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BOOK REVIEWS

A convergence of lives: Sofia Kovalevskaia, scientist, writer, revolutionary by Ann Hibner Koblitz. Birkhauser Verlag, CH-4010, Basel, Switzerland, 1984, pp. 304, S. Fr. 44. (Indian orders to Allied Publishers Pvt. Ltd.)

It was interesting for me (being a woman scientist myself) to read in the concluding chapter of the fascinating book *A convergence of lives* by Ann Koblitz giving a historical account of Sofia Kovalevskaia's life as a scientist, writer and revolutionary, that the mathematician Sophie Germain's name does not appear on the Eiffel Tower among the list of winners of the Grand Prize of the French Academy of Sciences, although she was the first woman to win the prize in 1816. The omission is particularly glaring since her work on the elasticity of metals was partly responsible for making the construction of the Tower itself possible. Another denigration of a woman's feat!

Similar is the case with the eminent woman mathematician Sofia Kovalevskaia. She has been often ignored by the historians in the past or has been referred in the biography of Weierstrass' as his favourite pupil (for example see the E.T. Bell's book *Men of mathematics*). Kovalevskaia's accomplishments cannot be spoken so slightly. She was the first woman in modern Europe to receive a doctorate in mathematics and the first woman in Europe outside of Italy to have a University Chair in any field. She was the first woman to be selected to the Russian Imperial Academy of Sciences. She won the Prize Bordin of the French Academy of Sciences. She was the first woman to become an editor of a major scientific journal, *Acta Mathematica.* Hers is one of the three classic cases of revolution of a solid body about a fixed point and her work on partial differential equations is basic to that field. A first biography in English presenting a unified historical portrait of this extraordinary woman scientist, therefore, is a most welcome and valuable addition-long overdue-in the history of science.

Kovalevskaia lived in a period of 1860's when women had to fight for personal independence and the right to a university education. She had to overcome the obstacles set at every step by the society to become a professional and a career woman. Therefore it is not surprising that the first six chapters of this exciting book, covering one third material in the book, were required by the author to tell us about the struggles Sophia had to go through to achieve her goal of getting a doctorate in mathematics.

In 1860's the movement for women's higher education had sprung up in Russia as a part of the general progressive egalitarian-democratic movement after the end of the Criméan war. But, due to student uprising in March 1861, the universities and other institutes of higher education closed their doors to all but officially enrolled students. Since many women were admitted on semi-official basis, this left Russian women with little possibility for higher education except private tutoring and study abroad. It was, however, not easy for women to travel outside Russia either as women were listed on their father's or husband's internal passport, and therefore, could not work, study or even live apart from them without express permission, which traditionally was not forthcoming.

One of the most common ways of circumventing parental authority came to be the 'fictitious marriage'. A young woman desirous of leaving home to work or study would come to an agreement with a man who would go through the marriage ceremony and then, theoretically at least, leave the woman to pursue her own life. Her father had no further authority over her, and her 'husband' was honour bound to keep their relationship platonic. Sofia Korvin-Krukovskaia became one of the first to avail herself of this method.

Determined to pursue her higher studies in mathematics and natural sciences, little Sofia Korvin-Krukovskaia became Sofia Kovalevskaia in late September 1868. At the age of eighteen she 'married' Vladimir Kovaleskii and left her childhood behind her ready to embark upon a new life. Later we find that this fictitious marriage turned into a real one and Sofia had a daughter born to her on October 17, 1878 at the age of twenty-eight. This was her only child.

Vladimir and Sofia travelled abroad in search of education. Inspite of all the difficulties a woman had to face in those days. Sofia managed to get training, at the best schools of mathematics of that time in Heidelberg and Berlin. At Berlin she was recommended by her professors to work with the world-famous mathematician Karl Teodore Weierstrass (1815-1897). It did not take long for Weierstrass to recognize genius in her. Weierstrass called Kovalevskaia as his most gifted disciple; this was no ordinary compliment. Ann Koblitz gives an interesting account of Sofia's struggles and about the difficulties Weierstrass had to face to get her thesis recognized by some university. Finally, in August 1874 Sofia was awarded the degree of doctor of philosophy in mathematics by the Gottingen Philosophical Faculty. She was the first woman in the world outside of Renaissance Italy to receive her doctorate in that field.

The years of apprenticeship were finally over. Sofia and Vladimir prepared to return to Russia, full of ambitious plans for their personal and professional lives. They both expected to be appointed immediately to prestigious teaching posts in St. Petersburg. Also, they seem to have been thinking serious of embarking on their 'new life' together, as real husband and wife. Unfortunately, both Sofia and Vladimir soon learned that the reality of life in Russia was far different from their rosy imaginings. The next nine years are described in the following three chapters by the author as times of increasing disillusionment, frustration for the couple leading to eventual tragedy of death of Vladimir, by committing suicide.

In November 17, 1883 Kovalevskaia arrived in Stockholm to take up position she had worked so hard to achieve - university teacher. Sofia did some of her best creative work, literary as well as mathematical, during her Stockholm period and the following years were crowned by triumph in her scientific career the likes of which she had never expected: an extraordinary professorship at Stockholm University, and the Prize Bordin of the French Academy of Sciences.

Sofia died in 1891 when she was just forty-one years of age. The author gives a very moving account of her death and her funeral, which was attended by many Swedish intellectuals, as well as a large number of students and ordinary citizens. Memorial meetings were held in mathematical societies of several countries. Obituaries and commemorative articles of particular interest are listed individually in the body of the bibliography at the end of the book.

Kovalevskaia's mathematics and literary works are summarized in the thirteenth and fourteenth chapters. The list of scientific and literary works and bibliographical notes strengthen these chapters.

Ann Hibner Koblitz teaches at the University of Washington. Her interests include the history of science, Russian intellectual history and the role of women in science. The author's experience of Russian and European culture, the grasp of times, understanding of women's difficulties in her career life has led her in the production of this excellent biography in the history of science of a first great woman of mathematics and a revolutionary, fighting for the rights of women till the end of her life.

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The genesis of the abstract group concept by Hans Wussing. The MIT Press, 28, Carleton Street, Cambridge, Massachusetts 02142, 1984, pp. 272, $ 34.50. (Indian orders to Allied Publishers Pvt. Ltd.)

As the title suggests it is 'a contribution to the history of the origin of abstract group theory' and the volume is a translation of the original in German.

The history of mathematics during the last 150 years or so presents a large number of fascinating fields. Group theory is undoubtedly one of them. Unfortunately there has not been a very well-knit presentation of this great subject. It is fair to say that a comprehensive historical record of modern mathematics has not yet been made available excepting the works of Klein and Bourbaki. In the context of older studies on the history of group theory, H. Poincare in his paper 'On the foundation of geometry' in 1898 stated that the actual basis of ancient Euclidean proofs was the concept and properties of a group and that the group concept is as old as mathematics. This kind of assertion is the result of a backward projection of modern group theoretic thought. Again in 1921, G.A. Miller, a leading American group theorist, published a short sketch of a history of group theory, according to which the beginning of group theory dates back to the origin of mathematical thoughts, *i.e.* essentially the human civilization. As an example, it was known to the earliest mathematicians that positive rational numbers are closed under multiplication and the difficulties associated with the concepts of zero and negative numbers in the early stages of mathematics were due to the traditional adherence to the multiplicative group of numbers. "As a result of well-known investigations of A. Speiser we know of modes of thought associated with geometric ornaments that are thousand years old admit a group theoretic interpretation. Again, in connection with the study of magic squares, Manual Moschopulos, a Byzantine scholar, employed operations resembling permutations". These and many other instances of group heoretical way of thinking certainly do not lead to a coherent line of development of the ıbstract group concept. In this regard the present author has correctly spelled out that the levelopment of group theory is based on three equally important roots, namely the theory of ılgebraic equations, number theory and geometry. The interactions of these three historical ·oots gave birth to the present day abstract group theory. It is usually asserted that this :oncept arose out of the concept of permutation group which obviously derived its origin from the theory of algebraic equation. *The development of mathematics* by E.T. Bell and *A concise history of mathematics* by D.J. Struik do certainly maintain this view. But certainly this is just one of the historical roots of group theory.

