# INVESTIGATIONS ON ARTIFICIAL DIELECTRICS AT MICROWAVE FREQUENCIES

## PART I

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ABSTRACT

An expression for the phase change suffered by a $H_{01}$ wave in passing through a parallel metallic plate medium for different angles of incidence has been derived from the Carlson and Heins (1947) formula for the transmission coefficient. The shape of the phase change *vs.* angle of incidence curve obtained experimentally shows fair agreement with the theoretical curve. The difference between theoretical and experimental values of the magnitude of phase shift is considered to be due to diffraction effects and the presence of higher order modes inside the metallic plate medium. An expression for the phase change on transmission with different spacings of parallel plates has also been derived. Experimental results show that for the size of the dielectric employed, the diffraction effects start when the angle of incidence becomes greater than 8°. For normal incidence, the minimum size of the sample of dielectric which is necessary to produce no diffraction effects should be greater than $57 \times 69$ cm.

## INTRODUCTION

When an electromagnetic wave is incident on a dielectric material, the wave suffers a change in its velocity depending on the refractive index of the material. The increasing use of optical technique for microwave work has led to the evolution of artificial dielectrics which can simulate the effects produced by actual dielectrics on microwave propagation. The artificial dielectric used in the present investigations consists of an array of equidistant metallic plates placed in air; the distance between successive plates of the array being more than half a wavelength and less than one wavelength of the incident radiation. The artificial dielectric may also be formed by a number of other arrangements. The reason for selecting a parallel plate dielectric is that a rigorous theoretical solution is available. The phase velocity inside a parallel plate dielectric being greater than the free space velocity, the refractive index is less than unity. An inhomogeneous medium of this nature differs fundamentally from any homogeneous isotropic dielectric due to the supplementary boundary conditions required to be satisfied at the surface of the parallel plate assembly. The basic theoretical work on the propagation of microwaves through a parallel plate structure placed in a homogeneous, isotropic medium has been done by Carlson and Heins (1947), Berz (1951) and Whitehead (1951).

Carlson and Heins have treated the idealised problem of reflection and transmission of plane waves at an infinite plane interface formed by the edges of a periodic structure of a set of parallel equidistant metallic plates having infinite conductivity and zero thickness; the stagger angle of the plates and the angle of incidence being arbitrary. Formulæ for the transmission and reflection coefficients of the waves are established in the restricted case when there is only one reflected and one transmitted wave, the possibility of diffraction being thus excluded. Whitehead (*loc. cit.*) has extended the scope of the theory by removing the restrictions on the number of reflected waves. Lengyel (1951) has treated the theory in a more general way by taking into account the effects of diffraction.

Carlson and Heins (*loc. cit.*) have derived an expression for the phase change on reflection of an electromagnetic wave incident on a parallel plate dielectric. The phase change on transmission of electromagnetic waves through such a dielectric though implicit in Heins' formula for transmission coefficient is not explicitly given. Considerable work is necessary to bridge the gap between the theoretical formula of Carlson and Heins and an explicit form of expression for the phase change which can be readily computed for experimental verification. The object of the present paper is (i) to derive an explicit form of expression for the phase change suffered by a wave on transmission through a parallel plate medium as a function of angles of incidence, (ii) to derive an expression for the phase change as a function of spacing of the parallel plates at normal incidence from Carlson and Heins formula for the transmission coefficient and (iii) to verify the above relations experimentally with the help of a microwave interferometer.

## THEORETICAL

### (i) *Transmission of* $H_{01}$ *wave through a parallel plate medium*

When a plane monochromatic electromagnetic wave is incident normally on an array of equidistant semi-infinite metallic plates having infinite conductivity and zero thickness, a portion of the wave is reflected from the interface separating the two media and the other portion is transmitted through the array. The magnitude of reflection and transmission depends on the ratio of the separation $a$ of successive plates to the wavelength $\lambda$ of the incident radiation. If the following restrictions are imposed, *viz.*, (i) the electric vector of the incident wave is parallel to the edges of the plates, (ii) all the other modes except the $H_{01}$ are evanescent within the plates and (iii) there is only one reflected and one transmitted ray, the transmission coefficient T is given by the following expression (Carlson and Heins, *loc. cit.*).

$$T = |T|\,\iota^{\iota\Phi} = \frac{\pi e^{\iota n(kp-\gamma b)}\,(-)^{n}\,[1 + e^{\iota(kp-\gamma b)}]\,K_{+}(k\cos\theta)}{(\gamma - k\cos\theta)\,a^{2}\gamma\,K_{+}(\gamma)} \qquad (1)$$

where,

$\Phi$ = Phase change on transmission

$b$ = Step of staggering of plates

$\theta$ = Direction of the incident beam with respect to the direction of propagation

$\gamma = \sqrt{k^2 - (\pi/a)^2}, \qquad k = 2\pi/\lambda$

$\rho = b \cos\theta + a \sin\theta$

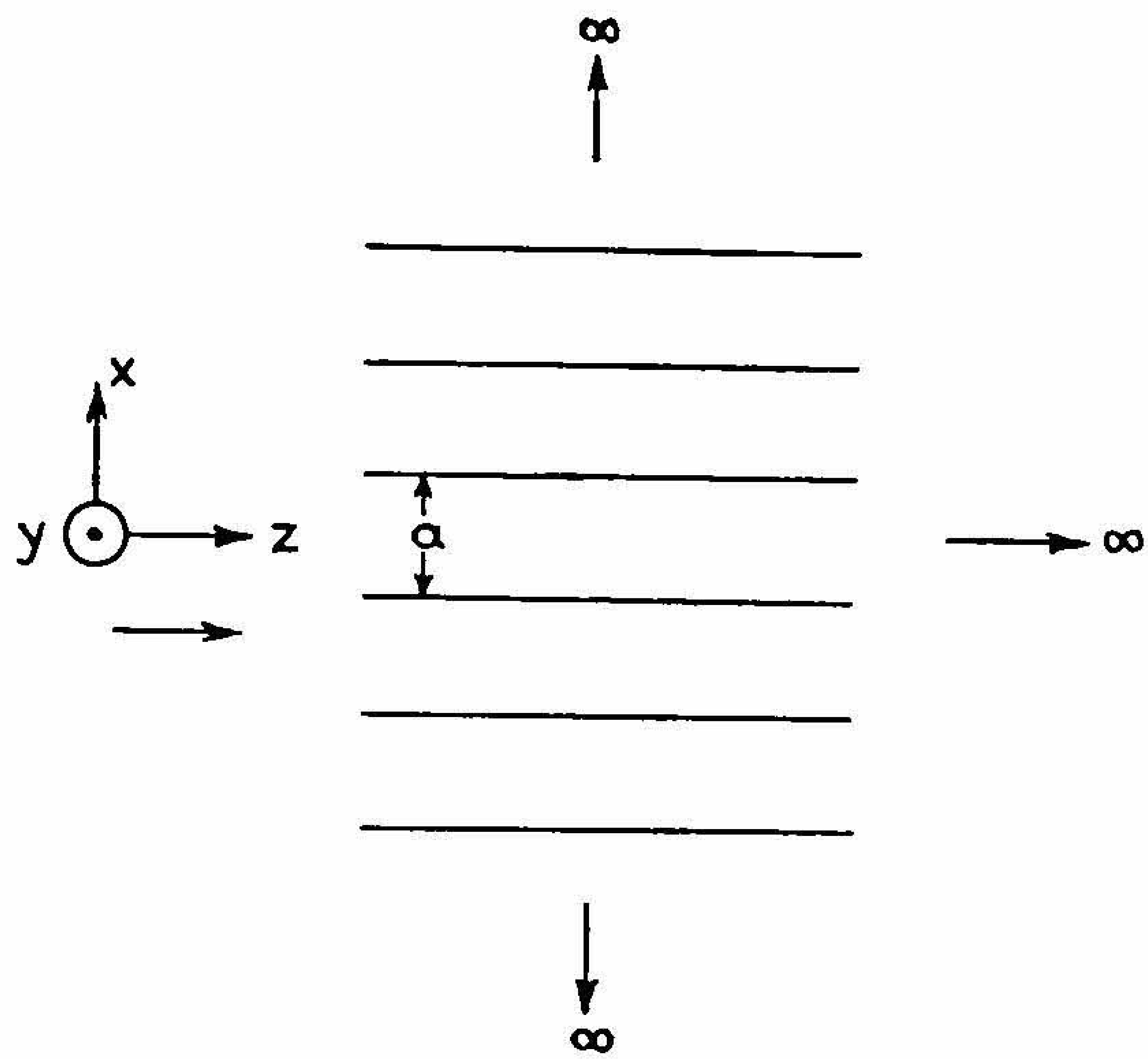



FIG. 1. Parallel plate dielectric.

If the edges of the parallel plates lie in the same plane, $b = 0$ (Fig. 1), the following symbols $\alpha$, $\sigma_2$, etc., in Carlson and Heins relations for $K_+$ $(k \cos\theta)$ assume the following values:

$\sigma_2 = -k \cos\theta, \; \alpha = 90°$

$\rho = a \sin\theta$

$$\Delta_n = \sqrt{1 - \frac{k^2a^2}{4\pi^2n^2}\cos^2\theta - \frac{ka\sin\theta}{\pi n}} \tag{1 a}$$

$$\psi_n = \frac{\omega a}{2\pi n}, \qquad \chi(\omega) = \frac{ia\omega}{\pi} \ln 2$$

