg–m/s
L_n	Nett output	kg–m/s
L_s	Specific output	HP/kg–s
m	Fraction of process air used for expansion	..
n	Index of compression or expansion	..
N	Speed of unit	RPM
p_e	Pressure in expansion cylinder when expansion begins	kg/cm², abs
p_1	Compressor suction pressure	kg/cm², abs
p_2	Compressor discharge pressure	kg/cm², abs
p_3	Expansion motor inlet pressure	kg/cm², abs
p_4	Expansion motor discharge pressure	kg/cm², abs
Q_i	Heat input into unit	kcal/s
R	Universal gas constant	kg–m/kg–°C
T_e	Temperature in expansion cylinder when expansion begins	° K
T_1	Compressor suction temperature	° K
T_2	Heat exchanger inlet temperature	° K
T_3	Heat exchanger outlet temperature	K
$T_{3\ min.}$	Value of T_3 when engine can just overcome its losses	K
T_4	Temperature at end of expansion	K
v	Constant volume heating period	s

Symbol	Significance	Units
V_a	Volume in expansion cylinder when expansion begins	m^3
V_{1c}	Volume of compression cylinder	m^3
V_{2c}	Clearance volume in compressor	m^3
V_{1e}	Volume of expansion cylinder	m^3
V_{HE}	Volume of compressed air space in heat exchanger	m^3
W_{HE}	Weight of compressed air filling heat exchanger tubes	kg
α	Ratio of v to $(e+c)$	..
γ	Adiabatic constant	..
Δh	Parasite losses	kcal/s
Δp	Pressure drop in heat exchanger	kg/cm^2
Δp_c	Rate of pressure rise in heat exchanger due to compressor discharge	kg/cm^2-s
Δp_e	Rate of pressure drop in heat exchanger due to expansion motor inlet opening	kg/cm^2-s
Δp_v	Rate of pressure rise in heat exchanger due to constant volume heating	kg/cm^2-s
ΔT	Temperature rise in heat exchanger	°C
η	Overall thermal efficiency	per cent.
η_a	Ideal efficiency	per cent.
η_B	Combustion efficiency	per cent.
η_c	Compression efficiency	per cent.
η_e	Expansion efficiency	per cent.
η_L	Factor used to express $J\Delta h$ as a fraction of L_n	per cent.
η_M	Mechanical efficiency	per cent.
η_R	Heat exchanger efficiency	per cent.
π_{2a}	Ratio of p_a to p_2	..
π_{23}	Ratio of p_3 to p_2	..
π_{41}	Ratio of p_1 to p_4	..

10. LIST OF TABLES

11. LIST OF FIGURES

TABLE I

Ideal cycle efficiency

E	η_a	E	η_a
2	0·2422	9	0·5848
3	0·3556	10	0·6018
4	0·4256	11	0·6168
5	0·4747	12	0·6298
6	0·5116	13	0·6416
7	0·5408	14	0·6520
8	0·5647		

TABLE II

Overall thermal efficiency

E	$T_3 = 600$	700	800	900	1000	1100	1200° K
2	4·4592	6·5203	7·5727	8·0248	8·5646	8·8345	8·9930
3	2·1540	7·3958	9·8098	11·1490	11·9797	12·5239	12·8743
4	..	5·9628	10·0916	12·2941	13·6118	14·4380	14·9815
5	..	2·9228	9·2734	12·4331	14·2746	15·4510	16·2705
6	..	..	7·7653	12·0634	14·4605	16·0134	17·0350
7	..	..	5·5811	11·1810	14·2535	16·1506	17·3749
8	..	..	2·8899	9·9371	13·8128	16·0877	17·6142
9	..	..	..	8·5999	13·1315	15·8632	17·6223
10	..	..	..	6·9155	12·3106	15·4352	17·5050
11	..	..	..	5·0469	11·3782	14·9872	17·3186
12	..	..	..	2·8243	10·2808	14·4311	17·0080
13	..	..	..	0·4277	9·0759	13·7739	16·6597
14	..	..	..	..	7·7889	13·0548	16·2655

