EFFECT OF COMPONENT TOLERANCES IN LOW FREQUENCY SELECTIVE AMPLIFIERS—AN ANALYSIS

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Summary

Two types of resistance-capacitance networks, namely the Wien bridge and the twin-T, used in low frequency selective amplifiers are analyzed in respect of the effect of the error in each component value on the null of the network and consequently on the selectivity of the amplifier. It is shown that when networks are designed for obtaining maximum selectivity with a given amplifier, the Q-factor of the amplifier is relatively sensitive to small errors in the network components. This sensitivity can be reduced by suitably choosing the ratios of the components of the network, though, as a consequence, a higher amplifier gain would be necessary for obtaining the same selectivity. Quantitative relations are developed between the rate of variation of Q with the component error factor and the relevant component ratios of the networks.

Introduction

The application of frequency selective feedback in an amplifier to obtain selective amplification at low frequencies is a well-known method. By introducing into the feedback loop a resistance-capacitance network which rejects a certain frequency, the full gain of the amplifier is realized at the rejection frequency, while the gain falls off at other frequencies due to finite negative feedback. Selective amplifiers of this type are specially suitable for low frequencies when conventional L-C circuits have disadvantages of bulk and cost. A further advantage is that the variation of the rejection frequency is readily accomplished by the simultaneous variation of a set of condensers or resistances of the R-C network, thereby "tuning" the amplifier to any desired frequency within a band. With suitable components, a ratio of the maximum to minimum frequency obtainable may be made as great as 20: 1.

2. Principle of Operation

The block diagram of the basic elements of a selective amplifier is shown in Fig. 1. If A is the gain of the amplifier without feedback and β the transmission

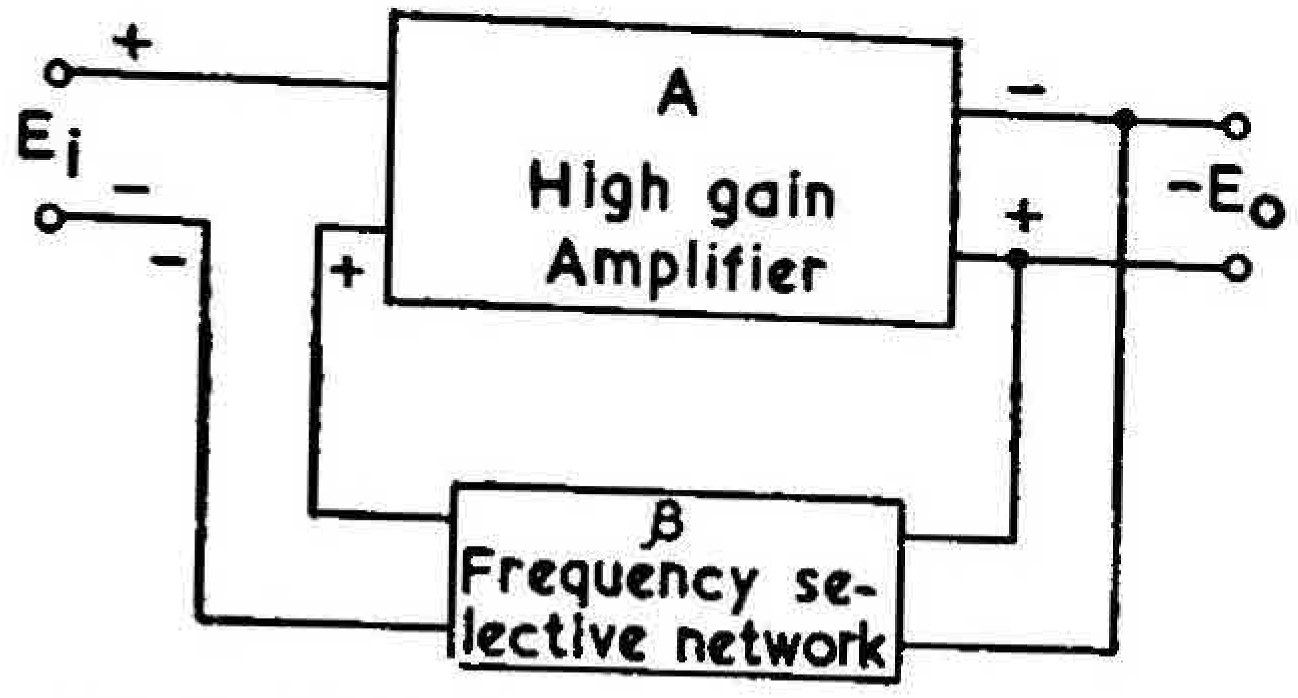

FIG. 1. Block diagram of Selective Amplifier.

of the frequency selective network, assuming that the output signal is 180° out of phase with the input signal, the gain with feedback is

$$\frac{E_0}{E_i} = G = \frac{A}{1 + A\beta} \tag{1}$$

For the types of R-C networks analyzed in this article, the transmission at the angular frequency ω is of the form

$$\beta = \frac{jy}{a + jby} \tag{2}$$

where

$$y = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}, \tag{3}$$

ω_0 = rejection frequency of the network and a and b are constants. Substituting the expression for β in (1)

$$G = \frac{A\,(a + jby)}{a + j\,(A + b)\,y} = \frac{A\left(1 + j\,\frac{b}{a}\,y\right)}{1 + j\,\frac{A + b}{a}\,y} \tag{4}$$

