A PRECISE METHOD OF DETERMINING THE RATIO AND PHASE ANGLE ERRORS IN HIGH POTENTIAL INSTRUMENT TRANSFORMERS

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SUMMARY

An analysis of the available null methods indicates that difficulties are encountered with the accurate determination of the ratio and phase angle errors of potential transformers in which voltages in excess of 30 KV are involved. The use of a high voltage condenser with a negligible loss angle has become universal. The uncertainties of the auxiliary apparatus in the different methods together with the circuit arrangement set a definite limit to the overall accuracy of the measurements.

This paper describes a simple set-up which is capable of high sensitivity and accuracy of observation and derives theoretical expressions for accurate calculations. Vector diagrams are presented for balance conditions. The factors influencing convergence and sensitivity are also considered. A brief description of the character of measuring equipment and technique employed precedes an investigation into errors of typical single-phase and three-phase high potential transformers under various load conditions and some comparative data are also obtained. The paper concludes with a discussion of test results.

1. Introduction

The growing tendency of transmitting power at extra-high voltages necessitates a constant review of measurement techniques of high voltages. An important measurement in high voltage work is the determination of the ratio and phase angle errors in high potential instrument transformers since the accuracy of the tests enters directly into the charges for electrical energy. The various methods of testing potential transformers are conveniently divided into the following two classes: Comparison Method and Absolute Method. For purposes of routine testing, it is common practice to employ the comparison method requiring a standard transformer of the same nominal ratio as the transformer under test. The ratio and phase angle of the standard transformer are themselves determined by an absolute method. The absolute method of measurement of errors is, therefore, of greater significance than the comparison method and the null methods in this respect offer the most convenient means of assessing the performance characteristics of potential transformers.

The object of this paper is two-fold: firstly, to review the laboratory methods for the measurements of the ratio and phase angle errors in potential transformers in excess of 30 KV; secondly, to present the study of an accurate and precise method of determining the errors using a condenser-potential-divider in conjunction with a thermionic-amplifier-detector.

2. General Considerations

The potentiometer principle forms the basis of null methods of testing potential transformers. A fraction of the secondary voltage is set in opposition to an equal portion of the primary voltage derived from some form of potential divider. Since the voltages differ in phase on account of the impedance drops in the windings, a suitable phase shifting device is incorporated in the system. The measuring circuit is adjusted to indicate a balance of magnitude as well as of phase on the detector. The ratio and phase angle are found from the settings of the circuit in this condition.

For low and moderately high voltages the impedance on the high voltage side can conveniently be a resistance-potential-divider. At all points of the divider the current should be proportional to and in phase with the voltage. In potential transformer testing where resistances above 100,000 ohms are involved, capacitance presents a serious problem. For instance, the impedance of a resistor having a d.c. resistance R shunted by a capacitance C is given by:

$$Z = R(1 - \omega^2C^2R^2) - j\omega CR^2(1 - \omega^2C^2R^2)$$

where

$$\omega = 2\pi \times \text{frequency}$$

so that the a.c. resistance is

$$R_{a.c.} = R(1 - \tfrac{1}{2}\omega^2C^2R^2)$$

and the phase angle is

$$\theta = \arctan(-\omega CR)$$

The phase error is thus nearly proportional to the frequency, the capacitance and the resistance, while the fractional error in the a.c. resistance is of the same order of magnitude as the square of the phase angle in radians. In practice, the self-capacitance is not lumped between the resistor ends: each element of the resistor has a capacitance to earth and to the conductor. Nevertheless, the above expressions show the trend in the relationship between the phase angle and the magnitude of the resistor.

In order to keep down the dissipation of a large amount of power it is, however, necessary to use a resistor of a large value. In any form of practical construction this is accompanied by an increase in capacitance so that the phase angle will thereby increase in proportion to the square of the resistance. The only practical remedy is to have an elaborate shielding system to reduce the phase error to a reasonable limit. In the 132 KV shielded potentiometer described by Welles the net phase angle at a frequency of 60 cycles per second ranges from $-7'$ at 44 KV to $-4'$ at 132 KV. Shielded resistors are installed at the Bureau of Standards and with some refinements at the National Physical Laboratory, England, for voltages up to 30 KV. At the National Physical Laboratory, England, Davis has designed a shielded resistor with a phase angle of $0{\cdot}7'$ at 40 KV and the resistor due to Bowdler is suitable for the measurement of magnitudes up to 100 KV. At high voltages the determination of time constants of such resistors poses a difficult problem and in any case the apparatus is cumbersome and costly. In precise work a resistor cannot be recognised as the best proposition for the present purpose.

The success of high voltage condensers has led to several methods of testing potential transformers in which condenser potential-dividers are employed. Two types are of interest—air condenser and compressed gas condenser. The air condenser has a dielectric of air at atmospheric pressure, while the compressed gas condenser has as the dielectric gas held under high pressure. The former is readily influenced by dust and humidity and its bulk is a serious defect. The latter is compact and portable, constant in capacitance, completely shielded and practically free from losses. It is built even for a working voltage of the order of 500 KV. A modern compressed gas condenser possesses the necessary characteristics of a reference standard principally for voltages in excess of 30 KV.

3. METHODS FOR LABORATORY TESTS

The method developed by Churcher is shown in Fig. 1. The high and low tension windings of the transformer to be tested are joined together so that the applied primary voltage acts in the same sense as the induced secondary voltage, as shown by the arrows. The junction of the two windings is earthed. The condenser C_1 and C_2 form a condenser-potential-divider, their junction being connected through a shunted vibration galvanometer. The secondary of a small mutual inductor M and one end of an auxiliary mutual inductor m are connected to the earthed point. The condenser C_1 is a standard air condenser and the condenser C_2 usually consists of several fixed mica standards in parallel with a variable air condenser. The necessary phase compensation is effected by the

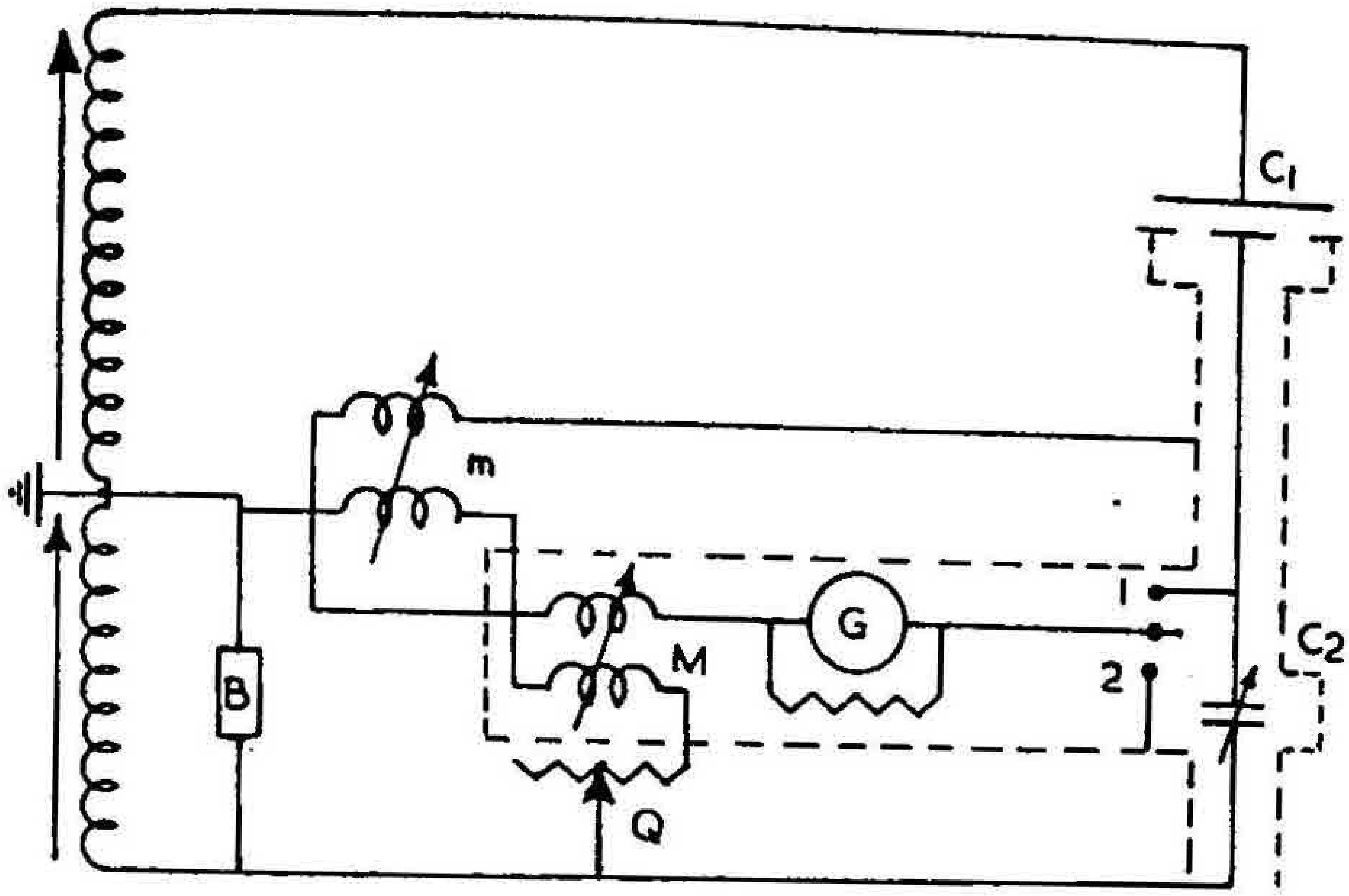

FIG. 1

mutual inductor M, the primary of which, in series with a resistance Q, is joined across the auxiliary terminals. Shields are provided for M, the vibration galvanometer G and its shunt and C_2; the lead joining C_1 and C_2 is also shielded. All the shields are joined together through the secondary of *m* to the earthead point. Balance is secured by regulation of C_2, M, varying Q also if necessary, with the test burden B. The switch contact 2 serves to effect subsidiary balance.

