AN ANALYSER FOR ELLIPTICALLY POLARISED LIGHT

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Abstract

A simple analyser for elliptically polarised light in which the half shade principle is incorporated in the determination of both the azimuth and the ratio of the axes of the ellipse is described.

1. Introduction

During the course of some experimental and theoretical investigations on the magneto-optic rotation in birefringent solids it became necessary to construct a fairly accurate analyser for elliptically polarised light using the simple materials available. It was possible to make a simple, yet accurate analyser in which the half shade principle could be incorporated in the determination of both the azimuth and the ratio of the axes of the ellipse. As the author has not come across a similar analyser in the literature, it was thought worthwhile to describe it in this paper.

2. Description of the Instrument

The analysis of elliptically polarised light usually consists in determining the azimuths and the ratio of the principal axes of the ellipse. The Stokes method of analysing elliptically polarised light is based on the principle that when such a beam of light passes through a quarter wave plate whose principal axes are parallel to those of the ellipse, it is converted into plane polarised light. In practice the elliptically polarised light is allowed to traverse a quarter wave plate and then a linear analyser (a nicol or polaroid). By a series of trials, that position of the quarter wave plate at which the emergent light from it can be completely crossed by the final analyser is determined. In such a position the principal axes of the quarter wave plate are parallel to those of the ellipse and the inclination of the principal plane of the linear analyser to the axis of the quarter wave plate, β, is a measure of the ratio of the axes; $\tan \beta = b/a$. The positions of the axes of the ellipse can, however, be located without resort to the trial and error method, by first setting the principal plane of the linear analyser parallel to a principal axis of the quarter wave plate. On rotating both together the position of minimum corresponds to an axis of the ellipse. Unfortunately excepting for large values of ellipticity this method proves to be quite unsatisfactory.

The half shade method could be incorporated for the accurate determination of both the azimuth and ellipticity in the following manner. For this, two half shades are necessary. They could either be optically active bi-quartz half shades (Nakamura plates) or birefringent half shades to be described later. The dispositions of the half shades with respect to the quarter wave plate are different