The gain has the maximum value of A for $y = 0$ and the limiting value of $Ab/(A + b)$ at zero and infinite frequencies. The impedance frequency relation of a parallel LCR circuit is given by

$$Z\,(y) = R\,\frac{1}{1 + jQy} \tag{5}$$

where $Z\,(y)$ is the impedance at an angular frequency ω, ω_0 being the resonance frequency of the LC circuit, and R the parallel resistance. Expression (4) is similar to equation (5) for small values of y and provided A is sufficiently large, the frequency response of the amplifier is similar to the resonance curve, Defining Q of the amplifier in terms of the values of y for which the gain falls by $1/\sqrt{2}$,

$$Q = \frac{1}{y} = \frac{A+b}{a}\left\{1 - 2\left(\frac{b}{A+b}\right)^2\right\}^{\frac{1}{2}}$$

$$\doteqdot \frac{A+b}{a}\left\{1 - \frac{b^2}{(A+b)^2}\right\} \tag{6}$$

Generally the value of A is so high that the second term within the brackets can be neglected and Q equated to $(A + b)/a$. This approximation will be made in all the subsequent analysis.

3. THE EFFECT OF ERRORS IN COMPONENT VALUES

The above analysis assumes that the frequency selective network has a perfect null or zero transmission at the rejection frequency. If the component values depart from the nominal values due, say to manufacturing tolerances, the rejection frequency departs from the nominal value and further a finite transmission may be present at the rejection frequency. In the case of fixed frequency selective amplifiers, it is possible, by the use of trimmer resistances or condensers, to obtain a null of the desired attenuation. By the use of components of good quality and of suitable temperature coefficients, variations in the null due to temperature and ageing may be minimized. In the case of amplifiers in which the frequency selected has to be varied over a range, an additional problem which arises is the need to hold the component ratios constant at the desired values. The variations due to component tolerances not only change the rejection frequency of the network from its nominal value but due to the finite transmission at the null, change the gain and selectivity of the amplifier. The factor of interest in this connection is the extent to which the gain and Q of the amplifier vary for a given small change in one of the component ratios. This factor, expressed as a derivative of the gain and Q with respect to the error in component ratio has been determined for several types of circuits by S. W. Punnet[3] and used as a basis for comparison of the merits of different types of rejection circuits. The purpose of this paper is to analyze the Wien bridge and the twin-T networks in their most general form and to show that the effect of component tolerances can be reduced by a proper choice of component ratios of rejection networks. In the following analysis it will be assumed that the amplifier without feedback has a constant, real gain in the frequency range of operation; that it is stable without feedback at all frequencies; that its output impedance is small compared with the impedance of the rejection network; and that the rejection network is terminated by an infinite load.

4. ANALYSIS OF A SELECTIVE AMPLIFIER USING THE WIEN BRIDGE

The Wien bridge in its general form is shown in Fig. 2. The conditions for null are given by the relations

