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PROBLEMS CONNECTED WITH THE RHOMBUS

II. Plastic Torsion

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INTRODUCTION

In a previous paper[1] we have considered the approximate solution of *elastic torsion* of a uniform isotropic bar with rhombus cross-section, employing the Rayleigh-Ritz and other methods in general and using the relaxation technique in the particular case where the acute angle of the rhombus is 60°. In this paper the relaxation technique is used to obtain a numerical solution in the case of plastic torsion.

When the bar is subjected to a purely twisting action, we have the equation for the (non-dimensional) stress function

$$\frac{\partial^2 \chi}{\partial \xi^2} + \frac{\partial^2 \chi}{\partial \eta^2} + 2 = 0 \tag{1}$$

The elastic stresses and the torque are given by (Ref. 1).

$$\tau_{xz} = \mu a a \frac{\partial \chi}{\partial \eta}, \quad \tau_{yz} = -\mu a a \frac{\partial \chi}{\partial \xi} \tag{2}$$

$$M = Da = 2\mu a a^4 \int\int \chi \, d\xi d\eta \tag{3}$$

The resultant shear stress at any point in the cross-section acts in the direction tangential to the level line of χ through the point and has magnitude $\tau = \mu a a |\text{grad } \chi|$.

2. Let us now assume that the shear stress-strain relation for the material of the bar is linear up to a value τ_0 of the shear stress and that an increase of shear strain does not increase the shear stress beyond τ_0 (Fig. 1). The straining is then

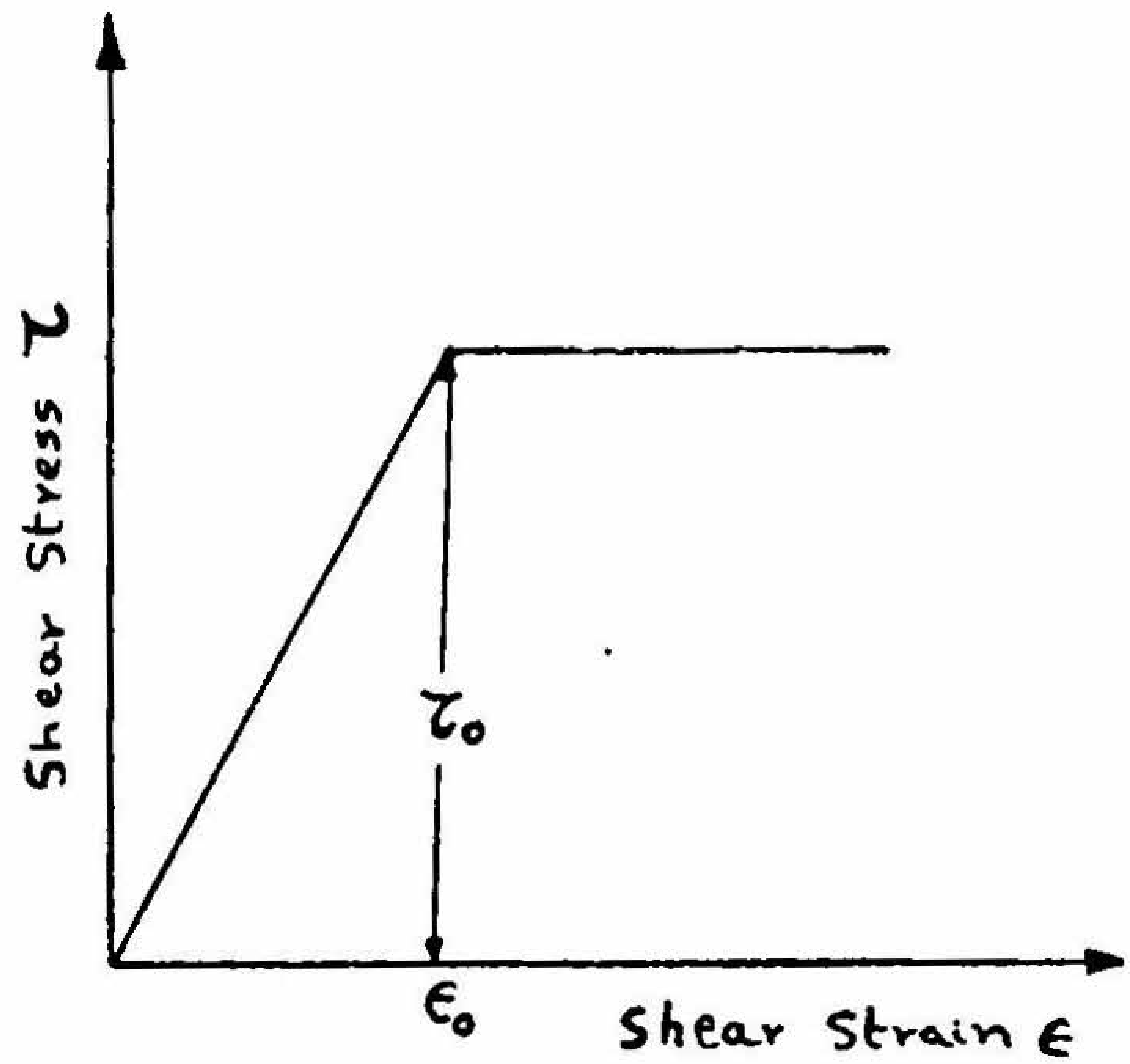

FIG. 1

plastic beyond ϵ_0 and we have $|\text{grad } X| \leqslant \tau_0/\mu a\alpha$ at all points in the cross-section. For large values of τ_0 or small values of a we may expect $|\text{grad } X| < \tau_0/\mu a\alpha$ and the straining is elastic everywhere. When $|\text{grad } X|$ has attained the limiting value, the straining is plastic and equation (1) no longer holds. In the case of elastic torsion we know that the maximum value of $|\text{grad } X|$ occurs at some point (or points) on the boundary. Now as the twisting action increases, plastic straining will commence only at such points and will spread from the boundary. In regions of plastic straining the X values lie on the *Nadai Roof*[2] *i.e.*, the surface of constant maximum gradient $\tau_0/\mu a\alpha$ drawn from $X = 0$ on the boundary.

