DERIVATION OF CONSTANTS OF A n-UNIT CASCADE TRANSFORMER

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SUMMARY

Expressions for the open-circuit admittance and short-circuit impedance of a n-unit cascade transformer have been derived. These enable the estimation of the above constants from a knowledge of the constants of any one transformer and the arrangement of the transformers.

1. INTRODUCTION

It is the standard practice in High Voltage Laboratories[1] to produce high voltage a.c. at power frequencies, above 500 kV, by Cascade Transformers.[2, 3] Cascading of transformers provides a connection whereby the h.t. windings of several similar transformers are virtually placed in series whilst only the primary of the first (ground end) transformer is connected to the supply, the power for the higher members of the group being provided by the first and intermediate transformers through a third winding known as the exciting winding. This winding has a 1:1 ratio with the primary, *i.e.*, low voltage winding, but is insulated from the tank upto the full voltage of the secondary, *i.e.*, high voltage winding. The connection diagram of a 3-unit cascade transformer to provide 1050 kV, which has been installed at the High Voltage Laboratory,[1] Indian Institute of Science, Bangalore, is shown in Fig. 1.

Since each unit of a cascade transformer is a three-winding transformer, for purposes of calculation of regulation, etc., it can be represented by an equivalent circuit[4, 5] shown in Fig. 2, in which the three impedances Z_{LV}, Z_E and Z_H represent the leakage impedances associated with the three windings, low voltage (LV), exciting (E) and high voltage (H). Y_x represents the excitation admittance of each transformer. Recently mention has been made by Hagenguth and others[6] that the ratio of transformers connected in cascade is calculable on the basis of a series connection of three-winding transformer equivalent circuits. In the present paper, with the idea of generalisation, the theory has been extended to a n-unit cascade transformer, in which 'n' identical transformers are connected in the Dessauer cascade connection.[2, 3] Expressions for the open-circuit admittance and short-circuit impedance have been derived. The results of tests conducted on the cascade transformers at the Indian Institute of Science will be published later.