The present author has traced the development of permutation group derived from the theory of algebraic equations and from Galois theory associated with the name of A. Vandermonde, J.-L. Lagrange, P. Ruffini, N.H. Abel, E. Galois, J.-A. Serret and C. Jordan. This is just one aspect of the history of development. Another aspect of the formulation of permutation group is through the researches on number theory which essentially began through the efforts of L. Euler. Euler's paper in 1761 on power residues proved a rich source of early implicit group theoretic thinking. Euler decided to investigate the remainders of the terms of a geometric sequence upon division by a prime. The results of this investigation culminated in the celebrated result of group theory that the order of a subgroup is a divisor of the order of the group. In this line of thought, the rich contribution of C.F. Gauss promoted the development simultaneously of the theory of algebraic equations and permutation group. This development ended finally with Kronecker's axiomatization of the implicit group concept following the works of Kummer who was his friend and teacher. The application of Kronecker's theorems to group theory was made in 1882 by his student E. Netto. The fact that the basic theorem for finite abelian group without using permutation group was established at the end of the nineteenth century indicates that the evolution of abstract group theory already came into existence. It is this development later that revolutionized the concept of group theory by the researches of Lie and others.

In 1871, J. Houël while giving a detailed review of the works of Jordan made an extremely interesting remark that 'Geometry is a new attraction to the theory of permutations'. When Houël wrote this, F. Klein was at work and he launched a far deeper penetration of the group theoretical view points into geometry than what Houël thought. This started the theory of invariants as a classification tool in geometry. For example, from the inception of the British geometric school centering around A. Cayley and J.J. Sylvester during 1840's it was strongly influenced by the analytic approaches to projective geometry characterized by the invariance of cross ratios under projections. G. Boole (1815-64) took up algebraic-number theoretic tools in geometry. His researches on 'The theory of analytic transformations' are the fore-runners of present day group theory. After this, Cayley entered the scene with his paper 'On the theory of linear transformations' in 1845, with a search for covariants under linear transformation. Under the influence of C.G.J. Jacobi, Cayley used calculus of determinants for decisive methods for geometric purposes. This is the beginning of the invariant theory and classification in geometry. The invariant theoretic classification in geometry is of historical interest as a transition to group theoretic classification of geometry particularly due to one mathematician namely Felix Klein who was an assistant to J. Plucker, from whom Klein learnt line geometry. Later he joined Clebsch in Gottingen and through him made contacts with Cayley and L. Cremona. From the end of August 1869 to March 1870 Klein stayed in Berlin and came in contact with S. Lie and Stolz. It is through Stolz that he learnt non-Euclidean geometry. In 1863, S. Lie learnt Galois theory from L. Sylow. The author has succinctly projected the works of these two great giants of mathematics leading to the present edifice of abstract group theory, which has been playing a great role in shaping modern science, particularly physical sciences. Klein believed that their researches played dominant roles in the formulation of special theory of relativity due to Einstein. In context to general theory of relativity Klein failed to appreciate the revolutionary physical content of this theory of relativity. He always regarded it as a mere physical interpretation of essentially mathematical theories that B. Reimann, Cayley, Sylvester, Klein himself and Minkowsky had developed much earlier. "The history of science and not least the history of mathematics, is an integrating component of the varied, objectively necessary and significant efforts to establish a science of science". In bringing out this history the author has divided his investigations into three different parts, namely (a) Implicit group-theoretic ways of thinking in geometry and number theory (b) Evolution of the concept of group as a permutation group and lastly (c) Transition to the concept of a transformation group and the development of the abstract group concept. It is a fascinating story of the birth of the theory of groups which now has emerged as a fundamental discipline in mathematics and shaping the things to come in physics in future. The present volume on the history of group theory is a complete one in all respects. It presents logical developments with complete references and quotations wherever necessary to make the book a perfect one. One derives a rare pleasure in reading such a masterpiece.

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Minimal surfaces and functions of bounded variation by Enrico Giusti. Birkhauser Verlag, CH-4010, Basel, Switzerland, 1984, pp. 240, S. Fr. 84 (Indian orders to Allied Publishers Pvt. Ltd.)

This book deals with the classical problem of finding minimal surfaces and their regularity properties. Called the problem of Plateau, the aim is to find a surface of least area among those bounded by a given curve. This monograph presents the techniques developed by De Giorgi and in general the Italian school to attack this problem.

In Part I, the author studies the problem in its general form. In the first two chapters, BV class functions are introduced and its properties like compactness, imbeddings, traces, etc., are studied. The results are generalisation of various properties known for $W^{1,1}(\Omega)$ space. The main differences lies in the fact that smooth functions are not dense in BV. Generalizations of this space have now been thoroughly investigated in other contexts like plasticity (cf. Temam[1].

The existence question to Plateau problem is settled rather easily but the qualitative properties and the regularity of minimal surfaces are rather difficult to prove. It needs a lot of machinery. This is developed in chapters 3-8. At the end of this preparation, we have a celebrated theorem of De Giorgi. The method outlined is very powerful when one looks for regularity almost everywhere. This is the main content of Part I.

The second part studies a particular case of the problem *viz.*, non-parametric minimal surface problem. In this form, the problem can be compared with another classical problem of Dirichlet for Laplacian operator. It is well-known that the Dirichlet problem is solvable for continuous boundary data if there exists a barrier at each point of the boundary. This is exactly the result of chapters 12 and 13 in the case of minimal surface equation. This is got as an application of the famous gradient estimates of Bombieri, De Giorgi, Miranda. Actually these estimates have been generalized to general quasi-linear equations and there exists now a well developed theory on the solvability of the associated Dirichlet problem (cf. Gilbarg and Trudinger[2]). So the material presented in these chapters can only be considered as a particular case.

In the next chapter, the author studies the problem through what are called direct methods in the calculus of variations. The corresponding set-up for Laplacian is provided by the space $H^1(\Omega)$. While the problem with Laplacian has always a solution, the minimal surface problem may not have. The author introduces a concept of generalized solution by relaxing the functional in question and then compares the relaxed problem with the original one. It is known that there are other methods of defining generalized solutions. For instance, one can use duality techniques to do this (cf. Ekeland and Temam[3]). In recent years, these procedures have been generalized and applied to more complicated problems (cf. Temam[1]).

