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PROPAGATION OF MICROWAVE THROUGH AN IMPERFECTLY CONDUCTING CYLINDRICAL GUIDE FILLED WITH AN IMPERFECT DIELECTRIC

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ABSTRACT

The propagation of hybrid wave $(EH)_{11}$ through an imperfectly conducting cylindrical guide filled completely with an imperfect dielectric has been treated as a boundary value problem. General expressions for attenuation and phase constants have been derived.

INTRODUCTION

The propagation of circularly symmetric wave through an imperfectly conducting guide has been treated by Carson, etc. (1936), Barrow (1936), Linder (1942), Hetrick (1950). But the problem of propagation of higher order waves through an imperfectly conducting guide filled with an imperfect dielectric has not yet been solved to the best knowledge of the author. It is the object of the present paper to discuss the propagation of higher order waves as a boundary value problem and derive general expressions for attenuation and phase constants for $(EH)_{11}$ wave by considering the losses contributed by the dielectric as well as the boundary wall of the guide.

FIELD COMPONENTS OF THE HYBRID $(EH)_{11}$ WAVE

In the case of a guide having infinitely conducting wall and filled with a perfect dielectric, the boundary conditions are simply $E_z = E_\phi = 0$ at the surface of the guide, and it is possible for either E or H wave to exist independently of the other. But, in the case of a guide having imperfectly conducting wall and containing an imperfect dielectric, the boundary conditions are the continuity of the tangential components of both electric and magnetic fields. This double set of boundary conditions show that the E or the H wave cannot exist alone unless the wave is circularly symmetric. In this case, both E_z and H_z are required for the analysis. Or, in other words, in the case of higher order waves we have to consider the co-existence of both the E and the H wave. Let us consider the case of $(EH)_{11}$ wave. The field components of E_{11} and H_{11} waves in the dielectric are

E_{11} Wave:

$$E_{z1} = A'' \cos\phi \, J_1(\chi_1{}^0 r) \exp(j\omega t - h^0 z)$$

$$E_{r1} = -A'' \frac{h^0}{\chi_1{}^0} \cos\phi \, J_1'(\chi_1{}^0 r) \exp(j\omega t - h^0 z)$$

$$E_{\phi 1} = A'' \frac{h^0}{\chi_1{}^0} \frac{1}{r} \sin\phi \, J_1(\chi_1{}^0 r) \exp(j\omega t - h^0 z) \tag{1}$$

$$H_{r1} = -\frac{jA''\omega\epsilon_1}{\chi_1{}^{0\,2}} \frac{1}{r} \sin\phi \, J_1(\chi_1{}^0 r) \exp(j\omega t - h^0 z)$$

$$H_{\phi 1} = -\frac{jA''\omega\epsilon_1}{\chi_1{}^0} \cos\phi \, J_1'(\chi_1{}^0 r) \exp(j\omega t - h^0 z)$$

H_{11} wave:

$$E_{r1} = \frac{jA^0\omega\mu_1}{h''^2 + k_1{}^2} \frac{1}{r} \sin\phi \, J_1(\chi''_1 r) \exp(j\omega t - h'' z)$$

$$E_{\phi 1} = \frac{jA^0\omega\mu_1}{\sqrt{h''^2 + k_1{}^2}} \cos\phi \, J_1'(\chi_1'' r) \exp(j\omega t - h'' z)$$

$$H_{z1} = A^0 \cos\phi \, J_1(\chi_1'' r) \exp(j\omega t - h'' z)$$

$$H_{r1} = -\frac{A^0 h''}{\sqrt{h''^2 + k^2_1}} \cos\phi \, J'_1(\chi_1'' r) \exp(j\omega t - h'' z)$$

$$H_{\phi 1} = \frac{A^0 h''}{h''^2 + k_1{}^2} \frac{1}{r} \sin\phi \, J_1(\chi_1'' r) \exp(j\omega t - h'' z) \tag{2}$$

In order to satisfy the energy conditions at infinity, the Bessel functions J's are to be replaced by the Hankel functions H's for the field components in the boundary wall of the guide. The field components for the E_{11} and H_{11} waves in the wall of the guide are