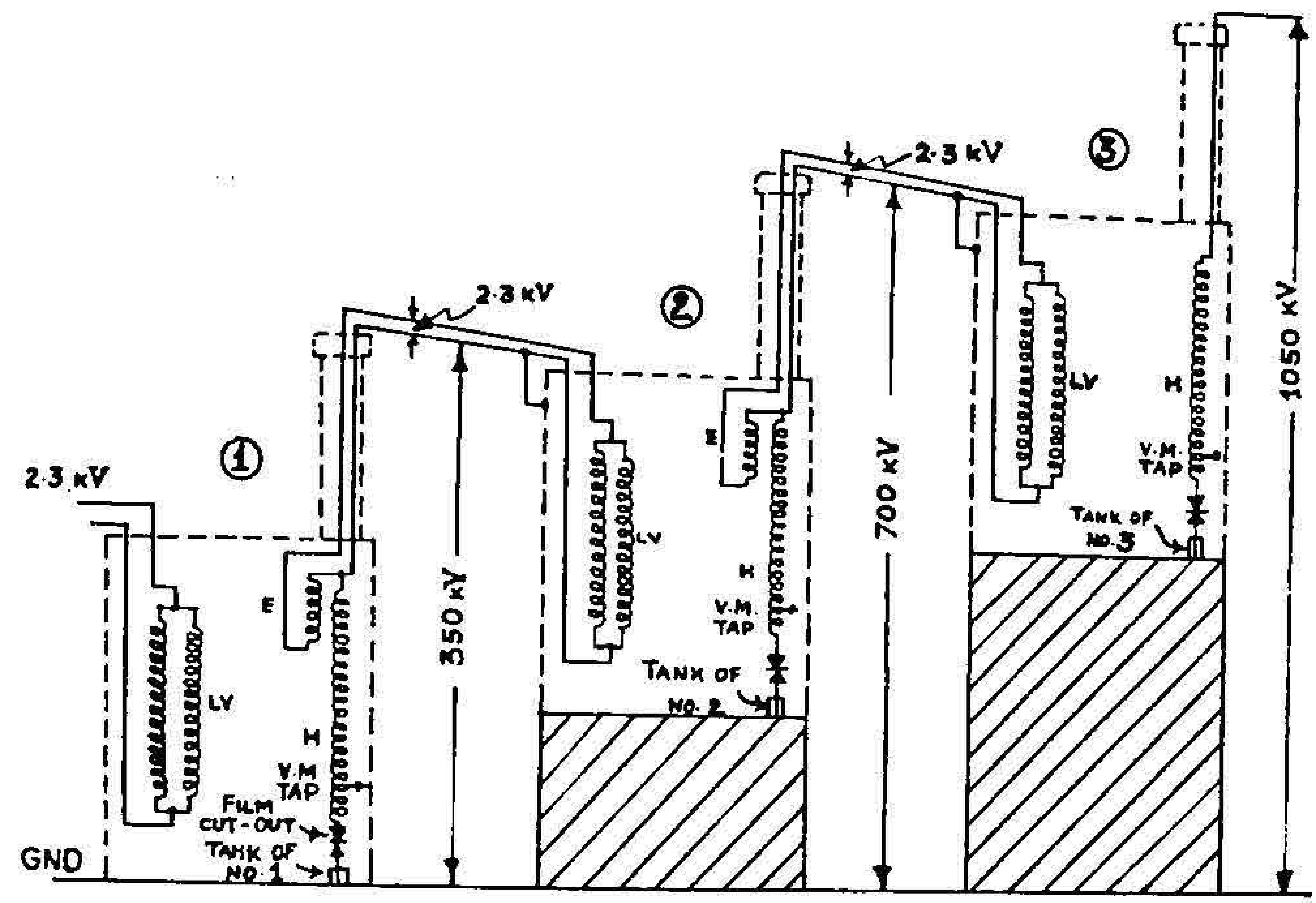

FIG. 1. CONNECTION DIAGRAM OF THE 3-UNIT CASCADE TRANSFORMER INSTALLED AT THE INDIAN INSTITUTE OF SCIENCE[1] FOR PRODUCING 1050 kV.

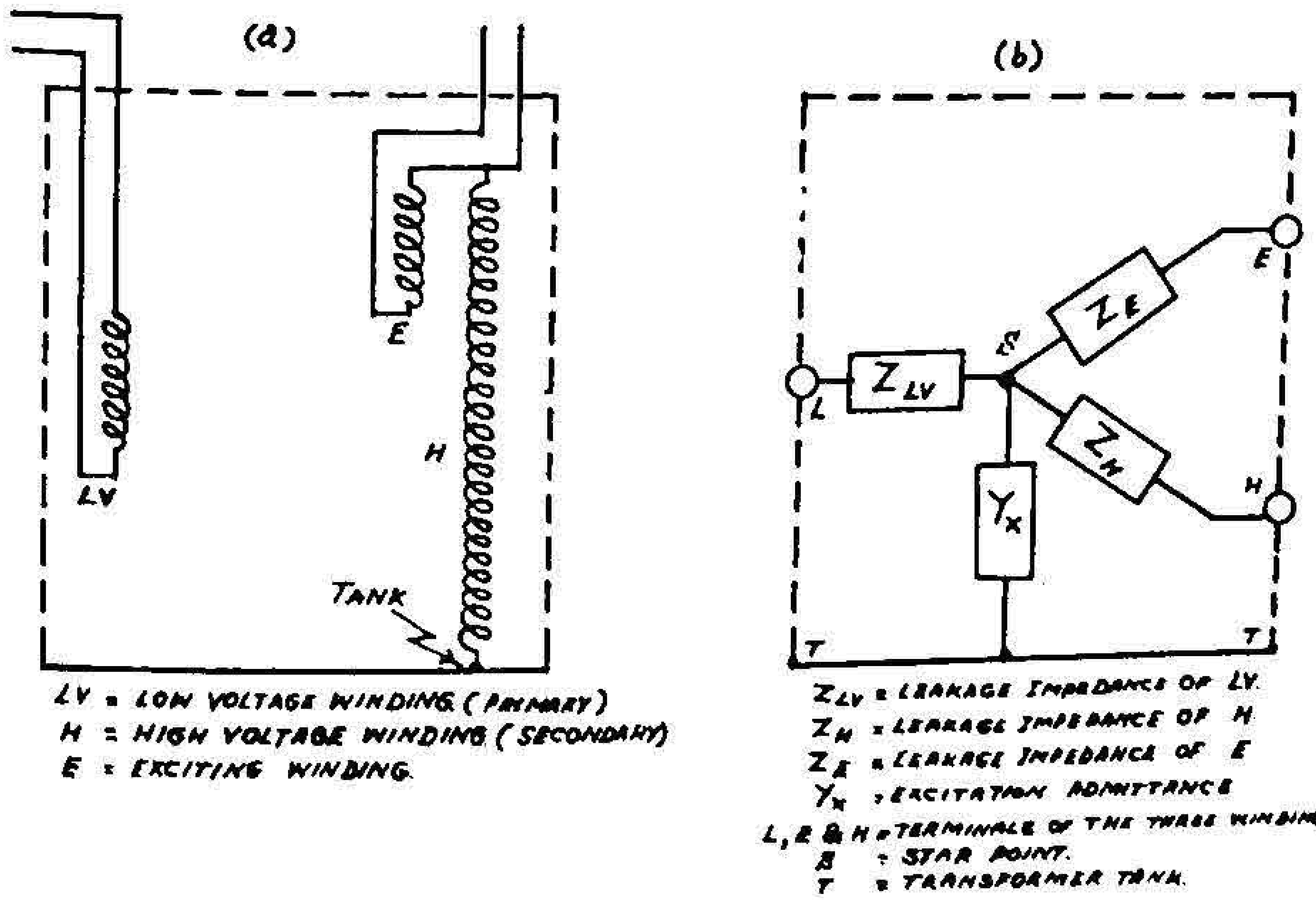

FIG. 2. (a) EACH UNIT OF A CASCADE TRANSFORMER AND (b) ITS EQUIVALENT CIRCUIT.