Yoganandam has described a method in which the arrangement utilises the Schering bridge equipment as shown in Fig. 2. The windings of the transformer

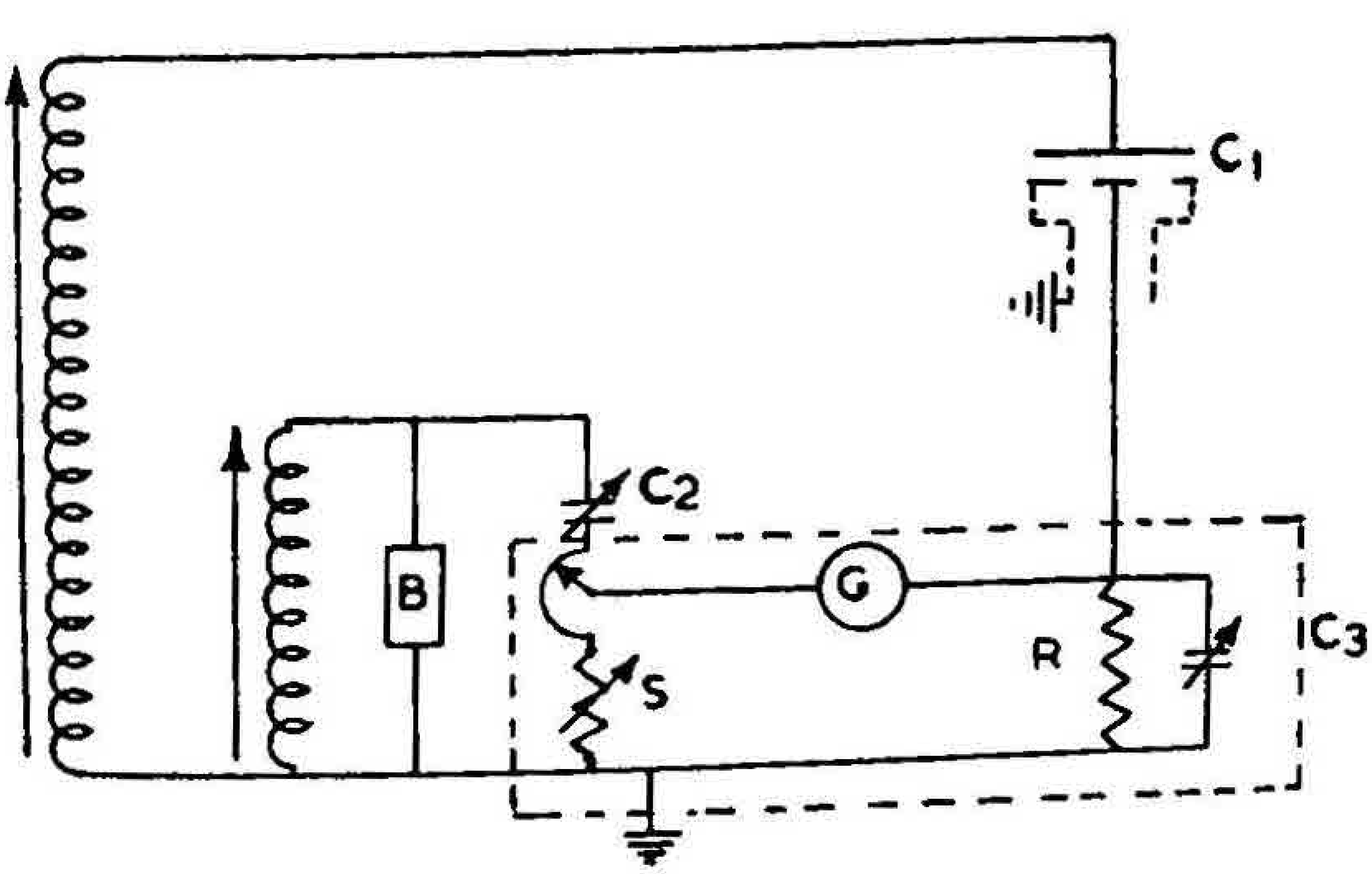

FIG. 2

are joined in opposition, the junction being earthed. The secondary with the test burden B closes on a branch composed of a variable mica condenser C_2

in series with a low resistance slide wire and a variable resistance S. The voltage divider across the primary side consists of the high voltage condenser C_1 in series with the resistance R shunted by a variable condenser C_3. The condenser C_2 is chosen so that C_2/C_1 is about equal to the nominal ratio of the transformer. As in the Schering bridge at 50 cycles per second, R may be fixed at $1{,}000/\pi$ ohms so that the reading of C_3 in microfarad gives the phase error in tenths of a radian. Earthed shields are provided as shown in the diagram. Balance is secured by adjustment of S and C_3 until the vibration galvanometer shows zero deflection.

Dannatt's arrangement is shown diagrammatically in Fig. 3. The standard air condenser C_1 is connected with a variable resistance R across the primary voltage, the secondary voltage divider consisting of two resistances r_1 and r_2 joined in series with the primary winding of a mutual inductor M. A variable condenser is placed across either r_1 and L_1, the self-inductance of M or r_2, the two portions retarding or advancing the voltage induced in the secondary of M respectively. The guard ring of C_1 is connected to a network y consisting

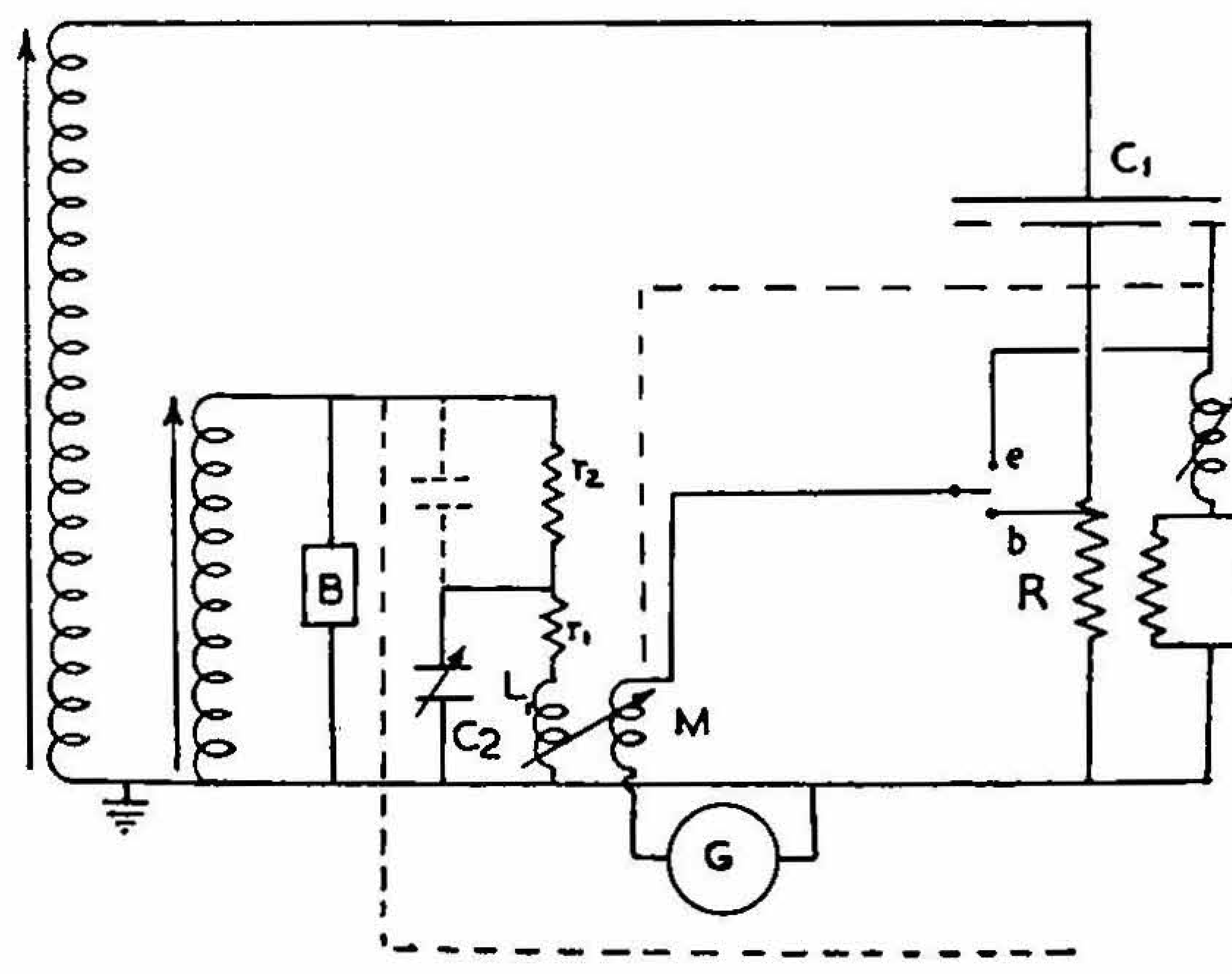

FIG. 3

of an inductance in series with a condenser-resistance parallel group. This network, as in Churcher's method, is adjusted successively until the guard circuit connected to e is at the same potential as b. Balance with the burden B is secured by varying R and C until the vibration galvanometer G shows zero deflection.

With a slight modification of Dannatt's method Jimbo and Sakimura have introduced the network illustrated in Fig. 4. C_1 is a high voltage compressed gas condenser with an effective series resistance r_1, and C_2 is a decade mica condenser. The leakage resistance of both the mica condenser and the lead connecting C_1 and C_2 is represented by R_l and the electrostatic capacitance of the cable C_l. C_2 is shunted by a fixed resistance r_0 in series with a slide wire. Across the secondary with a burden B is connected a fixed resistance R in series

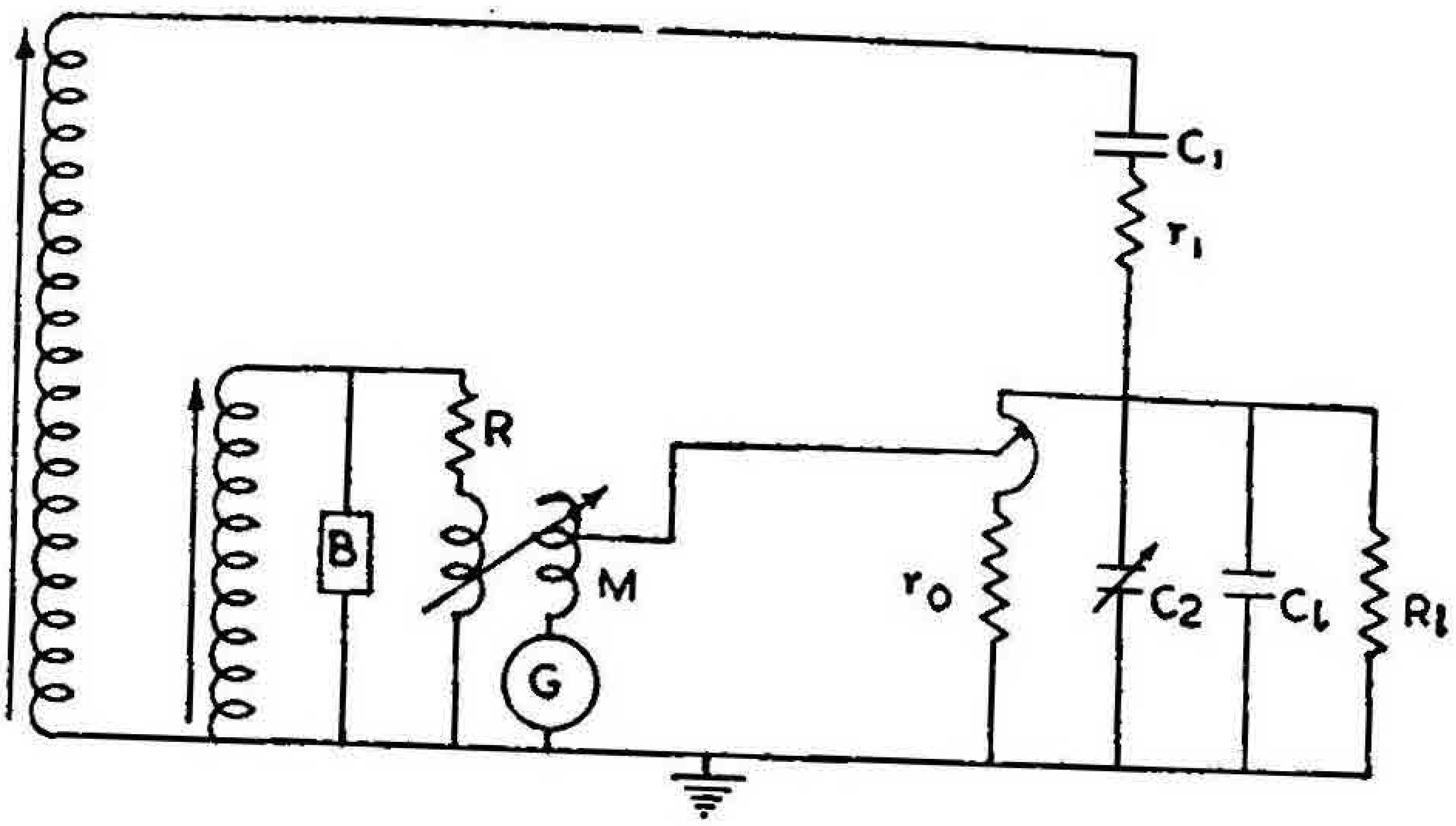

Fig. 4

with the primary of a mutual inductor M provided with taps on the secondary side to suit the nominal ratio of the transformer under test. Balance is obtained on the vibration galvanometer by adjustment of r_0 and C_2.