Using the above symbols (1 *a*), the expressions for $K_+(k\cos\theta)$ and $K_+(\gamma)$ in (1) are as follows:

$$K_+(k\cos\theta) = \pi k\cos\theta\, e^{\frac{iak\cos\theta}{\pi}\ln 2}\prod_1^\infty\left[\left\{1-\frac{k^2a^2}{4\pi^2n^2}\cos^2\theta - \frac{ka\sin\theta}{\pi n}\right\}^{\frac{1}{2}} - \frac{iak\cos\theta}{2\pi n}\right]$$

$$\times\left[\left\{1-\frac{k^2a^2}{4\pi^2n^2}\cos^2\theta + \frac{ka\sin\theta}{\pi n}\right\}^{\frac{1}{2}} - \frac{iak\cos\theta}{2\pi n}\right]$$

$$\div(\gamma + k\cos\theta)\, e^{\frac{iak\cos\theta}{\pi}}\prod_2^\infty\left[\left\{1-\left(\frac{ak}{\pi n}\right)^2\right\}^{\frac{1}{2}} - \frac{iak\cos\theta}{\pi n}\right] e^{\frac{iak\cos\theta}{\pi n}} \quad (2)$$

$$K_+(\gamma) = \frac{\pi(\gamma + k\cos\theta)\, e^{\frac{ia\gamma\ln 2}{\pi}}\prod_1^\infty\left[\left\{1-\frac{k^2a^2}{4\pi^2n^2}\cos^2\theta - \frac{ka\sin\theta}{\pi n}\right\}^{\frac{1}{2}} - \frac{i\gamma a}{2\pi n}\right]^2 e^{\frac{ka\sin\theta}{\pi n}}}{4\gamma\, e^{ia\gamma/\pi}\prod_2^\infty\left[\left\{1-\frac{a^2k^2}{\pi^2n^2}\right\}^{\frac{1}{2}} - \frac{ia\gamma}{\pi n}\right] e^{ia\gamma/\pi n}} \quad (3)$$

Substituting (2) and (3) in (1), taking logarithms of both sides, using the relation $\log(x+iy) = \log r + i\theta$, where $r = (x^2+y^2)^{\frac{1}{2}}$, $\theta = \arcsin y/r$, separating the real and imaginary parts and rearranging the terms, the following expressions for $\Phi$ and $|T|$ are obtained.

$$\frac{\Phi}{2\cdot 303} = \frac{1}{2\cdot 303}\, kan\sin\theta + n\pi + \arcsin\frac{\sin(ka\sin\theta)}{[2\{1+\cos(ka\sin\theta)\}]^{\frac{1}{2}}}$$

$$+\frac{1}{2\cdot 303}\, ka\cos\theta\ln 2 - \sum_1^\infty \arcsin\frac{ka\cos\theta/2\pi n}{\left[1-\frac{ka\sin\theta}{\pi n}\right]^{\frac{1}{2}}}$$

$$-\sum_1^\infty \arcsin\frac{ka\cos\theta/2\pi n}{\left[1+\frac{ka\sin\theta}{\pi n}\right]^{\frac{1}{2}}} + \frac{1}{2\cdot 303}\,\frac{ka\cos\theta}{\pi}$$

$$+\sum_2^\infty \arcsin\frac{ka\cos\theta/\pi n}{\left[1-\frac{k^2a^2}{\pi^2n^2}\sin^2\theta\right]^{\frac{1}{2}}} - \sum_2^\infty \frac{1}{2\cdot 303}\,\frac{ka\cos\theta}{\pi n}$$

$$-\frac{1}{2\cdot 303}\,\frac{\gamma a}{\pi}\ln 2 + 2\sum \arcsin\frac{\gamma a/2\pi n}{\left[1-\frac{k^2a^2}{4\pi^2n^2}\cos^2\theta - \frac{ka\sin\theta}{\pi n} + \frac{\gamma^2a^2}{4\pi^2n^2}\right]^{\frac{1}{2}}} \quad (4)$$

$$+\frac{1}{2\cdot 303}\,\frac{\gamma a}{\pi} + \sum_2^\infty \frac{1}{2\cdot 303}\,\frac{\gamma a}{\pi n} - \sum_2^\infty \arcsin\frac{\gamma a/\pi n}{\left[1-\frac{k^2a^2}{\pi^2n^2}+\frac{\gamma^2a^2}{\pi^2n^2}\right]^{\frac{1}{2}}}$$

$$|T| = \frac{4\pi k \cos\theta \left[2\{1+\cos(ka\sin\theta)\}\right]^{\frac{1}{2}} \prod_{1}^{\infty}\left[1-\frac{k^2a^2\sin^2\theta}{\pi^2n^2}\right]^{\frac{1}{2}} \prod_{2}^{\infty}\left[1-\frac{k^2a^2}{\pi^2n^2}+\frac{\gamma^2a^2}{\pi^2n^2}\right]^{\frac{1}{2}}}{a^2(\gamma-k\cos\theta)\prod_{2}^{\infty}\left[1-\frac{k^2a^2}{\pi^2n^2}\sin^2\theta\right]^{\frac{1}{2}}\prod_{1}^{\infty}\left(\frac{ka\sin\theta}{\pi n}\right)\left(1-\frac{k^2a^2}{4\pi^2n^2}\cos^2\theta-\frac{ka\sin\theta}{\pi n}+\frac{\gamma^2a^2}{4\pi^2n^2}\right)} \tag{5}$$

(ii) *Phase change at arbitrary incidence*

In order to find a convenient expression for the phase change as a function of angle of incidence $\theta$, some of the arc sines in (4) have been expanded and the terms have been regrouped after neglecting terms containing $\cos\theta$ and $\sin\theta$ of powers higher than seventh. The result is as follows:

$$\begin{aligned}
\frac{\Phi}{2\cdot303} &= \frac{1}{2\cdot303}\, kan\sin\theta + n\pi + \arcsin\frac{\sin(ka\sin\theta)}{[2\{1+\cos(ka\sin\theta)\}]^{\frac{1}{2}}} \\
&+ 2\sum_{1}^{\infty}\arcsin\frac{\gamma a/2\pi n}{\left[1-\frac{k^2a^2}{4\pi^2n^2}\cos^2\theta-\frac{ka\sin\theta}{\pi n}+\frac{\gamma^2a^2}{4\pi^2n^2}\right]^{\frac{1}{2}}} \\
&+ \frac{ka\cos\theta}{\pi}\left[\cdot4064-\cdot3512\,\frac{k^2a^2\sin^2\theta}{\pi^2}+\cdot0134\,\frac{k^4a^4\sin^4\theta}{\pi^4}\right] \\
&+ \left(\frac{ka\cos\theta}{\pi}\right)^3\left[-\cdot0179-\cdot072\,\frac{k^2a^2\sin^2\theta}{\pi^2}+\cdot0026\,\frac{k^4a^4\sin^4\theta}{\pi^4}\right] \\
&+ \left(\frac{ka\cos\theta}{\pi}\right)^5\left[-\cdot0021-\cdot0191\,\frac{k^2a^2\sin^2\theta}{\pi^2}+\cdot0005\,\frac{k^4a^4\sin^4\theta}{\pi^4}\right] \\
&+ \left(\frac{ka\cos\theta}{\pi}\right)^7\left[\cdot0003+\cdot0003\,\frac{k^2a^2\sin^2\theta}{\pi^2}+\cdot0001\,\frac{k^4a^4\sin^4\theta}{\pi^2}\right] \\
&+ 2\cdot5244\,\frac{\mu a}{\lambda}-\left[2\cdot0449\,\frac{2\mu a}{\lambda}+\cdot0446\left(\frac{2\mu a}{\lambda}\right)^3\right. \\
&\left.+\cdot0053\left(\frac{2\mu a}{\lambda}\right)^5+\cdot0009\left(\frac{2\mu a}{\lambda}\right)^7\right].
\end{aligned} \tag{6}$$

The refractive index $\mu$ in eq. (6) of the parallel plate medium is given by the expression

$$\mu = \frac{\lambda}{2a}\left[\frac{4a^2}{\lambda^2}-1\right]^{\frac{1}{2}} \tag{7}$$

and

$$\gamma = \frac{2\pi\mu}{\lambda} \text{ as } k = \frac{2\pi}{\lambda} \tag{8}$$

For $a/\lambda = 0\cdot625$, $\mu = 0\cdot599$ and eq. (6) reduces to

$$\frac{\Phi}{2\cdot303} = 2\cdot0497 + \frac{1}{2\cdot303}\, kan \sin\theta + n\pi$$

$$+ \arc\sin \frac{\sin(ka\sin\theta)}{[2\{1+\cos(ka\sin\theta)\}]^{\frac{1}{2}}}$$

$$+ \cdot1682\cos^2\theta + \cdot6804\sin\theta - \cdot0004\cos^4\theta + \cdot3934\sin^2\theta$$

$$+ \cdot2166\cos^2\theta\sin\theta + \cdot0168\cos^6\theta + \cdot5854\sin^3\theta$$

$$+ \cdot530\sin^2\theta\cos^2\theta + \cdot1632\sin\theta\cos^4\theta$$

$$+ \frac{ka\cos\theta}{\pi}\left[\cdot4064 - \cdot3512\,\frac{k^2a^2\sin^2\theta}{\pi^2} + \cdot0134\,\frac{k^4a^4\sin^4\theta}{\pi^4}\right]$$

$$+ \left(\frac{ka\cos\theta}{\pi}\right)^3\left[-\cdot0179 - \cdot072\,\frac{k^2a^2\sin^2\theta}{\pi^2} + \cdot0026\,\frac{k^4a^4\sin^4\theta}{\pi^4}\right]$$

$$+ \left(\frac{ka\cos\theta}{\pi}\right)^5\left[-\cdot0021 - \cdot0191\,\frac{k^2a^2\sin^2\theta}{\pi^2} + \cdot0005\,\frac{k^4a^4\sin^4\theta}{\pi^4}\right]$$

$$+ \left(\frac{ka\cos\theta}{\pi}\right)^7\left[\cdot0003 + \cdot0003\,\frac{k^2a^2\sin^2\theta}{\pi^2} + \cdot0001\,\frac{k^4a^4\sin^4\theta}{\pi^4}\right] \qquad (9)$$

For $\lambda = 3\cdot2$ cm., eq. (9) further reduces to

$$\frac{\Phi}{2\cdot303} = n\pi + 2\cdot0497 + \arc\sin\frac{\sin(3\cdot925\sin\theta)}{[2\{1+\cos(3\cdot925\sin\theta)\}]^{\frac{1}{2}}}$$

$$+ \sin\theta\,[\cdot5428\,n\pi + \cdot6804 + \cdot2166\cos^2\theta + \cdot1632\cos^4\theta]$$

$$- \sin^2\theta\,[\cdot6860\cos\theta - \cdot530\cos^2\theta + \cdot2197\cos^3\theta + \cdot0909\cos^5\theta$$

$$- \cdot0024\cos^7\theta - \cdot3934] + \cdot5854\sin^3\theta$$

$$+ \sin^4\theta\,[\cdot0409\cos\theta + \cdot0123\cos^3\theta + \cdot0037\cos^5\theta + \cdot0009\cos^7\theta]$$

$$+ [\cdot5080\cos\theta + \cdot1682\cos^2\theta - \cdot0350\cos^3\theta - \cdot0004\cos^4\theta$$

$$- \cdot0064\cos^5\theta + \cdot0168\cos^6\theta + \cdot0014\cos^7\theta]. \qquad (10)$$