As already pointed out, the material of the book is very classical. There are other well-known books which contain generalization of many results of Part II. But, since the book extensively deals with one example of a quasi-linear equation, the analysis is less cumbersome and the students can see easily what is going on before taking up more general cases. But the material presented in Part I (except chapters I and II) is not easily available elsewhere. Even though the experts working in the field know about it, it is getting published for the first time to benefit a wider audience. The ideas have been expressed in a beautiful way and in a self-contained manner. The techniques are found to be useful in other contexts as well. This book is recommended as a basic reference for students and researchers who initiate in areas like minimal surfaces, plasticity, hyperbolic equations, etc.

References

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2. GILBARG, D. AND TRUDINGER, N.S. — *Elliptic partial differential equations of second order*, Springer-Verlag, 1977.
3. EKELAND, I. AND TEMAM, R. — *Convex analysis and variational problems*, North-Holland, 1976.

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Topics in operator theory, systems and networks (proceedings of workshop on applications of linear operator theory to systems and networks, 1983) edited by H. Dym and I. Gohberg. Birkhauser Verlag, Basel, 1984, pp. 384, S. Fr. 74 (Indian orders to Allied Publishers Pvt. Ltd.).

It is a common feature in most branches of mathematics that the subject itself starts necessitated by applications but later the mathematicians take over the subject for generalization and the study of the structural aspects. In the process of this generalization, invariably, for a while, the contact with applications is lost and the mathematician and the applied practitioners like the engineer or the physicist work in isolation of each other. Periodically, they make attempts through workshops to communicate and benefit mutually from the works and ideas of each other. In this century the subject of operator theory with its wide variety of applications has gone through the same process. The volume under review contains the proceedings of one such workshop on applications of linear operator theory to systems and networks, which was held at the Weizmann Institute in 1983. The volume contains some very interesting papers by active researchers in this area and has a wealth of information which will certainly be found useful by the mathematicians as well as the engineers. It is difficult to provide a detailed review of each paper appearing in this volume as each paper deals with a speciality and would require a specialist to do the review. An attempt is made to provide brief reviews to provide an idea of the type of results that one can find in these papers.

Paper 1: Invariant subspace representations, unitary interpolants and factorization indices by J.A. Ball

In this paper unitary $n \times n$ matrix valued functions $F(\zeta) = \sum_{j=-\infty}^{\infty} F_j \zeta^j$ on the unit circle Π with prescribed matrix Fourier coefficients $F_j = K_j$ for $j < 0$ are described. Let H_k be the infinite Hankel matrix $[K_{-(j+k-l)}]$, $j, k, = 1,2, \ldots$ on $\mathbb{C}^2_n$. It is known that $\| K_k \| < 1$ is a necessary condition for the existence of such an F. With the additional assumption that $I - H_k H_k^*$ is Fredholm, the set of unitary interpolant is characterized. For the unitary interpolant to be unique and to have a generalized Wiener-Hopf factorization necessary and sufficient conditions are obtained. The results are obtained by combining the invariant subspace representation ideas of Ball and Helton and the results are motivated by the works of Dym and Gohberg on factorization indices for unitary interpolants.

Paper 2: The coupling method for solving integral equations by H. Bart, I. Gohberg and M.A. Kaashoek

Certain classes of integral equations are shown to reduce to simple operators, mostly to just finite matrices; consequently the analysis of such integral equations reduces to the analysis of finite matrices. The most interesting application of this method is to the systems of Wiener-Hopf integral operators and singular operators with rational matrix symbols. This method is also applied to integral equations of the second kind with finite rank or semiseparable kernels.

Paper 3: The expansion theorem for Hilbert spaces of analytic functions by Louis de Branges

Some results on the spectrum of nearly self-adjoint transformations are presented in this paper. Let T be a linear operator on a Hilbert space H with domain dense in H with T^* having same domain as T, and $T-T^*$ the restriction to this common domain, of a completely continuous operator. While the existence of invariant subspaces is not known for the resolvent of T it is shown that if $T-T^*$ is of Macaev type, then the invariant subspaces exist which cleave the spectrum of T. (A nonnegative operator A is said to be a Macaev type if with the usual notations $A = \Sigma\, \phi_n\, \phi_n$ for an orthogonal set of vectors $\{\phi_n\}_{n\in N}$ such that $|\phi_n| <$ $|\phi_{n+1}|$ for every n and such that $\Sigma\, n^{-1}\, |a_n|^2 < \infty$. A generalization of the Fourier transformation is also obtained.

Paper 4: The lossless inverse scattering problem in the network-theory context by P.M. Deivilde

The author uses the reproducing kernel Hilbert space method to describe a global method to construct lossless inverse scattering solutions. Also discussed are connections with applications, some classical interpolation problems and relation of the results to maximum entropy approximation theory.

Paper 5: Subisomorphic dilatiôns and the commutant lifting theorem by R.G. Douglas and C. Foias

Let S be a contraction on a Hilbert space G and let $D_s = (I - S^*S)^{1/2}$; let G_s be the closure of the range of D_s. Let H be a Hilbert space containing G and T an operator on H such that $PT = SP$, P being the orthogonal projection of H on to G. T has the form

$$T = \begin{pmatrix} S & O \\ X & T' \end{pmatrix} \qquad \text{where } H = G + H'.$$

T is called a dilation of S. The dilation T is called contractive if T is a contraction, and X is of the form $X = D_{T'^*}\, C D_s$ where C is a contraction from G to $G_s T'^*$. T is called minimal if $(T'^*)^n \to 0$ strongly as $n \to \infty$; and is called isometric if T' and C are isometric. It is known that two of the most important properties possessed by minimal isometric dilations is their uniqueness up to isomorphism and the commutant lifting property.

Uniqueness: If T_1 and H_1 and T_2 on H_2 are two minimal isometric dilations of S on G then there exists a unitary operator W from H_1 on to H_2 such that $WT_1 = T_2 W$ and $W/G = I_G$.

Commutant lifting property: For every contraction A commuting with S there exists a contraction B commuting with T which dilates A, that is $PB = AP$. In this paper the authors obtain these properties for a class of dilations for which they use the natural terminology of subisomorphic dilations. The operator part added on is a uniform Jordan model.

Paper 6: Positive definite extensions, canonical equations and inverse problems by H. Dym and A. Iacob

The intimate connections between a form of the covariance extension problem and spectral theory of canonical equations are developed and exploited in order to deduce a representation formula for the set of all solutions to the extension problem in an intuitively pleasing way. Enroute, an expository account of the forward and inverse spectral problem for canonical equations and the theory of Hilbert spaces of matrix valued entire functions is given and a number of related applications are discussed (Authors' summary).