E_{11} wave:

$$E_{z2} = B'' \cos\phi \, H_1^{(1)}(\chi_2^0 r) \exp(j\omega t - h^0 z)$$

$$E_{r2} = -B'' \frac{\chi_2^0}{h^0} \cos\phi \, H_1^{(1)'}(\chi_2^0 r) \exp(j\omega t - h^0 z)$$

$$E_{\phi 2} = B'' \frac{h^0}{\chi_2^0} \frac{1}{r} \sin\phi \, H_1^{(1)}(\chi_2^0 r) \exp(j\omega t - h^0 z) \tag{3}$$

$$H_{r2} = -\frac{B''\sigma_2}{\chi_2^{0^2}} \frac{1}{r} \sin\phi \, H_1^{(1)}(\chi_2^0 r) \exp(j\omega t - h^0 z)$$

$$H_{\phi 2} = -\frac{B''\sigma_2}{\chi_2^0} \cos\phi \, H_1^{(1)'}(\chi_2^0 r) \exp(j\omega t - h^0 z)$$

H_{11} wave:

$$E_{r2} = \frac{jB^0\omega\mu_2}{h''^2 + k_2^2} \frac{1}{r} \sin\phi \, H_1^{(1)}(\chi_2'' r) \exp(j\omega t - h'' z)$$

$$E_{\phi 2} = \frac{jB^0\omega\mu_2}{\sqrt{h''^2 + k_2^2}} \cos\phi \, H_1^{(1)'}(\chi_2'' r) \exp(j\omega t - h'' z)$$

$$H_{z2} = B^0 \cos\phi \, H_1^{(1)}(\chi_2'' r) \exp(j\omega t - h'' z) \tag{4}$$

$$H_{r2} = -\frac{B^0 h''}{\sqrt{h''^2 + k_2^2}} \cos\phi \, H_1^{(1)'}(\chi_2'' r) \exp(j\omega t - h'' z)$$

$$H_{\phi 2} = \frac{B^0 h''}{h''^2 + k_2^2} \frac{1}{r} \sin\phi \, H_1^{(1)}(\chi_2'' r) \exp(j\omega t - h'' z)$$

where r, ϕ, z are the cylindrical co-ordinates, h^0 and h'' are the propagation constants for the E_{11} and the H_{11} wave respectively; μ_1, ϵ_1, σ_1 and μ_2, ϵ_2, σ_2 refer to the properties of the dielectric and the boundary wall of the guide respectively

$$k_1^2 = \omega^2\mu_1\epsilon_1 \qquad k_2^2 \simeq -j\omega\mu_2\sigma_2$$

$$\chi_1^0 = \sqrt{h^{0^2} + k_1^2} \qquad \chi_2^0 = \sqrt{h^{0^2} + k_2^2} \tag{5}$$

$$\chi_1'' = \sqrt{h''^2 + k_1^2} \qquad \chi_2'' = \sqrt{h''^2 + k_2^2}$$

The subscripts 1 and 2 refer to the dielectric and metal wall of the guide respectively. A″, B″ and A^0, B^0 are constants depending on excitation for the E_{11} and the H_{11} wave respectively. ω is the angular frequency of excitation.

The hybrid wave is formed by the superposition of the E_{11} and H_{11} waves. The field components for the $(EH)_{11}$ wave in the dielectric and the wall of the guide are respectively as follows:—