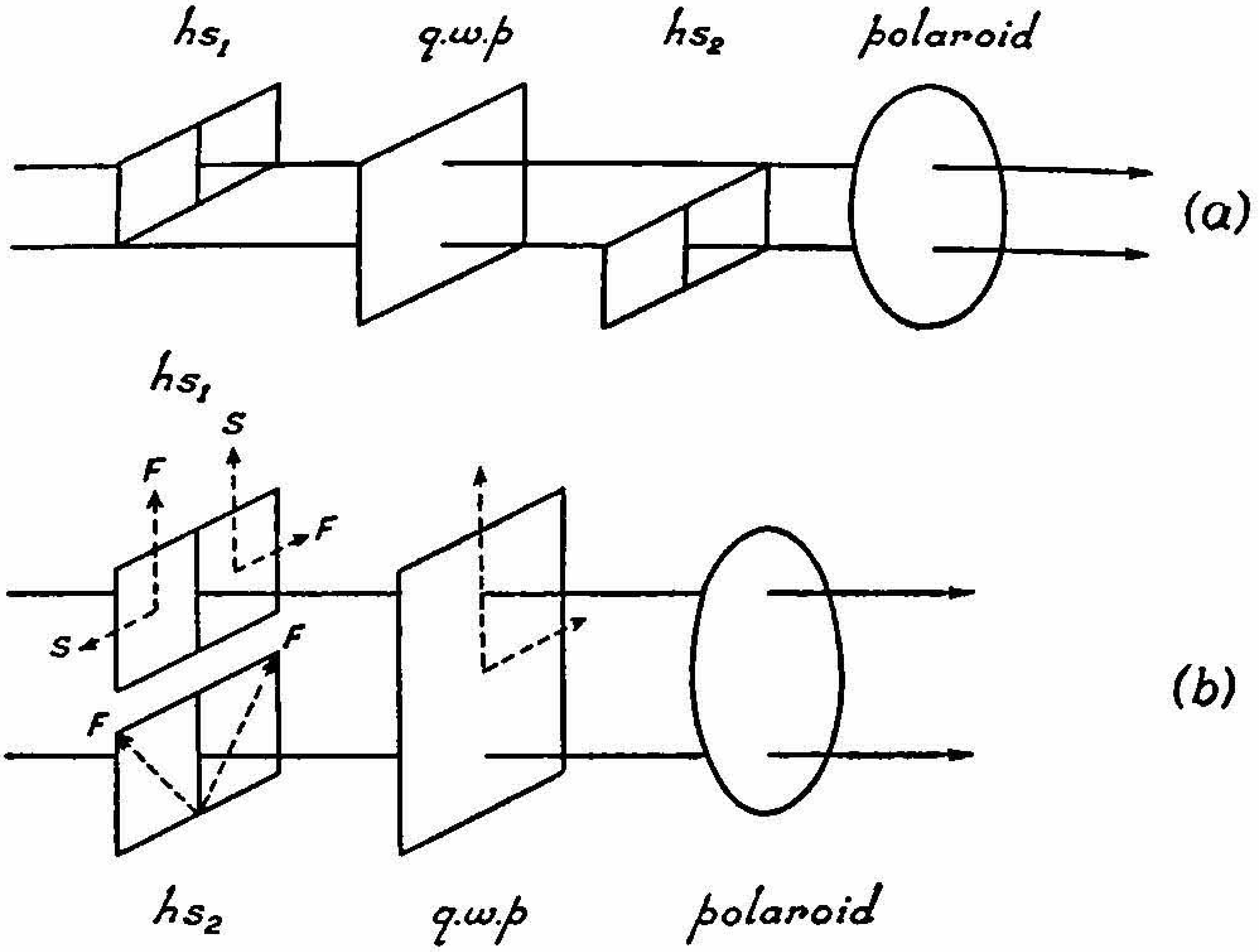


FIG. 1. The optical arrangement for the elliptic analyser.
(a) with bi-quartz half shades. (b) with birefringent half shades.

for the two cases. We shall first deal with the instrument with bi-quartz half shades.

To the quarter wave plate the two half shades are attached in the manner shown in Fig. 1 *a*. In the upper field the elliptically polarised light (to be analysed) passes through the half shade hs_1 and then through the quarter wave plate and finally through the analysing polaroid. In the lower field the light passes through the quarter wave plate, the half shade and then through the same polaroid. We shall call the unit consisting of the half shades hs_1, hs_2 and the quarter wave plate as the *q.w.p. unit.*

The procedure of analysing the incident light consists of first setting the analysing polaroid at an arbitrary azimuth to the quarter wave plate. The *q.w.p. unit* together with the polaroid is rotated till there is equality in hs_1. In case the half shade is not at its sensitive position at equality, the polaroid is rotated till the field in hs_1 is almost dark so that the setting could be accurately made. The axes of the quarter wave plate in this position are in exact coincidence with the axes of the ellipse. Further it will be found that the half shade hs_2 would necessarily be in a sensitive position although the two fields would be of unequal intensity. The polaroid is now rotated independent of the *q.w.p. unit* till exact equality is obtained in hs_2. If β is the angle between the principal plane of the polaroid and the principal axis of the quarter wave plate, $\tan \beta = b/a$. Two sliding slits are provided in the instrument so that hs_1 and hs_2 could be viewed independently.