The principal defect of a high voltage air condenser is its dimensions on account of the large clearances required to avoid dielectric breakdown and losses: proper care is to be taken to maintain cleanliness of the surfaces and to avoid high humidities. Jimbo and Sakimura have overcome these difficulties by employing a compressed gas condenser in their method. These investigators have shown that the defect in quadrature of such a condenser is less than 1 part in a millionth of a radian at 50 cycles per second at 80 KV. With the improved construction especially the present-day compressed gas condenser forms a most reliable reference standard in high voltage testing.

In the methods of Churcher and Yoganandam the accuracy of the transformer ratio rests chiefly on the ratio of the condensers C_2 and C_1. The high voltage condenser C_1 presents no serious uncertainty. In precise work, when the variable condenser C_2 consists of several mica condensers in parallel, it is a matter of great difficulty to determine and allow for the capacities and residuals of several units under the experimental conditions. Dannatt, in introducing a refinement by substituting a resistor for the auxiliary condenser, avoids its importance in ratio errors. Jimbo and Sakimura achieve a similar result by placing a low valued resistor across the high impedance auxiliary condenser. In all the set-ups, however, with capacitances below $0{\cdot}01\,\mu F$, if considerable care is not exercised in calibration, the power factor may be widely in error and the corrections especially in a phase angle measurement may be quite important. So far as resistance is concerned, the medium-valued resistors used in the circuits pose no special difficulty in ensuring the highest accuracy in the measurements.

To accomplish phase compensation the mutual inductor is undoubtedly a convenient standard and for accurate work it is necessary to know its phase defect and also the resistance and self-inductance of the primary winding at the operating conditions. However, methods containing a condenser standard are less liable to inductive interference than those using mutual inductance standards.

In these methods of testing, a vibration galvanometer is employed as the detector. Since it is a tuned instrument the frequency of the a.c. supply must be very steady if high sensitivity is to be maintained. However, by increasing the damping of the galvanometer a flattened resonance curve is obtained, thereby the loss of sensitivity with small changes in frequency becomes much less important; but the available sensitivity is reduced. Moreover, the sensitivity conditions involve explicitly the frequency. Hence the vibration galvanometer is nowadays tending to be superseded by the extremely sensitive thermionic-amplifier-detector whose sensitivity is substantially constant over the working range.

Where the guard electrode of a high voltage condenser is directly earthed and not through a Wagner earthing device, an endeavour must be made to eliminate the effect of the shunting capacitance between the low voltage electrode and the guard electrode and also of the leakage resistance and electrostatic capacitance of the shielded lead from the low voltage electrode to the measuring system. In this respect Yoganandam's network is open to some error.

4. PRESENT INVESTIGATION

The idealized arrangement of the network for positive phase angle is illustrated in Fig. 5, the phase angle being considered positive when the reversed secondary voltage vector V_2 leads the primary voltage vector V_1. The primary and secondary windings of the transformer under test are connected in opposition

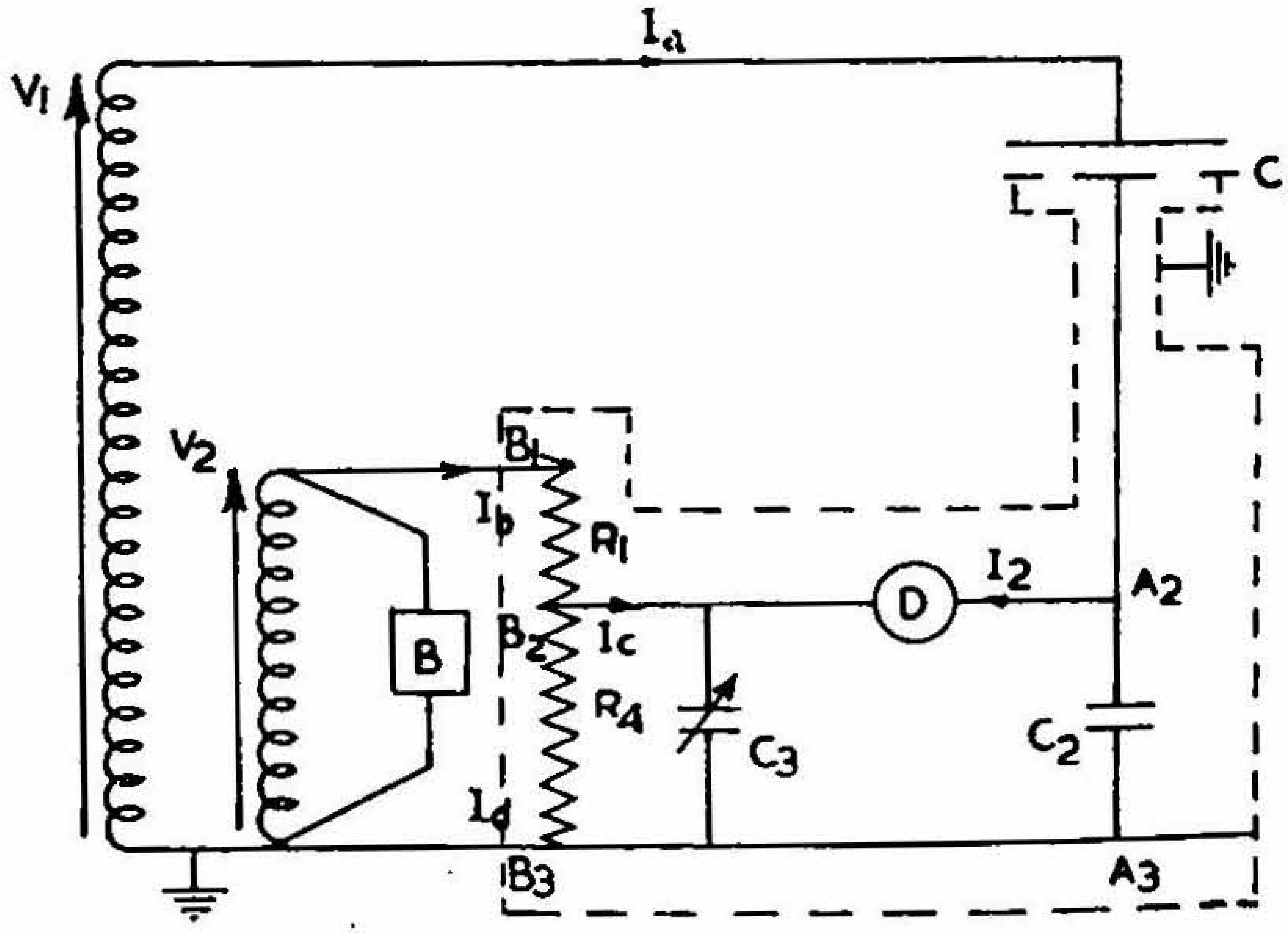

FIG. 5

as indicated by the arrows. The junction of the two windings is earthed. The standard high voltage compressed gas condenser C_1 is in series with a standard fixed mica condenser C_2, the circuit being in parallel with the primary of the transformer. The secondary is provided with the usual test burden B, in parallel with which is a branch consisting of a high fixed resistance R_1 in series with a variable resistance R_4. The necessary phase compensation is effected by a variable decade mica condenser C_3 placed across R_4. D is a highly sensitive thermionic-amplifier-detector connected through a screened lead and a one-to-one ratio transformer to the network. To avoid difficulty due to stray fields and any tendency to instability, the amplifier system is efficiently shielded and appropriately earthed and kept at some little distance from the network. The lead connecting C_1 and C_2 is also screened. Every component is enclosed in a metallic sheild connected to earth. By providing an earthed metallic enclosure to the measuring system the influence of the electric field of the high voltage electrode of C_1 is eliminated. Balance is secured by adjustment of R_4 and C_3.

In the case when the reversed secondary voltage V_2 lags on the primary voltage V_1, that is, for negative phase angles it is necessary to transfer the position of the condenser C_3 in the set-up as shown in Fig. 6. In order to

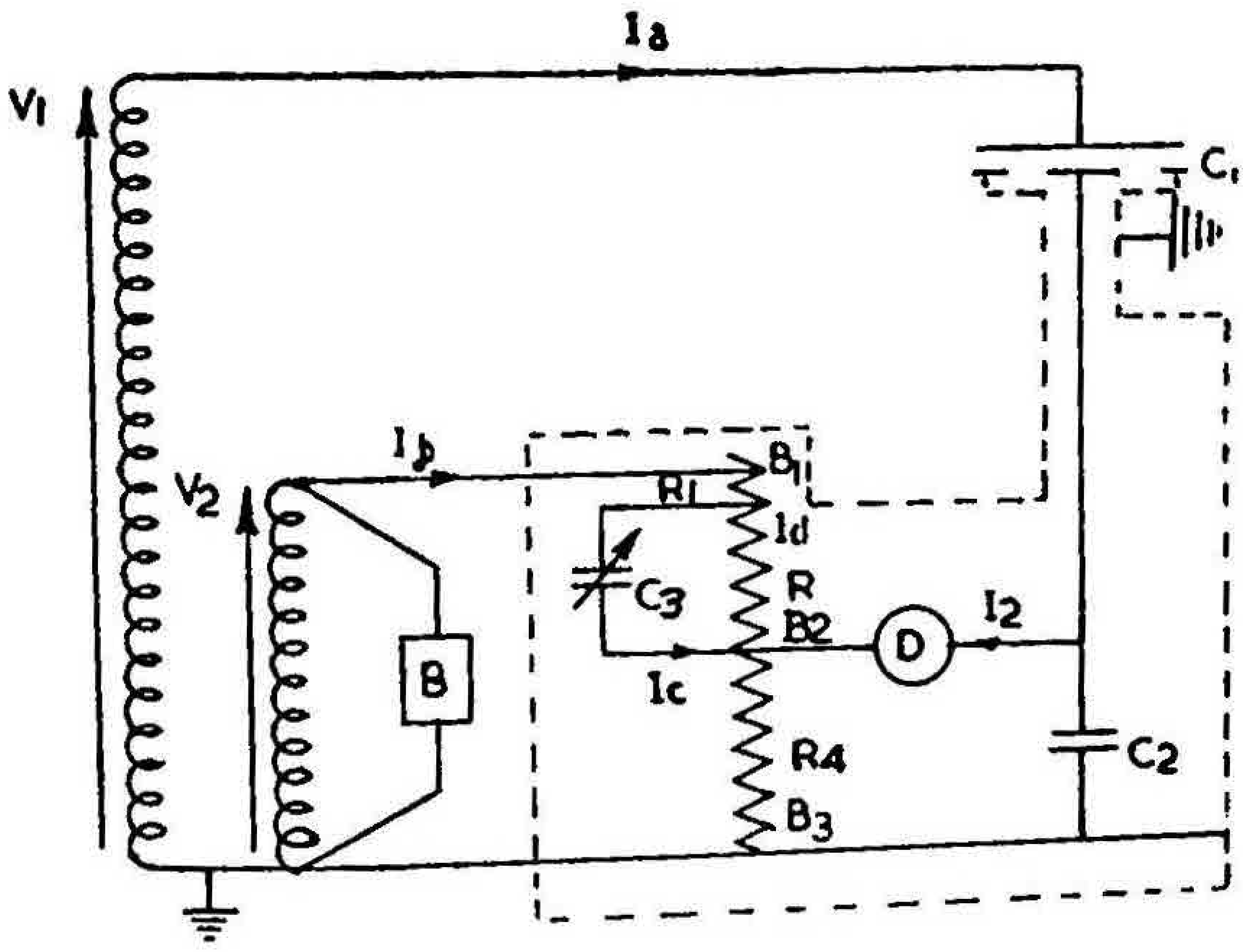

FIG. 6

obtain a fine phase angle adjustment the condenser C_3 is connected across a section of the high resistance contained in the branch.