2. THE n-UNIT CASCADE TRANSFORMER

In a n-unit cascade transformer, 'n' identical transformers are connected in cascade as shown in Fig. 3. The first unit is the ground unit, its tank and one end of h.v. winding being solidly grounded. The tanks of the intermediate and the last units are suitably insulated from ground for the proper voltages. Fig. 4 shows the equivalent circuit diagram of a n-unit cascade transformer, in which each unit is represented by a three-winding transformer equivalent circuit[4, 5] and

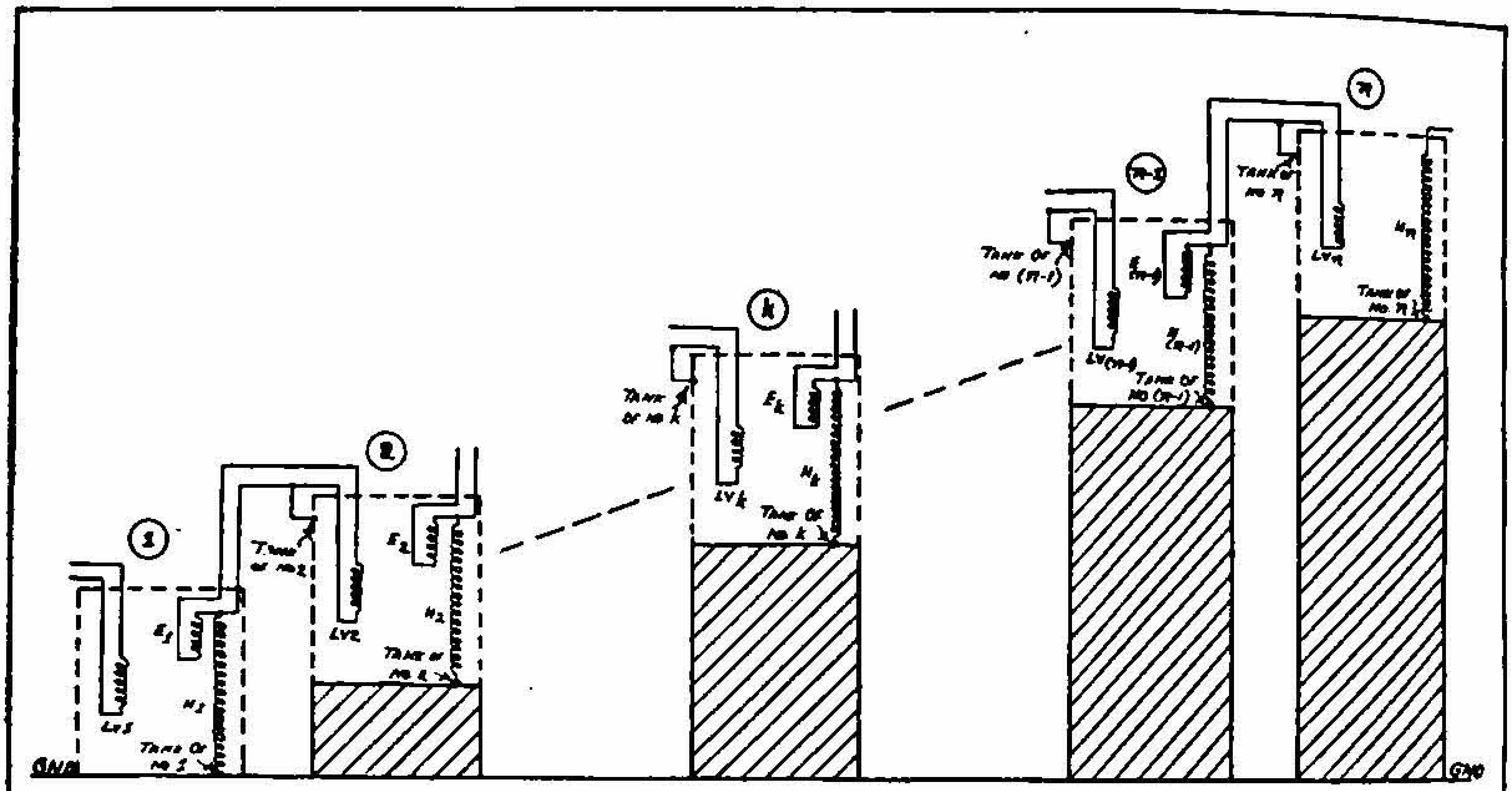

FIG. 3. CONNECTION DIAGRAM OF A n-UNIT CASCADE TRANSFORMER.

is coupled to the next by an ideal 1 : 1 transformer to preserve the voltage and current relations. The nth unit is represented by a two-winding transformer equivalent circuit.