TABLE III

Specific output

E	$T_s = 600$	700	800	900	1000	1100	1200° K
2	18·2	38·4	58·7	79·0	99·2	119·5	139·8
3	7·1	37·5	67·9	98·3	128·7	159·2	189·6
4	..	26·2	63·1	100·0	136·9	173·8	210·7
5	..	11·4	52·9	94·4	136·0	177·2	219·1
6	..	..	40·4	85·5	130·6	175·7	220·7
7	..	..	26·9	74·9	122·8	170·7	218·7
8	..	..	13·0	63·3	113·6	163·9	214·2
9	..	..	..	51·3	103·6	156·0	208·2
10	..	..	..	39·0	93·2	147·2	201·3
11	..	..	..	27·0	82·6	138·2	193·8
12	..	..	..	14·4	71·5	128·4	185·4
13	..	..	..	2·5	60·7	118·9	177·0
14	..	..	..	..	50·0	109·3	168·5

TABLE IV

Overall thermal efficiency for different values of expansion ratio

E_c	E_e	$T_3 = 600$	700	800	900	1000	1100	1200° K
2	2	4·2357	6·1991	7·2049	7·8033	8·1825	8·4392	8·6293
3	2				0·8269	2·4816	3·6321	4·4690
	3	[illegible]	[illegible]	[illegible]	[illegible]	11·4748	[illegible]	[illegible]
4	2							[illegible]
	3			[illegible]	[illegible]	[illegible]	7·9957	8·9505
	4		[illegible]	[illegible]	[illegible]	12·7261	13·5924	14·2021
5	3	..				2·5133	4·6199	6·1032
	4	..	..	[illegible]	6·5719	8·9414	10·4971	11·5754
	5	..	2·7759	8·7828	11·7952	13·5809	14·7390	15·5250
6	3	..	..	..	..	..	1·2713	3·3197
	4	..	..	..	1·5504	5·1070	7·4094	8·9982
	5	..	..	2·1534	7·1320	9·9948	11·8397	13·0968
	6	..	..	7·3280	11·4094	13·7405	15·2348	16·2377
7	3	..	..	..	..	..	..	0·6596
	4	..	..	..	..	1·3315	4·4177	6·5374
	5	..	..	..	2·4063	6·4632	9·0287	10·7797
	6	..	..	0·7615	6·9583	10·3959	12·5591	14·0308
	7	..	..	5·2980	10·6221	13·5613	15·4021	16·6476
8	4	..	..	..	..	..	1·5082	4·1669
	5	..	..	..	..	2·9678	6·2962	8·5479
	6	..	..	..	2·4292	7·0907	9·9656	11·9053
	7	..	..	..	6·3210	10·4092	12·9190	14·6076
	8	..	..	2·7823	9·5495	13·1620	15·3689	16·8493
9	4	..	..	..	..	..	..	1·8478
	5	..	..	..	..	..	3·5947	6·3572
	6	..	..	..	..	3·7587	7·3975	9·8130
	7	..	..	..	1·8734	7·2277	10·4583	12·5945
	8	..	..	..	5·2970	10·1053	12·9974	14·9019
	9	..	..	0·0013	8·4079	12·7202	15·3046	16·9985
10	5	..	..	..	..	..	0·9146	4·2194
	6	..	..	..	..	0·3947	4·8545	7·7776
	7	..	..	..	..	4·0195	8·0257	10·6414

TABLE IV—*Contd.*

E	E_e	$T_3 = 600$	700	800	900	1000	1100	1200° K
10	8	..	..	..	0·8777	7·0263	10·6563	13·0171
	9	..	..	..	4·1767	9·7586	13·0467	15·1759
	10	..	..	..	6·6303	11·7906	14·8244	16·7814
11	5	..	..	..	..	..	..	2·1000
	6	..	..	..	..	..	2·2994	5·7565
	7	..	..	..	..	0·7448	5·5751	8·6996
	8	..	..	..	..	3·8930	8·2925	11·1410
	9	..	..	..	..	6·7242	10·7617	13·3595
	10	..	..	..	2·3230	8·8443	12·5981	15·0094
	11	..	..	..	4·7950	10·8620	14·3476	16·5813
12	6	..	..	..	..	..	..	3·7418
	7	..	..	..	..	..	3·1042	6·7704
	8	..	..	..	..	0·6399	5·9155	9·2828
	9	..	..	..	..	3·6149	8·4701	11·5657
	10	..	..	..	..	5·8275	10·3700	13·2635
	11	..	..	..	0·3702	7·9355	12·1800	14·8811
	12	..	..	..	2·7339	9·8341	13·8103	16·3386
13	6	..	..	..	..	..	..	1·7616
	7	..	..	..	..	..	0·6544	4·8657
	8	..	..	..	..	..	3·5548	7·4416
	9	..	..	..	..	0·4691	6·1904	9·7819
	10	..	..	..	..	2·7720	8·1505	11·5224
	11	..	..	..	..	4·9660	10·0179	13·1805
	12	..	..	..	..	6·9421	11·6999	14·6740
	13	..	..	..	0·4474	8·6693	13·1700	15·9794
14	7	..	..	..	..	..	..	2·9884
	8	..	..	..	..	..	1·1929	5·6295
	9	..	..	..	..	..	3·9117	8·0294
	10	..	..	..	..	..	5·9337	9·8143
	11	..	..	..	..	1·9432	7·8600	11·5148
	12	..	..	..	..	4·0004	9·5951	13·0464
	13	..	..	..	..	5·7988	11·1116	14·3851
	14	..	..	..	..	7·4674	12·5187	15·6272

