



Buckling Analysis of Variable Angle Tow Composite Plates Using Differential Quadrature Method

Gangadharan Raju*, Zhangming Wu and Paul M. Weaver

Abstract | Variable Angle Tow (VAT) placement allows the designer to tailor the composite structure to enhance the structural response under prescribed loading conditions. VAT technology allows curvilinear placement of tows within the plane of a structure and gives freedom for altering pointwise in-plane, coupling and flexural stiffnesses of a plate. This stiffness tailoring improves the buckling performance of VAT plates by allowing re-distribution of loads from the critical regions of the plate. In the present work, the Differential Quadrature Method (DQM) is investigated for performing buckling analysis of VAT panels. The governing differential equations are derived for the in-plane and buckling analysis of symmetric VAT plate structure based on classical laminated plate theory. DQM was applied to solve the buckling problem of simply supported VAT plates subjected to uniform edge compression. To show the accuracy and robustness of DQM, the results obtained using DQM are compared with finite element analysis. In this work, Non-Uniform Rational B-Splines (NURBS) curves are used to model the fibre path and the fibre orientation can be designed by modifying the control points within the domain of the plate. The NURBS representation allows general fibre angle variation of tow resulting in wider design space of VAT panels. Also, the number of design variables for VAT panels are reduced by using NURBS curves and the fibre manufacturing constraints can be handled easily. Genetic Algorithm (GA) has been coupled with DQM to determine the optimal tow path for improving the buckling performance.

Keywords: Variable Angle Tow composites, buckling, Differential Quadrature Method.

1 Introduction

Conventional design of composite structures uses a standard combination of straight fibre laminae with constant thickness resulting in a constant stiffness structure and limited tailorability options. By allowing the fibre orientation to change in the plane of the structure, the stiffness of the structure can be varied resulting in redistribution of loads away from critical regions. This concept of VAT placement provides

the designer a wider design space for tailoring the composite structure for enhanced structural performance under prescribed loading conditions. Hyer *et al.*¹ demonstrated the improvement of buckling resistance of composite plate with a hole using curvilinear fibre placement. Gurdal and his coworkers^{2,3} have demonstrated the superior buckling performance of VAT panels with linear fibre angle variation over straight fibre composites. Weaver *et al.*⁴ employed an

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embroidery machine for tow steering and design of VAT plates with nonlinear angle distribution. Most of the works reported in literature for analysis of VAT panels use the Finite Element Method (FEM) and when FEM is coupled with optimisation algorithms analysis becomes computationally expensive. To overcome this drawback, new methods are required which are fast, accurate and easily integrable with optimisation algorithms for design of VAT panels. In this work, numerical methodology based on the Differential Quadrature Method (DQM) is developed for buckling analysis of VAT panels. The stability and robustness of DQM in computing the buckling performance of VAT panels with linear and nonlinear fibre orientations are studied.

Different methods have been proposed in the literature to represent tow path variations in VAT plates. Gurdal *et al.*² introduced a linear fibre angle variation definition based on three fibre angle parameters (ϕ , T_0 , T_1) which have subsequently been widely used for analysis, design and manufacture of VAT plates. This definition is simple and restricts the design space for tailoring the stiffness of VAT plates. Wu *et al.*⁵ used Lagrangian polynomial functions to represent the general fibre angle variation for VAT plates where the coefficients of polynomials are equal to the designed fibre angle at specified control points. The fibre angle based definition of tow path is suitable only for variation of fibre angle in a particular direction. When the fibre angle based definition is applied to two dimensional variations, it becomes quite tedious to represent the tow path corresponding to the fibre angle distribution and also presents difficulties for manufacturing. The drawbacks of fibre angle variation can be overcome by using mathematical functions to represent directly the fibre path variation in the VAT plate. Nagendra *et al.*⁶ used NURBS curves for tow path definition and they restricted their design of VAT plates to only five basic shape variations of NURBS curves for optimization studies. Parnas *et al.*⁷ employed bicubic Bezier surfaces and cubic Bezier curves for layer thickness and fibre angle for design of the VAT plates. Kim *et al.*⁸ used piecewise quadratic Bezier curve for defining the tow path in VAT plates. In the present work, the NURBS curves were used for tow path definition and this representation allows a general variation of fibre angle distribution across the x - y plane by moving the control points. This approach of tow path definition involves less design variables and the manufacturing constraints like minimum radius of curvature can be readily modeled.