The values of impedances (and admittances), currents and voltages are all those referred to the low voltage winding.

3. LIST OF SYMBOLS

1. $V_{L1}, V_{L2} \ldots V_{Lk} \ldots V_{L(n-1)}, \quad V_{Ln}$ = Low-side voltages of the units 1 to n.
2. I_0 = Current in the low voltage winding of the ground (No. 1) unit when the cascade set is open.
3. $I_{12} \ldots I_{(k-1)k} \ldots I_{(n-2)(n-1)}, I_{(n-1)n}$ = Currents in the low voltage windings of the units 2 to n = currents in the exciting windings of the units 1 to $(n-1)$.
4. $I_{X1}, I_{X2} \ldots I_{Xk} \ldots I_{X(n-1)}, \quad I_{Xn}$ = Exciting currents of the units 1 to n.

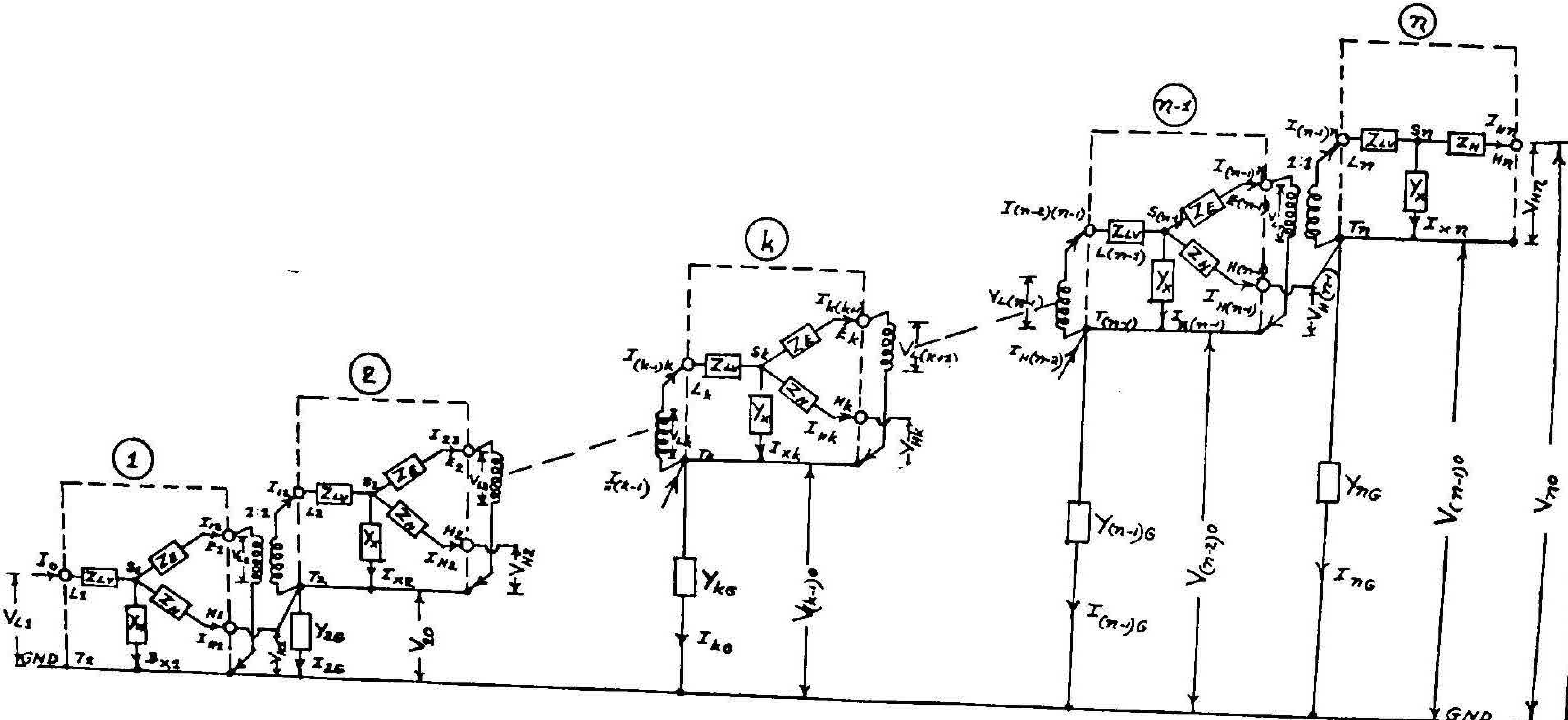

FIG. 4. EQUIVALENT CIRCUIT DIAGRAM OF A n-UNIT CASCADE TRANSFORMER.