Now in order to obtain the numerical solution of plastic torsion of a prismatic bar whose cross-section is a rhombus of acute angle 60°, we start with the solution when the straining is elastic everywhere. This is given in Fig. 2 and is taken from Ref. 1.

We modify this solution for the case when the straining is partly plastic, by supposing the value of the maximum gradient to be $8\sqrt{3}/27$. We can write down the maximum attainable values of X at all nodal points of the triangular net with mesh size $1/9\sqrt{3}$. Figure 3 records the roof values (with multiplier 6318) for the limiting gradient $8\sqrt{3}/27$.

Comparing Figs. 2 and 3 we notice that the values in Fig. 2 are less than the corresponding values in Fig. 3 except at three nodes. At these nodes we naturally

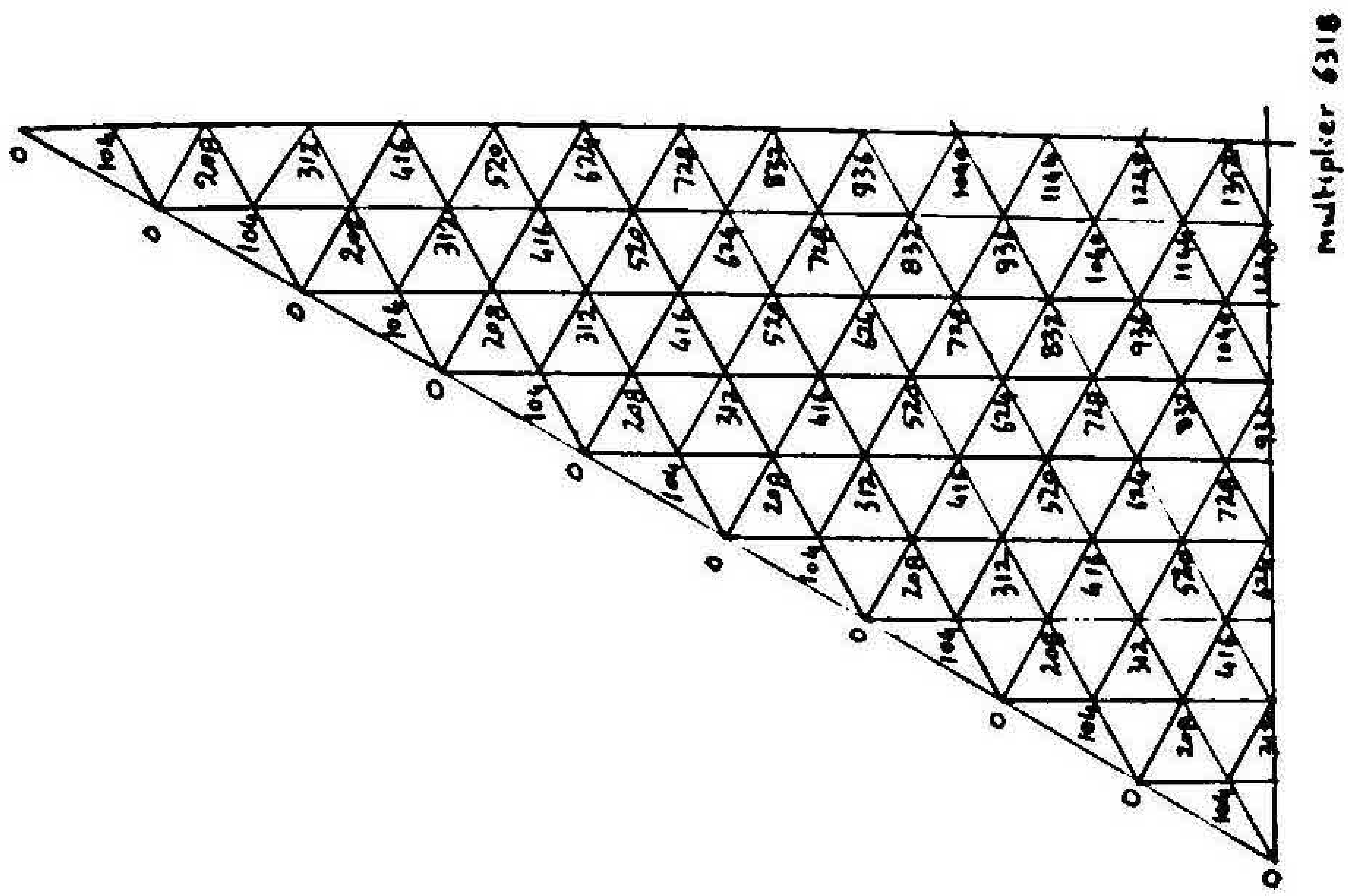

FIG. 3

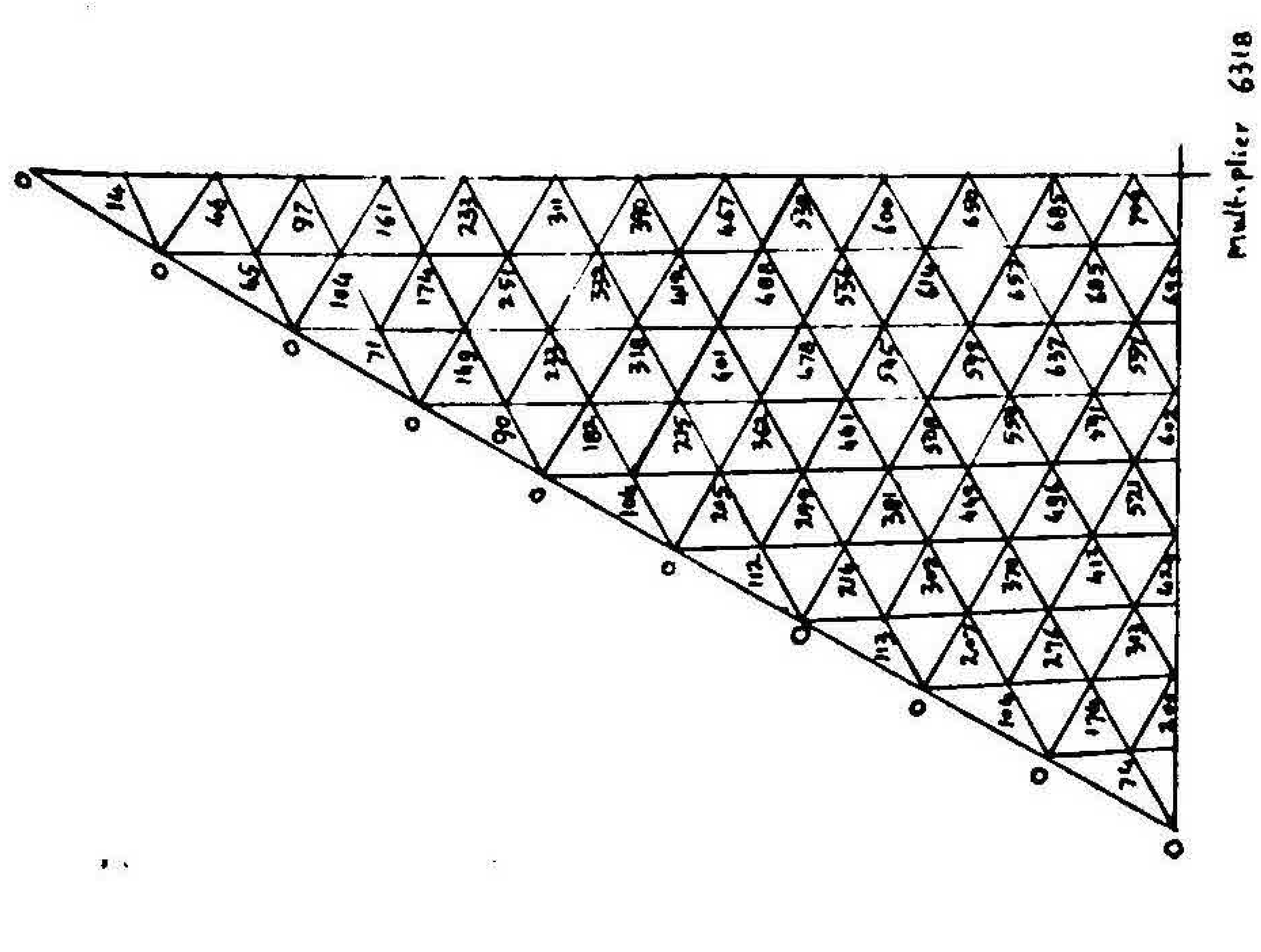

FIG. 2

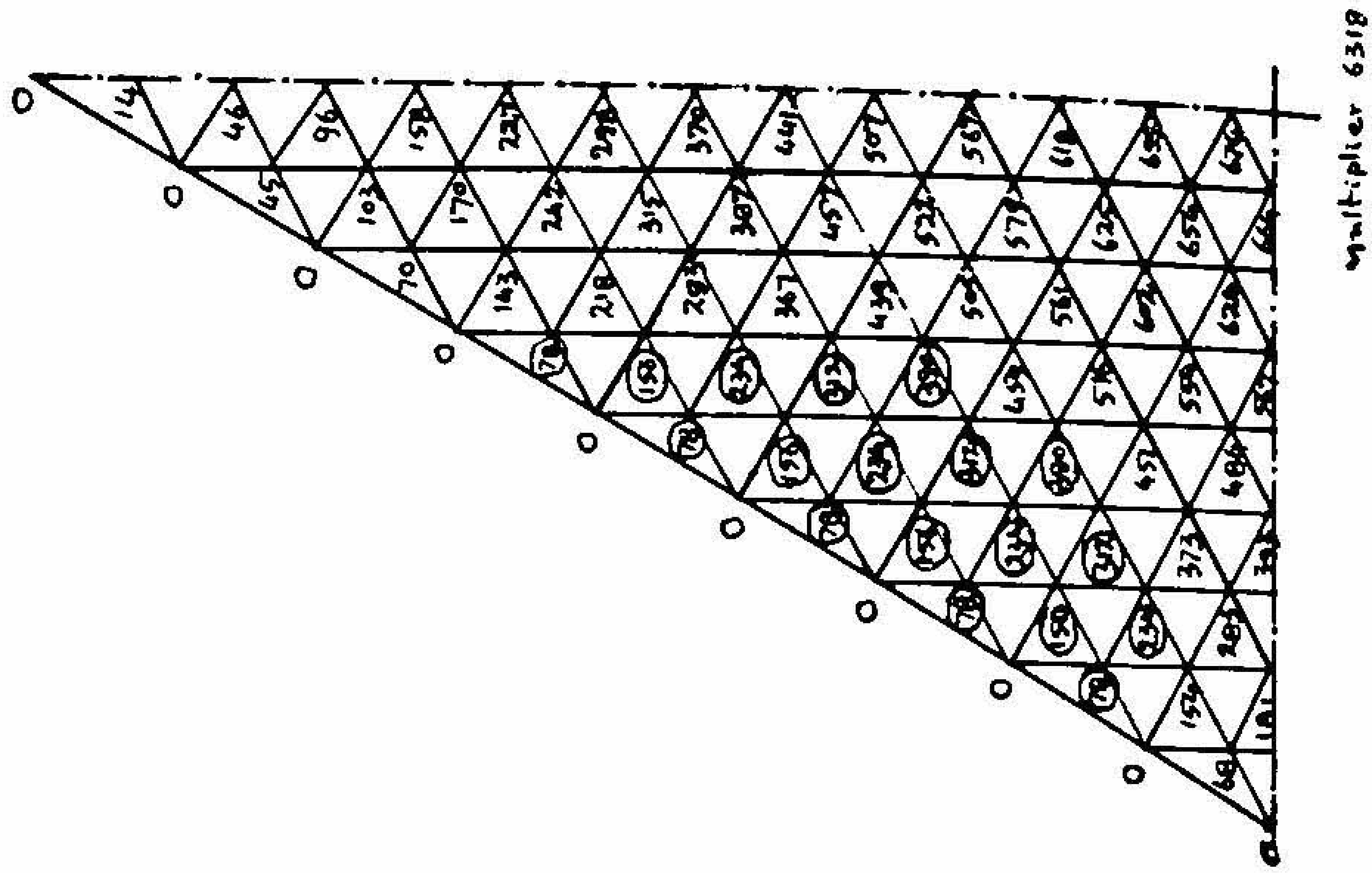

FIG. 5

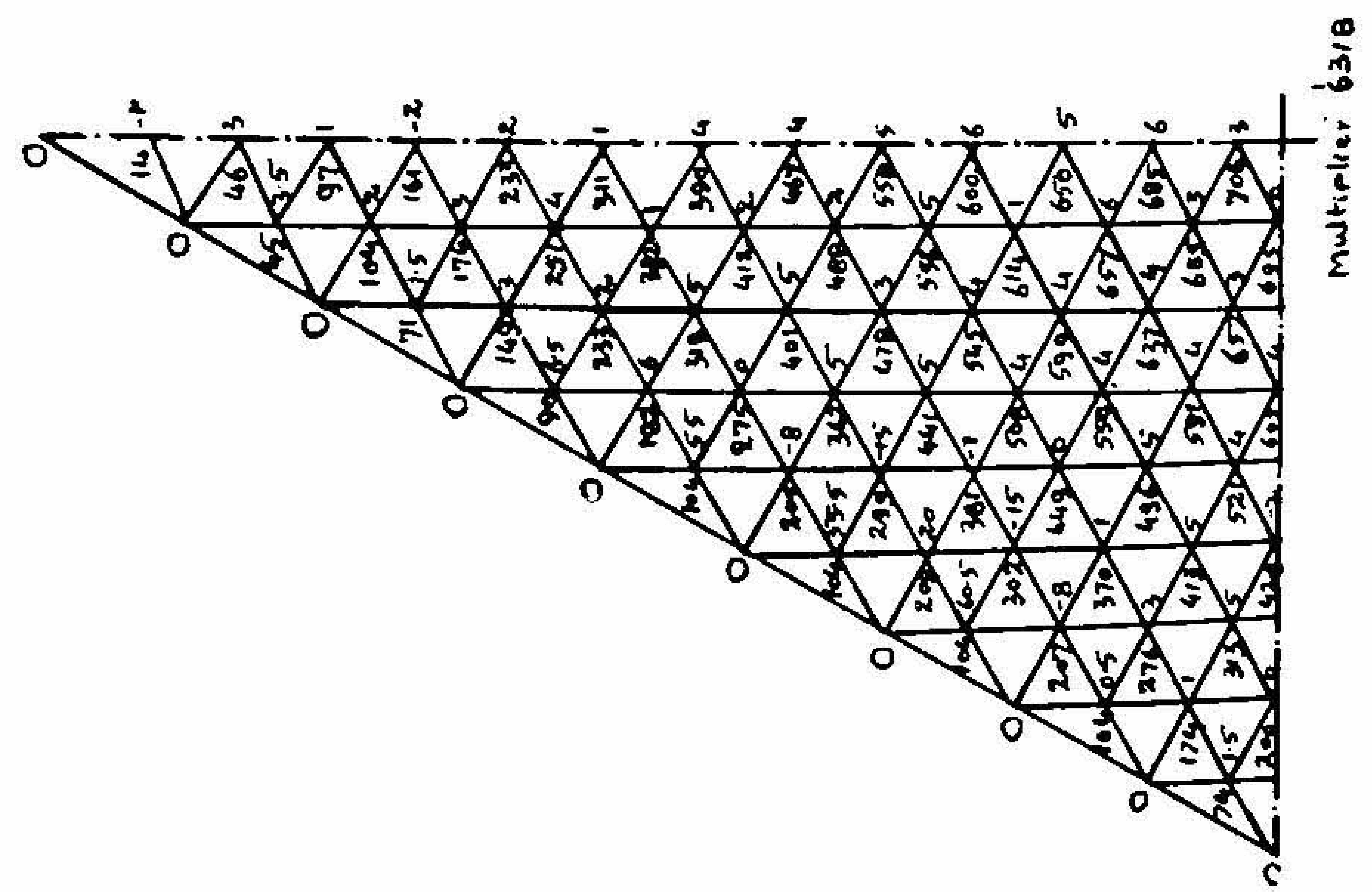

FIG. 4

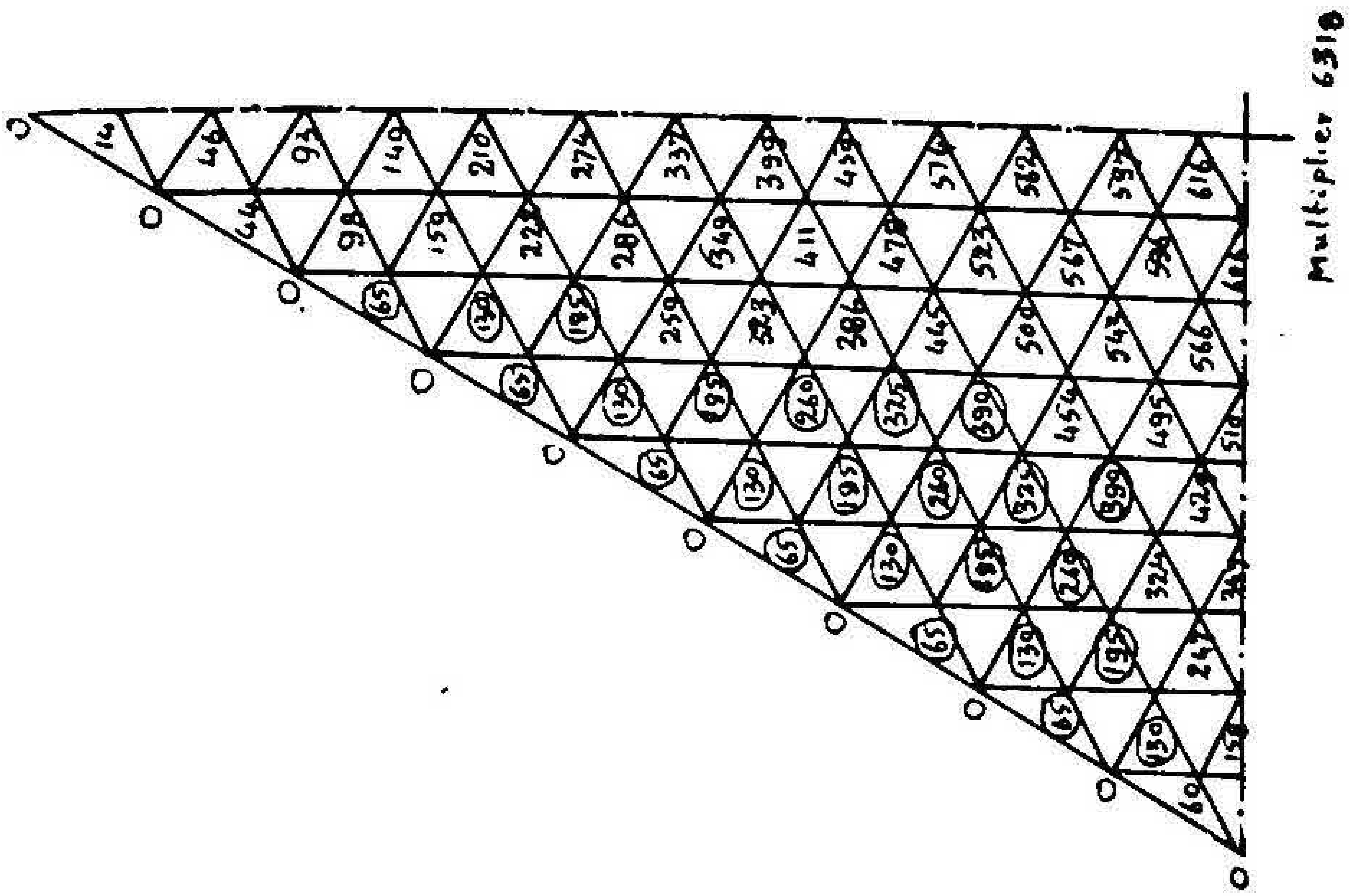

Fig. 7

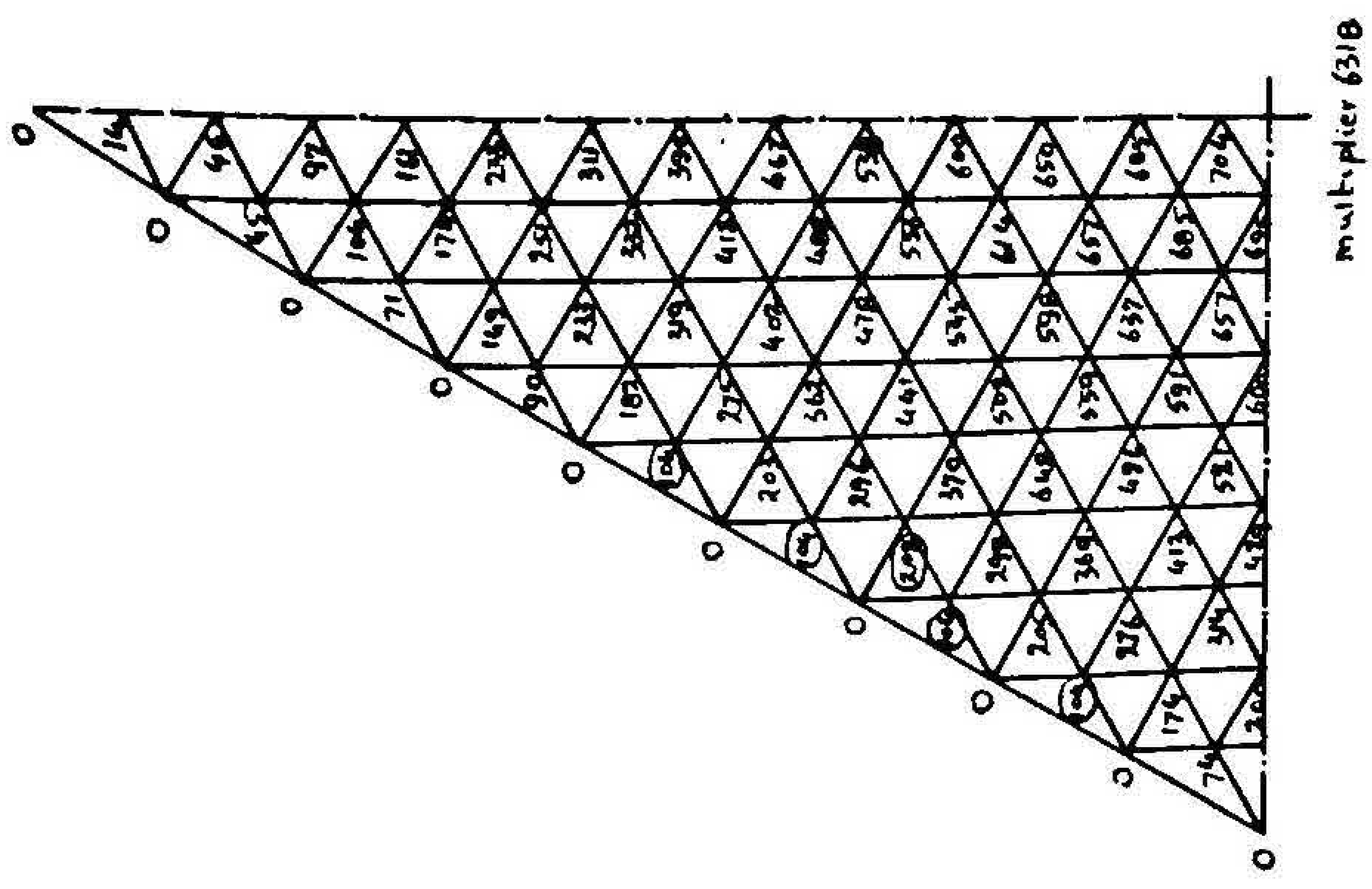

Fig. 6

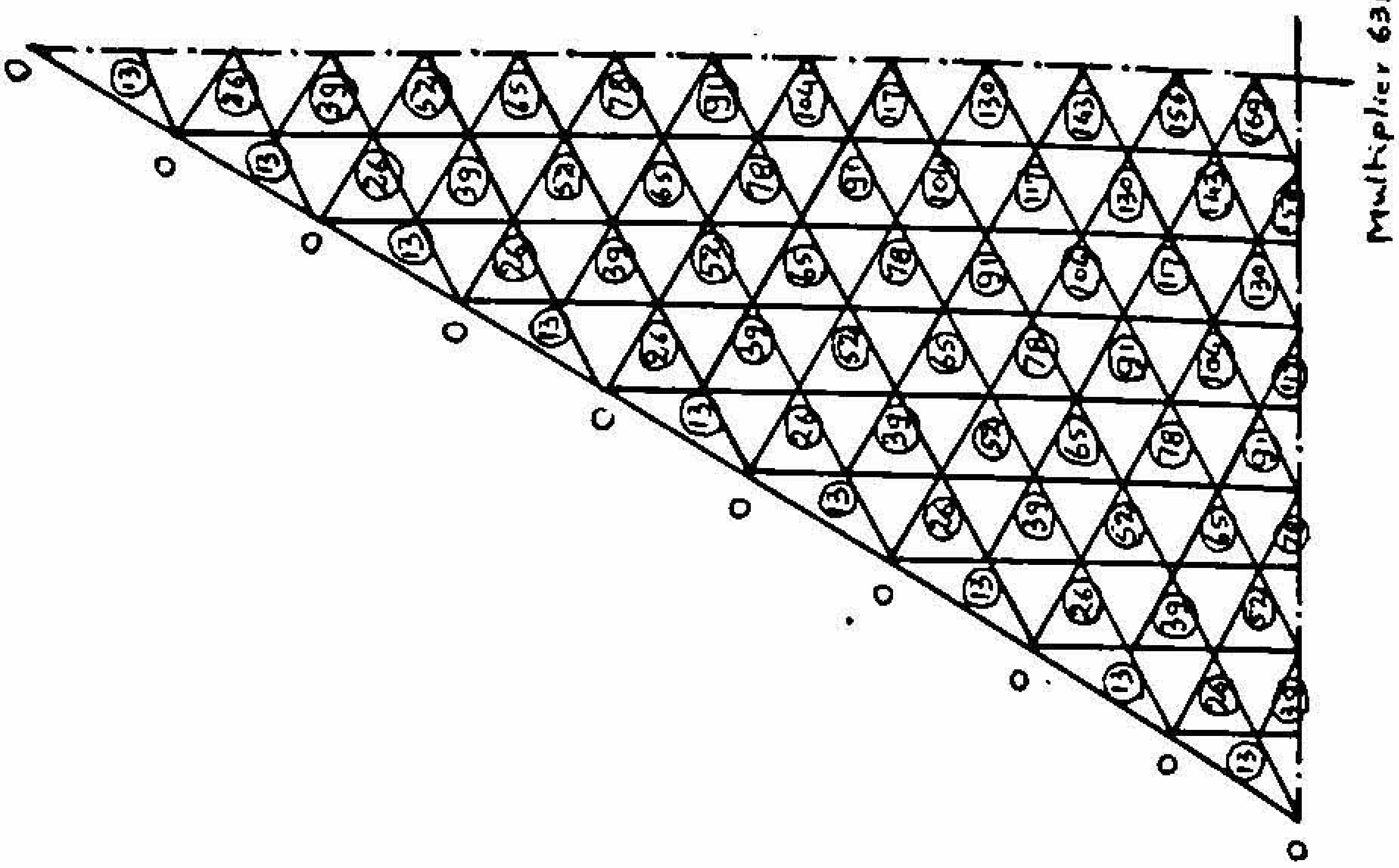

FIG. 9

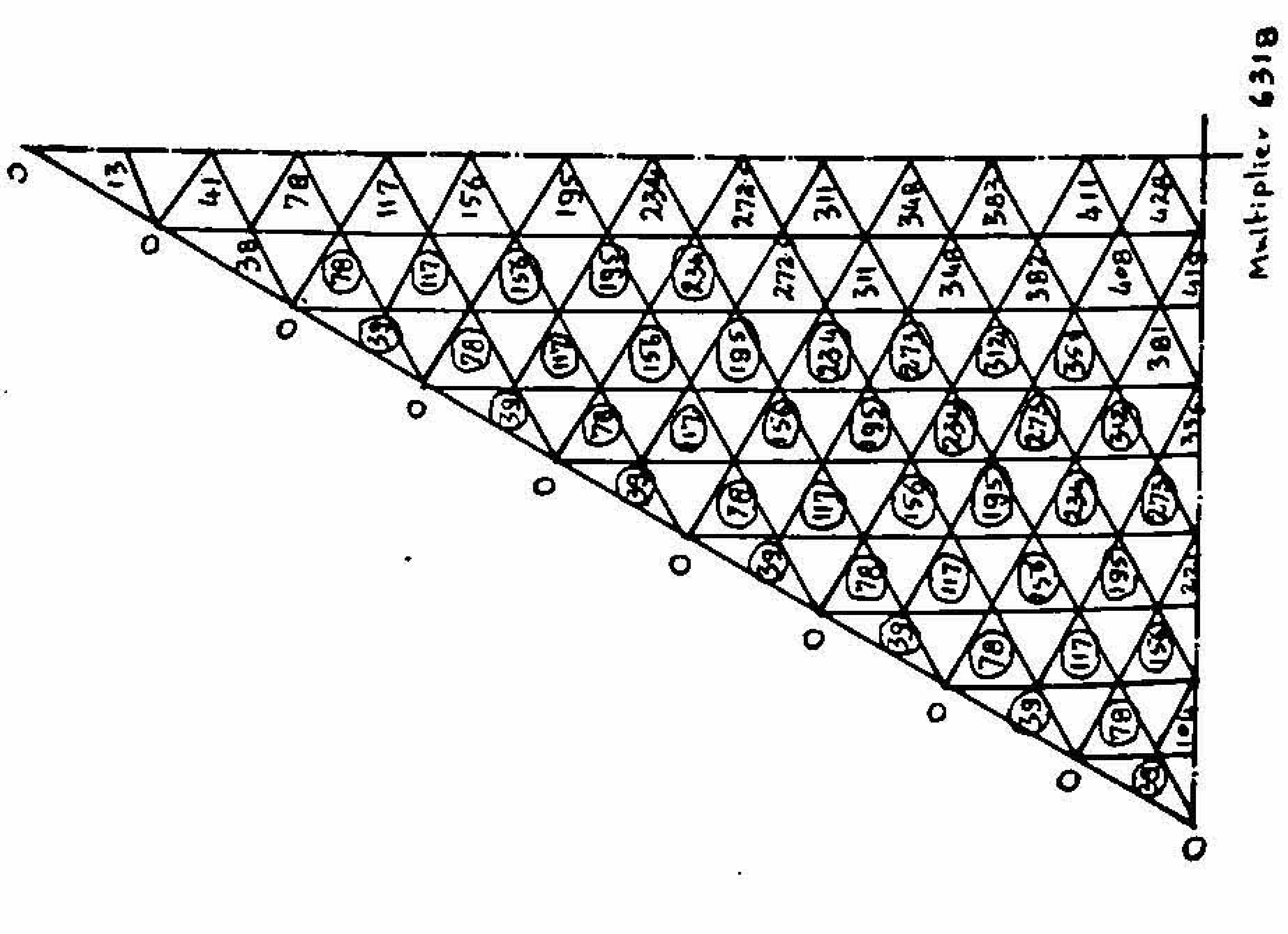