4.1 *Expressions for Errors*

Let Z_d and I_2 be the detector impedance and current respectively. Then, with the aid of the mesh equations, the detector current may be written as, for Fig. 5,

$$I_2 = \frac{\begin{vmatrix} -\dfrac{j}{\omega C_1} - \dfrac{j}{\omega C_2} & V_1 & 0 \\ \dfrac{j}{\omega C_2} & 0 & \dfrac{j}{\omega C_3} \\ 0 & -\dfrac{V_2 R_4}{R_4 + R_1} & R_4 - \dfrac{j}{\omega C_3} - \dfrac{R_4^2}{R_4 + R_1} \end{vmatrix}}{\begin{vmatrix} -\dfrac{j}{\omega C_1} - \dfrac{j}{\omega C_2} & \dfrac{j}{\omega C_2} & 0 \\ \dfrac{j}{\omega C_2} & -\dfrac{j}{\omega C_3} - \dfrac{j}{\omega C_2} + Z_d & \dfrac{j}{\omega C_3} \\ 0 & \dfrac{j}{\omega C_3} & R_4 - \dfrac{j}{\omega C_3} - \dfrac{R_4^2}{R_4 + R_1} \end{vmatrix}} \tag{1}$$

Under balance conditions, I_2 will vanish, that is, the numerator of the determinant is zero. This gives

$$\left(-\frac{j}{\omega C_1} - \frac{j}{\omega C_2}\right)\left(\frac{j}{\omega C_3}\right)\frac{V_2 R_4}{R_4 + R_1} - V_1\left(\frac{j}{\omega C_2}\right)\left(R_4 - \frac{j}{\omega C_3} - \frac{R_4^2}{R_4 + R_1}\right) = 0 \tag{2}$$

from which, to a high degree of approximation, the voltage ratio

$$\left|\frac{V_1}{V_2}\right| = \frac{C_2 + C_1}{C_1} \cdot \frac{R_4}{R_4 + R_1} \tag{3}$$

and the positive phase angle

$$\alpha = \text{arc tan} \frac{\omega C_3 R_4}{1 + \dfrac{R_4}{R_1}} \tag{4}$$

Similarly, for Fig. 6,

$$I_2 = \frac{\begin{vmatrix} -\dfrac{j}{\omega C_1} - \dfrac{j}{\omega C_2} & V_1 & 0 \\ \dfrac{j}{\omega C_2} & 0 & -R_4 \\ 0 & -V_2 & R_4 + R - \dfrac{R^2}{R - \dfrac{j}{\omega C_3}} + R_1 \end{vmatrix}}{\begin{vmatrix} -\dfrac{j}{\omega C_1} - \dfrac{j}{\omega C_2} & \dfrac{j}{\omega C_2} & 0 \\ \dfrac{j}{\omega C_2} & R_4 - \dfrac{j}{\omega C_2} + Z_d & -R_4 \\ 0 & -R_4 & R_4 + R - \dfrac{R^2}{R - \dfrac{j}{\omega C_3}} + R_1 \end{vmatrix}} \tag{5}$$

At balance I_2 will be zero, giving

$$\left(-\frac{j}{\omega C_1}-\frac{j}{\omega C_2}\right)-(V_2R_4)-V_1\left(\frac{j}{\omega C_2}\right)\left(R_4+R-\frac{R^2}{R-\frac{j}{\omega C_3}}+R_1\right)=0 \qquad (6)$$

so that, to a high degree of approximation, the voltage ratio

$$\left|\frac{V_1}{V_2}\right|=\frac{C_2+C_1}{C_1}\cdot\frac{R_4}{R_4+R_1+R} \qquad (7)$$

and

$$\text{the negative phase angle } \alpha = \text{arc tan}\,\frac{\omega C_3R^2}{R_4+R_1+R} \qquad (8)$$

4.2 *Vector Diagrams*

Fig. 7 illustrates the vector diagram for the balance condition for the circuit of Fig. 5. In this diagram, the current I_a leads on the primary voltage V_1, the

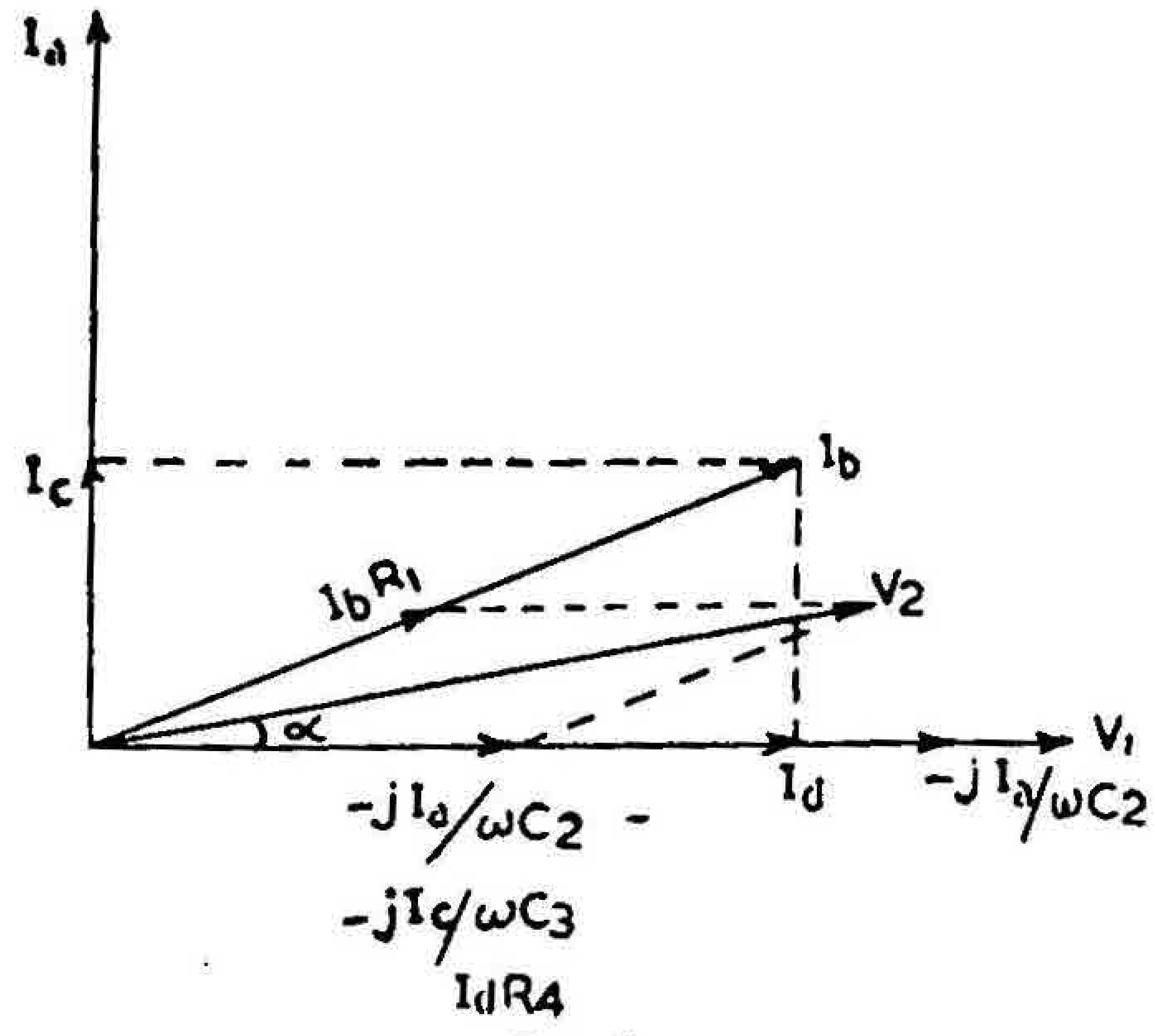

FIG. 7

reference vector by nearly a right angle. The voltage drop $-jI_a/\omega C_2$ between the points A_2 and A_3, is in lagging quadrature with this current. It is also the voltage I_dR_4 between B_2 and B_3 since A_2, B_2 and also the earthed points A_3, B_3 are at the same potential. The current I_c will be in quadrature with this voltage, while the current I_d is in phase with it. The drop I_bR_1 is in phase with the current I_b, the sum of I_c and I_d. V_2, compounded of the vectors of voltages I_bR_1 and I_dR_4, leads on V_1 by the angle α.

For the balanced circuit of Fig. 6 the vector diagram is illustrated in Fig. 8. The primary voltage V_1 is taken as the reference vector, the current I_a leading on this by nearly a right angle. The voltage drop $-jI_a/\omega C_2$ is in lagging quadrature with the current I_a and it is also the voltage drop I_bR_4 in the branch B_2B_3.

The current I_b in phase with the voltage drop I_bR_4 is resolved into its components I_c and I_d. The voltage drop I_dR is in phase with the latter and the voltage drop across BB_1 is in phase with the current I_b. The resultant of I_dR, I_bR_4 and I_bR gives the voltage vector V_2 lagging the voltage V_1 by the angle α.