(iii) *Phase change on transmission at normal incidence with different spacings of parallel plates*

For normal incidence ($\theta = 0^\circ$) of $H_{01}$ wave, the phase change on transmission is obtained from eq. (6) as follows:

$$\frac{\Phi}{2\cdot 303} = n\pi + \frac{a}{\lambda}\left[\cdot 8128 + 5\cdot 6326\,\mu\right] + \left(\frac{a}{\lambda}\right)^3\left[-\cdot 1432 + 1\cdot 19\mu - 1\cdot 1534\mu^3\right]$$
$$+ \left(\frac{a}{\lambda}\right)^5\left[-\cdot 0672 + \cdot 777\,\mu - 1\cdot 0362\,\mu^3 + \cdot 245\,\mu^5\right]$$
$$+ \left(\frac{a}{\lambda}\right)^7\left[\cdot 0384 + \cdot 63\,\mu - 1\cdot 26\,\mu^3 + 3\cdot 528\,\mu^5 - \cdot 4032\,\mu^7\right]$$
$$+ \left(\frac{a}{\lambda}\right)^9\left[\cdot 728\,\mu^3 - 1\cdot 5288\,\mu^5 + 1\cdot 1866\,\mu^7 - \cdot 3858\,\mu^9\right]$$
$$+ \left(\frac{a}{\lambda}\right)^{11}\left[1\cdot 008\,\mu^5 - 2\cdot 304\,\mu^7 + 1\cdot 584\,\mu^9 - \cdot 288\,\mu^{11}\right]$$
$$+ \left(\frac{a}{\lambda}\right)^{13}\left[1\cdot 288\,\mu^7 - 3\cdot 864\,\mu^9 + 3\cdot 864\,\mu^{11} - 1\cdot 288\,\mu^{13}\right]. \qquad (11)$$

(iv) *Effect of thickness of plates on phase change*

The phase change $\Phi$ has been derived with the assumption that the plates of the array are of zero thickness. This amounts to the existence of only the principal mode $H_{01}$ within the array. Due to the finite thickness of the plates, the array may be considered as a series of parallel plate waveguides. The non-vanishing field components inside such a guide for $H_{01}$ mode are $E_y$, $H_x$ and $H_z$. The propagation inside such a guide is explained by resolving the principal wave into Brillouin's elementary waves. Inside the array the wave proceeds by multiple reflections from the two boundary plates of waveguides, whereas, according to the assumption made in deriving expressions for $\Phi$, there is only one reflected beam and one refracted beam. If the boundary conditions at the two interfaces (*i.e.*), the continuity of $\vec{E}$ and $\vec{H}$ are to be satisfied, the presence of the higher modes inside the parallel plate medium cannot be ignored. It is known that the higher order modes are evanescent and hence do not contribute to the amplitude of the non-evanescent $H_{01}$ mode. The higher order H and E waves, however, being reactive store energy inside the parallel plates. The evanescent H modes, however, store more magnetic energy than electric energy. Hence, when the wave is incident normally with the electric vector having only the $E_y$ component, the edges of the parallel plate dielectric may be considered as inductive, the reactance of which is given by the following relation (Macfarlane, 1946).

$$jX = j\frac{a}{\lambda}\sqrt{\frac{\mu_0}{\epsilon_0}}\left[\ln\csc\left(\frac{\pi w}{2a}\right) + F\left(\frac{a}{\lambda}\right)\right], \qquad (12)$$

where $w$ = width of parallel plates in the direction of propagation of the wave, and

$$F\left(\frac{a}{\lambda}\right) = \sum_{n=1}^{\infty}\frac{1}{n}\left[\left(1 - \frac{a^2}{n^2\lambda^2}\right)^{-\frac{1}{2}} - 1\right]. \qquad (13)$$

The inductive susceptance B contributed by the edges of the parallel plates is therefore given by the following expression

$$B = \frac{1}{Z_0}\,\frac{\lambda}{a}\,\frac{1}{\ln \csc\left(\frac{\pi w}{2a}\right) + \sum_{n=1}^{\infty} \frac{1}{n}\left[\left(1 - \frac{a^2}{n^2\lambda^2}\right)^{-\frac{1}{2}} - 1\right]}. \tag{14}$$

The presence of the inductive susceptance B will modify only the phase of the transmitted wave by an amount say, $\delta$. The impedance of free space for normal incidence is $Z_0 = E_y/H_z$. If the elementary waves inside the guide make an angle $a$ with the boundary plates, the impedance of the guide is $Z_p = E/H_z \cos a$. But it is known that $\cos a = \mu$, the refractive index of the parallel plate medium. Hence the parallel plate dielectric immersed in free space may be represented by an equivalent circuit consisting of the free space impedance $Z_0$, the edge susceptance $-jB$ and the guide impedance $Z_0/\mu$ all in parallel. The phase angle contributed by the edge susceptance is therefore given by

$$\delta = \operatorname{arc\,tan} BZ_0/(1 + \mu) \tag{15}$$

The phase change on transmission is, therefore, due to the refractive index of the material being different from unity and also due to the periodic nature of the medium (*i.e.*), $\Phi + \delta$, where $\delta$ is given by

$$\delta = \operatorname{arc\,tan} \frac{\lambda}{a(1+\mu)}\,\frac{1}{\ln \csc\left(\frac{\pi w}{2a}\right) + \sum_{1}^{\infty} \frac{1}{n}\left[\left(1 - \frac{a^2}{n^2\lambda^2}\right)^{-\frac{1}{2}} - 1\right]} \tag{16}$$

## EXPERIMENTAL

(i) *Description of Dielectric.*—The parallel plate dielectric has been constructed with commercially available tinned copper strips of 1/64 inches thickness and 0·95 cm. in width. There are forty-three strips, the distance between adjacent strips being 1·95 ± 0·008 cm. The size of the parallel plate dielectric is 83 × 97 cm. The strips are fixed in milled grooves in a wooden frame, care being taken to place the strips parallel and equidistant from each other. The edges of the plates are adjusted to lie in the same plane. After constructing the dielectric it was found that the front edges of the plates deviate slightly from being in a line. A quantitative estimation of the deviation is rather difficult. A photograph of the dielectric sample is shown in Fig. 2.

(ii) *Equipments.*—The measurement of phase change on transmission through the sample has been made with the help of a microwave interferometer (Chatterjee, etc., 1954). The design details and the characteristics of the electronic equipments used have been reported elsewhere (Chatterjee, etc., 1954). The stability characteristics of the interferometer with respect to supply voltage variation and time are shown in Figs. 3 and 4 respectively. It will be observed that for one volt variation of the supply voltage when it is at the operating voltage of 220 volts the frequency variation is $\pm$ 0·375 mc/s. Fig. 5 shows the resonance curve of the interferometer with and without the artificial dielectric. It will be noticed that the change in the detector galvanometer reading is about 4·5 divisions for a change of 1 mm. in the position of the receiving horn in both the cases. This signifies that the loss of the

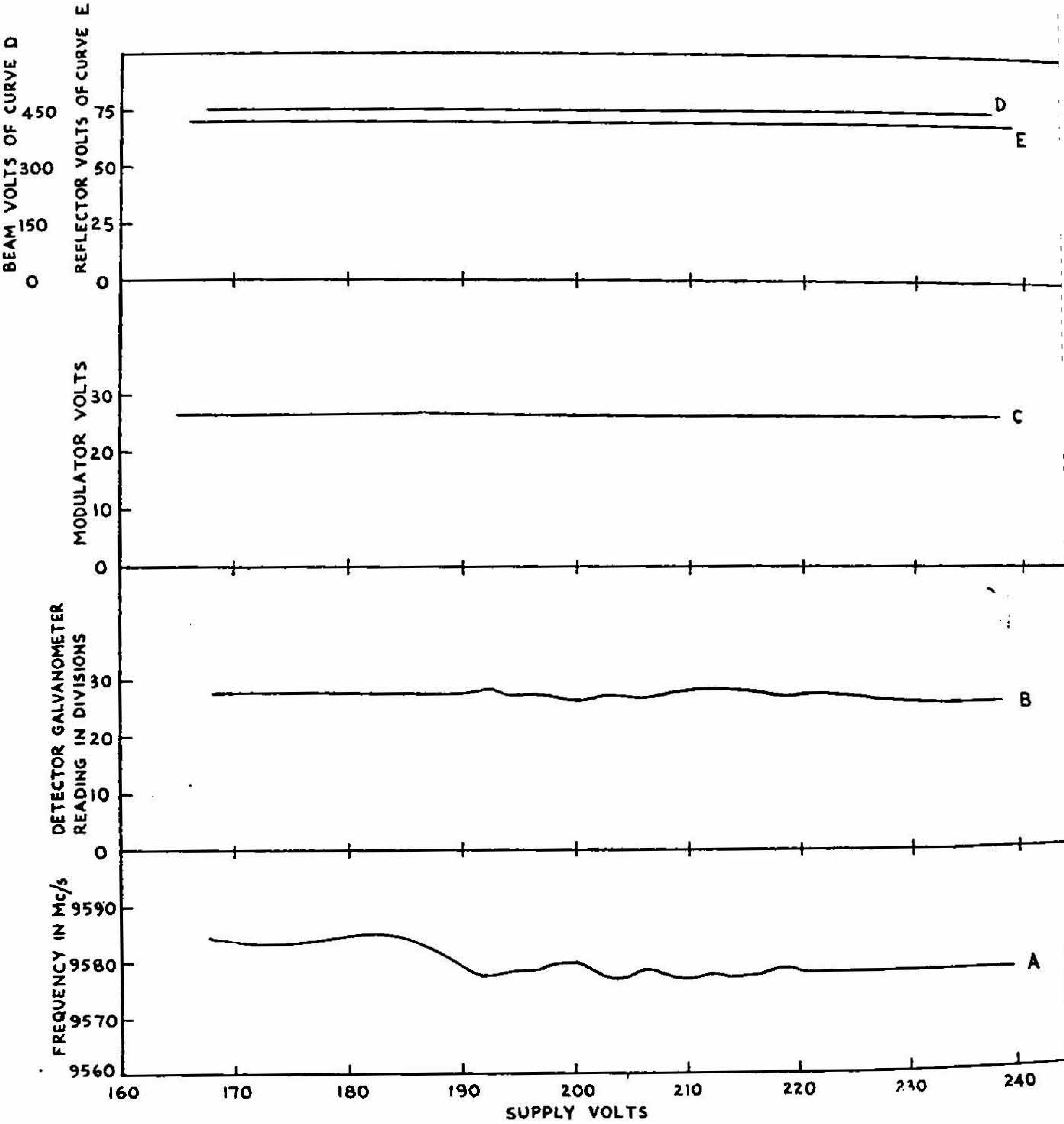



FIG. 3. Stability characteristics of the microwave interferometer with respect to supply voltage variation.