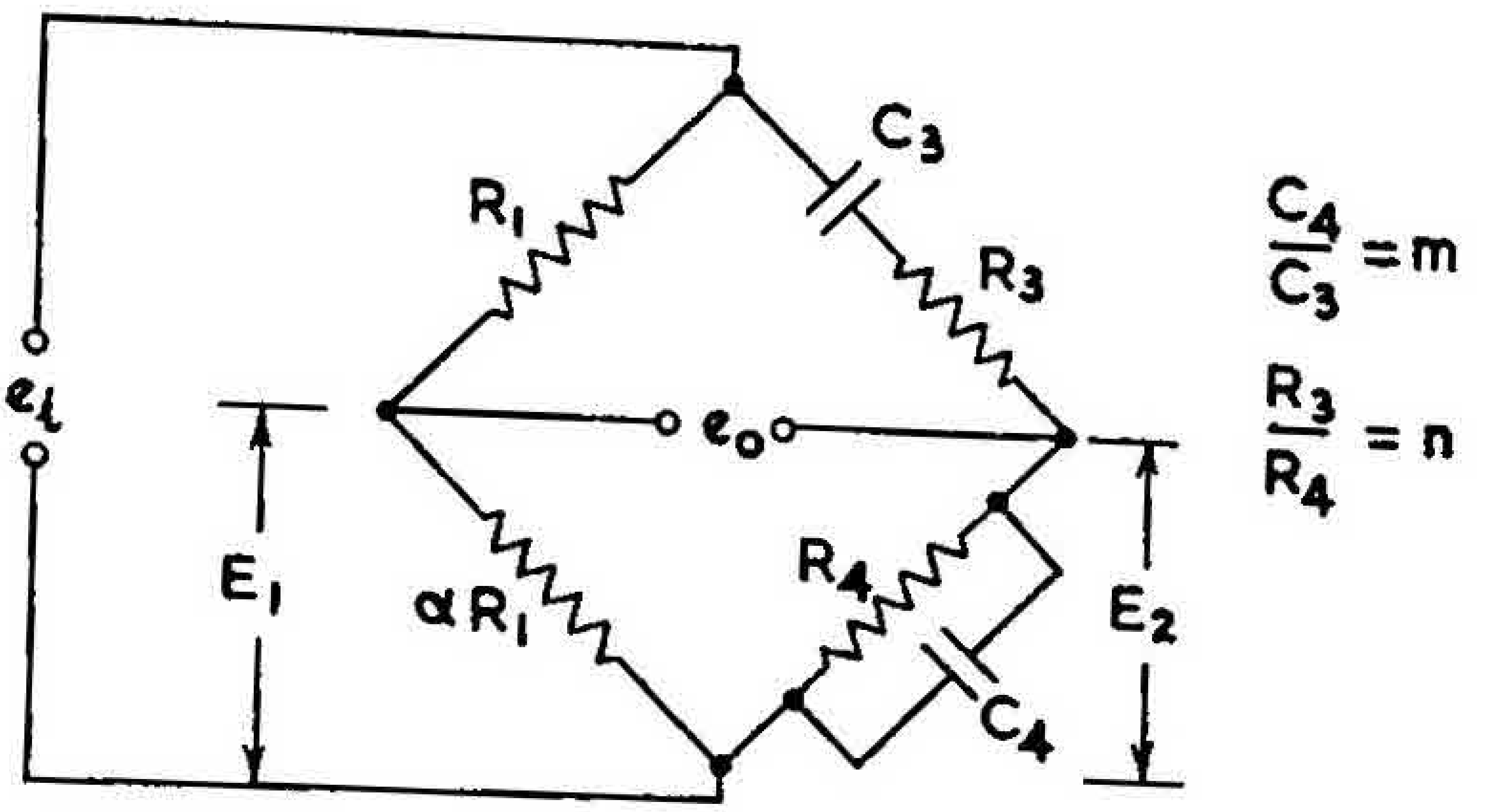

FIG. 2. The Wien Bridge.

$$\frac{1}{\alpha} = \frac{R_3}{R_4} + \frac{C_4}{C_3}\,; \quad \omega_0{}^2 = \frac{1}{R_3R_4C_3C_4} \tag{7}$$

Putting $R_3/R_4 = n$, $C_4/C_3 = m$, and substituting, this reduces to

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{m}{n}} \cdot \frac{1}{C_4R_4} \quad \text{and} \quad \frac{1}{\alpha} = m + n \tag{8}$$

where f_0 is the rejection frequency. The transmission of the network at the angular frequency ω is given by

$$\frac{e_0}{e_i} = \beta = \frac{E_1 - E_2}{e_i} = \frac{1}{1 + n + m} - \frac{1}{1 + n + m + j\sqrt{mn}\,y}$$

$$= \frac{j\dfrac{\sqrt{mn}}{(1 + n + m)^2}\,y}{1 + j\dfrac{\sqrt{mn}}{1 + n + m}\,y} \tag{9}$$

where

$$y = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$$

The output e_0 of the bridge may be used for frequency selective feedback. It may be noted however that there is no common point between the input and the output which can be earthed. So a transformer will be required for feedback. This is a drawback of the Wien bridge circuit but this has been overcome in one instance[4] by the use of a difference amplifier in the place of a transformer. Hence, in the following analysis, it will be assumed that the output voltage e_0 given by (9) is added to the input voltage to the amplifier by some means. The gain G of the amplifier with frequency selective feedback is then

$$G = \frac{A}{1 + A\beta} = \frac{A\left(1 + j\dfrac{\sqrt{mn}}{(1+n+m)}\,y\right)}{1 + j\dfrac{\sqrt{mn}\,(A+1+n+m)}{(1+n+m)^2}\,y} \quad (10)$$

comparing (10) with (4), the Q factor of the selective amplifier is

$$Q_0 = \frac{\sqrt{mn}\,(A+1+n+m)}{(1+n+m)^2} \quad (11)$$

For a given value of A, the maximum value of Q obtainable is for a value of $m = n \doteqdot \frac{1}{2}$. So the maximum Q obtainable with a selective amplifier using Wien bridge is

$$Q_{max.} \doteqdot \frac{A+2}{8} \quad (12)$$

5. EFFECT OF COMPONENT TOLERANCES

The effect of component tolerances on the gain and selectivity may be found by assuming that one of the (variable) components changes by a small amount from the nominal value required for perfect balance. Let the value of R_3 change to $R_3(1+k)$, where k is small (Fig. 3). The null frequency is then given by

$$\omega_r^2 = \frac{m}{n(1+k)\,C_4^2R_4^2} \quad \text{or} \quad \omega_r = \sqrt{\frac{m}{n(1+k)}} \cdot \frac{1}{C_4R_4} \quad (13)$$