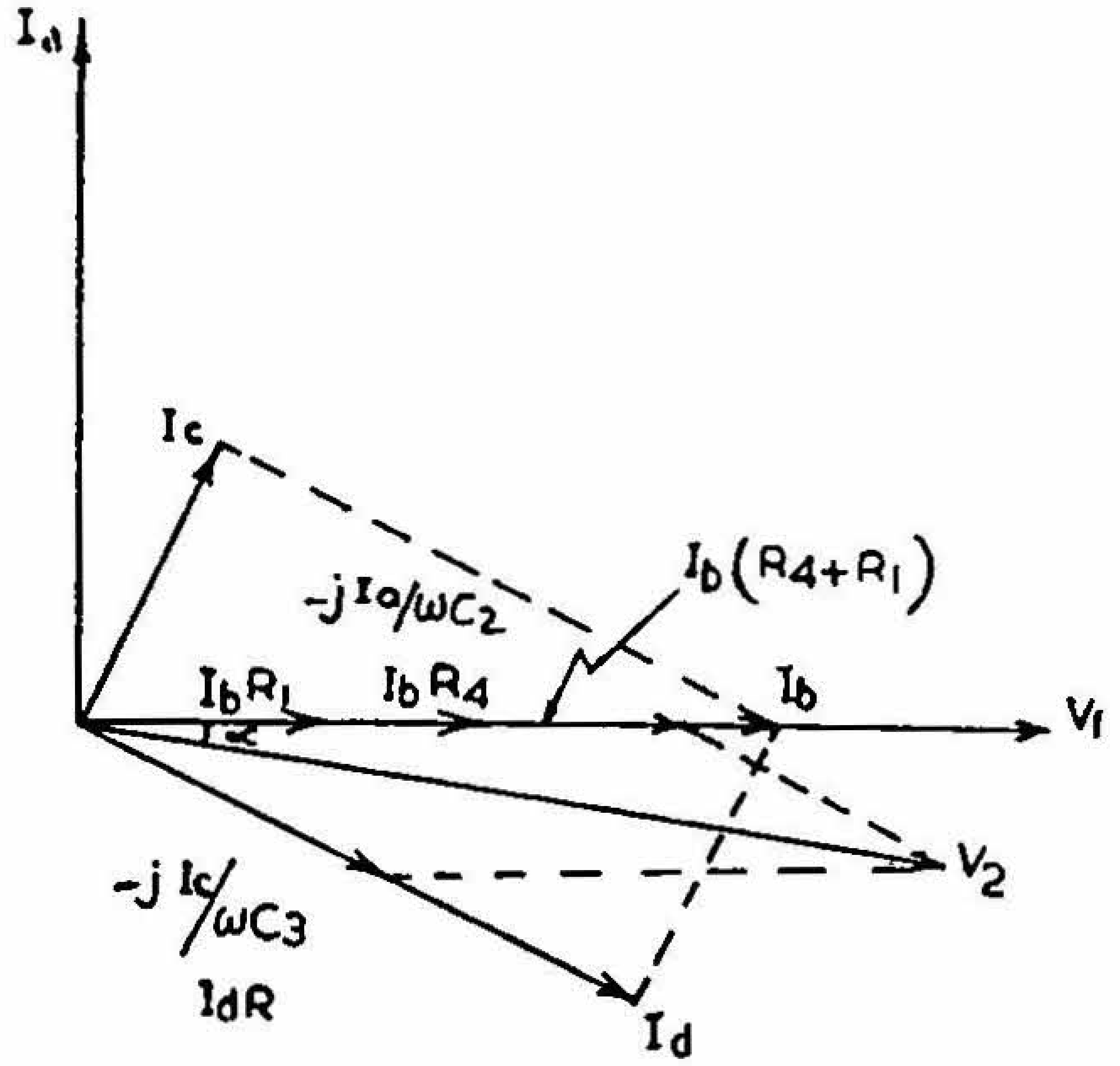

FIG. 8

4.3 *Convergence*

Reducing and clearing the numerator of inverse terms in the expression (1) to get only linear variations the detector current, in the first case, may be expressed as

$$I_2 = C_2 \cdot \frac{N}{\Delta} \tag{9}$$

where

$$N = R_4 (V_2C_2 + V_2C_1 - V_1C_1) - R_1V_1C_1 - jV_1\omega C_1C_3R_4R_1 \tag{10}$$

and

$$\Delta = (C_2 + C_1) [j(R_4 + R_1) - \omega C_2Z_d (R_4 + R_1) + \omega C_1C_3R_4R_1$$

$$-jC_1 (R_4 + R_1) - \omega R_4R_1 (C_2 + C_1) [C_2 + C_3 + j\omega C_2C_3Z_d \tag{11}$$

At balance I_2 will be zero and consequently N will be zero. The process of adjusting balance consists in making successive adjustments of the two parameters R_4 and C_3 independently until no current flows through the detector. It is desirable that the detector current shall rapidly converge towards zero.

The rate of change of I_2 with respect to R_4 is

$$\therefore \quad \frac{\partial I_2}{\partial R_4} = C_2 \frac{\Delta \dfrac{\partial N}{\partial R_4} - N \dfrac{\partial \Delta}{\partial R_4}}{\Delta^2} \tag{12}$$

which at balance is

$$_{R_4}\left[\frac{\partial I_2}{\partial R_4}\right]_0 = \omega C_2 \cdot \frac{1}{\Delta} \cdot \frac{\partial N}{\partial R_4} \tag{13}$$

and similarly

$$_{C_3}\left[\frac{\partial I_2}{\partial C_3}\right]_0 = \omega C_2 \frac{1}{\Delta} \cdot \frac{\partial N}{\partial C_3} \tag{14}$$

Differentiating N with respect to R_4 and C_3 gives

$$\frac{\partial N}{\partial R_4} = V_2C_2 + V_2C_1 - V_1C_1 - jV_1\omega C_1C_3R_1 \tag{15}$$

$$\doteqdot V_2C_2 + V_2C_1 - V_1C_1 \tag{16}$$

and

$$\frac{\partial N}{\partial C_3} = -jV_1\omega C_1R_4R_1 \tag{17}$$

Substituting these values in the expressions (13) and (14), it is seen that the loci of I_2 for small variations in R_4 and C_3 at balance cut normally, making it possible to obtain a rapid convergence by the pair (R_4, C_3).

In the second case, by using the expression (5), the detector current may be put in the form

$$I_2 = \omega C_2 \cdot \frac{N}{\Delta} \tag{18}$$

where

$$\begin{aligned} N &= R_4(V_2C_2 + V_2C_1 - V_1C_1) - V_1C_1(R_1 + R) \\ &+ j\,[V_2R_4R\omega C_3(C_2 + C_1) - V_1R\omega C_1C_3(R_4 + R_1)] \end{aligned} \tag{19}$$

and

$$\begin{aligned} \Delta &= (C_2 + C_1)\,[j(R_4 + R_1 + R) - \omega C_2R_4(R + R_1 + Z_d) \\ &- \omega C_3R(R_4 + R_1) - j\omega^2C_2C_3R(R_4R_1 + Z_dR_4 + Z_dR)] \\ &+ \omega C_1C_3R(R_4 + R_1) - jC_1(R_4 + R + R_1) \end{aligned} \tag{20}$$

Then

$$\frac{\partial N}{\partial R_4} = V_2C_2 + V_2C_1 - V_1C_1 + j\omega RC_3(V_2C_2 + V_2C_1 - V_1C_1) \tag{21}$$

$$\doteqdot V_2C_2 + V_2C_1 - V_1C_1 \tag{22}$$

and

$$\frac{\partial N}{\partial C_3} = j\,[V_2R_4R\omega(C_2 + C_1) - V_1R\omega C_1(R_4 + R_1)] \tag{23}$$

As before these relations indicate that the pair (R_4, C_3) are practically ideal for a rapid balance.

4.4 *Sensitivity*

Equations (13) and (14) express the rate of change of the detector current I_2 with respect to variations in R_4 and C_3 at balance.

In the first case, using equations (15) and (17)

$$_{R_4}\left[\frac{\partial I_2}{\partial R_4}\right]_0 = \omega C_2 \cdot \frac{1}{\Delta} (V_2C_2 + V_2C_1 - V_1C_1 - jV_1\omega C_1C_3R_1) \tag{24}$$

and

$$_{C_3}\left[\frac{\partial I_2}{\partial C_3}\right]_0 = \omega C_2 \cdot \frac{1}{\Delta} (-jV_1\omega C_1R_4R_1) \tag{25}$$

If $\partial R_4/R_4$ and $\partial C_3/C_3$ are fractional changes in R_4 and C_3 when near balance, the change of detector current is

$$I_2 = C_2 \cdot \frac{1}{\Delta} (V_2C_2 + V_2C_1 - V_1C_1 - jV_1\omega C_1C_3R_1)\, R_4 \cdot \frac{\delta R_4}{R_4}$$

$$= C_2 \cdot \frac{1}{\Delta} \cdot R_1V_1C_1 \cdot \frac{\delta R_4}{R_4} \tag{26}$$

from equation (10) with R_4 varied and

$$I_2 = \omega C_2 \cdot \frac{1}{\Delta} (-jV_1\omega C_1R_4R_1)\, C_3 \cdot \frac{\delta C_3}{C_3} \tag{27}$$

with C_3 varied. For a given detectable change of current with both adjustments

$$\frac{\delta R_4}{R_4} = \omega C_3R_4 \cdot \frac{\delta C_3}{C_3} \tag{28}$$

numerically.

In the second case, with the equations (21) and (23) the rate of change of the detector current with a small unbalance of δR_4 in R_4 and δC_3 in C_3 at balance

$$_{R_4}\left[\frac{\partial I_2}{\partial R_4}\right]_0 = C_2 \cdot \frac{1}{\Delta} [V_2C_2 + V_2C_1 - V_1C_1 + j\omega RC_3 (V_2C_2 + V_2C_1 - V_1C_1)] \tag{29}$$

and

$$_{C_3}\left[\frac{\partial I_2}{\partial C_3}\right]_0 = C_2 \cdot \frac{1}{\Delta} \cdot j\,[\omega V_2R_4R\,(C_2 + C_1) - V_1R\omega C_1\,(R_4 + R_1)] \tag{30}$$

If $\delta R_4/R_4$ and $\delta C_3/C_3$ are fractional changes in R_4 and C_3 when near balance, the change of dectetor current is

$$\delta I_2 = \omega C_2 \cdot \frac{1}{\Delta} \Big[V_2C_2 + V_2C_1 - V_1C_1 + j\omega RC_3 (V_2C_2 + V_2C_1 - V_1C_1) \times R_4 \cdot \frac{\delta R_4}{R_4}\Big]$$

$$\doteqdot \omega C_2 \cdot \frac{1}{\Delta} \cdot (R + R_1)\, V_1C_1 \cdot \frac{\delta R_4}{R_4} \tag{31}$$

from equation (19) with R_4 varied and

$$\delta I_2 = \omega C_2 \cdot \frac{1}{\Delta} \cdot j\left[V_2 R_4 R\omega (C_2 + C_1) - V_1 R\omega C_1 (R_4 + R_1) C_3 \cdot \frac{\delta C_3}{C_3}\right]$$

$$\doteqdot \omega C_2 \cdot \frac{1}{\Delta} \cdot \omega C_3 R^2 V_1 C_1 \cdot \frac{\delta C_3}{C_3} \tag{32}$$

from equation (31) with C_3 varied.

For a given detectable change of current with both adjustments

$$\frac{\delta R_4}{R_4} = \omega C_3 R \cdot \frac{R}{R + R_1} \cdot \frac{\delta C_3}{C_3} \tag{33}$$

numerically.

It is thus seen that the sensitivity (1) is proportional to the voltage applied V_1 and to the frequency; (2) increases with C_1 and C_2; (3) increases with the resistance in the secondary potential divider; and (4) increases with a low detector impedance (assuming detector of constant sensitivity). The circuits are more sensitive to R_4 adjustments for ratio than to changes of C_3 for phase angle balance. Also, the detector current due to a certain ratio unbalance is equal to that due to the same amount of phase unbalance.