TABLE V

Cut-off ratio for different ratios of compression to expansion cylinder volumes

C	E	$\Delta T = 300$	400	500	600° C
0·50	2	1·8124	2·0828	2·3542	2·6246
	3	1·1460	1·3056	1·4658	1·6010
	4	0·8312	0·9412	1·0512	1·1612
	5	0·6466	0·7286	0·8110	0·8932
	6	0·5276	0·5124	0·6574	0·7222
	7	0·4450	0·4582	0·5514	0·6044
	8	0·3840	0·4286	0·4732	0·5178
	9	0·3372	0·3754	0·4138	0·4520
	10	0·3002	0·3336	0·3672	0·4006
	11	0·2704	0·3000	0·3294	0·3590
	12	0·2458	0·2722	0·2986	0·3248
	13	0·2252	0·2488	0·2728	0·2964
	14	0·2076	0·2292	0·2508	0·2724
0·40	2	2·2655	2·3535	2·9427	3·2807
	3	1·4325	1·6320	1·8322	2·0012
	4	1·0390	1·1765	1·3140	1·4515
	5	0·8082	0·9107	1·0137	1·1165
	6	0·6595	0·7405	0·8217	0·9027
	7	0·5562	0·6227	0·6892	0·7555
	8	0·4800	0·5357	0·5915	0·6472
	9	0·4215	0·4692	0·5172	0·5650
	10	0·3752	0·4170	0·4590	0·5007
	11	0·3380	0·3750	0·4117	0·4487
	12	0·3072	0·3402	0·3732	0·4060
	13	0·2815	0·3110	0·3410	0·3705
	14	0·2595	0·2865	0·3135	0·3405
0·30	2	3·0206	3·4713	3·9236	4·3743
	3	1·9100	2·1760	2·4430	2·6683
	4	1·3853	1·5686	1·7520	1·9353
	5	1·0776	1·2143	1·3516	1·4887
	6	0·8793	0·9873	1·0957	1·2037
0·30	7	0·7417	0·8303	0·9190	1·0073
	8	0·6400	0·7143	0·7887	0·8630
	9	0·5620	0·6257	0·6897	0·7533
	10	0·5003	0·5560	0·6120	0·6677
	11	0·4506	0·5000	0·5490	0·5983
	12	0·4097	0·4533	0·4977	0·5413
	13	0·3753	0·4147	0·4880	0·4940
	14	0·3460	0·3820	0·4180	0·4540

TABLE V—*Contd.*

C	E	$\Delta T = 300$	400	500	600° C
0·20	2	4·5310	5·2070	5·8855	6·5615
	3	2·8650	3·2640	3·6645	4·0025
	4	2·0780	2·3530	2·6280	2·9030
	5	1·6165	1·8215	2·0275	2·2330
	6	1·3190	1·4810	1·6435	1·8055
	7	1·1125	1·2455	1·3785	1·5110
	8	0·9600	1·0715	1·1830	1·2945
	9	0·8430	0·9385	1·0345	1·1300
	10	0·7505	0·8340	0·9180	1·0015
	11	0·6145	0·6805	0·7465	0·8120
	12	0·6145	0·6805	0·7465	0·8120
	13	0·5630	0·6220	0·6820	0·7410
	14	0·5190	0·5730	0·6270	0·6810
0·10	2	9·0620	10·4140	11·7710	13·1230
	3	5·7300	6·5280	7·3290	8·0050
	4	4·1560	4·7060	5·2560	5·8060
	5	3·2330	3·6430	4·0550	4·4660
	6	2·6380	2·9620	3·2870	3·6110
	7	2·2250	2·4910	2·7570	3·0220
	8	1·9200	2·1430	2·3660	2·5890
	9	1·6860	1·8770	2·0690	2·2600
	10	1·5010	1·6680	1·8360	2·0030
0·10	11	1·3520	1·5000	1·6470	1·7950
	12	1·2290	1·3610	1·4930	1·6240
	13	1·1260	1·2440	1·3640	1·4820
	14	1·0380	1·1460	1·2540	1·3620
0·05	2	18·1240	20·8280	23·5420	26·2460
	3	11·4600	13·0560	14·6580	16·0100
	4	8·3120	9·4120	10·5120	11·6120
	5	6·4660	7·2860	8·1100	8·9320
	6	5·2760	5·1240	6·5740	7·2220
	7	4·4500	4·5820	5·5140	6·0440
	8	3·8400	4·2860	4·7320	5·1780
	9	3·3720	3·7540	4·1380	4·5200
	10	3·0020	3·3360	3·6720	4·0060
	11	2·7040	3·0000	3·2940	3·5900
	12	2·4580	2·7220	2·9860	3·2480
	13	2·2520	2·4880	2·7280	2·9640
	14	2·0760	2·2920	2·5080	2·7240