The design of variable stiffness plates is more complex compared to straight fibre composites because of the number of design variables involved and the manufacturing constraints on designing continuous tow paths without any discontinuities for design of VAT plates. Ghiasi *et al.*⁹ presented a good review of the different optimization algorithms for design of variable stiffness plates. Seetodeh *et al.*¹⁰ used nodal based fibre angles as design variables and used a generalized reciprocal approximation technique for design of VAT plates for maximum buckling load. The limitations of this approach are the increase in number of design variables with mesh size and the difficulty in imposing fibre continuity across the plate structure. Lamination parameters were used by Jsselmuiden *et al.*¹¹ for designing VAT plates for maximizing the buckling load. This approach reduces the number of design variable, but requires considerable effort to construct manufacturable tow paths from the optimal lamination parameters. The design problem based on the proposed NURBS curve representation to determine the optimal fibre path for maximizing the buckling load is non-convex and genetic algorithms have been applied to solve it. The speed, efficiency and accuracy of the proposed DQM approach to solve buckling analysis of VAT plates reduce the computational effort required by GA to determine the optimal tow path.

2 Differential Quadrature Method

The Differential Quadrature Method (DQM) was introduced by Bellman *et al.*¹² to solve initial and boundary value problems. In DQM, the derivative of a function, with respect to a space variable at a given discrete grid point is approximated as a weighted linear sum of function values at all of the grid points in the entire domain of that variable. The n th order partial derivative of a function $f(x)$ at the i th discrete point is approximated by,

$$\frac{\partial^n f(x_i)}{\partial x^n} = A_{ij}^{(n)} f(x_j) \quad j = 1, 2, \dots, N \quad (1)$$

where x_i = set of discrete points in the x direction; and $A_{ij}^{(n)}$ is the weighting coefficients of the n th derivative and repeated index j means summation from 1 to N . The weighting coefficients and the grid distribution determine the accuracy of the DQM results. For determining the weighting coefficients, the function $f(x)$ is approximated using test functions. Lagrange interpolation polynomial and Fourier series expansion are the commonly used test functions in DQM. In this work,

Lagrange interpolation polynomials are chosen as test functions for the computation of weighting coefficients and are given by

$$g_k(x) = \frac{M(x)}{(x-x_k)M^{(1)}(x_k)} \quad k=1, 2, \dots, N \quad (2)$$

where

$$M(x) = \prod_{j=1}^N (x-x_j),$$

$$M^{(1)}(x_i) = \frac{\partial M(x_i)}{\partial x} = \prod_{j=1, j \neq i}^N (x_i - x_j) \quad (3)$$

The weighting coefficient for the first order derivative are explicitly defined by

$$A_{ij}^{(1)} = \frac{M^{(1)}(x_i)}{(x_i - x_j)M^{(1)}(x_j)}, \quad i \neq j, \quad i, j = 1, 2, \dots, N$$

$$A_{ii}^{(1)} = - \sum_{j=1, j \neq i}^N A_{ij}^{(1)} \quad (4)$$

where x_i are the coordinates of the grid points. The second and higher order weighting coefficients can be obtained from $A_{ij}^{(1)}$ using matrix multiplication and is explained in detail by Shu.¹³ In this work, the non-uniform grid distribution given by the Chebyshev-Gauss-Labotto points are used for the computation of weighting matrices and is given by

$$X_i = \frac{1}{2} \left[1 - \cos \left(\frac{i-1}{N-1} \pi \right) \right], \quad i = 1, 2, \dots, N \quad (5)$$

where N is the number of grid points. Different methods have been reported for proper implementation of boundary conditions using the DQM. In this work, the direct substitution method proposed by Shu *et al.*^{14,15} has been used to implement the different plate boundary conditions. Using the above approach, the essential boundary conditions are implemented along the boundary points and the force boundary conditions are discretized using DQM and applied to grid points adjacent to the boundary points. In this work, DQM is applied to solve the buckling problem of VAT plates under simply supported boundary conditions.

3 Variable Angle Tow Panels

The concept of VAT placement provides the designer with a wider design space for tailoring composite laminated structures for enhanced structural performance under prescribed loading conditions. Potential benefits may be achieved using VAT

placement, for example, by blending (minimising) stiffness variations between structural components (e.g. stiffener to skin) to reduce inter-laminar stresses. In VAT panels, stiffness (A , B , D matrices) vary with x , y coordinates resulting in non-uniform in-plane stress distribution under constant edge loads or displacements.³ A stress function formulation for in-plane analysis and displacement formulation for buckling analysis was employed to derive the Governing Differential Equations (GDE) of VAT plates based on classical laminated plate theory.