It is of interset to examine in what respects the sensitivity conditions are modified by the use of a high impedance thermionic-amplifier-detector. In general, the open-circuit voltage across the detector arm may be expressed as

$$v = \frac{N_t}{\Delta_t} \tag{34}$$

The rate of change of v with respect to R_4 is

$$\frac{\partial v}{\partial R_4} = \frac{\Delta_t \frac{\partial N_t}{\partial R_4} - N_t \frac{\partial \Delta_t}{\partial R_4}}{\Delta_t^2} \tag{35}$$

and with respect to C_3

$$\frac{\partial v}{\partial C_3} = \frac{\Delta_t \frac{\partial N_t}{\partial C_3} - N_t \frac{\partial \Delta_t}{\partial C_3}}{\Delta_t^2} \tag{36}$$

At balance $N_t = 0$, so that

$$_{R_4}\left[\frac{\partial v}{\partial R_4}\right]_0 = \frac{1}{\Delta_t} \cdot \frac{\partial N_t}{\partial R_4} \tag{37}$$

and

$$_{C_3}\left[\frac{\partial v}{\partial C_3}\right]_0 = \frac{1}{\Delta_t} \cdot \frac{\partial N_t}{\partial C_3} \tag{38}$$

On solving the circuit equations

$$N_t = R_4 (V_2C_2 + V_2C_1 - V_1C_1) - R_1V_1C_1 - jV_1\omega C_1C_3R_4R_1 \tag{39}$$

$$\Delta_t = -(C_1 + C_2)(jR_1R_4\omega C_3 + R_1 + R_4) \tag{40}$$

in the first case and

$$N_t = (V_2C_2 + V_2C_1 - V_1C_1)R_4 - V_1C_1(R_1 + R) + j[V_2R_4R\omega C_3(C_2 + V_1) - C_1R\omega C_1C_3(R_4 + R_1) \tag{41}$$

$$\Delta_t = (C_1 + C_2)[R_4 + R_1 + R + jR\omega C_3(R_4 + R_1)] \tag{42}$$

in the second case.

Let $\delta R_4/R_4$ and $\delta C_3/C_3$ represent fractional changes in R_4 and C_3 when near balance. Then the voltage appearing across the detector when the balance is slightly upset is, in the first case,

$$\delta v \doteq -\frac{R_1V_1C_1}{(C_1 + C_2)(jR_1R_4\omega C_3 + R_1 + R_4)} \cdot \frac{\delta R_4}{R_4} \tag{43}$$

$$\doteq -\frac{V_1C_1}{C_2} \cdot \frac{\delta R_4}{R_4} \tag{44}$$

with R_4 varied and

$$\delta v = j\frac{R_1V_1C_1\omega C_3R_4}{(C_1 + C_2)(jR_1R_4\omega C_3 + R_1 + R_4)} \tag{45}$$

with C_3 varied. For a given detectable change of current with both adjustments

$$\frac{\delta R_4}{R_4} = \omega C_3R_4 \cdot \frac{\delta C_3}{C_3} \tag{46}$$

numerically.

In the second case

$$\delta v = \frac{(R + R_1)V_1C_1}{(C_1 + C_2)[R_4 + R_1 + R + jR\omega C_3(R_4 + R_1)]} \cdot \frac{\delta R_4}{R_4} \tag{47}$$

$$= \frac{V_1C_1}{C_2} \cdot \frac{\delta R_4}{R_4} \tag{48}$$

with R_4 varied and

$$\delta v \doteq \frac{\omega C_3R^2V_1C_1}{(C_1 + C_2)[R_4 + R_1 + R + jR\omega C_3(R_4 + R_1)]} \cdot \frac{\delta C_3}{C_3} \tag{49}$$

with C_3 varied. For a given detectable change of current with both adjustments

$$\frac{\delta R_4}{R_4} = \omega C_3R \cdot \frac{R}{R + R_1} \cdot \frac{\delta C_3}{C_3} \tag{50}$$

numerically.

The preceding theory indicates that, with a thermionic-amplifier-detector, the sensitivity (1) is proportional to the voltage applied V_1 and is practically independent of frequency; (2) increases with C_1; and (3) decreases with C_2. As before, the circuits are more sensitive to R_4 adjustments for ratio than to changes of C_3 for phase angle balance and also, the detector current due to a certain ratio unbalance is equal to that due to the same amount of phase unbalance.

4.5 *Character of Equipment and Experimental Procedure*

The high voltage compressed gas condenser C_1 used in the experiments had a capacitance of $49 \cdot 99\,\mu\mu F$, capable of operating up to 500 kV r.m.s. The power factor of this condenser was less than 1 part in 10,000 and was therefore regarded as a pure standard. From previous determinations it was known that its permeance was assured to the same accuracy over a wide range of applied voltage.

The standard fixed mica condenser C_2 of nominal value of $2\,\mu F$ had a capacitance of $1 \cdot 9988\,\mu F$ and power factor of $2 \cdot 9 \times 10^{-4}$ when calibrated at 20° C and 50 cycles per second. The condenser C_3 was a three-decade standard mica condenser totalling $1 \cdot 11\,\mu F$ by steps of $0 \cdot 001\,\mu F$. It was accurate at least to 1 part in 1,000 in capacitance and 2 minutes in phase angle at its various settings. Extreme accuracy in its value and power factor are not of importance. All the resistances were compared with known standards and their values were determined to an accuracy of 1 part in 10,000 at the working temperature. Prior calibrations of these specially wound resistors showed that their time-constants were less than 10^{-7} sec. The thermionic-amplifier-detector employed was sensitive to a tenth of a microvolt, the sensitivity being substantially constant over any working change in frequency.

The screens of the resistance boxes and the condensers C_2 and C_3 were suitably connected and earthed. Leads from the low potential electrode of C_1 and from the detector were run in earthed screens. Long and thin leads were avoided in the connections. All precautions were taken to effectively screen the low potential leads, detector and other auxiliary apparatus from any stray electric fields by providing an earthed metal enclosure, while it was noticed that the relative positions of the observer made no influence on the detector observations.

For comparison with Dannatt's method a Standard Mutual Inductor of $11 \cdot 1$ mH was used. Its accuracy was known to within 1 part in 10,000 and the phase defect was negligible at the operating frequency.

The source of power supply for the tests was an attenuator coupled to a direct current motor whose speed was capable of fine adjustment by means of a fine regulator incorporated into its field system. The voltage derived was of a pure sine wave form and was steady in its value. Any desired frequency between 15 and 60 c/s was obtainable and correctly maintained with this system. The voltage was stepped up by a high voltage power transformer, having one end earthed and the other connected to the high potential terminal of the transformer under test. For three-phase working three such units were appropriately connected. The application of a small initial voltage served to test the correctness of the circuit connections. In testing the three-phase potential transformer, it was ascertained that the maximum unbalance in the secondary voltages and consequently in the primary voltages was less than half a per cent. and also the phase sequence was verified by a Phase Sequence Indicator.

4.6 *Measurements*

The following results were obtained by tests on single-phase and three-phase potential transformers:—

(*a*) *Single-Phase Potential Transformer*

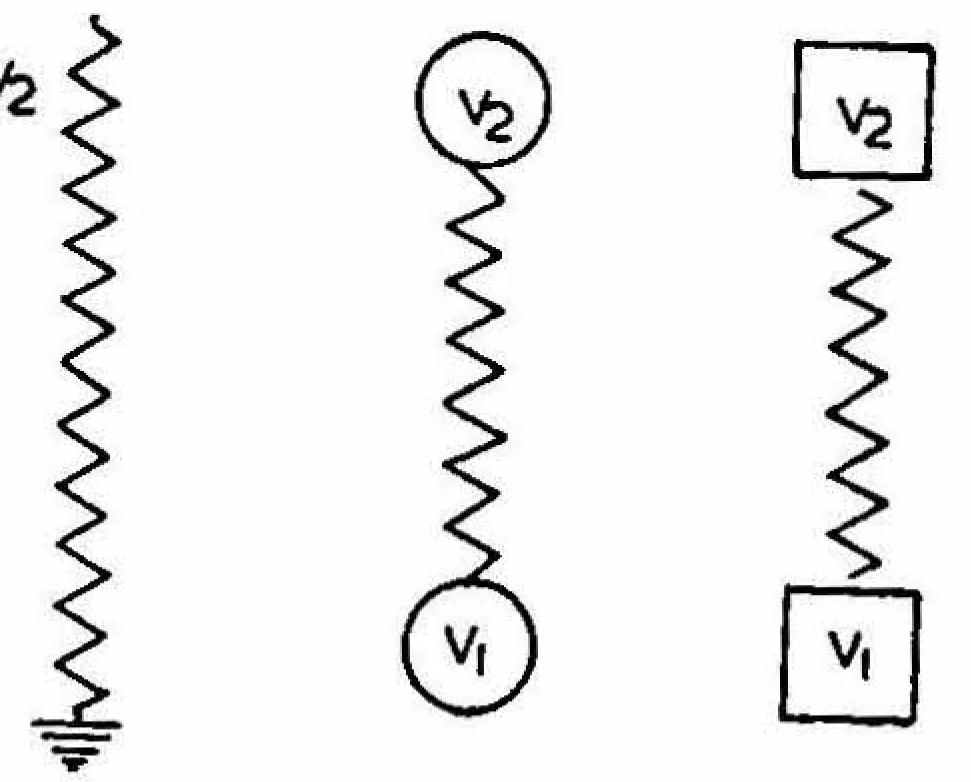

FIG. 9

Details

Voltage Ratio 13, 200/√3/110/√3/110/√3 volts system

Secondary Rating 500 VA

Tertiary Rating 200 VA

Accuracy Class B (B.S. 81–1936)

Frequency 50 cycles per second

(*b*) *Three-phase Five-limb Voltage Transformer*

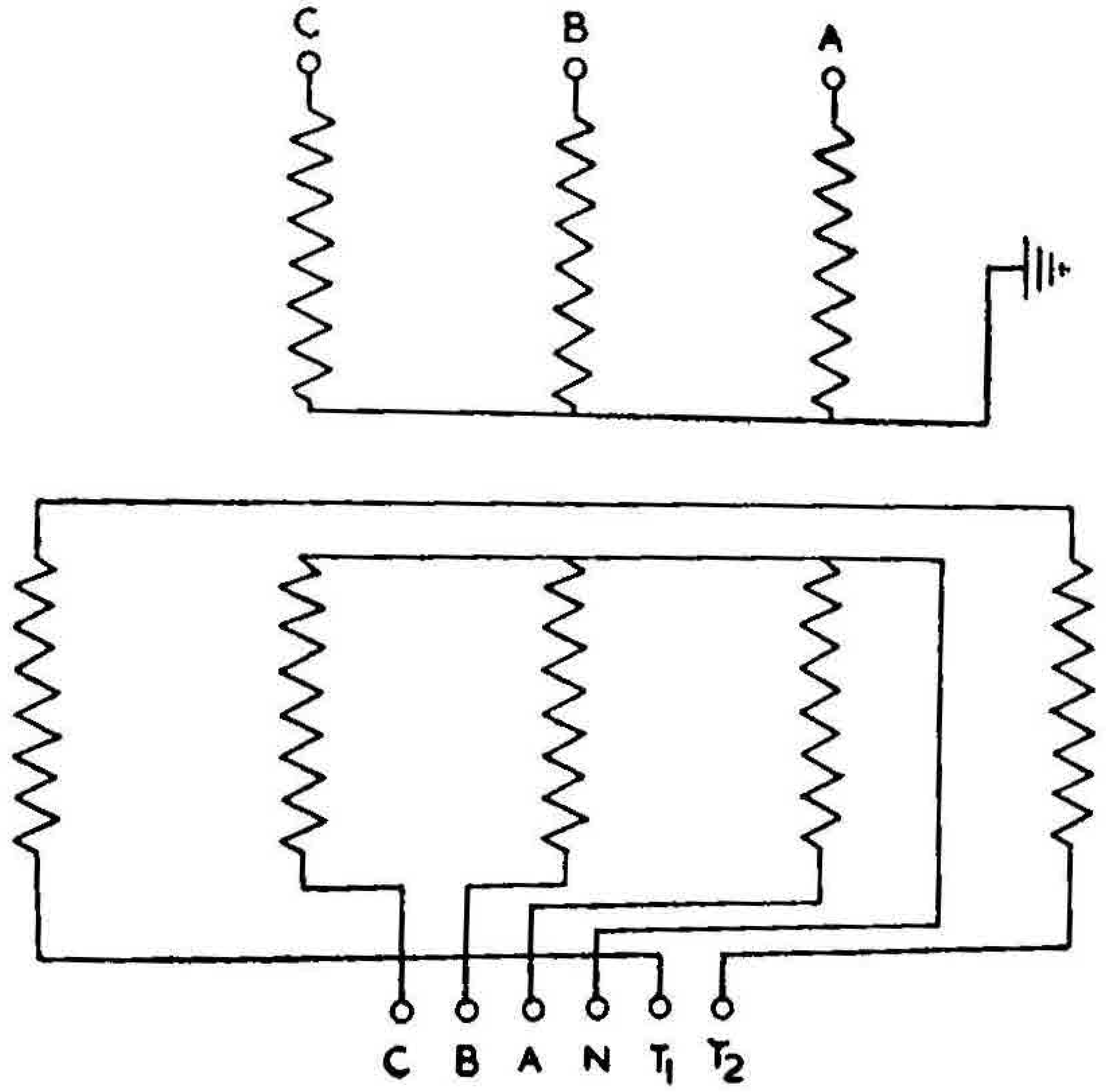

FIG. 10

Details

Voltage Rating 66,000/√3/110/√3

Secondary Rating 200 VA per phase

Accuracy Class B (B.S. 81-1936)

Frequency 50 cycles per second

The secondary and tertiary windings were identical. They were connected in parallel to form one effective secondary winding. The results are given in Table I.