Paper 7: Minimal divisors of rational matrix functions with prescribed zero and pole structure by I. Gohberg, M.A. Kaashoek, L. Lerer, and L. Rodman

Let $W(\lambda)$ be an $n\times n$ rational matrix function such that det $W(\lambda)$ does not vanish identically. For a complex number λ_o let $W(\lambda_0) = E_{\lambda_0}(\lambda)\, D_{\lambda_0}(\lambda)\, F_{\lambda_0}(\lambda)$ be the Smith-McMillan form of W at λ_0; where $E_{\lambda_0}(\lambda)$, $F_{\lambda_0}(\lambda)$ have no poles and invertible at λ_0 and $D_{\lambda_0}(\lambda)$ is a diagonal matrix with ith diagonal entry as $(\lambda-\lambda_0)^{\nu_i(\lambda_0)}$; where ν_i are integers and $\nu_1(\lambda_0)\geq \nu_2(\lambda_0)\geq ... \nu_r(\lambda_0)\geq 0\geq \nu_{r+1}(\lambda_0)\geq ...\geq \nu_n(\lambda_0)$. The integers $z(W,\lambda_0)= \sum_{j=1} \nu(\lambda_0)$; $p(W,\lambda_0)= \sum_{j=r+1}^{n} \nu_j(\lambda_0)$ are respectively called the zero and pole multiplicity of W at λ_0. If W is factored as $W(\lambda)= W_1(\lambda)\, W_2(\lambda)$, where $W_1(\lambda)$ and $W_2(\lambda)$ are rational matrix functions, then

$$z(W;\lambda_0) \leq z(W_1,\lambda_0) + z(W_2,\lambda_0) \text{ and}$$
$$p(W;\lambda_0) \leq p(W_1,\lambda_0) + p(W_2,\lambda_0).$$

If equality in one (and hence in both) holds then W_2 is called a right minimal divisor of W at λ_0. If equality holds at all λ_0 then W_2 is called a global minimal divisor of W. The purpose of this paper is to find necessary and sufficient conditions for a rational matrix function $W_2(\lambda)$ to be a minimal divisor of a given rational matrix function $W(\lambda)$. These conditions are expressed in terms of the zero and pole structure of $W(\lambda)$.

Paper 8: The linear-quadratic optimal control problem-The operator theoretic view point by E.A. Jonckheere and L.M. Silverman

The linear-quadratic optimal control problem is viewed from the operator theoretic point of view, and this approach shows the important role played by two bounded self-adjoint operators on appropriate Hilbert spaces of control. The existence of a stabilizing solution is shown to be related to the positive definiteness of a Weiner-Hopf operator while the existence of an antistabilizing solution to that of a perturbed Wiener-Hopf operator.

Paper 9: On the structure of invertible operators on a nest subalgebra of a von Neumann algebra by G.J. Knowles and R. Seeks

The aim of this paper is to study the topological structure of the set of all invertible elements in operator algebras. It is shown that in a certain nest subalgebra of a von Neumann algebra, under suitable boundary conditions, such a set is in the principal component of the identity with strong operator topology. With the uniform topology the set of invertible elements of a certain subalgebra of the above nest subalgebra, is in the principal component of the identity. A number of examples are provided exhibiting that the above subalgebra of the nest subalgebra is extensive.

Paper 10: On commuting integral operators by N. Kravitsky

Let $D(x_1,x_2)=0$ be an algebraic curve in C^2I. Let $S=\{(\lambda_1(x),\lambda_2(x)): 0\leq x\leq l\}$ be a continual set of points on the algebraic curve. The author constructs a pair of commuting triangular integral operators A_1, A_2 such that $D(A_1, A_2) = 0$ and having joint spectrum S. The results are related to the problem of constructing a triangular model for two commuting linear operators with continuous joint spectrum.

Paper 11: Infinite dimensional stochastic realizations of continuous-time stationery vector processes by A. Lindquist and G. Picci

An n-dimensional stationary Gaussian vector process (mean-square continuous, zero mean and purely nondeterministic) has a representation

$$y(t) = \int_{-\infty}^{\infty} e^{iwt} \, d\hat{y}(iw)$$

where $d\hat{y}$ is orthogonal stochastic measure with incremental covariance

$$E\{ d\hat{y}(iw) \, d\hat{y}(iw)^{+} \} = \frac{1}{2\pi} \phi(iw) \, dw,$$

ϕ being the $m \times m$ spectral density and $+$ denotes transpose conjugate. If ϕ is a rational function then y has the representation, called stochastic realization (with dimension n),

$$dx = Ax \, dt + B \, du$$

$$y = Cx$$

where $\{x(t): t \in R\}$ is a stationary vector process of dimension n and $\{u(t): t \in R\}$ is a vector Wiener process with incremental covariance $E\{ du(t) \, du(t)' \} = I dt$; where A, B, C are constant matrices and $'$ denotes transpose. The purpose of this paper is to construct stochastic realizations. The approach is based on the geometric theory of stochastic realization.

Paper 12: The algebraic Riccati equation by A.C.M. Ran and L. Rodman

This paper reviews some recent results on the Hermitian solutions of the algebraic Riccati equation

$$XDX + XA + A^{*}X = C$$

where A, C, D are $n \times n$ matrices with D, C Hermitian and X is $n \times n$ the unknown. Such equations have a wide variety of applications. Such solutions are described in terms of invariant subspaces of a matrix which is self-adjoint with respect to an indefinite scalar product. Along with the review the authors also present some new results.

Selected reading from this volume will immensely benefit any researcher in these areas. All the articles are presented in a lucid style. The volume is a must for any research library. The reviewer thoroughly enjoyed the experience of reading through the volume and has preferred to be close to the authors in the reviews presented.

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Systems of nonlinear partial differential equations edited by J.M. Ball. NATO ASI Series, C 111, D. Reidel Publishing Company, P.O. Box 17, 3300 AA Dordrecht, Holland, 1983, pp. 481, D. Fl. 150. (Indian orders to Allied Publishers Pvt. Ltd.)

Every publishing firm now has a fast publication series devoted to monographs and conference proceedings. As the number of conferences is indeed large we are flooded with books reporting on these. Often such books turn out to be of little value in a permanent collection for the papers in it reflect the nature of the talks which are usually short communications devoted to a special problem, giving only the main results and no details, and referring the interested reader to papers published in journals. It is often difficult to get an idea of the state of the art from such a collection.

In this context the volume reviewed here is likely to prove to be an agreeable exception. The book contains the proceedings of a NATO Advanced Study Institute held at Oxford in July 1982. The book consists of two parts-one on a series of expository lectures and another on papers presented on special problems.

The principal value of this book stems from the collection of expository lectures which have been delivered on a fairly wide range of topics in nonlinear partial differential equations by some of the leading experts in the various fields. These lectures are so written that they contain sufficient information to attract the reader and a large number of references to pursue the topics further. While the topics cover a wide range of problems, the talks are not totally disjoint from each other.

There is a set of three lectures on bifurcation theory which introduces one to this fascinating subject. Starting with the simple example of a buckling rod the method of Lyapunov-Schmidt and other basic tools in the study of bifurcations are introduced. The second lecture describes unfoldings or perturbed bifurcations. The final lecture discusses the Benard problem in particular and bifurcation in the presence of symmetries in general. Another related lecture is on topological invariants associated with the reaction-diffusion equations. Using concepts from algebraic topology, it is shown that new results can be gleaned on bifurcation, stability, etc.

A topic of active research in both theoretical and computational fluid dynamics is that of hyperbolic systems of conservation laws. The lecture devoted to this subject provides a very good introduction to it. It discusses models from continuum mechanics, non-existence of classical solutions, geometric structure of weak solutions, and their non-uniqueness; it also motivates the notion of shock, entropy and viscosity and constructs the random choice method of Glimm in the unidimensional case. A relatively recent technique in this subject is a novel idea from functional analysis called compensated compactness which develops an efficient method of using weak convergence in nonlinear problems. The method, originally devised to study homogenization problems, is also capable of being applied to conservation laws and other areas. These are described in a lecture on Compensated compactness.