In this work, VAT panels with fibre orientation angle variation along one direction and constant stiffness properties in the orthogonal direction are considered for analysis. A VAT plate with linear fibre angle variation is considered for analysis⁷ and the angle variation along the x direction is given by,

$$\theta(x) = \phi + \frac{2(T_1 - T_0)}{a} |x| + T_0 \quad (6)$$

where T_0 is the fibre orientation angle at the panel centre $x=0$, T_1 is the fibre orientation angle at the panel ends $x = \pm a/2$ and is the angle of rotation of the fibre path.

3.1 Pre-buckling analysis

The use of the Airy stress function to model the pre-buckling analysis of VAT plates considerably reduces the problem size, computational effort and provides generality to model pure stress and mixed boundary conditions. For symmetric VAT laminates, the mid-plane strains ϵ^0 are related to stress resultants \bar{N} in the following form,

$$\begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \epsilon_{xy}^0 \end{Bmatrix} = \begin{bmatrix} A_{11}^*(x, y) & A_{12}^*(x, y) & A_{16}^*(x, y) \\ A_{12}^*(x, y) & A_{22}^*(x, y) & A_{26}^*(x, y) \\ A_{16}^*(x, y) & A_{26}^*(x, y) & A_{66}^*(x, y) \end{bmatrix} \begin{Bmatrix} \bar{N}_x \\ \bar{N}_y \\ \bar{N}_{xy} \end{Bmatrix} \quad (7)$$

where A^* is the in-plane compliance matrix. The Airy's stress function Ω is used to define the stress resultants \bar{N} and is given by,

$$\bar{N}_x = \Omega_{,yy}, \quad \bar{N}_y = \Omega_{,xx}, \quad \bar{N}_{xy} = -\Omega_{,xy} \quad (8)$$

The compatibility condition in terms of mid-plane strains in a plane stress condition is given by

$$\epsilon_{x,yy}^0 + \epsilon_{y,xx}^0 - \epsilon_{xy,xy}^0 = 0 \quad (9)$$

Substituting the Eqns. 7 and 8 into Eqn. 9, the GDE for in-plane analysis of symmetric VAT plate is given by,

$$\begin{aligned}
 & A_{11}^*(x, y)\Omega_{,yyyy} - 2A_{16}^*(x, y)\Omega_{,xyyy} + (2A_{12}^*(x, y) + A_{66}^*(x, y))\Omega_{,xxyy} - 2A_{26}^*(x, y)\Omega_{,xxxy} \\
 & + A_{22}^*(x, y)\Omega_{,xxxx} + (2A_{11,y}^*(x, y) - A_{16,x}^*(x, y))\Omega_{,yyy} + (2A_{12,x}^*(x, y) - 3A_{16,y}^*(x, y) \\
 & + A_{66,x}^*(x, y))\Omega_{,xyy} + (2A_{12,y}^*(x, y) - 3A_{26,x}^*(x, y) + A_{66,y}^*(x, y))\Omega_{,xxy} + (2A_{22,x}^*(x, y) \\
 & - A_{26,y}^*(x, y))\Omega_{,xxx} + (A_{11,yy}^*(x, y) - A_{12,xx}^*(x, y) - A_{16,xy}^*(x, y))\Omega_{,yy} + (-A_{26,xx}^*(x, y) \\
 & - A_{16,yy}^*(x, y) + A_{66,xy}^*(x, y))\Omega_{,xy} + (A_{12,yy}^*(x, y) - A_{22,xx}^*(x, y) + A_{26,xy}^*(x, y))\Omega_{,xx} = 0
 \end{aligned} \tag{10}$$

where A^* is the in-plane compliance matrix and Ω is the Airy's stress function. Eqn. 10 is a fourth order elliptic partial differential equation in terms of stress functions with variable coefficients and derivatives of compliance terms which represent the additional degrees of freedom available for tailoring of VAT plates when compared to straight fibre composites. The DQM representation of Eqn. 10 is given by

is applied to numerically discretize the derivative terms in Eqn. 12 and they are reduced into a set of algebraic linear equations. Thus the DQM representation for in-plane analysis of VAT plates in discrete form can be expressed as,

$$\begin{bmatrix} K_{bb} & K_{bd} \\ K_{db} & K_{dd} \end{bmatrix} \begin{Bmatrix} \Omega_b \\ \Omega_d \end{Bmatrix} = \begin{Bmatrix} F_b \\ 0 \end{Bmatrix} \tag{13}$$