TABLE I

Temperature 16° C.

Secondary burden VA	Secondary burden Power factor	Frequency in c/s	Per cent. rated primary voltage	Nominal ratio	True ratio	Ratio* error ϵ_v in %	Phase angle in mins.
1	1	50	120	1,200	1195·5	+0·38	−1·2
			110		,,	,,	,,
			100		1195·6	+0·37	,,
			90		,,	,,	−1·4
			80		,,	,,	−1·5
25	1	50	120	1,200	1195·5	+0·38	−1·3
			110		,,	,,	,,
			100		1195·6	+0·37	,,
			90		,,	,,	−1·6
			80		,,	,,	,,
400	1	50	100	1,200	1196·6	+0·28	−3·6
25	0·9	50	120	1,200	1195·5	+0·38	−1·5
			110		,,	,,	−1·2
			100		1195·6	+0·37	−1·4
			90		,,	,,	,,
			80		,,	,,	−1·8
25	0·6	50	120	1,200	1195·5	+0·38	−1·2
			110		,,	,,	,,
			100		1195·6	+0·37	,,
			90		,,	,,	−1·3
			80		,,	,,	−1·4
25	0·2	50	120	1,200	1195·5	+0·38	−1·2
			110		,,	,,	,,
			100		1195·6	+0·37	,,
			90		,,	,,	,,
			80		,,	,,	−1·5

*Ratio Error ϵ_t in % $= 100\left(\frac{\text{Nominal Ratio}}{\text{True Ratio}} - 1\right)$

The neutral on the primary was solidly grounded and the tertiary winding was open. The results are recorded in Table II.

In the above experiments the phase angle correction was effected by a variable mica condenser C_3 shunted across one-tenth of a total of 10,000 ohms (actually 10,004·6 ohms) in the branch. A test check was carried out with an air condenser connected across the entire resistance and the following results were recorded in Table III.

To show the accuracy attainable, both the single phase and three-phase transformers were tested by the mutual inductance method of Dannatt and the following comparative results were secured in Table IV.

4.7 *Discussion of Results*

As already pointed out, phase compensation in the present method was effected by a three-decade mica condenser. A test check was made to ascertain whether the impurity of the condenser had any effect on the phase angle measurement by replacing it with an air condenser. The results are shown in Table III and the small difference in phase angle is probably due to observational error. As a matter of interest a phase defect of 0·1 per cent. of a radian in C_3 was assumed and the phase angle error was calculated. There was no modification of the error up to a tenth of a minute.

The figures in Table IV bear out a fairly close agreement between the mutual inductance method of Dannatt and the present method. The maximum discrepancy is about 1 part in 1,000 in ratio and about 5 parts in 10,000 of a radian in phase angle. Some difficulty was experienced in obtaining consistent results with the mutual inductance method. Each balance condition involved the determination of an average of four readings, the leads to the detector and mutuals being reversed alternately. The process, however, laid much emphasis on the correct screening and complete astaticism of the mutual inductor.

Some figures from Table I are plotted in Fig. 11 to illustrate the variation of ratio and phase angle errors with different power factors for a given volt-amperes secondary burden. Although an increase in ratio error is to be expected with decrease in power factor, there is no change probably because of the low burden in comparison with the large secondary rating of the trans-

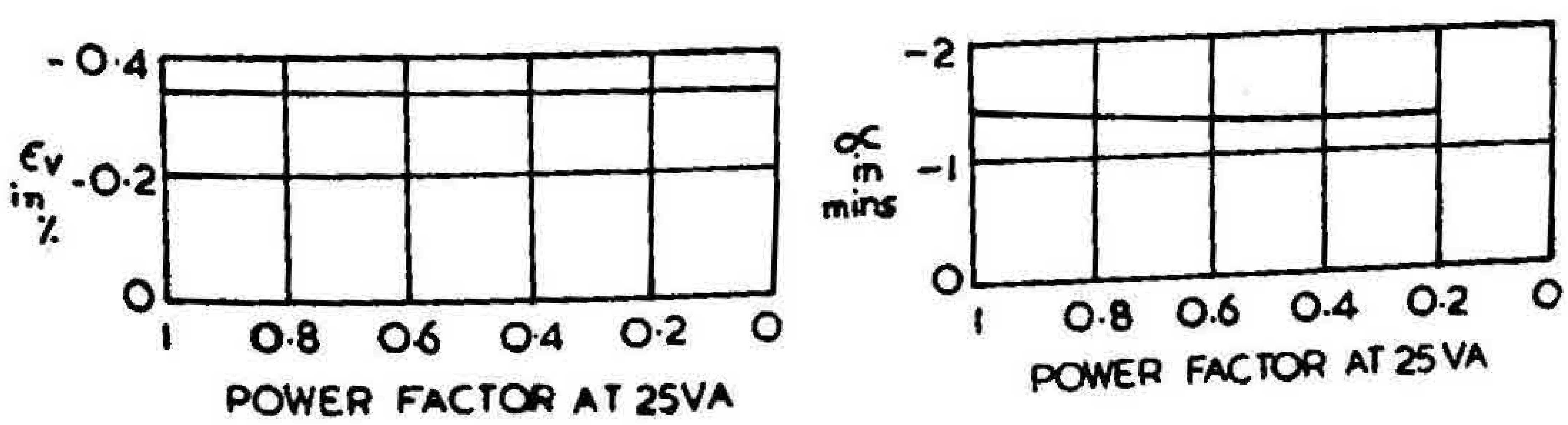

FIG. 11

TABLE II

Temperature 18° C.

Secondary Burden		Frequency in c/s	Per cent. Rated Primary Voltage	Nominal Ratio	True Ratio			Ratio Error ϵ_v in %			Phase Angle α in mins.		
VA/Phase	Power Factor				Phase A	Phase B	Phase C	Phase A	Phase B	Phase C	Phase A	Phase B	Phase C
0	..	50	110	600	597·1	597·2	597·4	+0·49	+0·47	+0·44	−0·5	+0·3	−0·8
			100		do	do	do	do	do	do	−0·6	+0·1	−1·2
			80		do	597·1	597·3	do	+0·45	+0·45	−0·7	−0·2	−1·4
			60		do	do	597·4	do	do	+0·44	−0·8	−0·3	−1·6
			40		597·2	597·2	597·5	+0·47	+0·47	+0·42	−0·8	−0·4	−1·7
0	..	60	110	600	596·8	596·9	597·3	+0·54	+0·52	+0·45	−1·0	−0	−1·0
			100		do	do	do	do	do	do	do	−0·3	−1·4
			80		do	do	do	do	do	do	do	−0·5	−1·5
50	1	50	110	600	597·1	597·2	597·5	+0·49	+0·47	+0·42	−1·8	−1·0	−2·3
			100		do	do	do	do	do	do	−1·9	−1·2	−2·4
			80		do	597·1	597·4	do	+0·49	+0·44	−2·3	−1·4	−2·6
			60		do	597·2	597·5	do	+0·47	+0·42	−2·2	−1·5	−2·8
			50		597·2	do	597·5	+0·47	do	do	do	−1·6	−2·9
50	1	60	110	600	597·1	597·1	597·4	+0·49	+0·49	+0·44	−2·4	−1·5	−2·8
			100		do	do	597·3	do	do	+0·45	−2·6	−1·7	do
			80		do	597·3	do	do	+0·45	do	do	−1·8	−2·9

100	1	50	110	600	597·2	597·6	597·5	+0·47	+0·40	+0·42	−3·2	−3·2	−3·5
			100		do	597·5	do	do	+0·42	do	do	−3·3	−3·6
			80		do	do	do	do	do	do	−3·3	−3·6	−3·8
			60		do	do	do	do	do	do	do	−3·8	−4·1
			40		597·3	597·6	597·6	+0·45	+0·40	+0·40	−3·4	−3·9	−4·2
100	1	60	110	600	597·1	597·2	597·5	+0·49	+0·47	+0·42	−4·0	−2·9	−4·1
			100		do	do	do	do	do	do	do	−3·1	do
			80		do	do	do	do	do	do	do	−3·2	−4·2
150	1	50	110	600	597·3	597·4	597·6	+0·45	+0·44	+0·40	−4·5	−3·4	−4·6
			100		597·2	597·3	597·5	+0·47	+0·45	+0·42	do	−3·7	−4·7
			80		do	do	do	do	do	do	−4·7	−3·9	−4·9
			60		597·3	do	do	+0·45	do	do	−4·8	−4·0	−5·1
			40		597·4	597·5	597·6	+0·44	+0·42	+0·40	do	−4·8	−5·4
150	1	60	110	600	597·2	597·3	597·5	+0·47	+0·45	+0·42	−5·2	−4·3	−5·5
			100		do	do	do	do	do	do	do	−4·5	do
			80		597·3	do	do	+0·45	do	do	−5·5	−4·6	−5·6
200	1	50	110	600	597·5	597·5	597·6	+0·42	+0·42	+0·40	−5·8	−5·0	−6·0
			100		597·4	597·4	do	+0·44	+0·44	do	do	−5·1	−5·9
			80		597·3	do	do	+0·45	do	do	−5·9	−5·2	−6·2
			60		do	do	do	do	do	do	−6·0	−5·3	−6·4
			40		597·5	597·5	do	+0·42	+0·42	do	−6·1	−5·4	do
200	1	60	110	600	597·4	597·5	597·6	+0·44	+0·42	+0·40	−7·0	−5·7	−7·0
			100		do	do	do	do	do	do	do	−5·9	−7·1
			80		do	do	do	do	do	do	do	do	do

TABLE III

Secondary burden	Frequency in c/s	Per cent. rated primary voltage	Phase tested	True ratio/Nominal ratio		Phase angle in mins.	
				Mica condenser C_3	Air condenser C_3	Mica condenser C_3	Air condenser C_3
0	50	100	C	0·9958	0·9958	−1·2	−1·0

TABLE IV

Type	Secondary burden	Frequency in c/s	Per cent. rated primary voltage	True ratio/Nominal ratio		Phase angle in mins.	
				Dannatt's method	Present method	Dannatt's method	Present method
Single phase transformer	25 VA at unity p.f.	50	100	0·9965	0·9967	−1·6	−1·3
	400 VA at unity p.f.	,,	,,	0·9975	0·9970	−2·1	−3·6
Three phase transformer	Zero	,,	,,	0·9947	0·9953	−1·6	+0·1
Phase tested B	200 VA/phase at unity p.f.	,,	,,	0·9967	0·9957	−5·0	−5·1

former. The phase angle, however, shows a slight tendency to fall with reduction in power factor.