5. $V_{H1}, V_{H2} \ldots V_{Hk} \ldots V_{H(n-1)}, \quad V_{Hn}$ = Secondary (high voltage winding) voltages of the units 1 to n (referred).

6. $I_{H1}, I_{H2} \ldots I_{Hk} \ldots I_{H(n-1)}, \quad I_{Hn}$ = Currents in the high voltage windings of the units 1 to n (referred).

7. $V_{10}, V_{20} \ldots V_{k0} \ldots V_{(n-1)0}, \quad V_{n0}$ = H.V. terminal voltages with respect to ground of the units 1 to n (referred).

8. $Y_{2G} \ldots Y_{kG} \ldots Y_{(n-1)G}, \quad Y_{nG}$ = Tank-to-ground admittances of the units 2 to n (referred).

9. $I_{2G} \ldots I_{kG} \ldots I_{(n-1)G}, \quad I_{nG}$ = Currents through the respective tank-to-ground admittances (referred).

10. I_{sh} = The short-circuit current flowing from the h.v. terminal of the nth unit to ground when the cascade set is shorted to ground (referred).

4. Open-Circuit Admittance of a n-Unit Cascade Transformer

While deriving the expression for the open-circuit admittance of a n-unit cascade transformer all series impedances are neglected and the modified equivalent circuit diagram is shown in Fig. 5.

Referring to the equivalent network shown in Fig. 5,

$$\text{Open-circuit admittance} = \frac{I_0}{V_{L1}} \tag{4.01}$$

$$\text{and by inspection, } I_{Hn} = 0 \tag{4.02}$$

Considering the junction S_n in the nth unit,

Similarly at

$$\left.\begin{array}{lllll} & I_{(n-1)n} & = & I_{Hn} & + I_{Xn} \\ S_{(n-1)}, & I_{(n-2)(n-1)} & = I_{(n-1)n} & + I_{H(n-1)} & + I_{X(n-1)} \\ \vdots & \vdots & & & \\ S_k, & I_{(k-1)k} & = I_{k(k+1)} & + I_{Hk} & + I_{Xk} \\ \vdots & \vdots & & & \\ S_2, & I_{12} & = I_{23} & + I_{H2} & + I_{X2} \\ S_1, & I_0 & = I_{12} & + I_{H1} & + I_{X1} \end{array}\right] \tag{4.03}$$