3. Analyser with Birefringent Half Shade

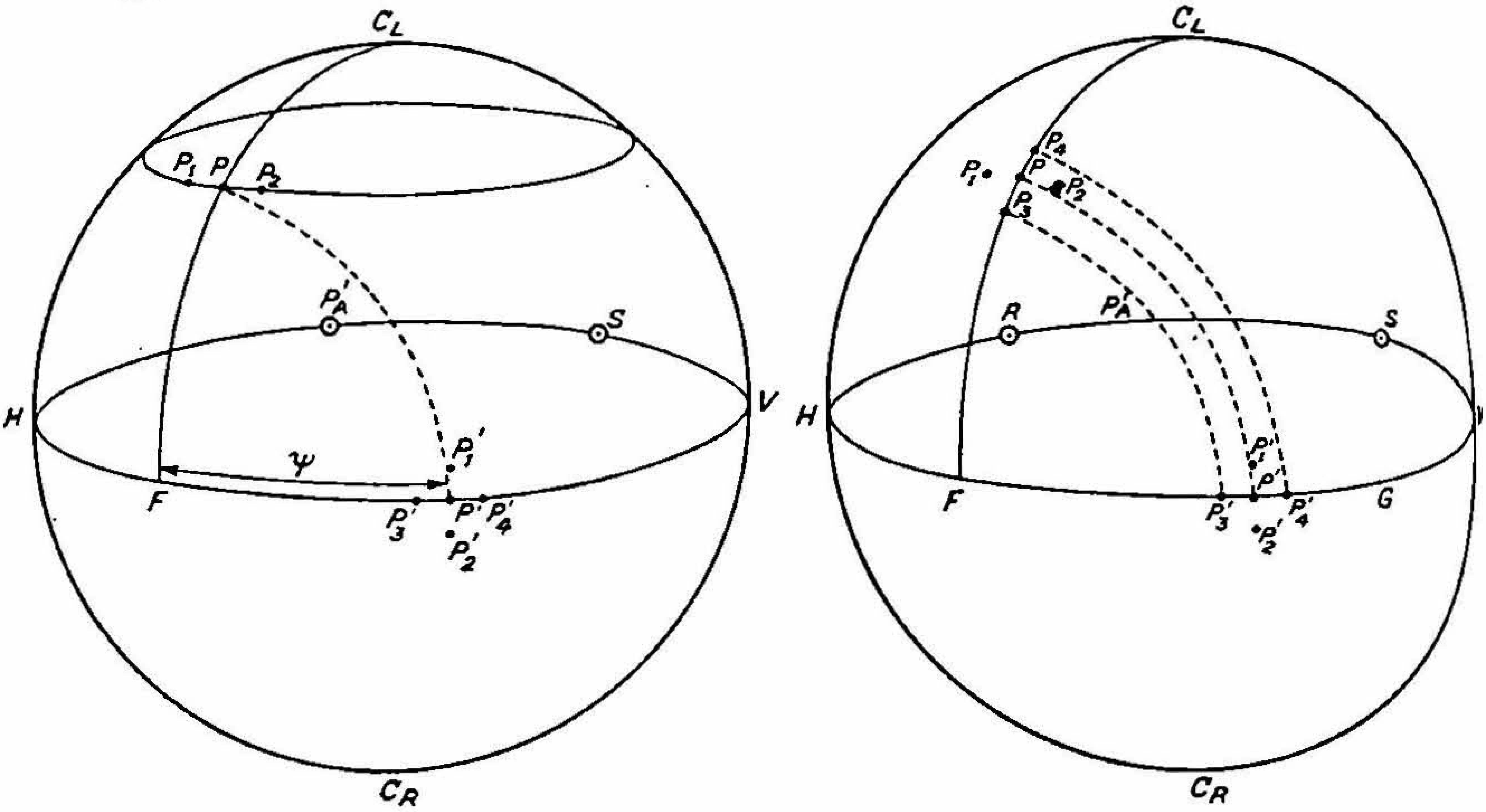
The birefringent half shade consists of two flakes of mica of equal retardation (about 3 to 4°) such that the slow and the fast axes are interchanged. The two flakes are fixed adjacent to each other with a sharp dividing line between them. In making this half shade, particular care must be taken to see that the two flakes are exactly of equal thickness and also that their axes are prefectly coincident.

In the analyser described in the last section the bi-quartz half shades could be replaced by half shades of this type with the dispositions shown in Fig. 1 *b*. The half shade hs_1 could either be in front or behind the quarter wave plate but its axes must be exactly parallel to those of the quarter wave plate. The half shade hs_2 must be in *front* of the quarter wave plate with its axes at 45° to those of the quarter wave plate. The procedure for the analysis of elliptic light is exactly the same as that given in the last section; firstly to get equality in hs_1 by rotating the *q.w.p. unit* and the polaroid, and later setting to equality in hs_2 by rotating the polaroid alone. It must be remembered that these two steps cannot be interchanged. The importance of allowing the light to pass exactly normal to the mica and quartz pieces must be remembered while constructing such an instrument.