The volume also contains lectures on ill-posed problems in thermo-elasticity and nonlinear systems in optimal control theory.

To those with a taste for the pure mathematical aspects of nonlinear partial differential equations there are lectures on regularity for nonlinear elliptic systems, systems in diagonal form and problems from variational integrals.

The second part of the book is devoted to the special sessions on applications of bifurcation theory to mechanics, non-elliptic problems and phase transitions, problems in nonlinear elasticity and dynamical systems. While the problems discussed are relevant to the expository lectures of the first part, the papers, naturally, are more terse in their style but indicate the wealth of mathematical techniques available in the treatments of nonlinear problems.

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Numerical methods for bifurcation problems edited by T. Kupper, H.D. Mittelmann, and H. Weber. Birkhauser Verlag, P.O. Box. 133, CH-4010, Basel, Switzerland, 1984, pp. 584, S. Fr. 88 (Indian orders to Allied Publishers Pvt. Ltd.).

Bifurcation theory is a subject of active research and there is a large output in terms of papers devoted to this topic, both from the theoretical and computational points of view. The present volume is such a collection of papers presented at a conference held at the University of Dortmund, West Germany, in August 1983.

While until recently most research in this area was confined to the use of the implicit function theorem and ramifications thereof and adaptation of those results to formulate computational algorithms, it is now evident that methods from algebraic topology and singularity theory have a lot to say in this subject. A large number of papers talk about homotopy methods, unfoldings, Morse index, etc.

The problems in bifurcation theory are the determination of singular points (like turning points, bifurcation points, etc.); determination of the number and direction of branches emanating from a bifurcation point, and, in practical problems, the global behaviour of these bifurcation branches.

Turning points are the simplest of the singularities encountered. Often by setting up a larger system (often called 'defining equations' in this book) these turn out to be regular points and can be determined fairly efficiently by Newton's method. Several papers discuss how to do this. Global branches can be calculated by continuation methods.

Several other papers are devoted to the classification and computation of singular points. Some of them heavily use singularity theory while others rely on more classical approaches.

Papers are also devoted to determination of bifurcation branches by methods like adaptive condensation, continuation, etc. The Keller-Langford method usually determines the directions of bifurcation branches by solving a system of polynomial equations whose solutions give the slopes of the tangents to the various arcs. However this works only when the roots are simple. A general technique to adapt this when two or more curves touch at a bifurcation point is discussed in this volume.

An important computational aspect in bifurcation theory is the extent to which the discrete system behaves like the original one. This is discussed in the context of Hopf bifurcation for systems of ODEs. Also a specific example where the discrete problems admit a bifurcation system like the original one in the case of a semilinear parabolic equation can be found in another paper.

Since most problems arising in bifurcation theory can be traced to problems in mechanics or engineering, it is natural that several papers discuss solutions of specific problems arising in these domains. Papers are devoted to Taylor's problem in fluid dynamics, the buckling of a visco-elastic rod, the buckling of shells and plates in the presence or absence of symmetries and to other problems in mechanical engineering.

While it is essential in a field of such intense activity like bifurcation theory to have books surveying the state of the art, it must be said that in constituting this volume its editors have taken an easier way out regarding the presentation. They have arranged the papers in alphabetical order of the authors' names while it would have been more helpful to the reader if the papers were classified according to the various sessions or the nature of the problems discussed. Presented as it is, it is a very large and bewildering collection of papers and quite difficult to find out those one is interested in.

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Gorge Polya: Collected Papers, Vol. IV: Probability, combinatorics, teaching and learning in mathematics edited by G.C. Rota, The MIT Press, Cambridge, USA, 1984, pp. 642, $ 69 (Indian orders to Allied Publishers Pvt. Ltd.).

The book under review is the final volume of the collected papers of G. Pólya, whose life and mathematical achievement span several generations. It contains 20 papers on probability, 17 on combinatorics and 18 on the teaching and learning of mathematics. Among the probability papers are several by-now classical ones: one on the central limit theorem (reputedly the first paper to use that term), marked by the first general theory of the method of moments to establish vague convergence of a sequence of probability distributions (on the line); one introduces the term 'random walk' and obtains the famous recurrence criteria for simple symmetric random walks in various dimensions, and the methods adopted there to solve a specific, special problem turned out to be sufficiently general and powerful to initiate major developments (especially the ones due to Chung-Fuchs and Port-Stone); one establishes the uniqueness theorem for characteristic functions (the inversion formula due to Lêvy was to come later), and one a characterization of the normal law, the precursor of the voluminous work since produced in the area. The 'urn models' due to Pólya and to Pólya-Eggenberger are of course to be found here. The well-known sufficient condition for a nonnegative, even function on the line to be a characteristic function and the formulae for the extremities of a function of bounded variation with compact support on the line in terms of its Laplace-Stieltjes transform (in the garb of the Fourier-Stieltjes transform extended to the complex plane) form two parts of another paper. Two papers are concerned with foundations of probability theory, and one of them even apparently foreshadows the later Neyman-Pearson theory. The evaluation of the (incomplete) normal probability integral in one and two dimensions is the subject of a paper.

Among the papers on combinatorics, one deals with doubly-periodic solutions of the problem of n queens on an $n \times n$ chess-board. Several are devoted to the determination of isomer-numbers, their generating functions and 'cycle indices' for several organic chemical compounds. The magnum opus is the paper, 'Combinatorial enumeration ('number-determination') for groups, graphs and chemical compounds'.

The last section is not the least interesting. As well-known, Pólya devoted several books and articles to the teaching and learning of mathematics. Among the former are: 'How to solve it?', 'Mathematics and plausible reasoning', and 'Mathematical discovery' (in two volumes). Pólya has always stressed the importance of problem-solving in mathematics (even as contrasted with increasing abstraction and generalization) even going to the extent of writing to a Department chairman for the promotion of a 'problem-solver'—letter reproduced on pp. 593-4 of this volume. He has allowed himself to be tempted into prescribing a code of 'Ten commandments' for teachers (pp. 525-533). Mathematical discovery induced by incidents like taking a walk in a park or by dreams forms the subject of a paper 'Two incidents' (pp. 582-585). The reader is referred to the volume for the full fare in this area.

There is no question that this volume in particular 'belongs' to every mathematical library, dealing as it does with not only the achievement in, but the philosophy of, mathematics of one of the great mathematicians of our time. The annotations at the end by experts, on selected papers, are a welcome feature of this collection.

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A. Ostrowski: Collected mathematical papers, Vol. 5, Birkhauser-Verlag AG, P.O. Box 133, CH-4010 Basel, 1984, pp. 560, S.Fr. 79. (Indian orders to Allied Publishers Pvt. Ltd.)

Alexander Ostrowski, born in 1892, is a very versatile and productive mathematician whose publications during the last 70 years have made important contributions to many areas of mathematics: Algebra, Number theory, Real and complex analysis, Differential equations, Numerical analysis. The present volume, the fifth of his six-volume *Collected mathematical papers*, contains almost all his papers on Complex function theory; a few papers on Conformal mappings and their numerical analytic aspects appear in Volume 6.