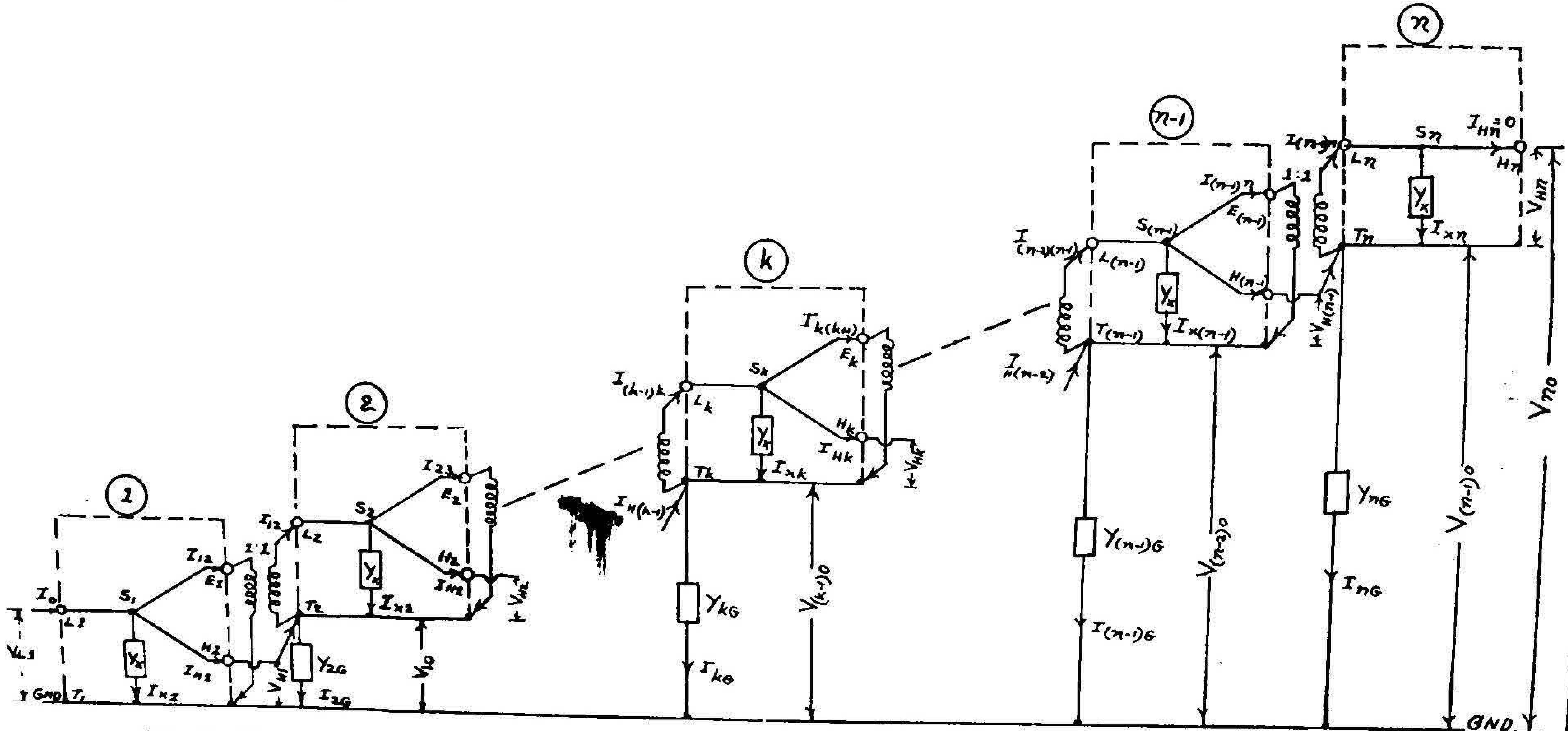

FIG. 5. MODIFIED EQUIVALENT CIRCUIT DIAGRAM OF A n-UNIT CASCADE TRANSFORMER.
[ALL SERIES IMPEDANCES ARE NEGLECTED FOR EVALUATING THE OPEN-CIRCUIT ADMITTANCE.]

Adding all the equations in (4.03)

$$I_0 = \sum_{1}^{(n-1)}{}_k I_{Hk} + \sum_{1}^{n}{}_k I_{Xk} \tag{4.04}$$

Again considering the junction T_n in the nth unit,

$$\left.\begin{array}{lllll} & I_{H(n-1)} & = I_{Hn} & + I_{nG} \\ \text{similarly at } T_{(n-1)}, & I_{H(n-2)} & = I_{H(n-1)} & + I_{(n-1)G} \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ T_k, & I_{H(k-1)} & = I_{Hk} & + I_{kG} \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ T_2, & I_{H1} & = I_{H2} & + I_{2G} \end{array}\right] \tag{4.05}$$

Adding all the equations in (4.05)

$$I_{H1} = I_{2G} + I_{3G} \ldots + I_{kG} \ldots + I_{nG} \tag{4.06}$$

$$\left.\begin{array}{llr} \text{similarly } I_{H2} & = & I_{3G} \ldots + I_{kG} \ldots + I_{nG} \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ I_{H(k-1)} & = & I_{kG} \ldots + I_{nG} \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ I_{H(n-1)} & = & + I_{nG} \end{array}\right] \tag{4.07}$$

Adding (4.06) and (4.07),

$$\sum_{1}^{(n-1)}{}_k I_{Hk} = \sum_{2}^{n}{}_k (k-1) I_{kG} \tag{4.08}$$

Substituting in (4.04)

$$I_0 = \sum_{2}^{n}{}_k (k-1) I_{kG} + \sum_{1}^{n}{}_k I_{Xk} \tag{4.09}$$

Further,

$$\sum_{2}^{n}{}_k (k-1) I_{kG} = \sum_{2}^{n}{}_k (k-1) V_{(k-1)0} . Y_{kG} \tag{4.10}$$

where $V_{(k-1)0} = V_{10} + V_{H2} \ldots\ldots + V_{H(k-1)}$

but since $V_{10} = V_{H2} = \ldots\ldots = V_{H(k-1)}$

$$V_{(k-1)0} = (k-1) V_{10}$$

$$\therefore \sum_{2}^{n}{}_k (k-1) I_{kG} = V_{10} \sum_{2}^{n}{}_k (k-1)^2 . Y_{kG} \tag{4.11}$$

And

$$\sum_{1}^{n}{}_k I_{Xk} = \sum_{1}^{n}{}_k V_{Lk} . Y_X \tag{4.12}$$

but since $V_{L1} = V_{L2} = \ldots\ldots = V_{Lk} \ldots\ldots = V_{Ln}$

$$\sum_{1}^{n}{}_k I_{Xk} = n . V_{L1} . Y_X \tag{4.13}$$

Substituting (4.11) and (4.13) in (4.09)

$$I_0 = V_{10} \sum_{2}^{n}{}_k (k-1)^2 . Y_{kG} + V_{L1} . (nY_X) \tag{4.14}$$

but since $V_{10} = V_{L1}$

$$I_0 = V_{L1} \left[(nY_X) + \sum_{2}^{n}{}_k (k-1)^2 . Y_{kG} \right] \tag{4.15}$$