FIG. 8

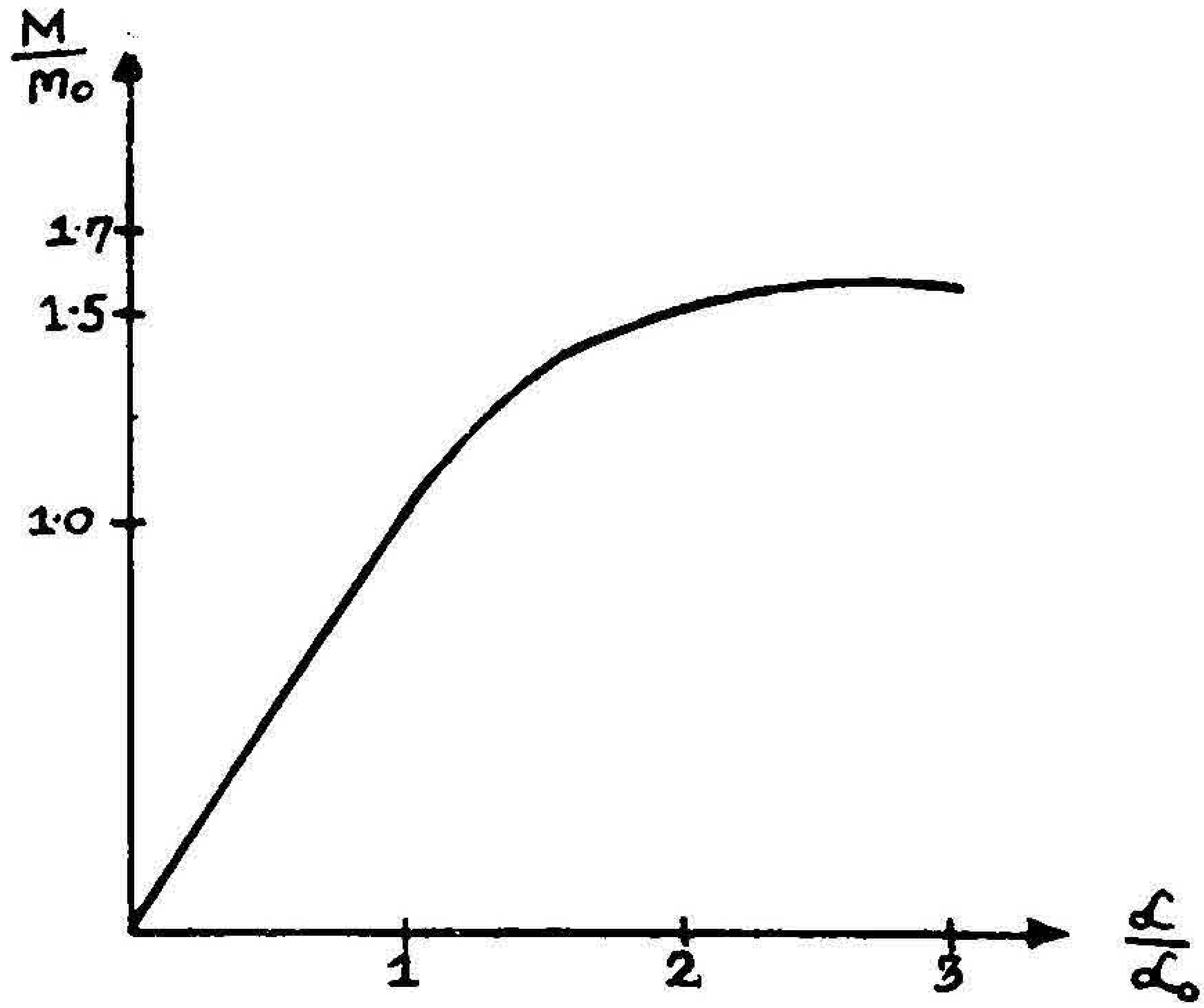

FIG. 10

take the roof values themselves. Figure 4 gives these initial values and the corresponding residuals obtained from the relation

$$R_0 = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 - 6X_0 + 78 \qquad (4)$$

Now by the usual relaxation process the residuals in Fig. 4 are liquidated. Here of course care must be taken