$$\begin{aligned}
E_{z1} &= A'' \cos\phi \, J_1(\chi_1{}^0 r) \exp(j\omega t - h^0 z) \\
E_{\phi 1} &= A'' \frac{h^0}{\chi_1{}^0} \frac{1}{r} \sin\phi \, J_1(\chi_1{}^0 r) \exp(j\omega t - h^0 z) \\
&\quad + \frac{jA^0 \omega \mu_1}{\chi_1{}''} \cos\phi \, J_1{}'(\chi_1{}'' r) \exp(j\omega t - h'' z) \\
E_{r1} &= - A'' \frac{h^0}{\chi_1{}^0} \cos\phi \, J_1{}'(\chi_1{}^0 r) \exp(j\omega t - h^0 z) \\
&\quad + \frac{jA^0 \omega \mu_1}{\chi_1{}''^2} \frac{1}{r} \sin\phi \, J_1(\chi_1{}'' r) \exp(j\omega t - h'' z) \qquad (6) \\
H_{r1} &= - \frac{jA'' \omega \epsilon_1}{\chi_1{}^{02}} \frac{1}{r} \sin\phi \, J_1(\chi_1{}^0 r) \exp(j\omega t - h^0 z) \\
&\quad - \frac{A^0 h''}{\chi_1{}''} \cos\phi \, J_1{}'(\chi_1{}'' r) \exp(j\omega t - h'' z) \\
H_{\phi 1} &= - \frac{jA'' \omega \epsilon_1}{\chi_1{}^0} \cos\phi \, J_2{}'(\chi_1{}^0 r) \exp(j\omega t - h^0 z) \\
&\quad + \frac{A^0 h''}{\chi_1{}''^2} \frac{1}{r} \sin\phi \, J_1(\chi_1{}'' r) \exp(j\omega t - h'' z) \\
H_{z1} &= A^0 \cos\phi \, J_1(\chi_1{}'' r) \exp(j\omega t - h'' z)
\end{aligned}$$

$$\begin{aligned}
E_{z2} &= B'' \cos\phi \, H_1{}^{(1)}(\chi_2{}^0 r) \exp(j\omega t - h^0 z) \\
E_{\phi 2} &= B'' \frac{h^0}{\chi_2{}^0} \frac{1}{r} \sin\phi \, H_1{}^{(1)}(\chi_2{}^0 r) \exp(j\omega t - h^0 z) \\
&\quad + \frac{jB^0 \omega \mu_2}{\chi_2{}''} \cos\phi \, H_1{}^{(1)\prime}(\chi_2{}'' r) \exp(j\omega t + h'' z) \\
E_{r2} &= - B'' \frac{h^0}{\chi_2{}^0} \cos\phi \, H_1{}^{(1)\prime}(\chi_2{}^0 r) \exp(j\omega t - h^0 z) \\
&\quad + \frac{jB^0 \omega \mu_2}{\chi_2{}''^2} \frac{1}{r} \sin\phi \, H_1{}^{(1)}(\chi_2{}''^2) \exp(j\omega t - h'' z) \qquad (7) \\
H_{r2} &= - \frac{B'' \sigma_2}{\chi_2{}^{02}} \frac{1}{r} \sin\phi \, H_1{}^{(1)}(\chi_2{}^0 r) \exp(j\omega t - h^0 z) \\
&\quad - \frac{B^0 h''}{\chi_2{}''} \cos\phi \, H_1{}^{(1)\prime}(\chi_2{}'' r) \exp(j\omega t - h'' z)
\end{aligned}$$

$$H_{\phi_2} = -\frac{B''\sigma_2}{\chi_2{}^0}\cos\phi\, H_1{}^{(1)'}(\chi_2{}^0 r)\exp(j\omega t - h^0 z)$$

$$+\frac{B^0 h''}{\chi_2{}''^2}\frac{1}{r}\sin\phi\, H_1{}^{(1)}(\chi_2{}'' r)\exp(j\omega t - h'' z)$$

$$H_{z2} = B^0\cos\phi\, H_1{}^{(1)}(\chi_2{}'' r)\exp(j\omega t - h'' z).$$

BOUNDARY CONDITIONS

At $r = a$, the inner radius of the guide, the following conditions are satisfied

$$E_{z1}(a) = E_{z2}(a) \qquad H_{z1}(a) = H_{z2}(a)$$

$$E\phi_1(a) = E\phi_2(a) \qquad H\phi_1(a) = H\phi_2(a) \tag{8}$$

CONDITIONAL EQUATIONS

Applying the boundary conditions (Equation 8) the following conditional equation is obtained from (6) and (7)

$$\frac{h^0 h''}{a^2\omega}\tan^2\phi\left[\frac{1}{\chi_1{}^0} - \frac{1}{\chi_2{}^0}\right]\left[\frac{1}{\chi_2{}''^2} - \frac{1}{\chi_1{}''^2}\right]$$

$$= \left[\frac{\omega\epsilon_1}{\chi_1{}^0}\frac{J_1'(\chi_1{}^0 a)}{J_1(\chi_1{}^0 a)} + j\frac{H_1{}^{(1)'}(\chi_2{}^0 a)}{H_1{}^{(1)}(\chi_2{}^0 a)}\right]\left[\frac{\mu_2}{\chi_2{}''}\frac{H_1{}^{(1)'}(\chi_2{}'' a)}{H_1{}^{(1)}(\chi_2{}'' a)} - \frac{\mu_1}{\chi_1{}''}\frac{J_1'(\chi_1{}'' a)}{J_1(\chi_1{}'' a)}\right] \tag{9}$$