TABLE VI

Cut-off ratio

E	$\pi_{41} = 0\cdot5$	0·6	0·7	0·8	0·9	1·0
2	0·869	0·755	0·668	0·604	0·550	0·509
3	0·579	0·503	0·446	0·403	0·367	0·339
4	0·435	0·378	0·334	0·302	0·275	0·255
5	0·348	0·302	0·267	0·242	0·220	0·204
6	0·289	0·252	0·223	0·201	0·184	0·169
7	0·248	0·216	0·191	0·173	0·157	0·146
8	0·217	0·189	0·167	0·151	0·138	0·127
9	0·193	0·168	0·149	0·134	0·122	0·113
10	0·174	0·151	0·134	0·121	0·110	0·102
11	0·158	0·136	0·122	0·110	0·100	0·093
12	0·144	0·126	0·111	0·100	0·092	0·084
13	0·134	0·117	0·103	0·093	0·085	0·078
14	0·124	0·108	0·095	0·086	0·078	0·073

TABLE VII

Amount of air necessary for expansion

ΔT, °C	E	C = 0·1	0·2	0·3	0·4	0·5	0·6
300	2	0·208	0·271	0·342	0·431	0·551	0·730
	3	0·327	0·426	0·538	0·673	0·869	1·149
	4	0·411	0·535	0·676	0·853	1·091	1·440
	5	0·475	0·619	0·782	0·980	1·159	1·670
	6	0·526	0·649	0·866	1·092	1·400	1·845
	7	0·570	0·742	0·936	1·181	1·521	1·999
	8	0·604	0·786	0·994	1·251	1·603	2·121
	9	0·638	0·833	1·050	1·324	1·700	2·245
	10	0·665	0·868	1·095	1·382	1·770	2·340
	11	0·693	0·904	1·138	1·439	1·840	2·435
	12	0·715	0·932	1·175	1·481	1·899	2·510
	13	0·736	0·958	1·211	1·528	1·955	2·590
	14	0·755	0·984	1·240	1·565	2·003	2·655