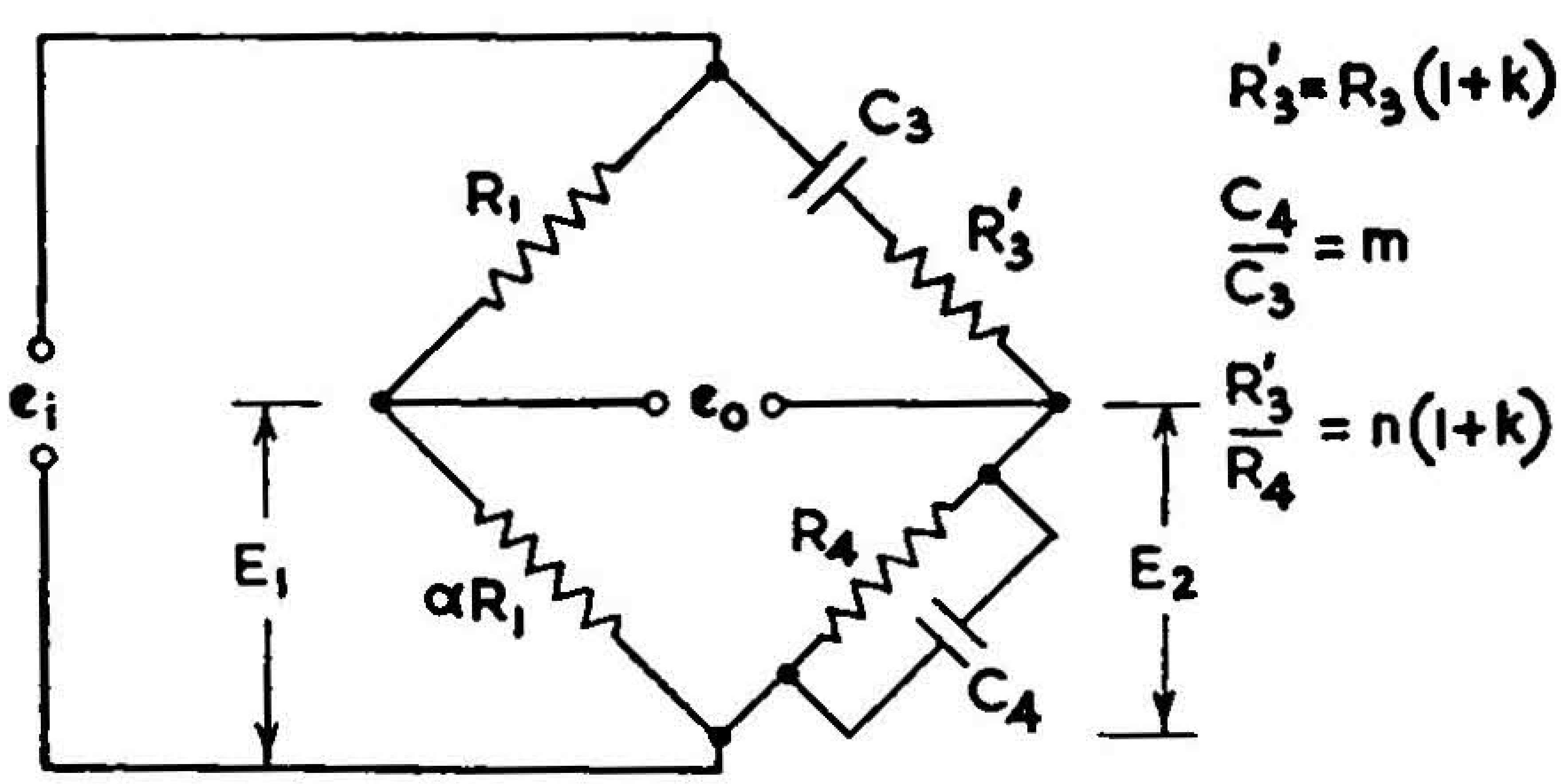

FIG. 3. The Wien Bridge with an error in component (resistance) value in one arm.

The transmission β' is given by

$$\frac{e_0'}{e_i} = \beta' = \frac{1}{1+n+m} - \frac{1}{1+n(1+k)+m+j\sqrt{mn}\,(1+k)\,y}$$

$$= \frac{nk + j\sqrt{mn}\,(1+k)\,y}{(1+n+m)\{1+n+m+nk+j\sqrt{mn}\,(1+k)\,y\}} \quad (14$$

where

$$y = \frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}$$

The gain of the amplifier as a function of y is now given by

$$G = \frac{G_r\left(1 + j\,\dfrac{\sqrt{mn\,(1+k)}}{(1+n+m+nk)}\,y\right)}{1 + j\,\dfrac{\sqrt{mn\,(1+k)}\,(A+1+n+m)}{(1+n+m)\,(1+n+m+nk)+Ank}} \tag{15}$$

where G_r, the gain at the null frequency, is

$$G_r = \frac{A}{1 + \dfrac{Ank}{(1+n+m)\,(1+n+m+nk)}} \doteqdot \frac{A}{1 + \dfrac{Ank}{(1+n+m)^2}} \tag{16}$$

The gain frequency relation is again of the same form as in (10) and so the new value of Q factor is, by comparison,

$$Q_r = \frac{\sqrt{mn\,(1+k)}\,(A+1+n+m)}{(1+n+m)\,(1+n+m+nk)+Ank} \tag{17}$$

By differentiation of (16) and (17) with respect to k,

$$\left(\frac{dG_r}{dk}\right)_{k=0} = -\frac{An\,(A+1+n+m)}{(1+n+m)^2} + \frac{An}{1+n+m} \tag{18}$$

$$\left(\frac{dQ_r}{dk}\right)_{k=0} = -\sqrt{\frac{n}{m}}\,Q_0^2 + \tfrac{1}{2}Q_0 \tag{19}$$

Provided A and Q are sufficiently large, the second term on the right-hand side of (17) and (18) can be neglected, and thus

$$\left.\begin{aligned} \frac{1}{G_r}\left(\frac{dG_r}{dk}\right)_{k=0} &= \frac{1}{A}\left(\frac{dG_r}{dk}\right)_{k=0} \doteqdot -\frac{n\,(A+1+n+m)}{(1+n+m)^2} = -\sqrt{\frac{n}{m}}\,Q_0 \\ \frac{1}{Q_r}\left(\frac{dQ_r}{dk}\right)_{k=0} &= \frac{1}{Q_0}\left(\frac{dQ_r}{dk}\right)_{k=0} \doteqdot -\sqrt{\frac{n}{m}}\,Q_0 \end{aligned}\right\} \tag{20}$$

The percentage change in gain and Q factor of the amplifier are thus seen to be a function of Q_0 and the component ratios m and n. If m and n are chosen for obtaining the maximum value of Q for a given gain, *i.e.*, if $m = n = \frac{1}{2}$, the above factors will numerically equal Q_0. However, the factor can be reduced by decreasing n and increasing m. This at the same time will necessitate a higher value of gain for maintaining the same Q factor. A numerical example will illustrate the point. With $n = m = \frac{1}{2}$, for a Q of 20, the amplifier gain required is, from (12), equal to 158. The rate of change of gain and Q factor, expressed as a fraction of gain and Q (henceforward referred to as the fractional rate of change) is, by (20), equal to -20. So a change in R_4 by a 0·5% changes Q_r by 10%, *i.e.*, from 20 to 18. If, on the other hand, $m = 2$ and $n = \frac{1}{2}$, the fractional rate of change is reduced to -10, and the same change in R_4 brings about a change in Q from 20 to 19. Thus there is reduction in the percentage change of gain and Q by a factor of 2. But the amplifier gain required for a Q of 20 is increased by (11), from 158 to 241·5, which is an increase by about 50%.