Almost all the papers in this volume were published during the years 1923 to 1933; the language is German. Although there are a number of isolated results, two main themes can be recognised. The first, to which Ostrowski perhaps made his most significant contributions in Function theory, is the analytic continuation of power-series and Dirichlet series beyond their circle (resp. half-plane) of convergence. Ostrowski, generalizing an isolated-looking result of Hadamard, proved the following theorem: in order that some subsequence of the sequence of partial sums of a complex power-series (of finite radius of convergence) converge uniformly in a neighbourhood of a point of regularity on the circle of convergence, it is necessary and sufficient that the series be the sum of one with bigger radius of convergence and a gap-series (a series $\Sigma a_i Z^{n_i}$ is a gap-series if lim sup $(n_{i+1}/n_i) > 1$).

Ostrowski also realised that theorems about analytic continuation of power-series become more natural and general in the context of Dirichlet series. One of his results in this direction is as follows. Let $f(s) = \Sigma a_n e^{-\lambda_n s}$ (λ_n increasing to infinity) be a convergent Dirichlet series, and g any function holomorphic in a neighbourhood of the origin. Then the series $\Sigma g(1/\lambda_n)$ $a_n e^{-\lambda_n s}$ (with $g(1/\lambda_n$ defined arbitrarily for small n) can be analytically continued along every path along which $f(s)$ itself can be continued.

The second theme, to which Ostrowski devoted a number of papers, is the Picard-Schottky theorem. In a long paper, Ostrowski examines in detail the notions of Julia curves and Julia sequences for functions meromorphic in a (punctured) neighbourhood of infinity (to say that a curve $\gamma(t)$ from the origin to infinity is a Julia curve for a meromorphic function f in the whole finite plane implies that, for some real θ, f assumes all but two values in the 'cone' $\{|Z - e^{i\theta}\, \nu(t)| < \alpha\, |\nu(t)|\}$ for every $\alpha > 0$. Meromorphic functions in the whole plane with no Julia curves at all are explicitly determined by Ostrowski, and for the remaining 'general' functions, it follows, that every curve from 0 to ∞ is a Julia line. To state Ostrowski's main contribution to Schottky's theorem, fix a in $\mathbb{C}$, $a \neq 0, 1$, and let S_a be the set of functions f holomorphic in the unit disc never assuming the values 0 and 1 there, normalised by $f(0)=a$; set $S(a,r) = \sup \{ |f(Z)| : |Z| = r, f \in S_a \}$. Then Ostrowski determines the precise asymptotic behaviour of $S(a,r)$ as r tends to 1, and carries out a similar analysis for $|\arg f(Z) \quad \arg f(0)|$.

Among the remaining papers, three are worth special mention. The first is a long paper in which Carleman's proof of his characterisation of quasi-analytic classes of functions is simplified, and Carleman's result on holomorphic functions in a disc (which is the essential part of the proof) is generalised to very general simply connected domains. The second is a paper in which it is proved that the field of all formal Dirichlet series (or that of all Dirichlet series with a half-plane of absolute convergence) is algebraically closed. Of course, other related questions of purely analytic interest are also considered in the paper, but the paper is of special interest because it gives analysts a glimpse of Ostrowski the algebraist; in particular, it contains a short, simple and purely algebraic proof of the Newton-Puisieux expansion theorem. And the last paper of Ostrowski I would like to mention (also the last in this volume, written in 1968) gives a simple and direct proof of Kuiper's Morse lemma for functions of finite order of differentiability; at a non-degenerate critical point, a function of class C^r ($r \geq 2$) is a quadratic form in suitable co-ordinates of class C^{r-1}. This result was proved in 1966 by Kuiper using rather sophisticated methods; the loss of one derivative is in general unavoidable.

This volume is a testimony to Ostrowski's insight and power in the domain of analysis. For workers in Complex analysis, its value is perhaps mainly historical since the questions considered by the author belong to what may be considered as a closed chapter of the subject.

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Vanishing theorems on complex manifolds (Progress in Mathematics, Vol. 56) by B. Shiffman and A.J. Sommese. Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, pp. 184, S. Fr. 46 (Indian orders to Allied Publishers Pvt. Ltd.).

This book is concerned with vanishing theorems for holomorphic vector-bundles on compact complex Kahler (usually projective) manifolds, a vanishing theorem being an assertion that certain cohomology groups of such a bundle vanish (*i.e.* reduce to the trivial group) under suitable assumptions.

This book is an essentially self-contained exposition, containing proofs of most of the known general vanishing theorems. Almost three-fourths of the book (Chs. I to III and Ch. VII) is devoted to vanishing theorems for line bundles. Complete proofs of the Kodaira-Akizuki-Nakano theorems and their various refinements (by Bochner's method) are presented in chapters I - III; the classical applications are stated. Particular emphasis is laid on the close connection between the Akizuki-Nakano theorems and the Leffchetz theorems on hyperplane sections, though the authors avoid stating explicitly that, in the projective case, the two theorems are in fact equivalent (as proved by C.P. Ramanujam in a paper cited frequently in the book). Chaper VII, in contrast to the rest of the book, has a purely geometric flavour, and is concerned mainly with C.P. Ramanujam's important generalizations of the Kodaira vanishing theorem.

Chapters V and VI treat the case of vector-bundles. The difficulty in the case of vector-bundles of rank >1 is that, unlike in the case of line bundles, there are many different candidates for the concept of ampleness or positivity. In chapter V, the Grothendieck-Grauert-Hartshorne notion of positivity which postulates that a certain line-bundle (on a different space) naturally associated with the given vector-bundle is ample, is used to derive, in a fairly formal way, vanishing theorems for vector-bundles from analogous theorems for line-bundles. In chapter VI, two other differential geometric notions of positivity of a rather technical nature are considered. These notions yield sharper vanishing theorems than those of chapter VI; however, the positivity that they require is very hard to verify in practice.

Finally, chapter IV is a useful collection of specific vanishing theorems, proved mainly by elementary and ad hoc methods.

The book is clearly written, and the first four chapters, together with the last, give a good account of many things which a geometer (algebraic or complex analytic) should know. The one drawback of the book is that there is no hint (not even references to the literature) as to why it is interesting to have such theorems as are proved in chapters V and VI.

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Electric properties of weakly nonideal plasmas by Klaus Gunther and Rainer Radtke. Birkhauser Verlag, P.O. Box 133, CH-4010, Basel, Switzerland, 1984, pp. 114, S.Fr. 46. (Indian orders to Allied Publishers Pvt. Ltd.).

Today, electric plasma of various types has gained vital importance in science and technology. This book on the electrical properties of weakly nonideal plasmas is very timely and fills an important gap in the experimental work and various technical applications as for example in high voltage power circuit breakers, welding arcs, high luminosity light sources, etc.

The book starts with a very lucid introduction followed by a neat description of the properties of ideal and weakly nonideal plasmas. Simple techniques for the production of the weakly nonideal plasma in the laboratory are given. The authors show in brief how the plasma parameters may be determined and evaluated.

The reviewer's only regret is that the treatment given is rather brief. However, the literature covered is very vast and it is impossible for any one to do reasonable justice in the space of 102 pages. References are quite numerous but mainly belong to the Soviet and East European world. There are a few from the Western world but many important works are omitted.

Notwithstanding the shortcomings noted above, the authors have done a good job in treating the complex subject of the weakly nonideal plasma in a condensed and highly readable way.