$$\begin{aligned}
 & A_{11}^*(x, y) \sum_{m=1}^{N_y} B_{jm}^{(4)} \Omega_{im} - 2A_{16}^*(x, y) \sum_{k=1}^{N_x} \sum_{m=1}^{N_y} A_{ik}^{(1)} B_{jm}^{(3)} \Omega_{km} + (2A_{12}^*(x, y) + A_{66}^*(x, y)) \sum_{k=1}^{N_x} \sum_{m=1}^{N_y} A_{ik}^{(2)} B_{jm}^{(2)} \Omega_{km} \\
 & - 2A_{26}^*(x, y) \sum_{k=1}^{N_x} \sum_{m=1}^{N_y} A_{ik}^{(3)} B_{jm}^{(1)} \Omega_{km} + A_{22}^*(x, y) \sum_{k=1}^{N_x} A_{ik}^{(4)} \Omega_{kj} + (2A_{11,y}^*(x, y) - A_{16,x}^*(x, y)) \sum_{m=1}^{N_y} B_{jm}^{(3)} \Omega_{im} \\
 & + (2A_{12,x}^*(x, y) - 3A_{16,y}^*(x, y) + A_{66,x}^*(x, y)) \sum_{k=1}^{N_x} \sum_{m=1}^{N_y} A_{ik}^{(1)} B_{jm}^{(2)} \Omega_{km} \\
 & + (2A_{12,y}^*(x, y) - 3A_{26,x}^*(x, y) + A_{66,y}^*(x, y)) \sum_{k=1}^{N_x} \sum_{m=1}^{N_y} A_{ik}^{(2)} B_{jm}^{(1)} \Omega_{km} \\
 & + (2A_{22,x}^*(x, y) - A_{26,y}^*(x, y)) \sum_{k=1}^{N_x} A_{ik}^{(3)} \Omega_{kj} \\
 & + (A_{11,yy}^*(x, y) - A_{12,xx}^*(x, y) - A_{16,xy}^*(x, y)) \sum_{m=1}^{N_y} B_{jm}^{(2)} \Omega_{im} \\
 & + (-A_{26,xx}^*(x, y) - A_{16,yy}^*(x, y) + A_{66,xy}^*(x, y)) \sum_{k=1}^{N_x} \sum_{m=1}^{N_y} A_{ik}^{(1)} B_{jm}^{(1)} \Omega_{km} \\
 & + (A_{12,yy}^*(x, y) - A_{22,xx}^*(x, y) + A_{26,xy}^*(x, y)) \sum_{k=1}^{N_x} A_{ik}^{(2)} \Omega_{kj} = 0, \quad i = 1 \dots N_x, j = 1, N_y
 \end{aligned} \tag{11}$$

where N_x, N_y are the number of grid points in the x and y directions respectively. The boundary conditions expressed using stress function for axial compression loading are given by

where F_b is the generalizes force vector. The subscript b denotes the boundary and adjacent grid points for applying boundary conditions. The subscript d refers the domain grid points. Equation 13 is solved for stress function Ω and the stress resultants can be evaluated using Eqn. 8.

$$\begin{aligned}
 & \left. \frac{\partial^2 \Omega}{\partial y^2} \right|_{x=0,a} = \sigma_x(y), \quad \left. \frac{\partial^2 \Omega}{\partial x^2} \right|_{y=0,b} = 0, \\
 & \left. \frac{\partial^2 \Omega}{\partial x \partial y} \right|_{x=0,a; y=0,b} = 0, \\
 & \Omega \Big|_{x=0,y=b} = \frac{\partial \Omega}{\partial x} \Big|_{x=0,y=b} = \frac{\partial \Omega}{\partial y} \Big|_{x=0,y=b} = 0
 \end{aligned} \tag{12}$$

where $\sigma_x(y)$ is the applied compression loading along the edge $x = 0, a$ of the VAT plate. The DQM

3.2 Buckling analysis

The moment equilibrium equation for symmetrical VAT plate is given by,

$$\begin{aligned}
 & \frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \bar{N}_x \frac{\partial^2 w}{\partial x^2} \\
 & + 2\bar{N}_{xy} \frac{\partial^2 w}{\partial x \partial y} + \bar{N}_y \frac{\partial^2 w}{\partial y^2} + q = 0
 \end{aligned} \tag{14}$$

where M_x , M_y , M_{xy} are the moment distributions and q is the load applied in transverse direction. The moment distributions are related to the mid-plane curvatures by the following relation,