Substituting (4.15) in (4.01)

$$\left.\begin{array}{l}\text{Open-circuit admittance of a} \\ n\text{-unit cascade transformer}\end{array}\right\} = \left[(nY_X) + \sum_{2}^{n}{}_k (k-1)^2 \, Y_{kG} \right] \tag{4.16}$$

5. Short-Circuit Impedance of a *n*-Unit Cascade Transformer

While deriving the expression for the short-circuit impedance of a *n*-unit cascade transformer all the shunt admittances are neglected and the modified equivalent circuit diagram is shown in Fig. 6. The high voltage terminal of the *n*th unit is shorted to ground and a current of I_{sh} (referred) flows through the h.v. winding of the *n*th unit. The current distribution in the different windings of the other units is also shown in Fig. 6 and a current of nI_{sh} flows through the low voltage winding of the ground unit.

Referring to the equivalent network shown in Fig. 6,

$$\text{Short-circuit impedance} = \frac{V_{L1}}{nI_{sh}} \tag{5.01}$$

Considering the loop $L_n S_n H_n T_n$ in the *n*th unit,

$$V_{Hn} = V_{Ln} - I_{sh} (Z_{LV} + Z_H) \tag{5.02}$$

Again considering the loop $H_{n-1} S_{n-1} E_{n-1} T_{n-1}$ in the $(n-1)$th unit

$$V_{Ln} = V_{H(n-1)} + I_{sh} . Z_H - I_{sh} . Z_E \tag{5.03}$$

Substituting (5.03) in (5.02)

$$V_{Hn} = V_{H(n-1)} + I_{sh} . Z_H - I_{sh} . Z_E - I_{sh} . Z_{LV} - I_{sh} . Z_H$$