TABLE VII—*Contd.*

ΔT, °C	E	C = 0·1	0·2	0·3	0·4	0·5	0·6
400	2	0·181	0·236	0·298	0·376	0·480	0·636
	3	0·287	0·374	0·472	0·596	0·761	1·008
	4	0·363	0·472	0·597	0·754	0·965	1·272
	5	0·422	0·550	0·684	0·869	1·028	1·481
	6	0·469	0·581	0·771	0·974	1·246	1·641
	7	0·509	0·663	0·837	1·055	1·359	1·789
	8	0·541	0·705	0·890	1·121	1·438	1·899
	9	0·573	0·747	0·944	1·190	1·528	2·015
	10	0·599	0·781	0·986	1·243	1·591	2·105
	11	0·625	0·815	1·025	1·299	1·660	2·195
	12	0·646	0·841	1·061	1·339	1·713	2·265
	13	0·665	0·867	1·095	1·380	1·770	2·340
	14	0·684	0·890	1·125	1·419	1·819	2·403
500	2	0·160	0·208	0·264	0·332	0·425	0·563
	3	0·256	0·334	0·421	0·531	0·680	0·899
	4	0·325	0·423	0·535	0·674	0·863	1·139
	5	0·379	0·494	0·623	0·780	0·924	1·329
	6	0·423	0·527	0·696	0·878	1·122	1·481
	7	0·459	0·599	0·756	0·955	1·229	1·614
	8	0·490	0·637	0·806	1·015	1·300	1·720
	9	0·520	0·678	0·855	1·080	1·382	1·830
	10	0·544	0·710	0·895	1·130	1·448	1·914
	11	0·569	0·742	0·934	1·180	1·510	1·996
	12	0·588	0·767	0·969	1·220	1·562	2·065
	13	0·607	0·791	1·000	1·260	1·615	2·136
	14	0·625	0·815	1·028	1·295	1·661	2·195
600	2	0·144	0·187	0·236	0·298	0·382	0·505
	3	0·234	0·305	0·386	0·479	0·622	0·823
	4	0·294	0·383	0·484	0·610	0·782	1·031
	5	0·344	0·448	0·566	0·710	0·839	1·209
	6	0·385	0·481	0·634	0·799	1·021	1·349
	7	0·419	0·547	0·690	0·870	1·120	1·471
	8	0·448	0·583	0·737	0·930	1·190	1·571
	9	0·476	0·621	0·784	1·013	1·268	1·672
	10	0·498	0·651	0·821	1·038	1·326	1·754
	11	0·521	0·681	0·856	1·082	1·388	1·833
	12	0·541	0·705	0·890	1·121	1·438	1·899
	13	0·509	0·727	0·920	1·160	1·485	1·966
	14	0·576	0·750	0·948	1·191	1·529	2·022

TABLE VIII

Cut-off ratio for necessary expansion

E	C = 0·1	0·2	0·3	0·4	0·5
2	1·8798	1·2245	1·0365	0·9761	0·9989
3	1·8737	1·2206	1·0331	0·9729	0·9957
4	1·7194	1·1201	0·9481	0·8928	0·9137
5	1·5351	1·0000	0·8464	0·7971	0·8158
6	1·3888	0·9047	0·7657	0·7211	0·7380
7	1·2665	0·8251	0·6983	0·6576	0·6731
8	1·1623	0·7572	0·6409	0·6035	0·6177
9	1·0741	0·6997	0·5923	0·5578	0·5708
10	1·0000	0·6514	0·5514	0·5193	0·5314
11	0·9339	0·6084	0·5149	0·4849	0·4963
12	0·8778	0·5718	0·4840	0·4558	0·4665
13	0·8277	0·5392	0·4554	0·4298	0·4398
14	0·7836	0·5104	0·4320	0·4069	0·4164

TABLE IX

Cut-off ratio for useful expansion

C	ΔT = 300	400	500	600° C.
0·1	6·870	7·932	8·983	10·043
0·2	4·374	4·545	5·166	5·725
0·3	2·904	3·303	3·700	4·072
0·4	2·447	2·710	2·903	3·145
0·5	..	2·223	2·488	2·690
0·6	..	..	..	2·226

TABLE X

Overall thermal efficiency for different values of compression and expansion indices

T_3, °K	*n*	E	*n* = 1·20	1·25	1·30	1·35	1·40
600	1·20	2	5·3417	4·2473	3·2728	1·7351	0·2328
		3	6·1516	3·7797	0·6958	..	..
		4	5·2674	0·7215	..	..	..
		5	3·4122	..	..	..	..
		6	0·8806	..	..	..	..
	1·25	2	6·0214	4·9611	4·0231	2·5316	1·0793
		3	6·7592	4·4482	1·4435	..	..
		4	5·9221	1·4817	..	..	..
		5	3·6683	..	..	..	..
	1·30	2	6·4164	5·3760	4·4592	2·9947	1·5714
		3	7·3367	5·0834	2·1540	..	..
		4	6·2765	1·8932	..	..	..
		5	3·7597	..	..	..	..
	1·35	2	7·1430	6·1389	5·2613	3·8462	2·4763
		3	7·9292	5·7353	2·8832	..	..
		4	6·6268	2·3000	..	..	..
		5	3·8329	..	..	..	..
	1·40	2	7·6217	6·6417	5·7898	4·4073	3·0726
		3	8·4387	6·2958	3·5101	..	..
		4	6·8854	2·6002	..	..	..
		5	3·8420	..	..	..	..
800	1·20	2	7·7171	7·1858	6·7433	6·0845	5·4693
		3	10·9440	9·9985	8·9005	7·5637	6·0030
		4	12·5887	11·0368	9·3152	7·0614	4·1407
		5	13·4715	11·5135	9·0059	5·5159	0·4913
		6	13·9147	11·4039	8·6355	2·7403	..
		7	14·0168	10·9133	6·3566	..	..
		8	14·1384	10·1630	4·3263	..	..
		9	14·0379	9·5751	2·2618	..	..
		10	13·3590	8·0623	..	..	..