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11}(x,y) & D_{12}(x,y) & D_{16}(x,y) \\ D_{16}(x,y) & D_{22}(x,y) & D_{26}(x,y) \\ D_{16}(x,y) & D_{26}(x,y) & D_{66}(x,y) \end{bmatrix} \begin{Bmatrix} \kappa_x^0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{Bmatrix} \quad (15)$$

where D_{ij} is the laminate bending stiffness matrix. Substitution of Eqn. 15 into Eqn. 14 results in the governing differential equation for buckling analysis of symmetric VAT plate given by,

$$\begin{aligned} &D_{11}(x,y)w_{,xxxx} + 4D_{16}(x,y)w_{,xxxy} + 2(D_{12}(x,y) + 2D_{66}(x,y))w_{,xxyy} + 4D_{26}(x,y)w_{,yyyx} + \\ &D_{22}(x,y)w_{,yyyy} + 2(D_{11,x}(x,y) + D_{16,y}(x,y))w_{,xxx} + (6D_{16,x}(x,y) + 2D_{12,y}(x,y) + 4D_{66,y}(x,y)) \\ &w_{,xxy} + (2D_{12,x}(x,y) + 4D_{66,x}(x,y) + 6D_{26,y}(x,y))w_{,xyy} + 2(D_{26,x}(x,y) + D_{22,y}(x,y)) \\ &w_{,yyy} + (D_{11,xx}(x,y) + 2D_{16,xy}(x,y) + D_{12,yy}(x,y))w_{,xx} + (2D_{16,xx}(x,y) + 4D_{66,xy}(x,y) + 2D_{26,yy}(x,y)) \\ &w_{,xy} + (D_{12,xx}(x,y) + 2D_{26,xy}(x,y) + D_{22,yy}(x,y))w_{,xyy} + \bar{N}_x w_{,xx} + 2\bar{N}_{xy} w_{,xy} + \bar{N}_y w_{,yy} = 0 \end{aligned} \quad (16)$$

where w is the out of plane displacement. The simply supported plate boundary conditions for the VAT plate are given by,

$$\begin{aligned} x=0, a; \quad w=0; \quad M_x &= -D_{11}(x,y) \frac{\partial^2 w}{\partial x^2} - D_{12}(x,y) \frac{\partial^2 w}{\partial y^2} - 2D_{16}(x,y) \frac{\partial^2 w}{\partial x \partial y} = 0, \\ y=0, b; \quad w=0; \quad M_y &= -D_{12}(x,y) \frac{\partial^2 w}{\partial x^2} - D_{22}(x,y) \frac{\partial^2 w}{\partial y^2} - 2D_{26}(x,y) \frac{\partial^2 w}{\partial x \partial y} = 0 \end{aligned} \quad (17)$$

The DQM is applied to numerically discretize the derivative terms in GDE and boundary conditions in Eqns. 16 and 17 and they are reduced into a set of algebraic linear equations,

$$\begin{bmatrix} K_{bb} & K_{bd} \\ K_{db} & K_{dd} \end{bmatrix} \begin{Bmatrix} w_b \\ w_d \end{Bmatrix} = N_{var} \begin{bmatrix} 0 & 0 \\ F_{db} & F_{dd} \end{bmatrix} \begin{Bmatrix} w_b \\ w_d \end{Bmatrix} \quad (18)$$

where N_{var} is the buckling load and w is the mode shape. Solving the eigenvalue problem presented by Eqn. (18) yields the buckling load and mode shapes of symmetrical VAT plates. The details regarding the numerical implementation of DQM for pre-buckling and buckling analysis of VAT plates can be found in this reference.¹⁶

4 Buckling Optimization Formulation

In this work, a GA is applied to determine the optimal tow path modeled using NURBS curves for maximizing the buckling load of VAT plates. In the

GA optimal search, a sufficiently large population and generation are used to avoid local optimisation results. Several trials of the GA search with different initial populations were performed to obtain converged results. The NURBS curve is defined by,

$$C(u) = \frac{\sum_{i=0}^n N_{i,p}(u)w_i P_i}{\sum_{i=0}^n N_{i,p}(u)w_i}, \quad a \leq u \leq b \quad (19)$$

where P_i are the control points, w_i are the weights and $N_{i,p}(u)$ are the p th degree B-spline functions defined on the non-uniform knot vector. The design of the tow path is done using a NURBS curve

passing through a set of control points as shown in Fig 1. The fibre angle distribution is computed from the tow path and is given as input for DQM buckling

analysis of VAT plates. The control points define the shape of the tow path and the co-ordinates of the control points are taken as the design variables in this formulation. The optimization problem can be cast in the mathematical programming form given by,

Maximize: Buckling coefficient K_{cr} ,
 Design Variables: $\{X_{11}, X_{12}, \dots, X_{1n}, X_{2n}\}$, n is the number of control points,
 Subject to: $X^l \leq X_{1i}, X_{2i} \leq X^U$ (bounds on design variables).

The optimal tow path results obtained using GA is presented in the next section.