The effect of component tolerance in respect of other components of the bridge may be found similarly. In the case of the resistance R_4, the fractional rate of change G and Q is numerically the same as in the case of R_3. In the case of the condenser elements, it can be shown that

$$\left| \frac{1}{A} \left(\frac{dG_r}{dk}\right)_{k=0} \right| \doteqdot \left| \frac{1}{Q_0} \left(\frac{dQ_r}{dk}\right)_{k=0} \right| \doteqdot \sqrt{\frac{n}{m}}\, Q_0 \tag{21}$$

In this case the above factor can be reduced by decreasing m and increasing n.

In both the above cases, the sensitivity of the amplifier to variations in the fixed element is increased in the same ratio as that with respect to the variable element is decreased. This however is not considered a serious drawback if high grade components are used as the fixed elements, though it should be reckoned with if the amplifier is designed to have more than one range and the range switching is done by changing the fixed elements. The component ratios of the fixed elements will have to be kept constant within closer limits than would be required in the case $m = n = \frac{1}{2}$.

6. Analysis of the Amplifier using Twin-T Network

The twin-T network in its most general form is shown in Fig. 4, where the various resistance and capacitance elements are expressed in terms of a resistance R, a capacitance C and the ratios m, n, p and q. The transmission of this network, when terminated by infinite impedance is given by[3]

$$\frac{e_0}{e_i} = \beta = \frac{1}{1 + \dfrac{2p\,(2mn + q + 1)\left\{1 - j\dfrac{1}{\alpha}\,\dfrac{2pq + n + 1}{2mn + q + 1}\right\}}{(n + 1) + j\,2mn\alpha - j\dfrac{p\,(q + 1)}{m\alpha} - \dfrac{2p^2 q}{m\alpha^2}}} \tag{22}$$

where m, n, p and q are the component ratios indicated in Fig. 4 and $\alpha = \omega CR$. A network of this type has a null provided

$$4mnpq = (n + 1)(q + 1) \tag{23}$$

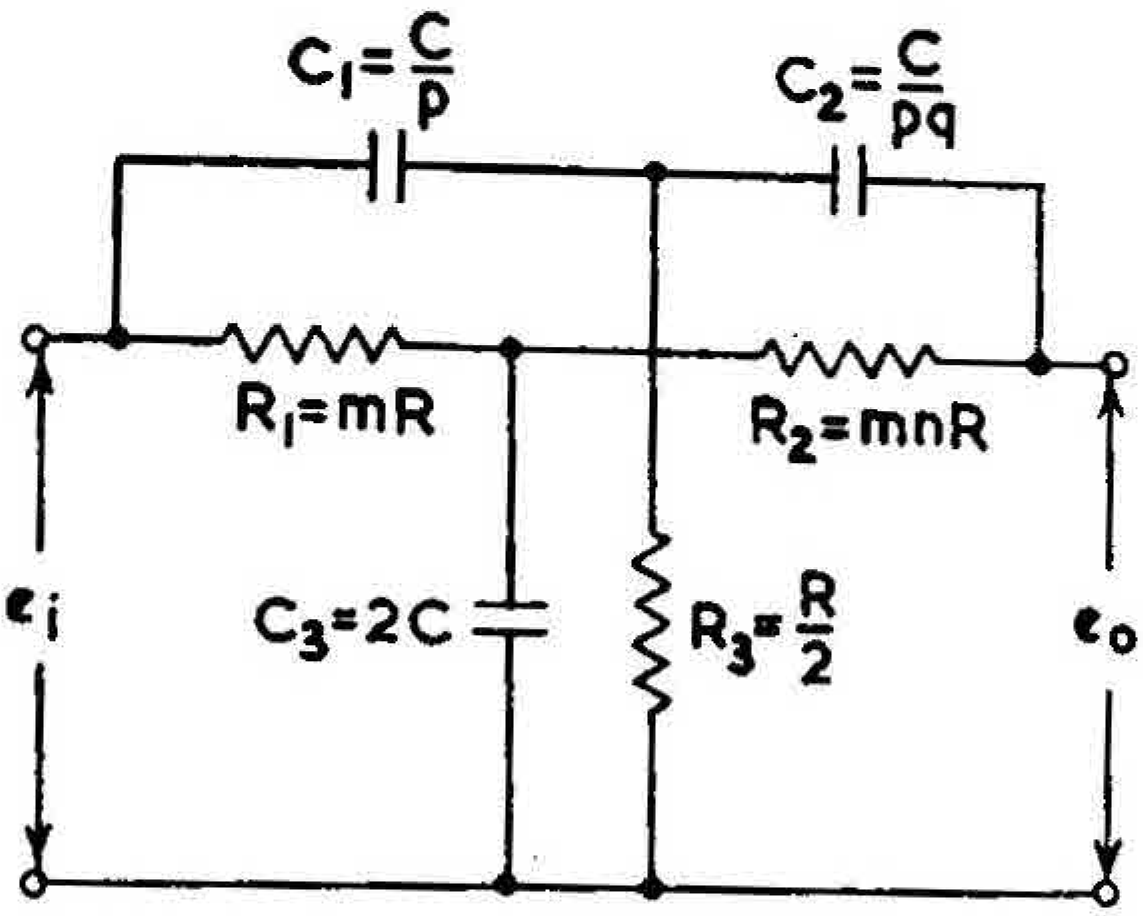

FIG. 4. A twin-T network of the general form.