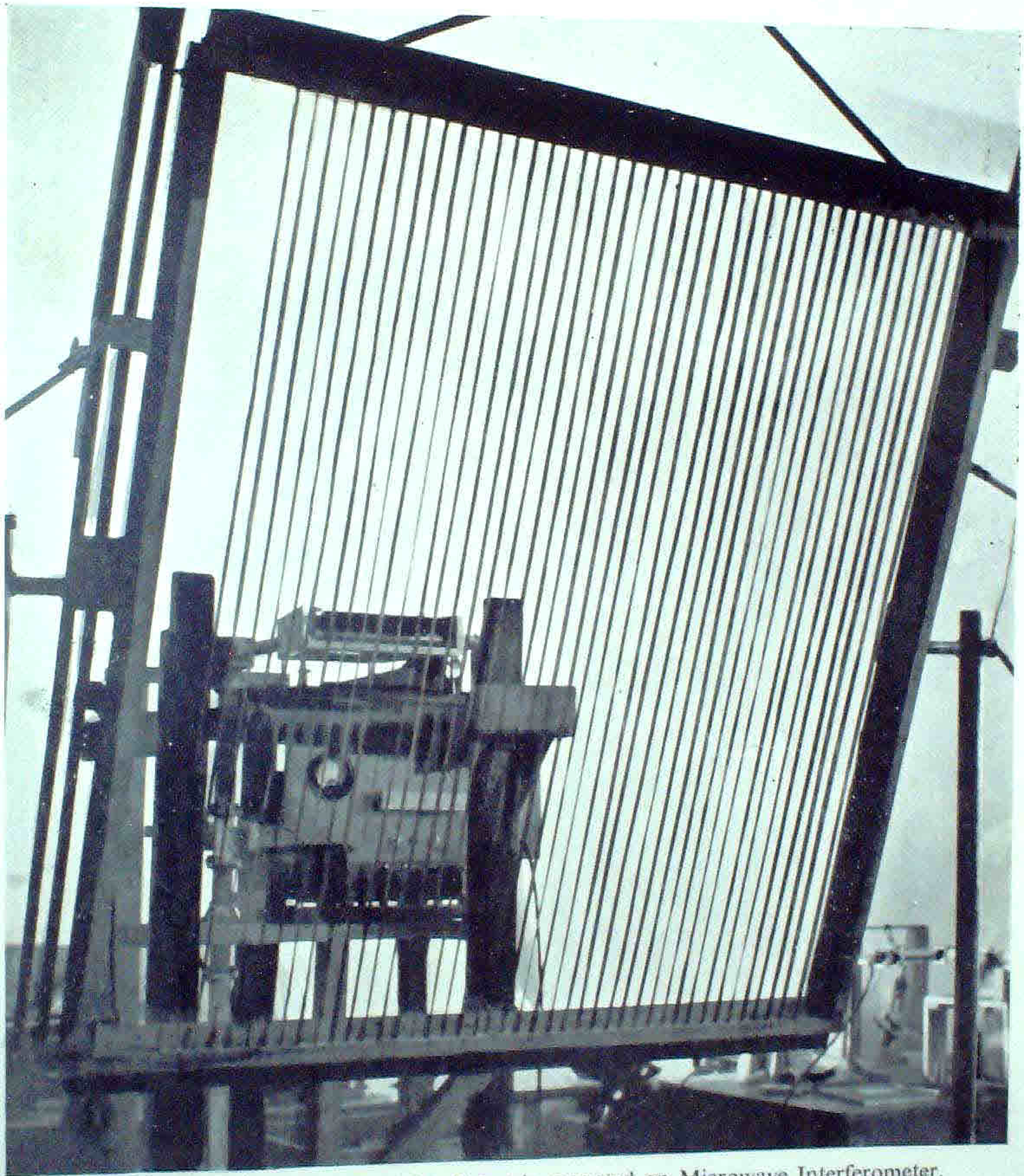


FIG. 2. Photograph of the artificial dielectric mounted on Microwave Interferometer.

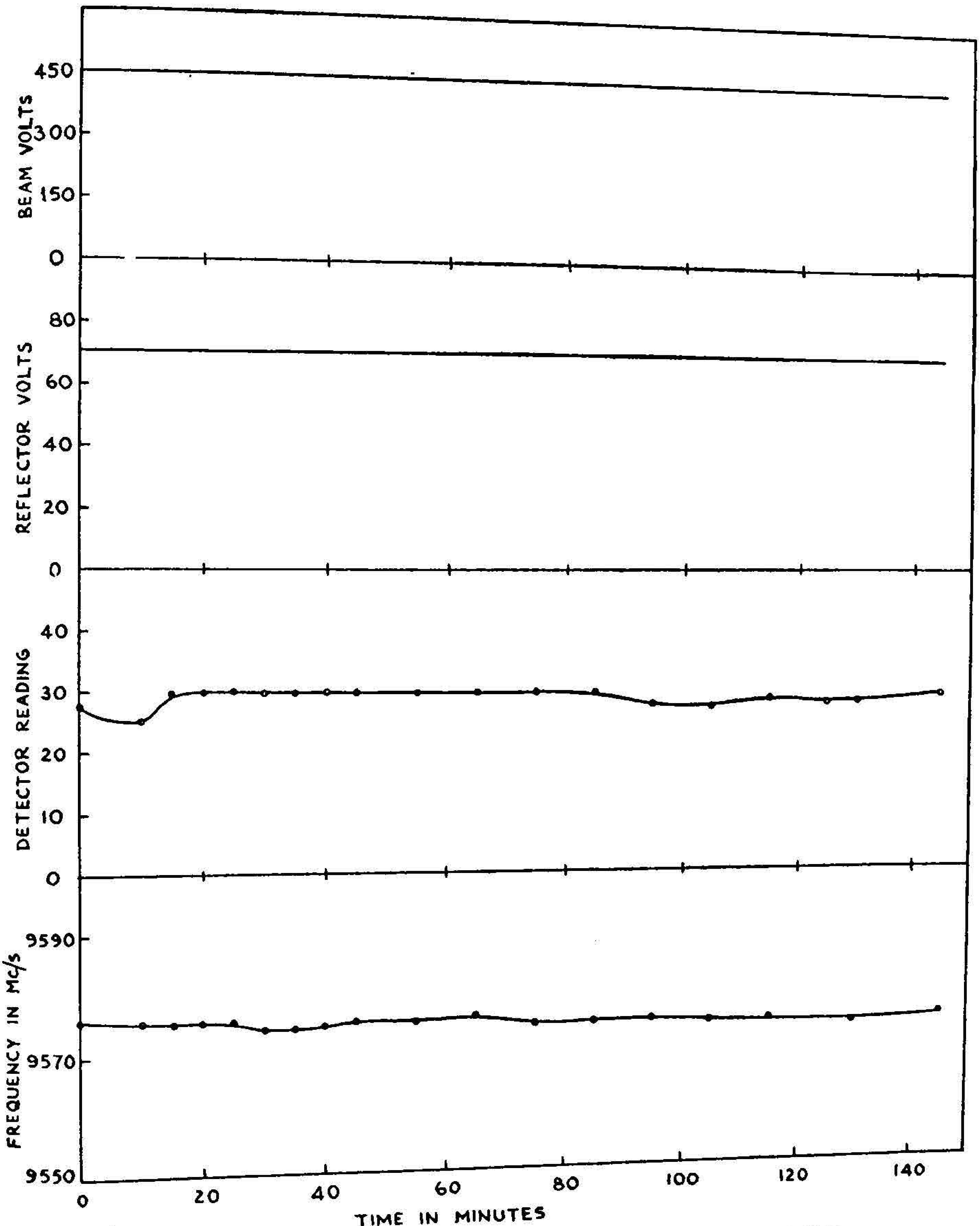



FIG. 4. Stability characteristics of the microwave interferometer with respect to time.