5 Results and Discussion

5.1 Model validation

The pre-buckling and buckling results obtained using DQM are presented in this section for VAT plates with linear fibre angle variation

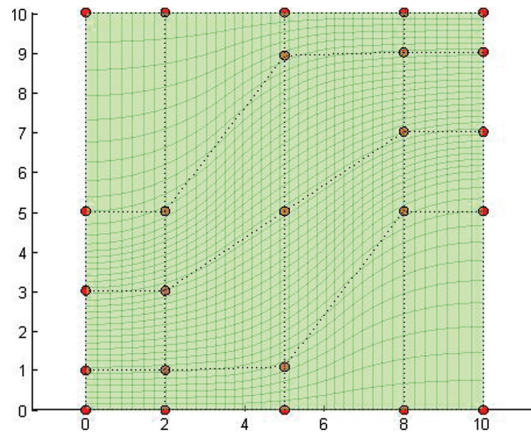


Figure 1: NURBS representation of tow path (red dots are the control points for shape control).

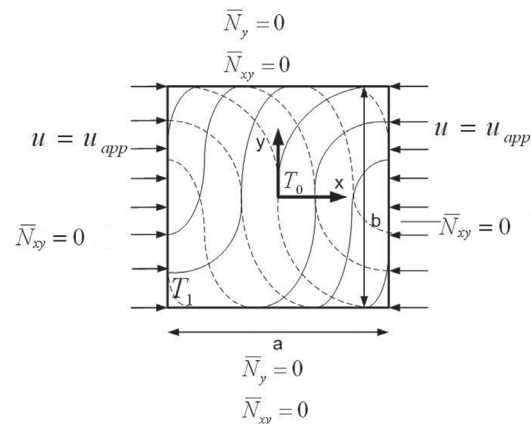


Figure 2: Square VAT plate subjected to uniform compression.

for validation purposes. The material properties for each lamina are chosen as, $E_1 = 181$ GPa, $E_2 = 10.27$ GPa, $G_{12} = 7.17$ GPa, $\nu_{12} = 0.28$ with thickness $t = 0.1272$ mm. FE modelling of the VAT panels was carried out using ABAQUS. The S4 shell element was chosen for discretization of the VAT plate structure and appropriate mesh size was selected to achieve the desired accuracy. A square VAT plate ($a = b = 1$ m) subjected to uniform axial compression along the edges $x = 0$; a and other stress boundary conditions are shown in Fig. 2. A symmetric VAT plate $[0 \pm <0|45>]_{3s}$ was chosen for the in-plane analysis and the number of grid points for DQM modelling was chosen to be $N_x = N_y = 30$.

The stress resultant distributions obtained using DQM and FEM are shown in Fig. 3 and match reasonably closely.

Subsequently DQM was applied to perform buckling analysis of square VAT plates under simply supported boundary condition with fibre orientation perpendicular to the loading direction $[90 \pm <T_0|T_1>]_{3s}$. The non-dimensional buckling coefficient and non-dimensional stiffness of VAT plates are evaluated by the following quantities,

$$K_{cr} = \frac{N_{vat} a^2}{Ebh^3}, E_{vat} = \frac{a \int_0^b N_x(a, y) dy}{bhu_{app}} \quad (20)$$

where N_{vat} is the lowest critical buckling load of the VAT plate. Non-dimensional values of buckling coefficient versus stiffness for various VAT plate configurations $[90 \pm <T_0|T_1>]_{3s}$ obtained