Substituting $\mu_2 = \mu_1 = \mu_0$ and $\epsilon_1 = \epsilon - j\frac{\sigma_1}{\omega}$ in (9) and separating the real and the imaginary parts, the following two equations are obtained.

$$\frac{h^0 h''}{\mu_0 a^2\omega}\tan^2\phi\left[\frac{1}{\chi_1{}^0} - \frac{1}{\chi_2{}^0}\right]\left[\frac{1}{\chi_2{}''^2} - \frac{1}{\chi_1{}''^2}\right]$$

$$= \left[\frac{\omega\epsilon_1}{\chi_1{}^0\chi_2{}''}\frac{J_1'(\chi_1{}^0 a)}{J_1(\chi_1{}^0 a)}\frac{H_1{}^{(1)'}(\chi_2{}'' a)}{H_1{}^{(1)}(\chi_2{}'' a)} - \frac{\omega\epsilon}{\chi_1{}^0\chi_1{}''}\frac{J_1'(\chi_1{}^0 a)}{J_1(\chi_1{}^0 a)}\frac{J_1'(\chi_1{}'' a)}{J_1(\chi_1{}'' a)}\right] \tag{10}$$

$$\frac{\chi_1{}''}{\chi_2{}''}\frac{H_1{}^{(1)'}(\chi_2{}'' a)}{H_1{}^{(1)}(\chi_2{}'' a)} = \frac{J_1'(\chi_1{}'' a)}{J_1(\chi_1{}'' a)} \tag{11}$$

PROPAGATION FACTOR

Using recurrence relations, the Equation 11 reduces to

$$\frac{\chi_1{}''}{\chi_2{}''^2 a} + \frac{\chi_1{}''}{\chi_2{}''}\frac{H_0{}^{(1)}(\chi_2{}'' a)}{H_1{}^{(1)}(\chi_2{}'' a)} = \frac{1}{\chi_1{}'' a} - \frac{J_0(\chi_1{}'' a)}{J_1(\chi_1{}'' a)} \tag{12}$$

Using the following expressions for large arguments

$$H_0^{(1)}(\chi_2''a) = \sqrt{\frac{2}{\pi\chi_2''a}}\, e^{j(\chi_2''a - \frac{\pi}{4})}$$

$$H_1^{(1)}(\chi_2''a) = \sqrt{\frac{2}{\pi\chi''a}}\, e^{j(\chi_2''a - \frac{3}{4}\pi)} \tag{13}$$

the Equation (12) yields

$$\frac{J_0(\chi_1''a)}{J_1(\chi_1''a)} = \frac{1}{\chi_1''a} - \frac{\chi_1''}{\chi_2''^2 a} - j\frac{\chi_1''}{\chi_2''} \tag{14}$$

Expanding J's in the above equation by complex Taylor series

$$J_0(\chi_1''a) = J_0(r_{01}) + (\chi_1''a - r_{01})\, J_0'(r_{01}) + \ldots\ldots..$$

$$J_1(\chi_1''a) = J_1(r_{01}) + (\chi_1''a - r_{01})\, J_1'(r_{01}) + \ldots\ldots\ldots \tag{15}$$

where $r_{01} = 2 \cdot 4048$. Substituting (15) in (14), using the relation $\chi_2''^2 = \chi_1''^2 - k_1^2 + k_2^2$, rearranging the terms, separating the real and the imaginary parts, we obtain the following two equations

$$a^2 r_{01}\chi_1''^4 - a r_{01}^2 \chi_1''^3 + \chi_1''^2 (a^2 r_{01} k_2^2 - a^2 r_{01} k_1^2 - 2a r_{01} + 2r_{01})$$
$$+ \chi_1''(a r_{01}^2 k_1^2 - a r_{01}^2 k_2^2 + a k_1^2 - a k_2^2) + 2r_{01}(k_2^2 - k_1^2) = 0 \tag{16}$$

$$a^2\chi_1''^2 [\chi_1'' - 2r_{01}] \sqrt{\chi_1''^2 - k_1^2 + k_2^2} = 0 \tag{17}$$

The propagation factor χ_1'' can be found from any of the above equations.