and the null frequency is given by

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi CR}\sqrt{\frac{2p^2q}{m\,(n+1)}} = \frac{1}{2\pi CR}\sqrt{\frac{p\,(q+1)}{2m^2n}} \tag{24}$$

Using the above relations, by suitable manipulation the transmission can be expressed as

$$\beta = \frac{1}{1 + \dfrac{2p\,(2mn + q + 1)}{\sqrt{2np\,(q+1)}\,jy}} = \frac{1}{1 + \dfrac{\sqrt{\dfrac{2m\,(n+1)}{q}} + \sqrt{\dfrac{2p\,(q+1)}{n}}}{jy}}$$

$$= \frac{jy}{D + jy} \tag{25}$$

where

$$y = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$$

and

$$D = \sqrt{\frac{2m\,(n+1)}{q}} + \sqrt{\frac{2p\,(q+1)}{n}}$$

If this network is used for frequency selective feedback in an amplifier, under the same assumptions as before

$$G = \frac{A}{1 + A\beta} = \frac{A\left(1 + \dfrac{jy}{D}\right)}{1 + j\dfrac{(A+1)}{D}\,y} \tag{26}$$

The Q of the amplifier, by comparison with (10), is

$$Q_0 = \frac{A+1}{D} \tag{27}$$

For a given gain, the maximum value of Q_0 is obtained when D is minimum. It can be shown that the condition for the minimum value of D is $m = (q+1)/2n$ and $p = (n+1)/2q$ so that

$$D = 2\sqrt{\left(1 + \frac{1}{n}\right)\left(1 + \frac{1}{q}\right)}$$

When $n = q = 1$, *i.e.*, when the network is symmetrical, $D = 4$ and $Q = (A+1)/4$. The maximum value of Q is thus obtained when $R_1 = R_2 = 2R_3$ and $C_1 = C_2 = C_3/2$, which is a well-known result.[2]

7. Effect of Component Tolerances in Twin-T Network

To analyze the effect of component tolerance on the null of the network, the change in the transmission of the network when one component is varied will be examined. Let the series resistance on the input side change to where k is a small quantity (Fig. 5). The resistance on the output side which is unchanged

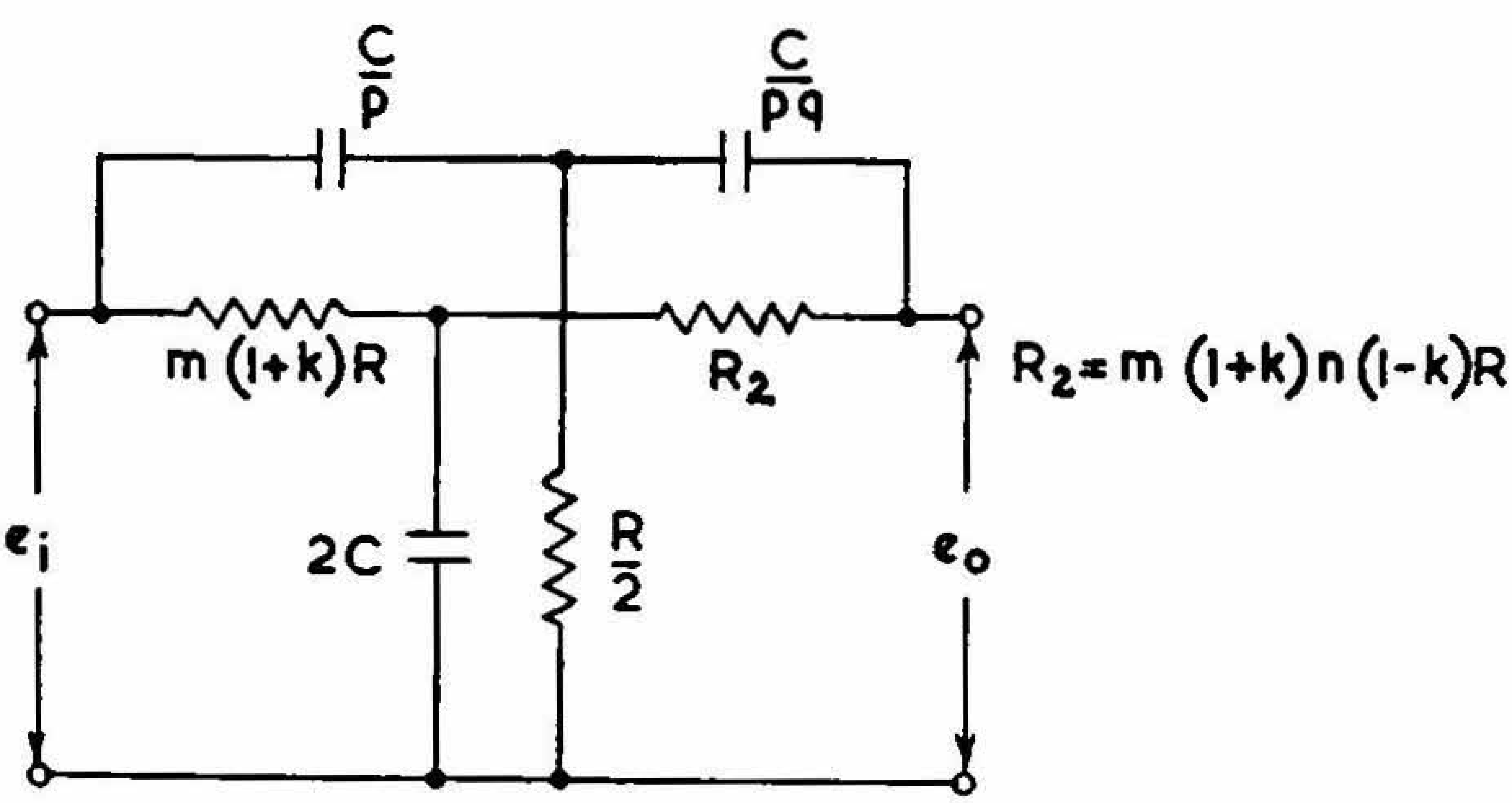

FIG. 5. A twin-T network with an error in the value of series resistance on the input side.