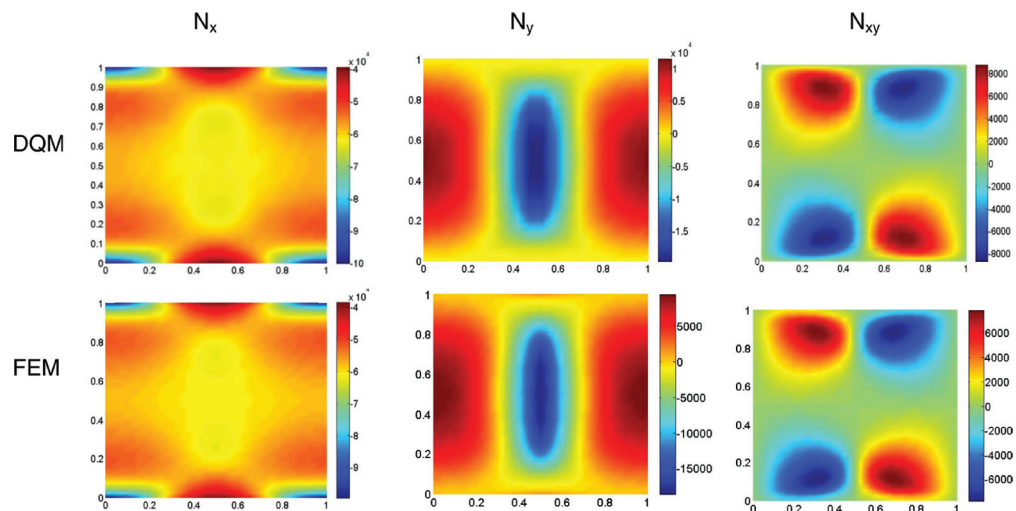


Figure 3: DQM and FEM in-plane stress resultant distributions (N_x , N_y , N_{xy}) results for a square VAT plate $[0 \pm <0|45>]_{3s}$ subjected to uniform axial compression.

using DQM and FE method are shown in Fig. 4. The results match closely and clearly show the variation in buckling load for different values of T_0 , T_1 . The maximum buckling coefficient value ($K_{cr} = 3.1$) was achieved for VAT plate configuration $[90 \pm <0|80>]_{3s}$ and has an improvement of 80% over the maximum buckling coefficient ($K_{cr} = 1.75$) of a straight fibre composite ($[\pm 45]_{3s}$). The mechanics behind the improvement of buckling performance of the VAT plate $[90 \pm <0|80>]_{3s}$ is due to redistribution of the applied compression load from the centre of the plate towards the edges by steering of fibre paths. The results also show the wider tailorability options available for design of VAT plates for a particular stiffness or buckling coefficient value. But, the linear fibre angle variation along a particular direction results in a restricted design space and limits the design options. The design space can be expanded by

allowing the fibre orientation to vary nonlinearly in both principle directions.

5.2 Optimisation results

A square symmetric and balanced VAT laminate ($[\pm\theta_1/\pm\theta_2]_s$) subjected to axial compression and simply supported boundary conditions is considered in the optimisation study. To observe the redistribution of the applied compression loading in the VAT plate, the change in fibre angle variation of the tow path was restricted to be perpendicular to the loading direction (y direction). For modelling and optimisation of the tow path, NURBS curves of order 3 and 7 control points were used. The x and y coordinates of the control point are the design parameters and for two ply design, the total number of variables is 28 to define the fibre path. Manufacturing constraints such as the maximum curvature of tow paths were not considered in this study. The population size and number of generations for GA optimisation were chosen to be 25 and 100 respectively. The optimal tow paths obtained for both the VAT laminas are shown in Fig. 5 and it illustrates the smooth and continuous variation of tow paths in the plate. Figure 6 shows the variation of buckling coefficient with number of generations and GA is shown to converge to optimal results after 30 generations. The buckling coefficient computed using DQM for the optimal VAT plate configuration is $K_{cr} = 3.5026$ and it shows an increase of 16.75% over the optimal VAT plate with linear fibre angle variation.³ The axial stress resultant distribution N_x and the buckling mode shape for the optimal VAT plate design is shown in Fig. 7 and redistribution of the applied compressive load was observed.

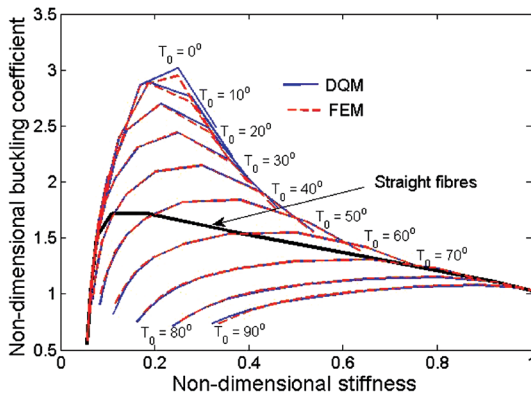


Figure 4: Square VAT plate $[90 \pm <T_0|T_1>]_{3s}$ subjected to axial compression and simply supported plate boundary conditions: Non-dimensional buckling coefficient versus non-dimensional stiffness.