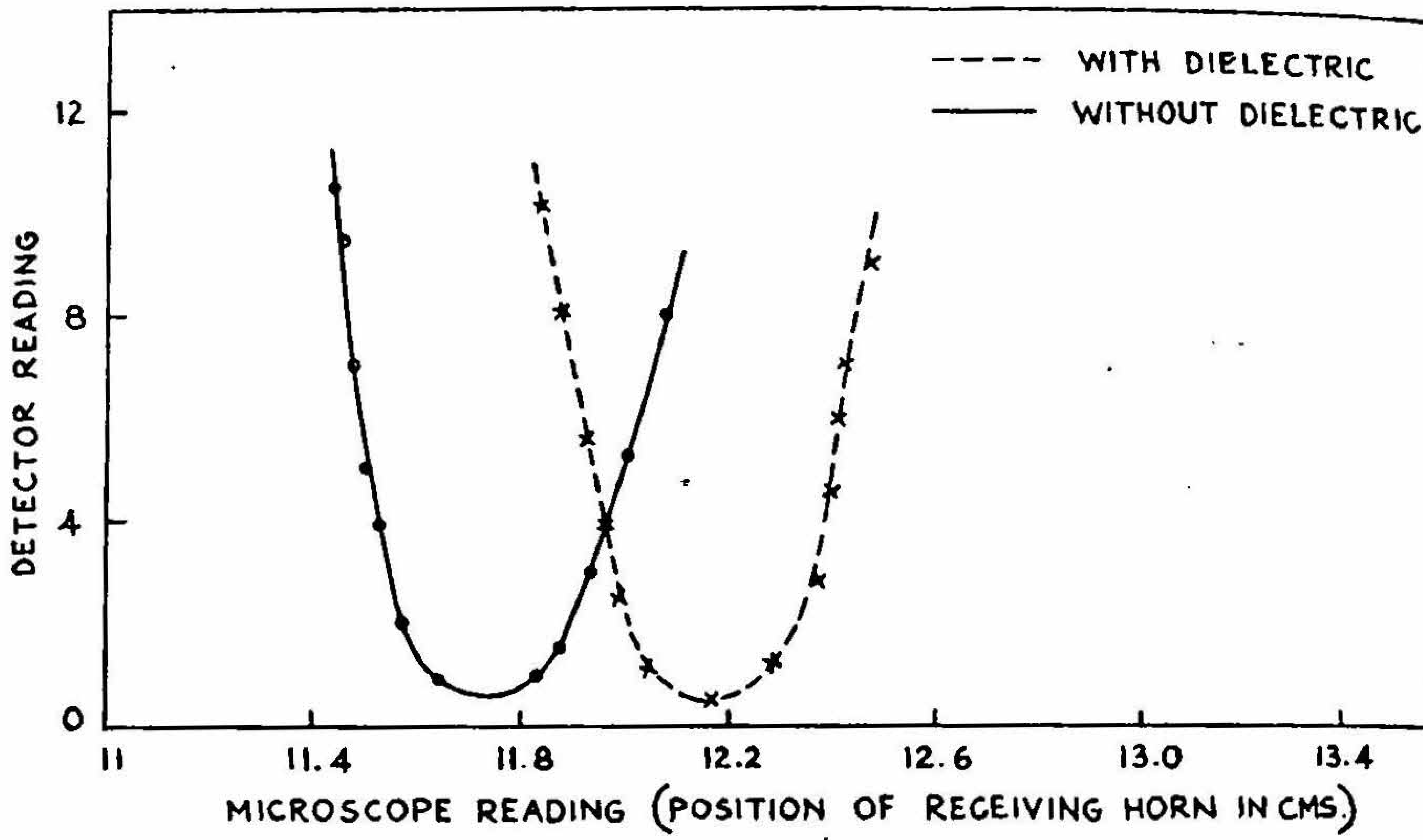



FIG. 5. Resonance curve of the interferometer with and without the artificial dielectric.

dielectric is not significant. The phase shift characteristics of the variable attenuator of the interferometer is shown in Fig. 6. The setting of the attenuator is fixed

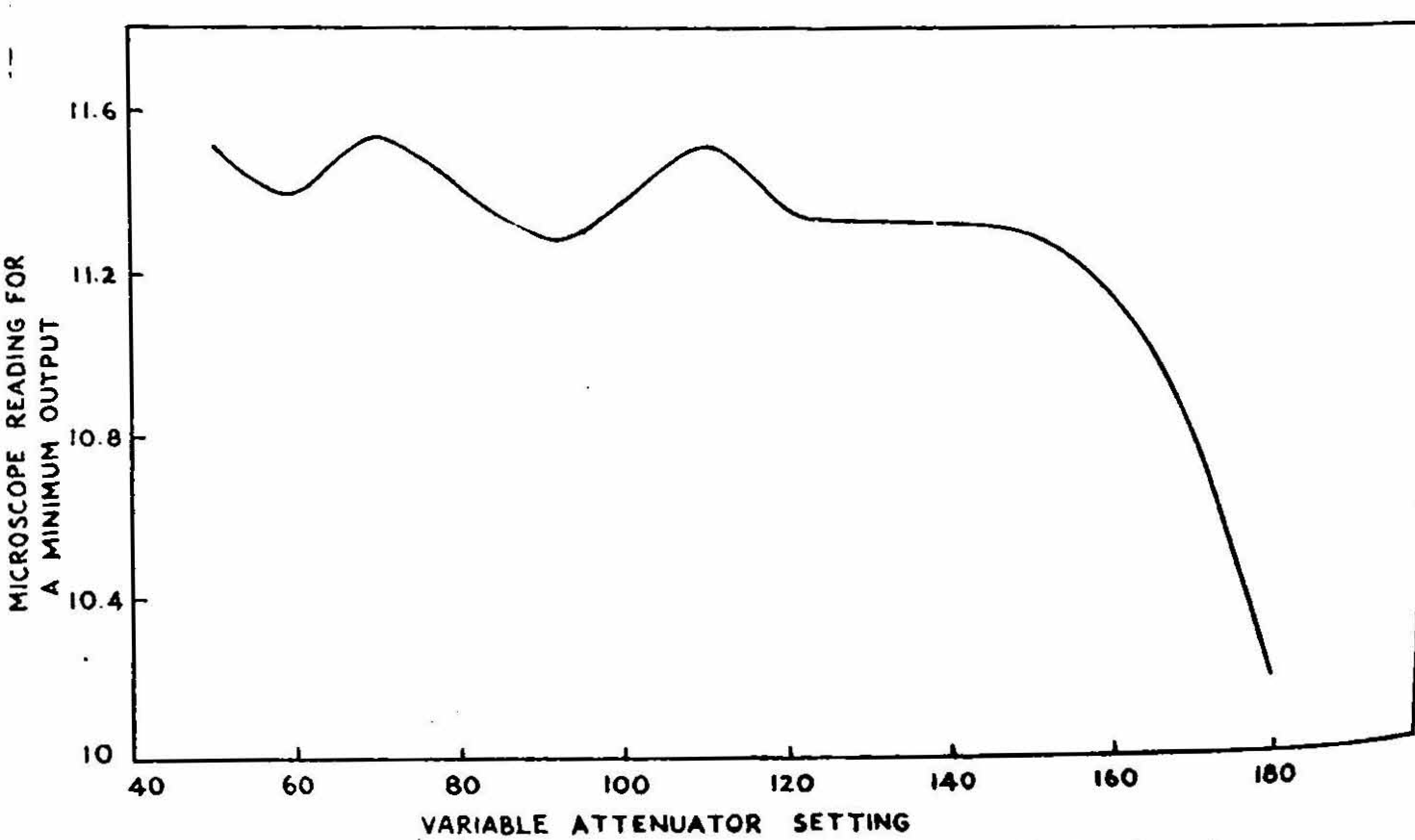



FIG. 6. Phase shift characteristics of the variable attenuator of the interferometer.

at 63 divisions during the measurements so as to obtain the minimum of the resonance curve as sharp as possible.

(iii) *Theory of the measurement of refractive index.*—For a homogeneous dielectric of thickness $w$ immersed in free space, the refractive index is determined from the following relations

$$\mu_{w\to\infty} \simeq \left(1 \pm \frac{\Delta}{w}\right) \tag{17 a}$$

$$\mu_{w\to 0} \simeq \left(1 \pm \frac{2\Delta}{w}\right)^{\frac{1}{2}} \tag{17 b}$$

The sign of the phase shift $\Delta$ depends on whether the length of the air-gap is shortened or extended after the sample is placed in the free space arm of the interferometer. In the present case, as $\mu < 1$, the air gap is shortened and hence the negative sign before $\Delta$ is chosen. When neither of the conditions $w \to \infty$ or $w \to 0$ hold good but the sample possesses a definite thickness $\mu$ is determined from the relation

$$\left(\mu + \frac{1}{\mu}\right) \tan\left(\frac{2\pi\mu w}{\lambda}\right) = 2 \tan \Phi \tag{18}$$

where $\Phi$ is the measured phase change on transmission.

The value of $\mu$ determined from (17 *a*) is only approximate, whereas $\mu$ can be determined from the more accurate relation (18) only after laborious computation. From (17 *a*) and (18), however, the desired value of the phase shift $\Delta_t$ is obtained as follows (Chatterjee, *loc. cit.*).

$$\Delta_m - \Delta_t = \frac{\lambda}{4} - \mu w - \frac{\lambda}{2\pi} \operatorname{arc\,tan}\left[\frac{2\mu}{1+\mu^2} \cot \frac{2\pi\mu w}{\lambda}\right]. \tag{19}$$

In practice, the measured value of $\Delta$ (*i.e.*), $\Delta_m$ is used to determine $\mu$ from (17 *a*). This value of $\mu$ is substituted in (19) to determine $\Delta_t$. The value of $\Delta_t$ substituted in (17 *a*) gives a different value of $\mu$ which is again substituted in (19) to determine $\Delta_t$. By successive application of this correction, the error converges rapidly and a true value of $\mu$ is found.

(iv) *Interaction between the transmitting horn and the dielectric sample.*—The refractive index determined from (17 *a*) and (19) takes no account of the effect of the standing wave created in the free space between the horns of the interferometer and the dielectric sample. The multiple reflections taking place between the sample and the horns are responsible for making $\Delta$ dependent on the distance $d$ of the sample from the transmitting horn. The received power for different positions of the sample varies as given by the following relation (Pfister and Roth, 1938; Redheffer, 1949)

$$\rho = \frac{P_0 T^2}{1 - 2\rho R \cos \frac{4\pi d}{\lambda} + \rho^2 R^2} \tag{20}$$

The above relation indicates a sinusoidal variation in the received power and hence in $\Delta$ with respect to $d$ with a period of $\lambda/2$. This is also evident from the experimental results (Fig. 7). Table I shows the ratio of the maximum to minimum phase shift for some angles of incidence when the sample is placed at different distances from the transmitting horn of the interferometer.

TABLE I

*Ratio of maximum to minimum phase shift at different angles of incidence for different positions of the dielectric*

| Angle of incidence | Positions of the dielectric | | |
|---|---|---|---|
| | Near the transmitting horn | Midway between the transmitting and receiving horn | Near the receiving horn |
| 0° | 1·14 | 1·11 | 1·26 |
| 8° | 1·14 | 1·08 | 1·24 |
| 22° | 1·00 | 1·00 | 1·00 |
| 26° | 1·00 | 1·00 | 1·00 |

It is found that the interaction is appreciable when the angle of incidence is 0° and 8°. For angles 22° and greater the interaction is vanishingly small. It is also observed that the interaction in the case 0° and 8° angles of incidence is more when the sample is placed near the transmitting and the receiving horn. At angle of incidence near the Brewster's angles given by

$$\theta_B = \text{arc} \tan \mu \tag{21}$$

there is practically no variation of $\Delta$ with $d$. This is due to the fact that near the Brewster's angles, the rays reflected from the interface of the sample and air do not reach the horns.