In potential transformers which are designed for reasonably close accuracy, the ratio and phase angle vary linearly with secondary burden with full voltage applied to the primary winding. The influence of exciting current predominates at no load and it becomes less important with increasing burden. Fig. 12 is derived from figures of Table II. It shows a decrease in ratio error with increasing volt-amperes burden, though this change is very small. Under the same conditions the effect of change in load on phase angle is considerable.

In the same figure the effect of an increase in frequency is depicted. An increase in frequency for a constant primary voltage causes a reduction in flux, with a corresponding reduction in exciting current. The ratio falls, thereby increasing the ratio error at no load. With the diminution in influence of the exciting current with increasing loads, the difference in ratio error narrows down

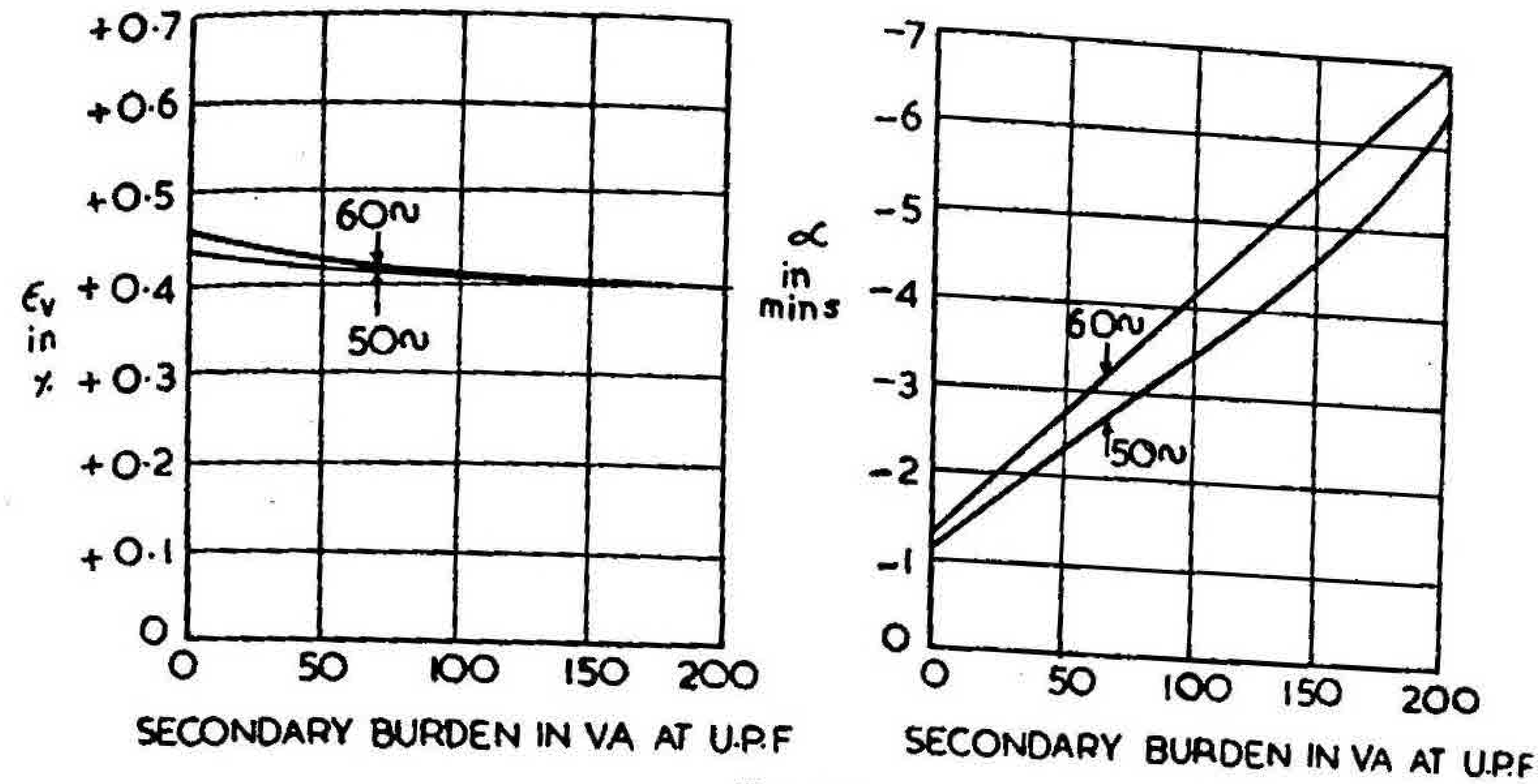

FIG. 12

to small limits. The phase angle, however, increases as the frequency is increased and the difference steadily increases with load.

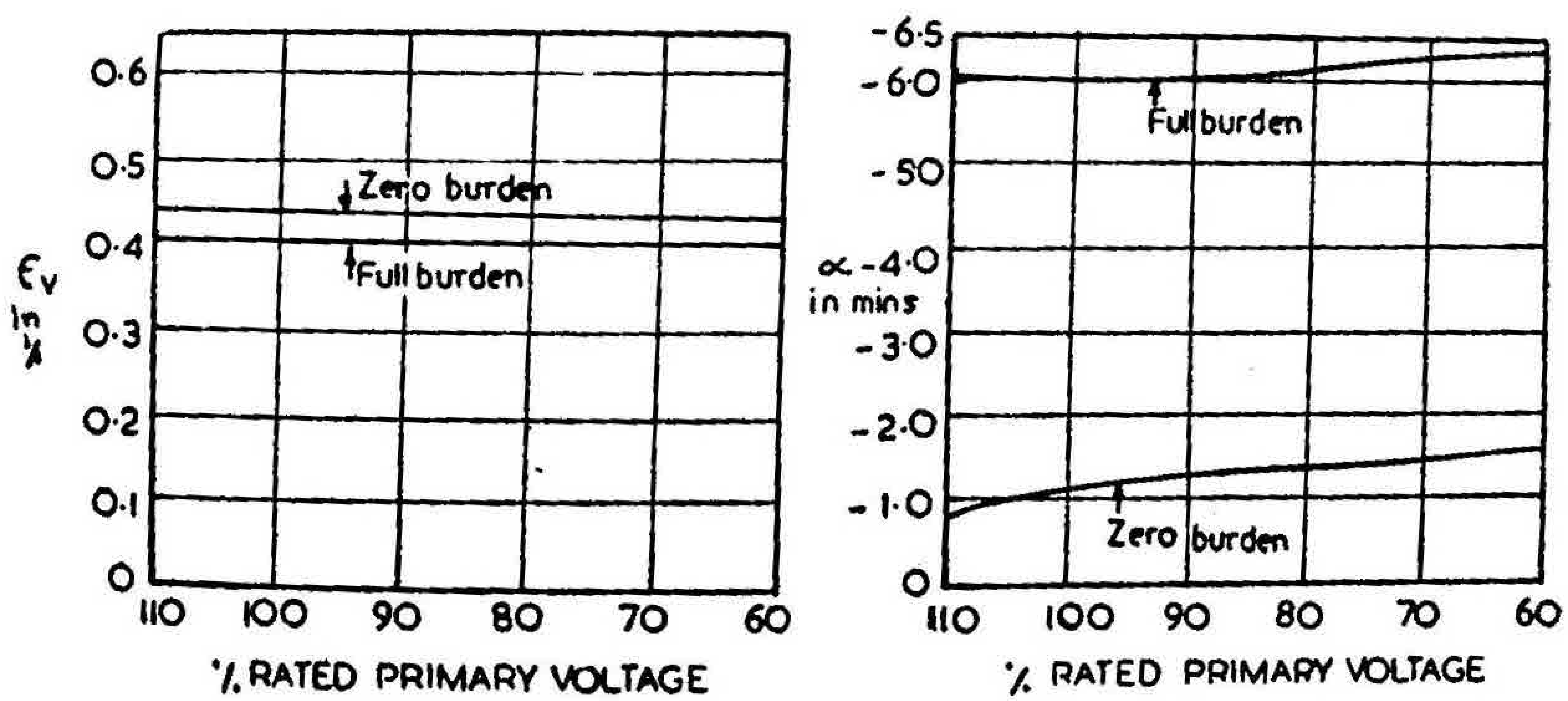

FIG. 13

Fig. 13 shows the ratio and phase angle curves plotted from Table II, the errors being plotted against the values of secondary voltage. As the applied voltage is reduced, the flux density in the core will be correspondingly reduced involving a fall of the exciting current. The ratio may change very slightly as the components of the exciting current, namely, the magnetizing and iron-loss components may not reduce in the same proportion. Actually there is very little change in the ratio as the voltage is reduced. The effect on the phase angle is, however, more pronounced. Even when the transformer is loaded, the general effects on the errors are not very much modified on account of the existence of the secondary current.

4.8 *Conclusion*

The present method enables potential transformers to be readily tested with an accuracy of 0·1 per cent. and 1 minute in phase angle at all voltages in excess of 30 KV.

5. ACKNOWLEDGMENT

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6. BIBLIOGRAPHY

1. Weller, C. T. .. "132 kV Shielded Potentiometer for Determining the Accuracy of Potential Transformers," *Trans. A.I.E.E.*, 1929, **48**, 312–16.
2. Silsbee, F. B. .. "A shielded Resistor for Voltage Transformer Testing," Scientific Papers of the Bureau of Standards, No. 516, 1925.
3. Davis, R. .. "The Design and Construction of a Shielded Resistor for High Voltages," *J.I.E.E.*, Aug. 1931, **69**, No. 416, 1028–034.
4. Bowdler, G. W. .. "A Resistor for High Voltage Measurements, *Ibid.*, July 1933, **73**, No. 349, 65-8.
5. Jimbo, S. and Sakimura, H. "A New Method of Testing Potential Transformers," *Res. Elect. Lab., Tokyo*, 1934, No. 363, 1–42.
6. Spilsbury, R. S. J. .. "A Precise Method of Voltage-Transformer Electrometer Testing, *J.I.E.E.*, 1929, **67**, 1143.
7. Dannatt, C. .. "Equipment for Ratios and Phase Angle Measurements of Voltage Transformers," *M.V. Gazette*, 1930, **12**, 45.
8. ——— .. "The Testing of Voltage Transformers for Ratio and Phase Angle, *Ibid.*, 1931, **13**, 62.
9. Churcher, B. G. .. "The Measurement of the Ratio and Phase Displacement of High Alternating Voltages," *J.I.E.E.*, 1927, **65**, 430–39.
10. Yoganandam, G. .. "A Ready Method of Measuring the Voltage Ratio and Phase Angle of High Voltage Transformers," *ibid.*, 1930, **68**, No. 397, 192–93.
11. Churcher B. G. and Dannatt, C. "The Use of Air Condensers as High Voltage Standards," *ibid.*, 1931, **69**, No. 416, 1019–27.
12. Hague, B. .. "Instrument Transformers," *Pitman*, 1936.
13. ——— .. "A.C. Bridge Methods," *Pitman*, 1946.
14. Golding, E. W. .. "Electrical Measurements and Measuring Instruments," *ibid.*, 1949.