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The life table and its applications by Chin Long Chaing. Robert E. Kreiger Publishing Company, Malabar, Florida, 1984, pp. 316, $ 29.50.

This publication is a follow-up on a previous work *Life tables and mortality analysis* brought out by the author as part of a publication series initiated by the World Health Organisation with the support of the United Nations Fund for Population Activities as teaching aids. The series is addressed to a wide spectrum of professionals in public health, medical research, statistics and demography who are less familiar with higher mathematics.

The popularity of the previous publication which soon got out of print is not only an indication of the increasing use made of life table analysis in several fields but also is a testimony to the lucidity with which the author explains what essentially is an intricate field in statistical science. The book under review is not merely a second edition of the original publication. It takes into account the application of life-table techniques to the 'staging process' met with in chronic disease conditions in which the disease advances from mild through intermediate stages to severe conditions and finally to death. In the application of such techniques, age intervals which play an important role in the classical type of life-tables worked out by actuaries become unimportant. The method of analysis to deal with the set of problems arising in the 'staging process' problems have not only been explained lucidly by the author but has been illustrated by application to a problem in human reproduction.

The Life table and its applications can be used as a treatise for self-instruction or can serve as a text-book for courses in health statistics, biostatistics or acturial science. The first three chapters are devoted to explaining the fundamental concepts of statistics. The next four chapters relate to discussions on death rates and their adjustment and the construction of complete and abridged life tables. Statistical inference regarding life-table functions, the application of life-table techniques to problems in a number of fields including ecological studies, family life cycle and medical follow-up studies, an exposition of the statistical theory underlying the construction and use of the life-table and a discussion of the 'staging process' technique already referred to form the subject matter of the other five chapters in the book.

The author is well known for his work in the application of stochastic theory to life-table functions and the book is filled with formulae for the determination of maximum likelihood estimates, variances and fiducial limits for various life-table functions. The basic assumption made in deriving these formulae, however, is that the age-specific death rate behaves like a binomial variable. This stand can be contended and the author himself raises the controversy by stating on page 78, "Since a death rate is often determined from the mortality experience of an entire population rather than from a sample, it is sometimes argued that there is no sampling error; and therefore the standard deviation if it exists, can be disregarded. The point of view, however, is static". To get further into the controversy in an elementary text book would have been injudicious; but at the same time the framework in which the author has developed his various formulae has to be recognised. From the point of view of actual application, it is the reviewer's view that the standard deviation and related functions given in the book are about the absolute minimum conceivable.

The book should be of great value to those without a rich mathematical background. The computer program for the construction of complete and abridged life tables prepared by the author's son Robert given at the end of the book should be of practical assistance for those interested in the application of the life-table techniques.

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Atoms of silence by H. Reeves. The MIT Press, 28, Carleton Street, Cambridge, Massachusetts, 02142, USA, 1984, pp. 250, $ 17.19. (Indian orders to Allied Publishers Pvt. Ltd.)

The author has expounded on the evolution of the universe in the framework of the presently most acceptable theory of the creation of the universe, the Big Bang, not caring to mention either the aborted or the contemporary theories. The book will probably be categorized in the popular astronomy section perhaps because it is all words and no mathematics. The missing mathematics, however, does not make it less technical and that is why it is a popular book on science with a difference. For once, the scientifically uninitiated reader has been taken into confidence. The reader is exposed to the descriptions derived from the workings of the underlying principles. The emphasis is on why it is the way it is.

The organization and the evolution of the universe has been presented as a repeated play of the basic forces of nature which exert themselves whenever the conditions are right. The chronology of the events has been correlated with the forces causing them. A very long time ago, quarks, the most fundamental constituents of matter glued together to form nucleons, the neutrons and the protons which combine to form helium nucleus. This is achieved through the action of nuclear forces. The nuclear evolution is interrupted due to the frigidity of helium. The expansion of the universe continuously takes it where the action of other forces is facilitated. It takes about a million years before the electromagnetic forces help the nuclei of hydrogen and helium to capture electrons. Thus the birth of the first atoms takes place. Electromagnetic era ends in the formation of hydrogen molecule which is so stable that it refuses to combine with anything else. Force of gravity is the main actor for the next hundreds of millions of years. In an otherwise expanding space, localized clumps of matter grow into galaxies and stars. The conditions being right, once more the nuleosynthesis begins in the stellar cores. The newly formed elements are then redistributed in the interstellar space by the massive stars which explode due to gravitational collapse. In the cold spaces between the stars, the chemical evolution begins and results in the formation of large molecules. The molecular combination occurs for a long time in the remnant of the exploded star and the puritanical crystals emerge in the confines of a strict order. Crystals form solids around which the hydrogenerated molecules form icy shells. Solid grains stick together to make planets, the trapped gases are released to generate atmospheres and the icy shells make the oceans. Complex molecules take refuge in the ocean beds to protect themselves from the breaking effects of the ultraviolet radiation of the Sun. In the deep deep serenity of the sea, the chemical and biological evolution is at its best. Life emerges in a variety of shapes and sizes, the amoeba, the fish and finally through Darwinism, the man, to wonder and marvel at his birth and future and to write about it.

The book contains several thumb rules, repeated now and then which help the reader to maintain a state of scientific consciousness. To give a few examples: The arrow of time points in the direction of increasing complexity and increasing organization; the forces act effectively and efficiently when the temperature is right: Given a long enough time chance achieves the impossible like the homecoming of three helium nuclei deciding in favour of the carbon-based civilization; heat destroys information; entities combine with release of energy and therefore bending reduces the burden of freedom, so on and so forth.

The book has its more than due share of storiesand similes *e.g.*, the birth of universe is compared with the birth of a mouse from a mountain, the motion of galaxies is illustrated by asking the reader to go atop the Arc de Triomphe in Paris and watch the motion of cars, Robinson Crusoe is called in to realize the extent of the universe, newly born atoms are compared with Walt Disney's Bambi; Autocatalysis is illustrated by a Du Pont couple giving dinner parties to help form more couples and many more. The author has also given airs to his literary talent (probably a secret desire of every scientist) by comparing the ebbing tide on the Ile d' Ouessant and the expanding universe. Mixing art, literature and science in a space of two hundred and odd pages certainly makes a good punch, but didn't the author say somewhere in the book that 'purity preserves'. Reeves certainly did not need the shaky support of analogies to make his point. He has said it simply, clearly and accurately. The scientific phenomena are interesting in themselves. We need not say it with flowers. Like an eminent scientist said "science should be made simple, but not simpler."

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A hundred billion stars by Mario Rigutti. The MIT Press, Cambridge, Mass, 02142, USA. 1984, $ 28.75. (Indian orders to Allied Publishers Pvt. Ltd.).