to see that the X values do not exceed the roof values. Figure 5 gives the final values corresponding to the limiting gradient $8\sqrt{3}/27$.

3. By a similar procedure, the X values are evaluated for the following values of maximum gradient, *viz.*,

$$\frac{6\sqrt{3}}{27}, \frac{5\sqrt{3}}{27}, \frac{3\sqrt{3}}{27}, \frac{\sqrt{3}}{27}$$

and Figs. 6, 7, 8, 9 give the X values in these cases. In the Figs. 5 to 9 the X values enclosed by circles indicate that the straining at those points is plastic.
In all these cases, the torque M is calculated by the formula

$$\begin{aligned} M &= 2\mu \alpha a^4 \int\int X d\xi d\eta \\ &= 2a^3 \frac{\tau_0}{max.\ \text{gradient}} \int\int X\, d\xi\, d\eta \end{aligned} \qquad (5)$$

The integral is evaluated numerically by noting that a single triangle contributes

$$\int\int X d\xi d\eta = \left(\frac{X_a + X_b + X_c}{3} + \frac{l^2}{6}\right) \frac{\sqrt{3}}{4} l^2 \qquad (6)$$

where l is the mesh length $1/9\sqrt{3}$

Figure 10 gives the graph of M/M_0 against α/α_0 where M_0, α_0 are the limiting torque and the corresponding twist for the elastic case.

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