In order to reduce the error in the determination of phase shift due to interaction, the phase shift has been measured in successive steps of $\lambda/4$ and the results are averaged. The effects of interaction is reduced if the measurements are done at arbitrary incidence especially near the Brewster's angle. For arbitrary angles of incidence $\theta$, the theoretical value of phase shift for a homogeneous dielectric is obtained from the following relation:

$$\Delta = w\,[-\cos\theta \pm (\mu^2 - \sin^2\theta)^{\frac{1}{2}}] \tag{22}$$

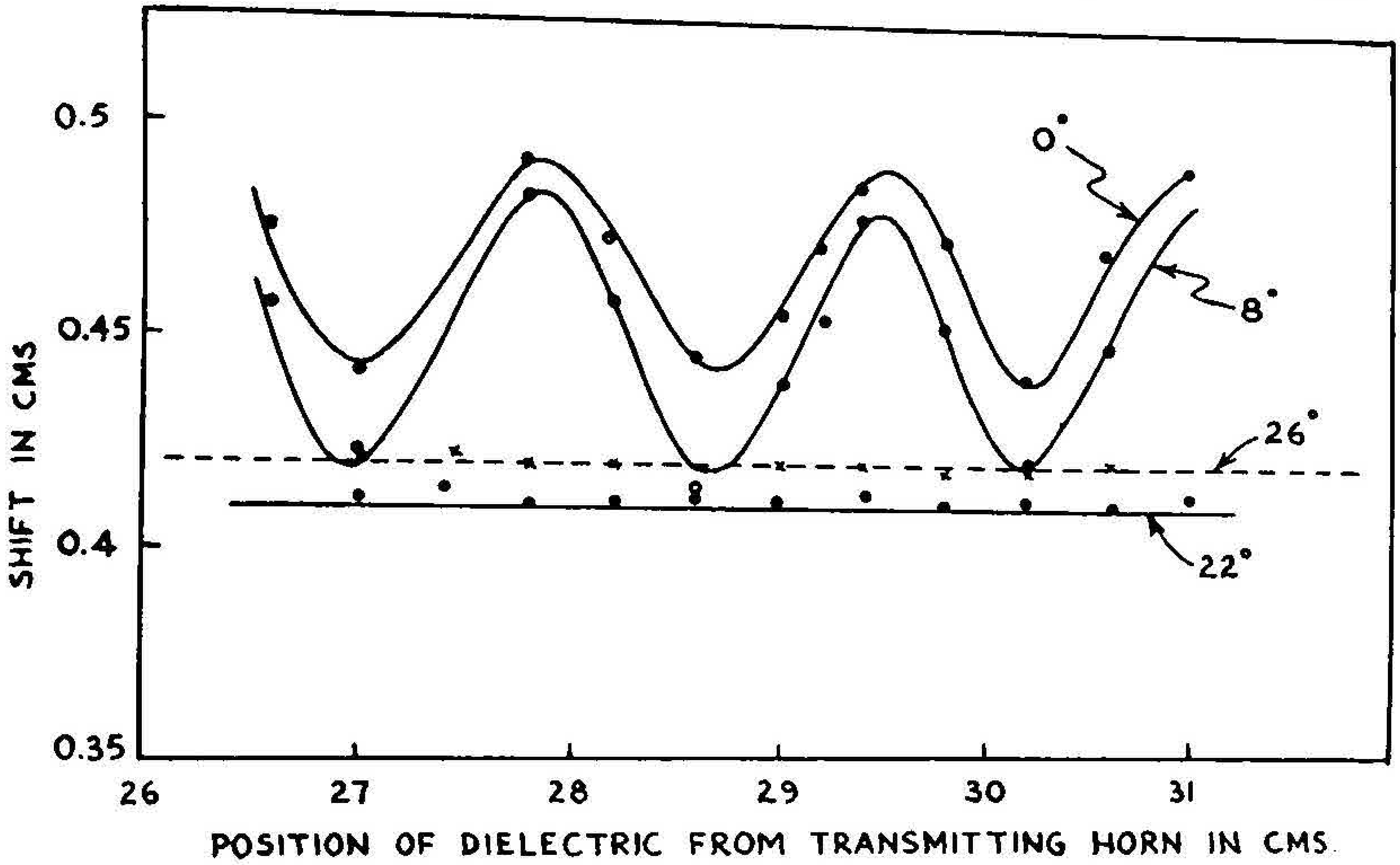
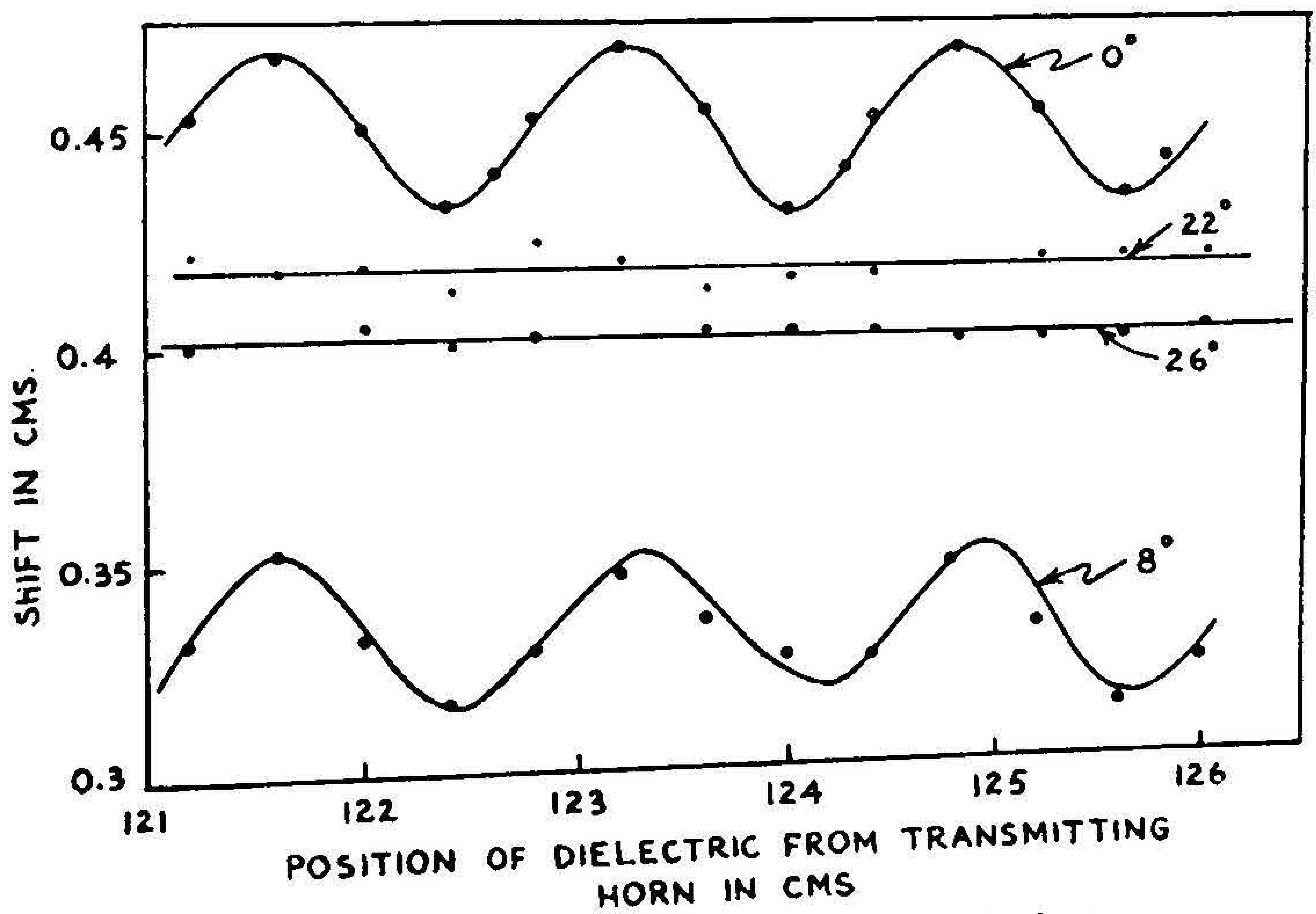
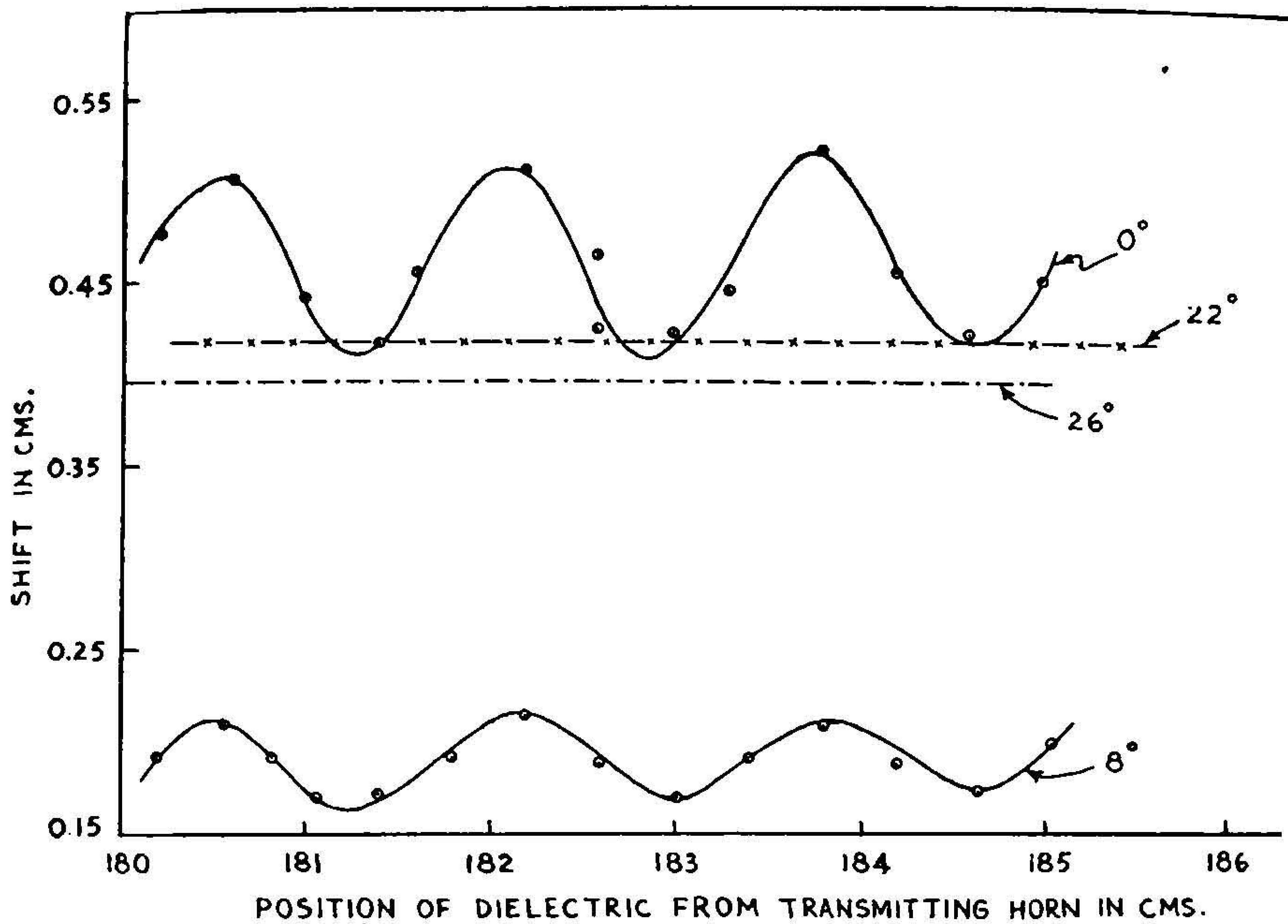