TABLE X—*Contd.*

T_3, ° K	n	E	n = 1·20	1·25	1·30	1·35	1·40
800	1·20	11	12·8947	6·5540	..	..	..
		12	12·3848	5·2572	..	..	..
		13	11·8064	3·5248	..	..	..
		14	11·1232	1·7425	..	..	..
	1·25	2	8·2156	7·6973	7·2678	6·6269	6·0279
		3	11·3652	10·4402	9·3668	8·0596	6·5349
		4	13·0207	11·5003	9·8189	7·6165	4·7612
		5	13·6327	11·6900	9·2030	5·7412	0·9291
		6	13·8488	11·3303	7·8422	2·6393	..
		7	13·7691	10·6298	6·0213	..	..
		8	13·4616	9·6075	3·6469	..	..
		9	13·0202	8·3677	0·7341	..	..
		10	12·4546	6·9664	..	..	..
		11	11·8902	5·3098	..	..	..
		12	11·0665	3·5925	..	..	..
		13	10·3270	1·6177	..	..	..
		14	9·4504	..	..	..	..
	1·30	2	8·5054	7·9947	7·5727	6·9422	6·3526
		3	11·7654	10·8601	9·8098	8·5308	7·0404
		4	13·2547	11·7512	10·0916	7·9171	5·0971
		5	13·6901	11·7530	9·2734	5·8217	0·8543
		6	13·7890	11·2634	7·7653	2·5474	..
		7	13·4440	10·2577	5·5811	..	..
		8	12·9304	8·9887	2·8899	..	..
		9	12·3099	7·5249	..	..	..
		10	11·5332	5·8499	..	..	..
		11	10·6868	3·8194	..	..	..
		12	9·7698	1·9550	..	..	..
		13	8·8145	..	..	..	..
		14	7·7587	..	..	..	..

TABLE X—*Contd.*

T_3, ° K	n	E	n = 1·20	1·25	1·30	1·35	1·30
800	1·35	2	9·0384	8·5415	8·1335	7·5220	6·9498
		3	12·1762	11·2909	10·2645	9·0144	7·5591
		4	13·4951	11·9991	10·3610	8·2141	5·4137
		5	13·7361	11·8035	9·3298	5·8861	0·9307
		6	13·6304	11·0862	7·5613	2·3040	..
		7	13·0880	9·8501	5·0990	..	..
		8	12·3800	8·3475	2·1056	..	..
		9	11·5896	6·6704	..	..	..
		10	10·6356	4·7622	..	..	..
		11	9·5881	2·4586	..	..	..
		12	8·4443	0·2812	..	..	..
		13	7·2248	..	..	..	..
		14	6·1011	..	..	..	..
	1·40	2	9·3896	8·9019	8·5030	7·9041	7·3434
		3	12·5293	11·6613	10·6555	9·4302	8·0052
		4	13·6565	12·1822	10·5599	8·4334	5·6741
		5	13·3828	11·8098	9·3368	5·8942	0·9403
		6	13·4090	10·8387	7·2766	1·9642	..
		7	12·7226	9·4319	4·6044	..	..
		8	11·8168	7·6913	1·3030	..	..
		9	10·8231	5·7609	..	..	..
		10	9·6904	3·6168	..	..	..
		11	8·4754	1·0805	..	..	..
		12	7·1584	..	..	..	..
		13	5·8595	..	..	..	..
		14	4·4623	..	..	..	..
1000	1·20	2	8·5065	8·1741	7·8713	7·4656	7·0792
		3	12·4600	11·8886	11·2481	10·4868	9·6704
		4	13·9975	13·9065	13·0093	11·8779	10·5088