A hundred billion stars is a translation of the Italian *Cento miliardi di stello* published first in 1978. As a translated piece of work it has (though small) its share of language anomalies. The author attempts to describe our galaxy, the milky homeland made up of hundred billion stars, dust and gas in a rather businesslike manner. Odorous comparisons and frivolous analogies considered to be an integral part of popular astronomy have no place in this book. The author wants to say it as it is; wants to popularize astronomy by informing and not by creating awe and wonder over the infinite and the infinitesimal and that requires an 'honest days work' and the author has done it. The book begins with the solar system, dwells on the Sun, one of the hundred billion stars, supporting a family of planets with their satellites. The importance of the magnetic field in accounting for the multitude of phenomena below and above the photosphere has been emphasized and this certainly raises the standard of a popular proposition. From here onwards it is all about stars and more. What should one know about stars? Their distances; their radii; a way to describe their brightness as they really are and as they appear to us; the secret of their brightness, their temperature, their emissions and ejections, their motion. Stars are seen single, in doubles, in multiples, as stellar associations, clusters and constellations. The star aggregates form due to common emergent qualities. This is highlighted in the Herzaprung Russel (HR) diagram in which, the main sequence stars, the giants, the supergiants, the white dwarfs, all depicting different phases of the life of a star are positioned. The HR diagram demonstrates the brightness or absolute magnitude of a star *vs* its temperature or spectral type. the last section of the book deals with stellar evolution. When a mass of interstellar gas contracts, the temperature rises, and hydrogen begins to burn into helium, the collapsed cloud begins to shine and a star is born. It spends 90% of its life on the main sequence. When the hydrogen in the stellar core is exhausted, the production of energy stops, the core starts to shrink under the gravitational pull. Now, there is an inert helium core surrounded by a burning hydrogen shell and a second hydrogen shell. Energy production goes on in the first hydrogen shell. This increased energy output expands the star which reduces the surface temperature. The star is bigger and cooler and ends up in the red giant section of the HR diagram. Further expansion of the star cools its surface so much that electrons and protons recombine to form a neutral, atomic environment and finally the envelope of the star is torn away from the core and the core goes through the white dwarf and later ends up as a fading cold black dwarf. A star more massive than the Sun burns up faster, has a shorter stay on the main sequence, becomes a blue giant, core contracts enough to initiate carbon burning, heavy elements are formed and with the delivery of iron, the star explodes into a supernova, enriching the interstellar medium with heavy element ejecta. The supernova is billion times brighter, the nuclei combines with other elements to facilitate the manufacture of the heaviest elements up to uranium. The remains of the explosion is a dense core which depending upon its mass would become a white dwarf, a pulsating neutron star or an invisible black hole. Invisible also in this book is the missing mass problem, a subject of much speculation and debate.

The printer's devil has been active all through the book. 'Galazy' is 'pratically' harmless, but a value of 98 cm/sec^2 for acceleration due to earth's gravity is certainly hazardous. A complete line is missing on page 88. There are printing mistakes in the mathematical form of

Kepler's third law. The author thinks, putting ideas in a mathematical form is a bad habit and I do not agree. If a picture is worth a thousand words, a mathematical relationship is certainly worth much more since it qualifies much with how much. May the prose and the precision be united.

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Cosmic discovery by Martin Harwit. The MIT Press, Cambridge, Massachusetts, 02142, USA, 1984, pp. 334, $ 11.44. (Indian orders to Allied Publishers Pvt. Ltd.)

In the last few decades astronomers have made a series of discoveries of a variety of objects and phenomena in the Universe, like radio galaxies, quasars, pulsars, the microwave background, gamma-ray bursts, etc. In *Cosmic discovery*, Martin Harwit has compiled a list of 43 such major phenomena discovered since antiquity (most of them in fact in the last 50 years) and taken a close look at the circumstances that lead to these discoveries. He also makes a scientific attempt to quantitatively predict the number of phenomena yet to be discovered and tells us how best to go about discovering them.

The case studies make fascinating reading and indicate some interesting trends. For instance, discoveries are often made by accident, by physicists and engineers not trained in astronomy, by the application of new technology and equipment (often originally developed for military use).

Predicting the number of discoveries yet to be made in any branch of experimental science is difficult and hazardous. It is less so, however, in the case of astronomy because an astronomer can only make passive observations of the Universe. The observations can only be made through a limited number of carriers of information (electromagnetic radiations, cosmic rays, neutrinos and gravitational waves) and of a limited number of parameters. It is therefore relatively easier to foresee the impact that future developments in technology could make. By considering the entire volume of phase space of all possible observations that can be made from Earth, and by noting the number of phenomena discovered independently through different information carriers, Harwit concludes that the known discoveries represent only about one third of the total number of phenomena that we can hope to discover. Most of the discoveries are therefore yet to be made.

The last chapter of the book stresses the need for more enlightened planning of astronomical research if astronomy is to stay active and healthy. Based on his analysis of the discoveries to date, Harwit makes a series of recommendations regarding the training of manpower, development of new instrumentation and techniques, the system of making research grants, etc.

Although one may not agree with Harwit on some points, the book is certainly thought-provoking and well worth reading for all those interested in astronomy, history of sciences and in science planning.

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Aspects of tectonic: Focus on South-Central Asia by K.S. Valdiya. Tata McGraw-Hill Co. Ltd., New Delhi, 1984, Rs. 180.

This book provides a synthesis of oceanographic, geophysical and geological data relating to the patterns and trends of evolution of the crustal structures. It focusses on South-Central Asia, particularly the Indian subcontinent covering modern developments, reflecting changes that have occurred over the last two decades. There are 15 chapters highlighting global tectonics and regional structural architecture in terms of modern plate tectonics. The attempts to relate sedimentation, rock deformation, igneous activities, regional metamorphism and pre-gnesis to the geodynamic conditions of the Earth's crust in lucid style by the author is commendable. The general pattern, design and evolution of the large-scale structures—mobile belts, sedimentary basins, petrological complexes, metamorphic facies, etc.—are discussed with reference to kinematics of crustal plates and mantle processes.

The 15 chapters in the book deal with various aspects of tectonics of South-Central Asia. The first chapter emphasizes the scope of tectonics. In the second chapter the author deals with crust and interior of the Earth while the third chapter explains thrust movements and their mechanism and consequences with illustrations. The vertical movements in continents and their manifestations and consequences are clearly brought out in the fifth chapter. The concept of sea-floor spreading and plate-tectonics—the revolutionary concept in earth science—is dealt in chapter 6. Mountain belts and orogeny is the topic of chapter 7. Tectonic design and evolution of the Himalaya in chapter 8 brings out the importance of plate tectonic concept for understanding the evolution of the mountain belt. The author describes the evolution of cratons in a lucid manner in chapter 9. His analysis of structural, petrochemical and geochemical studies carried out in crucial areas of the Indian craton indicates the existence of a number of tectonic provinces, each characterized by its distinctive structural setting, lithological composition and evolutionary history. Tectonics of continental margins is the interesting topic of chapter 10. It is shown that the history of the evolution of continental margin is the history of the growth of continental craton. Tectonics and sedimentation in chapter 11 deal with tell-tale marks of metamorphic evolution of the rocks in the orogenic belts and their progressive unroofing resulting from the quickened pace of erosion, which indeed reflects the diastrophic uplift of the provenance. There is a short chapter on deformation in metamorphism. Geotectonic settings of igneous activities (chapter 13) and the topic on mineralization in the cratonic domain (chapter 14) are highly instructive. Neotectonic movements in the last chapter show how the imperceptibly low and secular as well as episodic crustal movements that have been taking place since the beginning of the Quaternary Period some two million years ago are described as neotectonic activities.

The book is written in a lucid style and is interspersed with nice illustrations. There is no doubt that it would be of immense use to research workers in the field of earth science.

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