FIG. 7. Interaction between the transmitting horn of the interferometer and the artificial dielectric. Pcsition of the sample

(*a*) Near the transmitting horn.




(*b*) Midway between the transmitting and the receiving horn.




(*c*) Near the receiving horn.

The theoretical value of phase shift as a function of angles of incidence for a homogeneous dielectric having the same refractive index $\mu = \cdot 593$ and thickness $w = 0 \cdot 95$ cm. as the inhomogeneous parallel plate dielectric is shown in Fig. 8 for the sake of comparison.

(v) *Effect of diffraction.*—If the sample size is A, the effective size of the sample for arbitrary incidence is A cos $\theta$. It is obvious that for angles of incidence other than the normal ($\theta = 0°$), the effective size of the sample is reduced. Hence as $\theta$ increases, the effect of diffraction on phase shift becomes more pronounced. The effect of diffraction on phase shift for some angles of incidence is shown in Figs. 9 and 10, when the sample is placed near the receiving horn. It has been found that the effect of diffraction is more pronounced when the sample is placed near the transmitting horn. This is one of the reasons for placing the sample near the receiving horn during measurements.

(vi) *Wavefront error.*—The wave emerging from the transmitting horn even though fitted with a phase correcting lens, is not perfectly plane. This makes the path length of different rays through the sample different at places off the axis. Hence the attenuation suffered by the wave at different distances from the axis will be different. This non-uniform illumination of the sample will introduce an

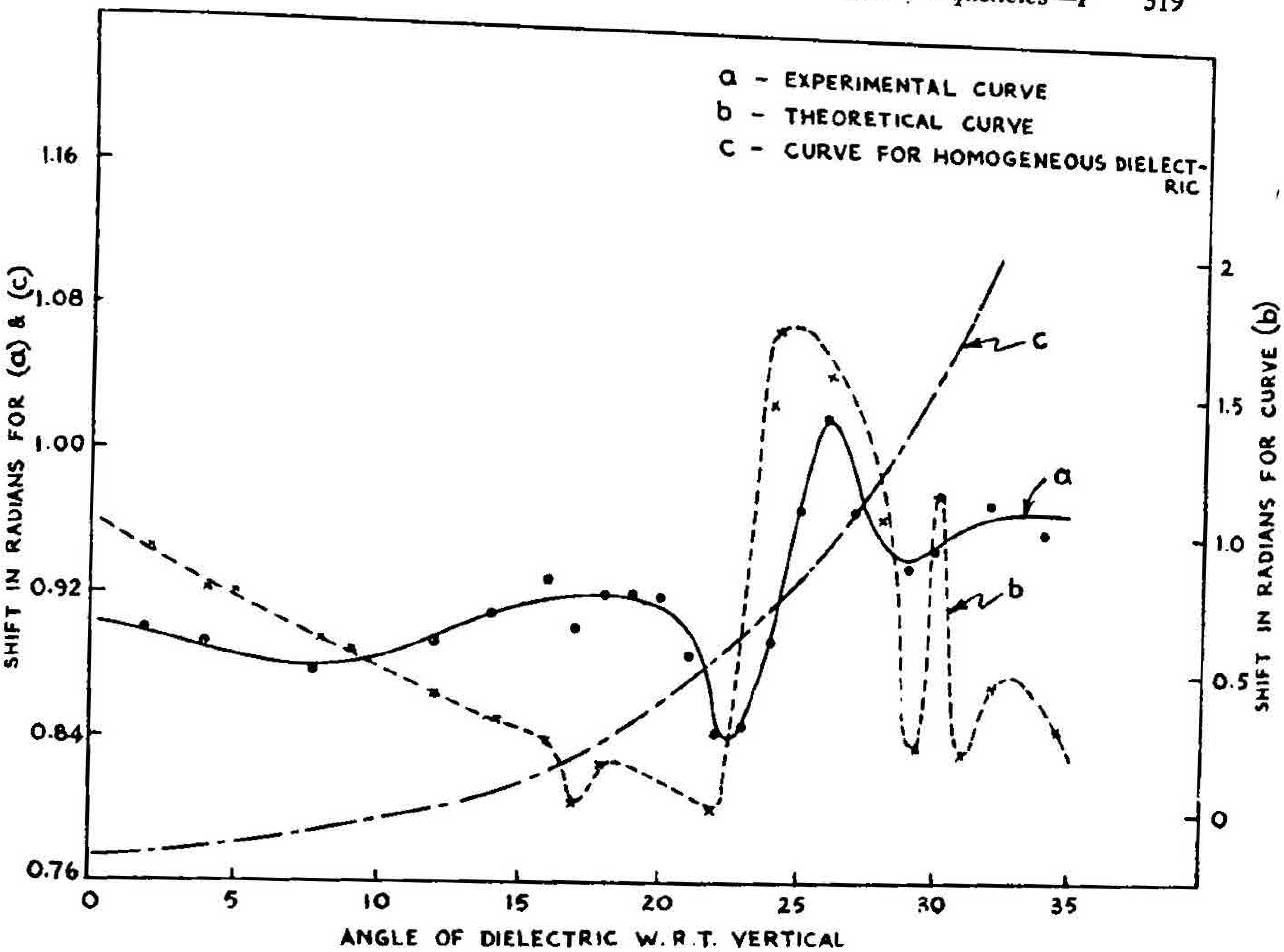



FIG. 8. Variation of phase shift on transmission with angles of incidence.
(*a*) Theoretical, eq. 10.
(*b*) Experimental.
(*c*) Theoretical for a homogeneous dielectric $\mu = \cdot 593$, $w = \cdot 95$ cm., eq. (22).

error in $\Delta_m$. The degree of error is reduced by placing the sample near the receiving horn during measurements as with increase of distance from the transmitting horn, the wave tends to become more and more plane. The values of $\mu$ measured by placing the sample at various distances from the transmitting horn are reported in Fig. 11.

It is obvious from Fig. 11 that the mean deviations in refractive index are $\pm 0\cdot025$, $\pm 0\cdot003$, and $\pm 0\cdot016$, depending on whether the sample is placed near the transmitting horn, midway between the transmitting and the receiving horn or near the receiving horn respectively. If the deviation is only due to the wave-front error, then the greater deviation near the receiving horn than at intermediate distance remains still to be explained.

A study of the diffraction curves (Figs. 9 and 10) at different angles of incidence shows that the effective sizes of the sample necessary to avoid diffraction effect at

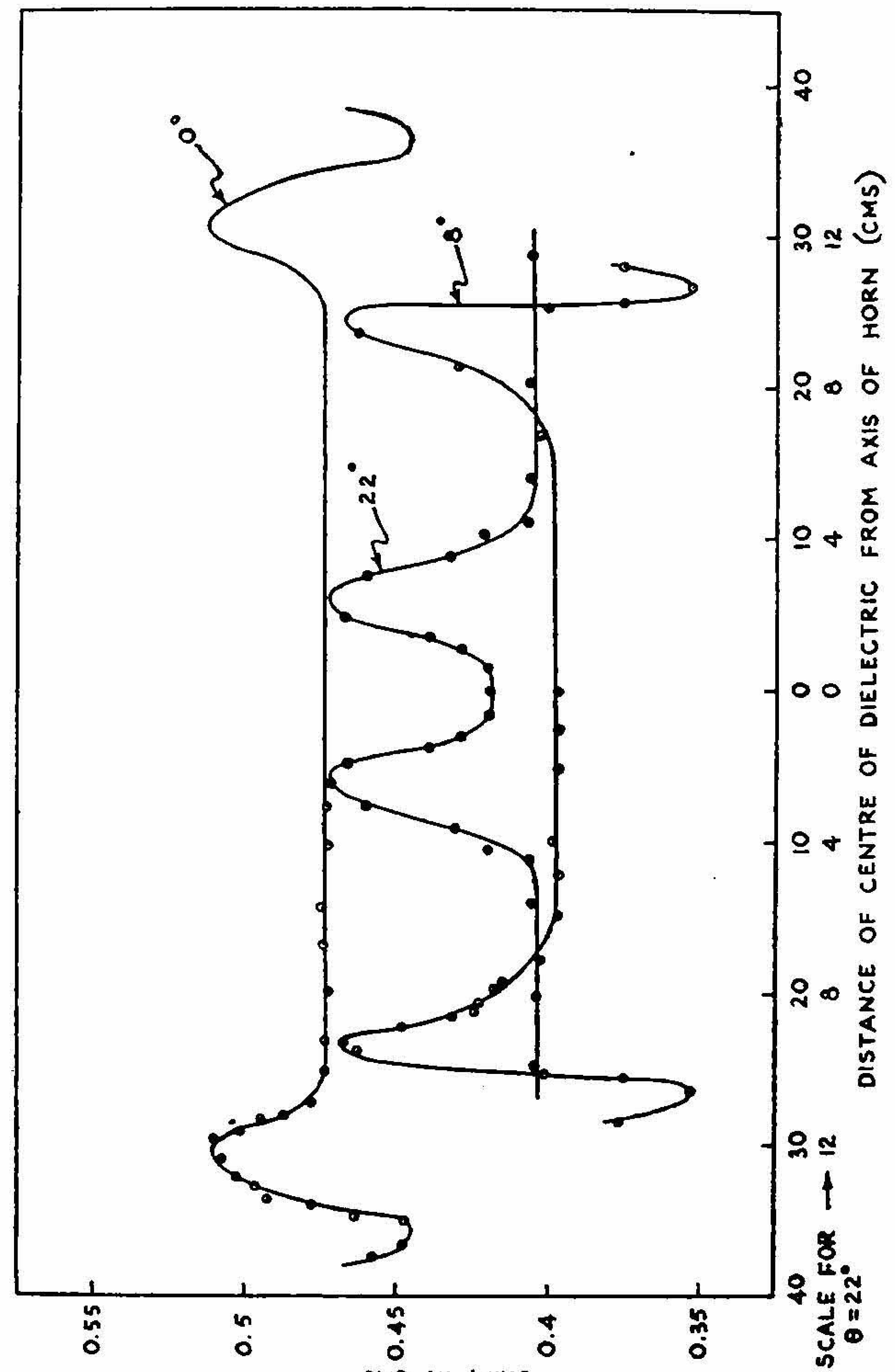



FIG. 9. Effect of diffraction on phase shift at various angles of incidence for vertical movement of the dielectric.