TABLE X—*Contd.*

T_3, °K	n	E	n = 1·20	1·25	1·30	1·35	1·40
1000	1·20	5	16·3732	15·3184	14·0747	12·5000	10·5678
		6	17·4571	16·1736	14·5768	12·4800	9·8567
		7	18·2053	16·7204	14·7918	12·2026	8·5459
		8	18·7721	17·0427	14·7736	11·5444	6·7138
		9	19·4471	17·4768	14·7946	10·8807	4·6403
		10	19·5371	17·2647	14·0934	9·2879	0·9253
		11	19·6457	17·1186	13·5684	7·8002	..
		12	19·8401	17·1079	12·9162	6·0626	..
		13	19·9287	16·8214	12·1409	3·9301	..
		14	19·9313	16·5823	11·2905	1·4133	..
	1·25	2	8·9308	8·6065	8·3097	7·9142	7·5359
		3	12·8110	12·2508	11·6232	10·8764	10·0777
		4	15·1157	14·2771	13·4002	12·2924	10·9500
		5	16·5031	15·4562	14·2220	12·6590	10·7425
		6	17·4047	16·1175	14·5159	12·4129	9·7808
		7	18·0109	16·5092	14·5590	11·9402	8·2395
		8	18·4021	16·6374	14·3191	11·0214	6·0790
		9	18·6679	16·6144	13·8152	9·7226	3·1773
		10	18·8508	16·4987	13·2103	8·2160	..
		11	18·8941	16·2661	12·5719	6·5560	..
		12	18·8639	15·9922	11·5876	4·3597	..
		13	18·8433	15·5711	..	1·9241	..
		14	18·7175	15·1691	..	..	..
	1·30	2	9·1774	8·8578	8·5646	8·1750	7·8014
		3	13·1445	12·5951	11·9797	11·2467	10·4648
		4	15·3070	14·4777	13·6118	12·5168	11·1889
		5	16·5494	15·5054	14·2746	12·7158	10·8049
		6	17·3571	16·0665	14·4605	12·3519	9·7118
		7	17·7558	16·2320	14·2535	11·5958	7·8373

TABLE X—*Contd.*

T_2, °K	n	E	n = 1·20	1·25	1·30	1·35	1·40
1000	1·30	8	17·9909	16·1859	13·8128	10·4347	5·3718
		9	18·1239	16·0124	13·1315	8·9142	2·1560
		10	18·1516	15·6894	12·3106	7·1238	..
		11	17·9936	15·2449	11·3782	5·0656	..
		12	17·9037	14·8948	10·2808	2·6847	..
		13	17·7336	14·2929	9·0759	..	..
		14	17·4900	13·7400	7·7889	..	..
	1·35	2	9·6309	9·3200	9·0333	8·6546	8·2897
		3	13·4808	12·9483	12·3455	11·6266	10·8620
		4	15·4960	14·6759	13·8210	12·7385	11·4249
		5	16·5866	15·5447	14·3167	12·7613	10·8548
		6	17·2310	15·9314	14·3137	12·1902	9·5289
		7	17·4763	15·9284	13·9188	11·2186	7·3969
		8	17·5644	15·7181	13·2882	9·8350	4·6391
		9	17·5724	15·4020	12·4383	8·0945	1·1205
		10	17·4605	14·9579	11·4341	6·0599	..
		11	17·1715	14·3135	10·2883	3·7048	..
		12	16·9221	13·7730	8·9450	0·9725	..
		13	16·5672	12·9494	7·4473	..	..
		14	16·2871	12·3396	6·0636	..	..
	1·40	2	9·9298	9·6246	9·3422	8·9706	8·6114
		3	13·7811	13·2521	12·6601	11·9533	11·2036
		4	15·6355	14·8222	13·9754	12·9022	11·5991
		5	16·5912	15·5497	14·3219	12·7669	10·8610
		6	17·0548	15·7427	14·1087	11·9645	9·2736
		7	17·1896	15·6169	13·5754	10·8315	6·9449
		8	17·1279	15·2393	12·7514	9·2173	3·8894
		9	16·9854	14·7524	11·7005	7·2222	0·0185
		10	16·7532	14·5721	10·5111	4·9396	..