PROPAGATION CONSTANT

Equation (17) leads to the following conclusions:—

(i) $\chi_1''^2 = 0$ which gives the propagation constant $h'' = jk_1$ which yields the attenuation constant $\alpha = 0$.

(ii) $\chi_1'' - 2r_{01} = 0$ which yields $h'' = j\sqrt{k_1^2 - 4r_{01}^2}$ or $h'' = -\sqrt{4r_{01}^2 - k_1^2}$. In the former case, $\alpha = 0$, $\beta = \sqrt{k_1^2 - 4r_{01}^2}$. In the latter case α is negative and $\beta = 0$.

(iii) $\chi_1''^2 = k_1^2 - k_2^2$ or $h'' = jk_2$ which gives $\alpha = 0$. So, the solution of Equation (17) for χ_1'' is physically inadmissible in the present case. Let us consider the Equation (16) which is quartic in χ_1''. The four roots of this equation obtained by Ferrari's method (Burnside and Panton, 1924) are as follows:—

$$
\begin{aligned}
{}^{1}\chi_1'' &= \tfrac{1}{2}\left[-\left(\frac{\alpha'}{2}-e\right)+\left\{\left(\frac{\alpha'}{2}-e\right)^2-(2y-4f)\right\}^{\frac{1}{2}}\right]\\
{}^{2}\chi_1'' &= \tfrac{1}{2}\left[-\left(\frac{\alpha'}{2}-e\right)-\left\{\left(\frac{\alpha'}{2}-e\right)^2-(2y-4f)\right\}^{\frac{1}{2}}\right]\\
{}^{3}\chi_1'' &= \tfrac{1}{2}\left[-\left(\frac{\alpha'}{2}+e\right)+\left\{\left(\frac{\alpha'}{2}+e\right)^2-(2y+4f)\right\}^{\frac{1}{2}}\right]\\
{}^{4}\chi_1'' &= \tfrac{1}{2}\left[-\left(\frac{\alpha'}{2}+e\right)-\left\{\left(\frac{\alpha'}{2}+e\right)^2-(2y+4f)\right\}^{\frac{1}{2}}\right]
\end{aligned}
\tag{18}
$$

where,

$$
\begin{aligned}
y &= \sqrt[3]{A}+\sqrt[3]{B}-\frac{\lambda}{3}\\
A &= -\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}\\
B &= -\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}\\
p &= \xi-\frac{\lambda^2}{3}\\
q &= \Gamma-\frac{\lambda\xi}{3}+\frac{2\lambda^3}{27}\\
\lambda &= -\beta'\\
\xi &= (\alpha'\gamma-4\delta)\\
\Gamma &= 4\beta'\delta-\alpha'^2\delta-\gamma^2\\
e^2 &= \frac{\alpha'^2}{4}-\beta'+y\\
f^2 &= -\delta+\frac{y^2}{4}\\
\alpha' &= -\frac{r_{01}}{a}\\
\beta^1 &= \left(k_2^2-k_1^2-\frac{2}{a}+\frac{2}{a^2}\right)
\end{aligned}
\tag{19}
$$

$$\gamma = \left(\frac{r_{01}}{a}k_1^2 - \frac{r_{01}}{a}k_2^2 + \frac{k_1^2}{ar_{01}} - \frac{k_2^2}{ar_{01}}\right)$$

$$\delta = \frac{2}{a^2}(k_2^2 - k_1^2)$$

In Equation (18) y is the real root of the cubic

$$y^3 - \beta' y^2 + (\alpha'\gamma - 4\delta)\,y + 4\beta'\delta - \alpha'^2\delta - \gamma^2 = 0$$

The other two roots of y being imaginary have been discarded. Applying the relation $h'' = \sqrt{\chi_1''^2 - k_1^2} = \alpha + j\beta$ in Equation (18) where α is the attenuation constant and β is the phase constant, we find that h'' in the case of the first and the second root assumes the form

$$h''_{1,2} = \pm\left[\left(\frac{s+t}{2}\right)^{\frac{1}{2}} \mp j\left(\frac{s-t}{2}\right)^{\frac{1}{2}}\right] \tag{20}$$