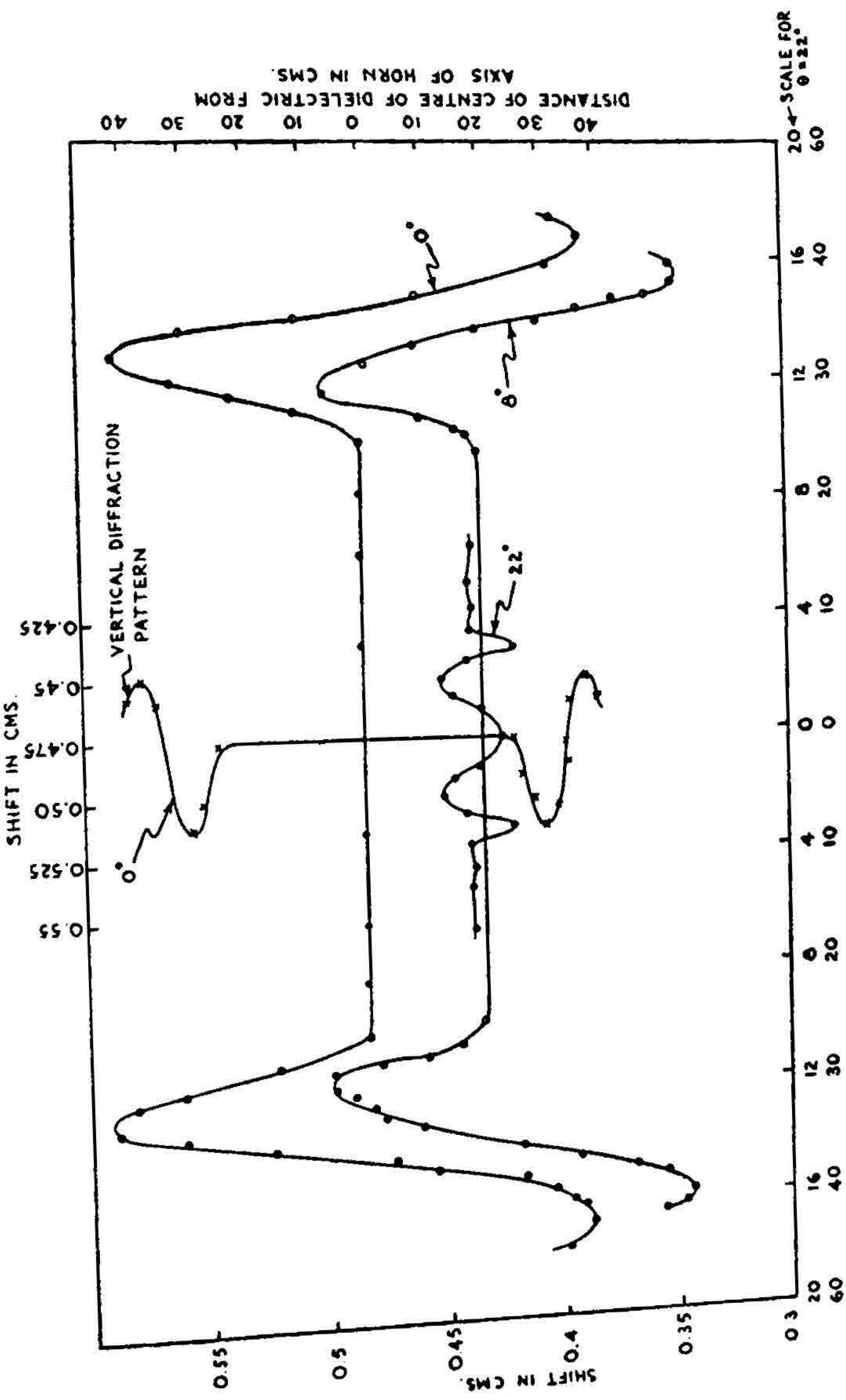



FIG. 10. Effect of diffraction on phase shift at various angles of incidence for horizontal movement of the dielectric.

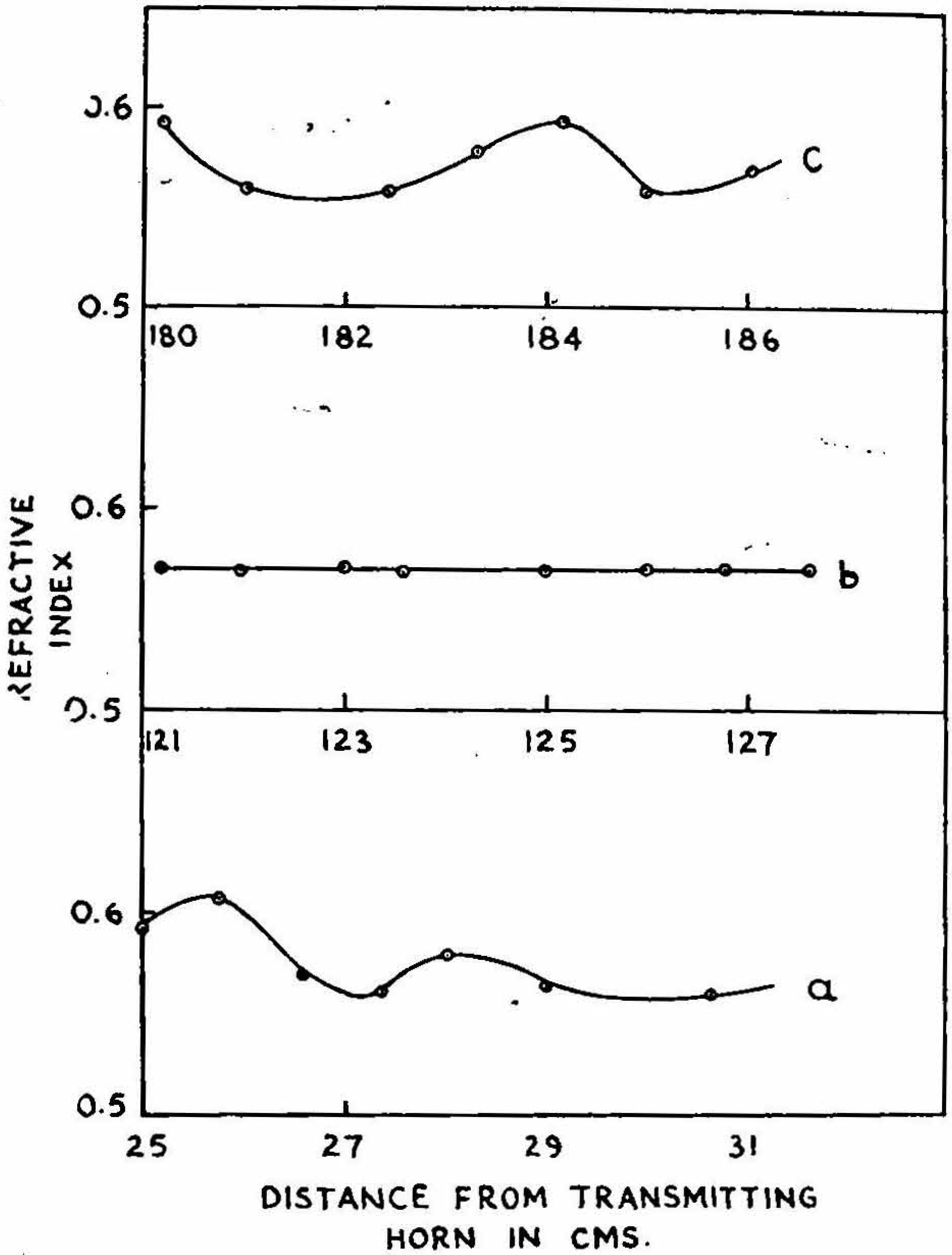



FIG. 11. Refractive index at normal incidence for different positions of the dielectric.
(*a*) Near the transmitting horn.
(*b*) Midway between the transmitting and the receiving horn.
(*c*) Near the receiving horn.

angles of incidence 0°, 8°, and 22° are greater than 0·40, 0·48, 0·82 sq. metres respectively. It is also noticed from the diffraction curves that the diffraction effect is practically absent for the size of the sample employed at normal incidence.

The variation of phase-shift with angles of incidence is shown in Fig. 8. The accuracy with which the phase shift has been measured is $\pm 0{\cdot}016$ radians. The divergence between the experimental and theoretical phase shift is probably due to (i) the fact that in deriving the theory the step of staggering $b$ has been assumed to be zero. But the construction of the dielectric shows that the edges of the plates deviate from being in a line, *i.e.*, $b \neq 0$; (ii) the limitations imposed by the theory in assuming the propagation of only the $H_{01}$ mode. The theory also assumes that the plates of the dielectric are of zero thickness. It is considered that the presence of higher order modes and the plates having definite thickness may contribute significantly to the phase change; (iii) the assumptions made in expanding the arc sines.

The experimental verification of the theory of the variation of phase shift with different spacings of the parallel plates is under progress.

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