TABLE X—*Contd.*

T_3, ° K	n	E	n = 1·20	1·25	1·30	1·35	1·40
1000	1·40	11	16·3389	13·3682	9·1846	2·3268	..
		12	15·9698	12·6847	7·6491	..	..
		13	15·5655	11·7956	6·0486	..	..
		14	15·0979	10·9551	4·3580	..	..
1200	1·20	2	8·8336	8·5606	8·3728	8·0926	7·8065
		3	13·0967	12·7033	12·2337	11·7289	11·2059
		4	15·7081	15·1260	14·4646	13·7655	12·9507
		5	17·6157	16·8713	16·1009	15·1042	13·9861
		6	18·9423	18·1217	17·1322	15·8748	14·4312
		7	19·9969	18·9785	17·8167	16·3950	14·5211
		8	20·7445	19·6660	18·3929	16·6648	14·4350
		9	21·6733	20·4268	18·9528	16·9481	14·2668
		10	21·9631	20·6393	18·9098	16·5654	13·3031
		11	22·3789	20·8093	19·0201	16·3205	12·4743
		12	22·7375	21·1724	19·0173	16·0387	11·4474
		13	23·0493	21·2643	18·9621	15·6296	10·1456
		14	23·2863	21·4455	18·8530	15·0747	8·6132
	1·25	2	9·2161	8·7947	8·7650	8·4921	8·2109
		3	13·4097	13·0240	12·5622	12·0670	11·5556
		4	16·0208	15·4501	14·8000	14·1159	13·3181
		5	17·7300	16·9902	16·2259	15·2359	14·1265
		6	18·8966	18·0737	17·0813	15·8204	14·3724
		7	19·9865	18·9674	17·8047	16·3821	14·5068
		8	20·4262	19·3262	18·0246	16·2607	13·9791
		9	21·0043	19·7098	18·1692	16·0758	13·2652
		10	21·3790	20·0073	18·2139	15·7802	12·3796
		11	22·1094	20·5167	18·6939	15·9478	11·4099
		12	21·9194	20·2696	18·0044	14·8628	9·9941
		13	22·1376	20·2609	17·8237	14·2837	8·4427
		14	22·2710	20·3185	17·5665	13·5277	6·5984

TABLE X—*Contd.*

T_2, °K	n	E	n = 1·20	1·25	1·30	1·35	1·40
1200	1·30	2	9·4384	9·1889	8·9930	8·7243	8·4459
		3	13·7071	13·3288	12·8743	12·3883	11·8879
		4	16·1901	15·6255	14·9815	14·3056	13·5170
		5	17·7708	17·0327	16·2705	15·2829	14·1767
		6	18·8551	18·0300	17·0350	15·7709	14·3188
		7	19·6062	18·5656	17·3749	15·9155	13·9927
		8	20·0716	18·9478	17·6142	15·8105	13·4711
		9	20·5373	19·2093	17·6223	15·4670	12·5660
		10	20·7839	19·3634	17·5050	14·9802	11·4398
		11	20·9736	19·2825	17·3186	14·3759	10·1347
		12	21·1039	19·3815	17·0080	13·7062	8·5645
		13	21·2054	19·2350	16·6597	12·9077	6·7017
		14	21·2442	19·1788	16·2655	11·9633	4·5609
	1·35	2	9·8472	9·6030	9·4122	9·1514	8·8782
		3	14·0124	13·6415	13·1946	12·7180	12·2289
		4	16·3575	15·7989	15·1610	14·4931	13·7136
		5	17·8034	17·0667	16·3062	15·3205	14·2168
		6	18·7451	17·9143	16·9122	15·6399	14·1770
		7	19·3633	18·3089	17·1003	15·6174	13·6643
		8	19·7042	18·5556	17·1891	15·3440	12·9448
		9	20·0639	18·7018	17·0677	14·8496	11·8571
		10	20·2041	18·7361	16·8143	14·2008	10·5242
		11	20·2743	18·5228	16·4718	13·4083	8·9705
		12	20·2758	18·4738	15·9896	12·5239	7·1033
		13	20·2256	18·1567	15·4363	11·4613	4·8718
		14	20·2380	18·0620	14·9907	10·4303	2·5643
	1·40	2	10·1166	9·8758	9·6885	9·4328	9·1630
		3	14·2748	13·9105	13·4700	13·0015	12·5221
		4	16·4810	15·9269	15·2934	14·6315	13·8587
		5	17·8075	17·0709	16·3107	15·3252	14·2218

TABLE X—*Contd.*

T_3, °K	n	E	n=1·20	1·25	1·30	1·35	1·40
1200	1·40	6	18·5914	17·7526	16·7408	15·4570	13·9790
		7	19·1141	18·0455	16·8185	15·3116	13·3273
		8	19·3283	18·1543	16·7540	14·8667	12·4063
		9	19·5599	18·1617	16·4775	14·1926	11·1026
		10	19·5662	18·0756	16·0871	13·3802	9·5601
		11	19·5662	17·7533	15·6144	12·4283	7·7915
		12	19·4725	17·5931	15·0015	11·3769	5·6856
		13	19·3842	17·2307	14·3856	10·2192	3·3002
		14	19·2433	16·9579	13·7304	8·9148	0·5905