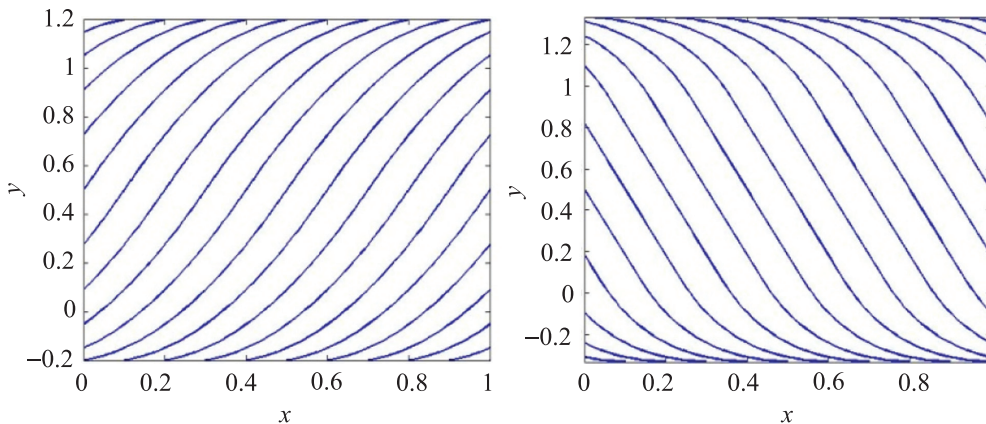


Figure 5: GA optimal tow path results: (a) VAT layup 1 (b) VAT layup 2.

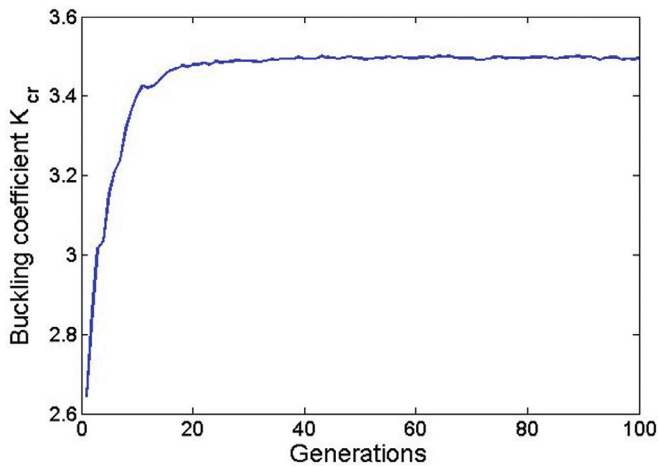


Figure 6: GA optimization results: Buckling coefficient (K_{cr}) versus generations.

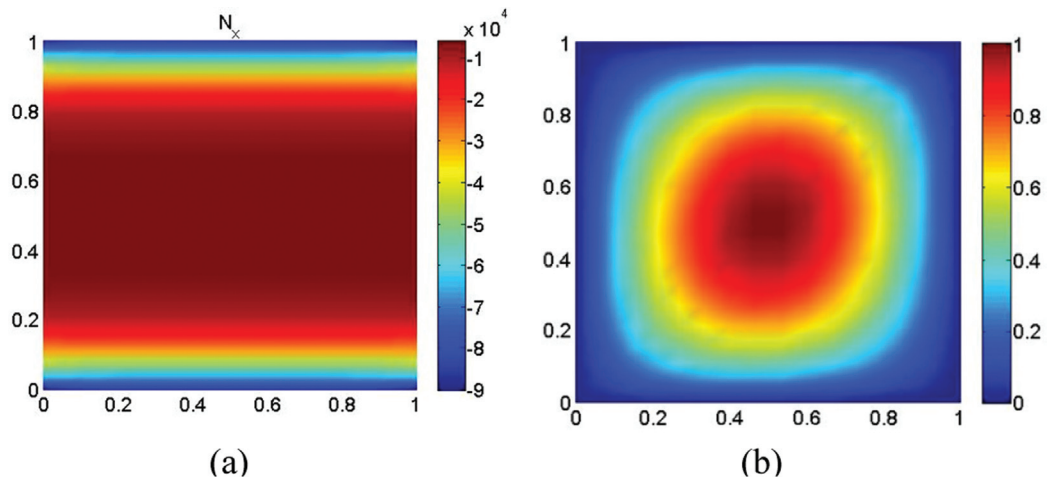


Figure 7: Stress distribution and buckling results for optimal VAT plate ($[\pm\theta_1/\pm\theta_2]_{2s}$) (Fig. 5) (a) N_x distribution (b) buckling mode shape ($K_{cr} = 3.5026$).

6 Conclusions

DQM was successfully applied to solve the buckling analysis problem of symmetric VAT composite panels with simply supported boundary conditions. The results obtained using DQM compared very well with FEM results. This study shows the stability and robustness of DQM in solving the buckling analysis of VAT plates with general fibre angle orientations. A NURBS representation for modelling of two paths is presented and this allows an extendable design space for VAT plates with general fibre angle distributions. GA was used with DQM to determine the optimal tow path for maximising the buckling load of VAT plates. The design results shows that the integration of DQM with GA is robust, efficient and requires relatively less computational effort. In future, the proposed optimisation strategy can be extended for designing of VAT plates for

maximising the buckling loads under shear and combined loading conditions.

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