In the case of the third and the fourth root h'' assumes the form

$$h''_{3,4} = \pm\left[\left(\frac{s'+t'}{2}\right)^{\frac{1}{2}} \mp j\left(\frac{s'-t'}{2}\right)^{\frac{1}{2}}\right]. \tag{21}$$

where the negative signs inside the brackets are for first and third roots only and

$$t = \tfrac{1}{2}\left\{\left(\frac{\alpha'}{2} - e\right)^2 - (y - 2f)\right\} - \omega^2\mu_0\epsilon_1$$

$$u = \tfrac{1}{2}\left(\frac{\alpha'}{2} - e\right)\left\{(2y - 4f) - \left(\frac{\alpha'}{2} - e\right)^2\right\}^{\frac{1}{2}}$$

$$s = [t^2 + u^2]^{\frac{1}{2}} \tag{22}$$

$$t' = \tfrac{1}{2}\left\{\left(\frac{\alpha'}{2} + e\right)^2 - (y + 2f)\right\} - \omega^2\mu_0\epsilon_1$$

$$u' = \tfrac{1}{2}\left(\frac{\alpha'}{2} + e\right)\left\{(2y + 4f) - \left(\frac{\alpha'}{2} + e\right)^2\right]^{\frac{1}{2}}$$

$$s' = [t'^2 + u'^2]^{\frac{1}{2}}$$

It will be observed from (20) and (21) that for h_1'' and h_3'', the attenuation constant α is positive when β is negative and *vice versa*. This means the existence of either a backward wave of decreasing amplitude or a forward wave of increasing amplitude. As we are concerned with a forward wave of decreasing amplitude, the two progapation constans h_1'' and h_3'' and hence ${}^1\chi_1''$ and ${}^3\chi_1''$ being physically inadmissible are discarded. In order to find which of the other two roots is valid let $\epsilon_1 = 2 \cdot 5$ and $\sigma_2 = 5 \cdot 7 \times 10^5$ ℧/cm., $a = 2$ cm. Substituting these values of ϵ_1, σ_2 and a in (19) it is found that both e and f are $>> \alpha'$. Also, $e^2 \simeq y$ and $f \simeq y/2$. Substituting the values of e and f and neglecting α' compared to e and f in the expression for ${}^2\chi_1''$, it is found that ${}^2\chi_1''$ nearly vanishes and $h'' \simeq jk_1$ which yields $\alpha = 0$ and $\beta = k_1$. The root ${}^2\chi_1''$, therefore, does not yield a valid solution. Considering the other root it is found that $\chi_1 \simeq \frac{1}{2} [-e - (e^2 - 4y)^{\frac{1}{2}}]$ which yields

$$h = [\tfrac{1}{2}(s_1 - t_1)]^{\frac{1}{2}} + j\,[\tfrac{1}{2}(s_1 + t_1)]^{\frac{1}{2}} \tag{23}$$

where the primes and the superscript in χ_1 and h have been dropped and

$$t_1 = k_1^2 + \frac{e^2}{2},\; u_1 = \frac{\sqrt{3}}{2} e^2,\; s_1' = (t_1^2 + u_1^2)^{\frac{1}{2}} \tag{24}$$

Attenuation and Phase Constants

From the value of the propagation constant h (Equation 23), the following expressions for the attenuation constant and the phase constant are obtained

$$\alpha = \frac{1}{\sqrt{2}} \left[\left\{ \left(k_1^2 + \frac{e^2}{2} \right)^2 - \frac{3}{4} e^4 \right\}^{\frac{1}{2}} - k_1^2 - \frac{e^2}{2} \right]^{\frac{1}{2}}$$

$$\beta = \frac{1}{\sqrt{2}} \left[\left\{ \left(k_1^2 + \frac{e^2}{2} \right)^2 - \frac{3}{4} e^4 \right\}^{\frac{1}{2}} + k_1^2 + \frac{e^2}{2} \right]^{\frac{1}{2}} \tag{25}$$

where e depends on the electrical properties of the boundary wall of the guide and the enclosed dielectric.

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