may now be represented by $m(1 + k)\,n(1 - k)$ R if the second order terms in k are neglected. The new value of β may be now obtained by substituting $m(1 + k)$ and $n(1 - k)$ for m and n in (22). Indicating by X the second term in the denominator of (22), the new value of X is

$$X = \frac{2p\,(2mn + q + 1)\left\{1 - j\,\frac{2pq}{(q+1)}\,(1 - ak)\,\frac{1}{a}\right\}}{(n + 1) - nk + j\,2mna - j\,\frac{p\,(q+1)}{m}\cdot\frac{1}{a}\,(1 - k) - \frac{2p^2q\,(1 - k)}{ma^2}} \tag{28}$$

where $a = n/(2pq + n + 1)$. In the above the second and higher order terms in k have been neglected and use has been made of the relation given by (23). By suitable manipulation, (28) can be expressed as

$$X = \frac{2p\,(2mn + q + 1)}{j\,2mn\alpha - j\,\dfrac{p\,(q+1)}{m}\,\dfrac{1}{\alpha}\,(1-k) - \dfrac{k.2pqn}{(2pq+n+1)}\;\dfrac{\left[1 + \dfrac{p}{m}\dfrac{1}{\alpha^2}\right]}{\left[1 - j\dfrac{1}{\alpha}\dfrac{2pq}{(q+1)}\,(1-\alpha k)\right]}} \tag{29}$$

The value of α which gives the maximum of X (*i.e.*, the null of the network) can be obtained by setting the imaginary part of the denominator equal to zero. While the exact analysis is complicated, the case $n = q$ is simple enough to be examined in detail. The ratio of the two series condensers would then equal the ratio of the two series resistances, though neither of the two ratios is unity. In this case, when $n = q = r$, say, the equation (23) reduces to

$$4mpr^2 = (r+1)^2$$

and therefore

$$\left(\frac{2pq}{q+1}\right)^2 = \frac{p}{m} \tag{30}$$

Making this substitution in the last term in the denominator of (29) and neglecting the quantity αk which is small and occurs in the error term which is itself small,

$$X = \frac{2p\,(2mr + r + 1)}{j\,2mr\alpha - j\,\dfrac{p\,(r+1)}{m}\,\dfrac{1}{\alpha}\,(1-k) - \dfrac{k.2pr^2}{(2pr+r+1)}\left[1 + j\dfrac{1}{\alpha}\,\dfrac{2pr}{(r+1)}\right]} \tag{31}$$

and the new null frequency is

$$\left.\begin{aligned} \omega_r &= \frac{1}{CR}\sqrt{\frac{p\,(r+1)}{2m^2r}}\;\gamma \\ \text{where}\quad \gamma^2 &= 1 - k\left[1 - \frac{r}{2pr + r + 1}\right] = 1 - k\,\frac{2pr+1}{2pr+r+1} \end{aligned}\right\} \tag{32}$$

The transmission at any frequency may be expressed in the form

$$\beta' = \frac{1}{1 + \dfrac{2p\,(2mr+r+1)\,/\,\gamma\sqrt{2pr\,(r+1)}}{jy - \dfrac{2p\quad\sqrt{2pr\,(r+1)}\;k}{(2pr+r+1)}}} \tag{33}$$

where

$$y = \frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}.$$

The expression reduces to the form of (25) when $k = 0$. At the new null frequency when $y = 0$

$$\beta_0 = \frac{1}{1 - \dfrac{(2mr + r + 1)(2pr + r + 1)}{kr^2}} = \frac{1}{1 - \dfrac{\eta}{k}}$$

where

$$\eta = \frac{(2mr + r + 1)(2pr + r + 1)}{r^2}$$

The gain at the null frequency is then

$$G_r = \frac{A}{1 + \beta_0 A} = \frac{A(k - \eta)}{(A + 1)k - \eta}$$

Differentiating the above

$$\left(\frac{dG_r}{dk}\right)_{k=0} = \frac{A^2}{\eta} = A^2 \frac{r^2}{(2mr + r + 1)(2pr + r + 1)} \tag{34}$$

By substituting (33) in the expression for gain with feedback, the equivalent Q of the amplifier is found to be

$$Q_r = \frac{(A + 1)\gamma\sqrt{2pr(r + 1)}}{2p(2mr + r + 1) - \dfrac{2pr^2k(A + 1)}{(2pr + r + 1)}} \tag{35}$$

and

$$\left(\frac{dQ_r}{dk}\right)_{k=0} = \frac{2pr^2}{(2pr + r + 1)\sqrt{2pr(r + 1)}} Q_0^2 - \frac{(2pr + 1)}{2(2pr + r + 1)} Q_0 \tag{36}$$

For high values of Q, the second term on the r.h.s. of (36) may be neglected. The fractional rates of change of the gain and Q may then be expressed thus:—

$$\left.\begin{aligned} \frac{1}{A}\left(\frac{dGr}{dk}\right)_{k=0} &= \frac{\gamma^2}{(2mr + r + 1)(2pr + r + 1)} A \doteq \frac{2pr^2}{(2pr + r + 1)\sqrt{2pr(r+1)}} \cdot Q_0 \\ \frac{1}{Q_0}\left(\frac{dQr}{dk}\right)_{k=0} &\doteq \frac{2pr^2}{(2pr + r + 1)\sqrt{2pr(r + 1)}} \cdot Q_0 \end{aligned}\right\} \tag{37}$$

The above relations can be expressed in other forms making use of the relation between m, p and r.