$$\therefore \; V_{Hn} = V_{H(n-1)} - I_{sh} (Z_{LV} + Z_E) \tag{5.04}$$

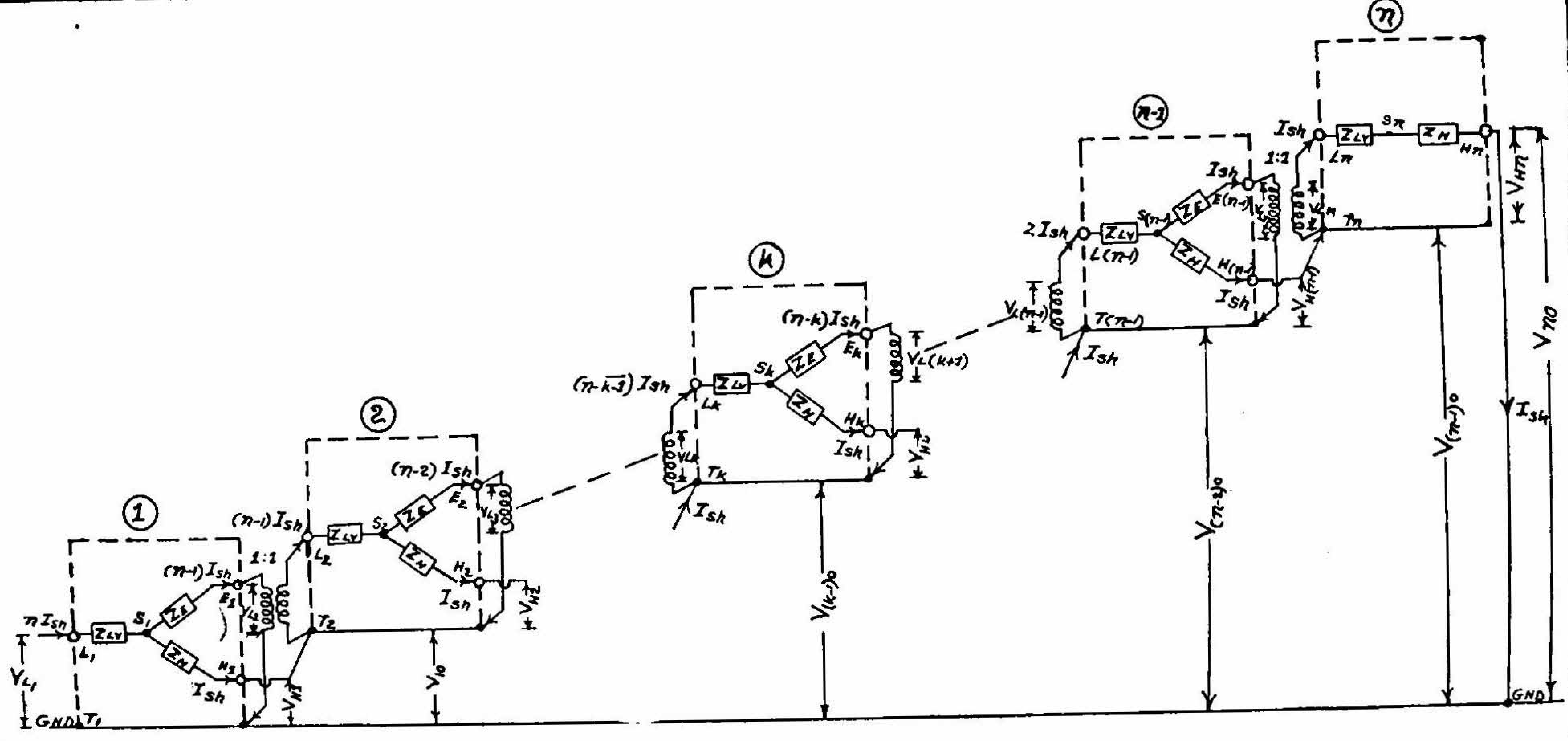

FIG. 6. MODIFIED EQUIVALENT CIRCUIT DIAGRAM OF A n-UNIT CASCADE TRANSFORMER
[ALL SHUNT ADMITTANCES ARE NEGLECTED FOR EVALUATING THE SHORT-CIRCUIT IMPEDANCE.]

similarly

$$\left.\begin{aligned} V_{H(n-1)} &= V_{H(n-2)} - 2\, I_{sh}\,(Z_{LV}+Z_E) \\ &\vdots \\ V_{H(k+1)} &= V_{Hk} - (n-k)\, I_{sh}\,(Z_{LV}+Z_E) \\ &\vdots \\ V_{H2} &= V_{H1} - (n-1)\, I_{sh}\,(Z_{LV}+Z_E) \end{aligned}\right\} \quad (5.05)$$

Adding (5.04) and (5.05)

$$V_{Hn} = V_{H1} - [1 + 2 \ldots + (n-k) \ldots + (n-1)]\, I_{sh}\,(Z_{LV}+Z_E) \quad (5.06)$$

similarly

$$\left.\begin{aligned} V_{H(n-1)} &= V_{H1} - [\quad 2 \ldots + (n-k) \ldots + (n-1)]\, I_{sh}\,(Z_{LV}+Z_E) \\ &\vdots \\ V_{H(k+1)} &= V_{H1} - [\quad (n-k) \ldots + (n-1)]\, I_{sh}\,(Z_{LV}+Z_E) \\ &\vdots \\ V_{H2} &= V_{H1} - [\quad (n-1)]\, I_{sh}\,(Z_{LV}+Z_E) \\ V_{H1} &= V_{H1} \end{aligned}\right\} \quad (5.07)$$

Adding (5.06) and (5.07)

$$\sum_{1}^{n}{}_k\, V_{Hk} = nV_{H1} - \sum_{1}^{(n-1)}{}_k\, k^2\, I_{sh}\,(Z_{LV}+Z_E) \quad (5.08)$$

But L.H.S. of (5.08) is zero since

$$\sum_{1}^{n}{}_k\, V_{Hk} = V_{n0} = 0 \quad (5.09)$$

$$\therefore\ V_{H1} = \frac{\sum_{1}^{(n-1)}{}_k\, k^2}{n}\, I_{sh}\,(Z_{LV}+Z_E) \quad (5.10)$$

Again considering the loop $L_1S_1H_1T_1$ in the first unit

$$V_{L1} = V_{H1} + I_{sh} . Z_H + nI_{sh} . Z_{LV} \quad (5.11)$$

Dividing throughout (5.11) by nI_{sh} and rearranging,

$$\frac{V_{L1}}{nI_{sh}} = Z_{LV} + \frac{Z_H}{n} + \frac{V_{H1}}{nI_{sh}} \quad (5.12)$$

Substituting for $\frac{V_{L1}}{nI_{sh}}$ and V_{H1} in (5.12) from (5.01) and (5.10)

$$\left.\begin{array}{l}\text{Short-circuit impedance of a}\\ n\text{-unit cascade transformer}\end{array}\right\} = Z_{LV} + \frac{Z_H}{n} + \frac{\sum_{1}^{(n-1)} {}_k k^2}{n^2} (Z_{LV} + Z_E) \qquad (5.13)$$

6. CONCLUSIONS

The constants of a n-unit cascade transformer can be derived from a knowledge of the constants of any one transformer and the arrangement of the transformers. It is expected that these derived constants could be used with advantage in a two-winding transformer equivalent circuit for rapid and approximate calculation of the ratio of output voltage to input voltage of transformers connected in cascade with different load capacitances.

7. ACKNOWLEDGEMENT

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