The effect of the variation in the capacitance on the input side may be calculated similarly, assuming that p and q change to $p(1 + k)$ and $q(1 - k)$. The fractional rates of change in this case are also found to be given by (37).

A small variation in the resistance on the output side namely mnR may be regarded as a change of the ratio n to $n(1+k)$, and the corresponding formulæ in this case may be shown to be

$$\left.\begin{aligned}\frac{1}{A}\left(\frac{dGr}{dk}\right)_{k=0} &= \frac{r}{(2mr+r+1)(2pr+r+1)}\,A \doteqdot \frac{2pr}{(2pr+r+1)\sqrt{2pr(r+1)}}\\ \frac{1}{Q_0}\left(\frac{dQr}{dk}\right)_{k=0} &= \frac{2pr}{(2pr+r+1)\sqrt{2pr(r+1)}}\end{aligned}\right\}\tag{38}$$

The formulæ for effect of capacitance variations are also precisely the same.

The effect of the variation of the shunt resistance R/2 is obtained by introducing an error factor mk into the ratio m, without any simultaneous change in n. This may also be regarded as a simultaneous change in the input and output resistances.* As may be expected, the fractional rate of change corresponding to the input and output resistances are:—

$$\left.\begin{aligned}\frac{1}{A}\left(\frac{dGr}{dk}\right)_{k=0} &= \frac{r(r+1)A}{(2mr+r+1)(2pr+r+1)} \doteqdot \frac{2pr(r+1)Q_0}{(2pr+r+1)\sqrt{2pr(r+1)}}\\ \frac{1}{Q_0}\left(\frac{dQr}{dk}\right)_{k=0} &= \frac{2pr(r+1)}{(2pr+r+1)\sqrt{2pr(r+1)}}\,Q_0\end{aligned}\right\}\tag{39}$$

The analysis of the variation in shunt capacitance gives the same formulæ for the fractional rates of change for the gain and Q.

An examination of (37) and (38) shows that the effect of tolerance in the components on the input side is r times that in the components on the output side. In a symmetrical network, however, $r = 1$ and the expressions (37) and (38) become identical. When $r = 1$, equation (30) reduces to the relation $mp = 1$, and so, in the case of tolerance in the series components,

$$\left.\begin{aligned}\frac{1}{A}\left(\frac{dGr}{dk}\right)_{k=0} &= \frac{Am}{4(m+1)^2} = \frac{Ap}{4(p+1)^2}\\ \frac{1}{Q_0}\left(\frac{dQr}{dk}\right)_{k=0} &= \frac{\sqrt{p}}{2(p+1)}\,Q_0 = \frac{\sqrt{m}}{2(m+1)}\cdot Q_0\end{aligned}\right\}\tag{40}$$

The corresponding factors for the shunt components will be twice the above figure.

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* The effect of component tolerance in the shunt impedances has not been considered by S. W. Punnet (*op. cit.*). As the fractional rate of change of Q in this case is twice that associated with the series components, the maximum possible figure for the simultaneous variation of two component ratios is thrice the figure for that of each of the series elements alone, and not twice as concluded by Punnet.

In the case of the symmetrical network, it follows from (25) and (27) that

$$Q = (A + 1) \frac{\sqrt{m}}{2(m + 1)} \tag{41}$$

Hence for a given value of A, maximum Q is obtained when $\sqrt{m}/2(m + 1)$ is maximum, whereas to have a minimum rate of change of Q with component errors, this factor is to be minimum. As these two are conflicting requirements, it follows that the reduced sensitivity to component errors may be obtained to any desired extent by suitable choice of m and p, but at the expense of the requirement of a higher gain in the amplifier to give the same value of Q. It is to be noted in this case, that the reduction in the fractional rate of change in gain and Q by half entails a two-fold increase in the gain of the amplifier. This may be compared with the corresponding result in the case of the Wien bridge circuit, (*vide* para 5), where an increase in gain of 50% is required in order to reduce the effect of component tolerance by half.

In the case of the twin-T network, there are three variable elements and hence two possible ratio errors. As one of these may be associated with the shunt element, which has twice the effect of the series element for the same percentage deviation of the component value, the maximum fractional rate of change of Q is three times that for the series element, *i.e.*, $3\sqrt{m}/2(m + 1) Q_0$. If the network is designed for maximum Q, $m = 1$ and this factor is $\frac{3}{4} Q_0$. (The corresponding factor for the Wien bridge is Q_0.) The factor $3\sqrt{m}/2(m + 1)$ varies slowly with m. For $m = 7$, it is nearly $\frac{1}{2}$ and hence the fractional rate of change is $Q_0/2$. The amplifier gain required is approximately $6Q_0$. This may be compared with the corresponding value for the Wien bridge, which is $12 Q_0$.

CONCLUSIONS

While feedback networks for low frequency selective amplifiers are generally chosen with a view to obtain the highest Q for a given gain, other factors may be considered in the case of those amplifiers in which the selected frequency has to be continuously variable over a range. Among these is the extent to which manufacturing tolerances of the variable components affect the constancy of gain and selectivity, expressed as the rate of change of the gain or Q with the component error factor. This may be considerably reduced below the value which prevails when the network is designed for maximum Q, by proper choice of component ratios. The change however necessitates an increase in the gain of the amplifier to obtain the same value of Q.

Experimental verification of the above analysis is in progress.

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