4. The Theory of the Instrument

In giving the theory of the instrument it is most convenient to make use of the Poincaré representation of the state of polarisation of any light beam by a point on a sphere. The details and the advantages of this geometrical method of representation have already been explained in the various communications from this laboratory published elsewhere (Ramachandran and Chandrasekharan, 1951; Ramachandran and Ramaseshan, 1952).

Figure 2 represents such a sphere, wherein C_L and C_R are the two poles of the sphere and represent respectively left and right rotating circular vibrations. The great circle HFVS is the equator and points on it represent linear vibrations of varying azimuths, H being the horizontal and V a point 180° away representing the vertical vibration. Points on the same latitude ω represent elliptic vibrations with the same ellipticity ($\tan \omega/2 = b/a$) but of varying azimuths. Similarly points on the same longitude 2ϵ represent vibrations of different ellipticities (from circular to plane) but with the same azimuth ϵ. All vibrations in the upper hemisphere are left rotating and in the lower, right rotating. Any two diametrically opposite points represent vibrations exactly crossed with respect to each other (*e.g.*, H & V). Further, if P is the incident light and S the analyser, then the intensity transmitted by S is given by $\cos^2 PS/2$ (Ramachandran and Ramaseshan, 1952).



(*a*) FIG. 2 (*b*)

When light is propagated in a birefringent or optically active medium, the state of polarisation of the emergent light can be easily described with the aid of the Poincaré sphere. Let the state of polarisation of the incident light be represented by the point P on the sphere. If the crystal is birefringent, then the sphere is rotated about FS (where F and S represent the azimuths of the fast and slow axes of the crystal) through an angle equal to the phase retardation introduced by the crystalline medium, so that the point P is brought to P′. In the particular case shown in the diagram, the azimuth of the incident light P coincides with the orientation of the principal axes of the crystal and the phase retardation introduced by the crystal is $\lambda/4$. On the other hand if the crystal had been optically active and not linearly birefringent, then the sphere has to be rotated about the axis C_L C_R through an angle 2ρ, ρ being the rotation introduced by the crystal; so that the point P is brought to say P_2.

The analyser with optically active half shades.—As before, let the state of polarisation of the incident light be represented by P (Fig. 2 *a*). In the upper field (Fig. 1 *a*) of the analyser the light passes first through the half shade hs_1, then through the quarter wave plate and finally through the polaroid. On entering hs_1, P is divided into P_1 and P_2 by equal rotations about C_L C_R where $P_1\hat{C}_l P = P_2\hat{C}_l P = 2\theta/2$ where θ is the half shade angle. The action of the quarter wave plate is to rotate the points P_1 and P_2 about a diametral axis lying in the equatorial plane which corresponds to the principal axes of the quarter wave plate. If these coincide with the principal axes of the ellipse then the rotation about FS will bring points P_1 and P_2 to points P_1' and P_2' where $P_1\hat{F}P_1' = P_2\hat{F}P_2' = 90°$ and P_1' will be as much above the equator as P_2' would be below it. Hence a linear analyser which is arbitrarily placed on the equator would transmit equal amounts of light from P_1' and P_2'. But the most sensitive position for the half shade is when the linear analyser is near about P_A' the point antipodal to P'. It must be noted that even if there is the slightest difference in the directions of the axes of the quarter wave plate and the elliptic vibrations P_1' and P_2' will be asymmetrically situated with respect to the equator and hence hs_1 cannot show equality. Let us now consider the passage of light in the lower field of the analyser (Fig. 1 *a*). In this case the elliptically polarised light first passes through the quarter wave plate, *i.e.*, P is brought to a point P'. Then the half shade hs_2 splits P' into P_3' and P_4' by equal and opposite rotations about the axis C_L C_R. Hence when the two enter the final polaroid they will show exact equality only when the polaroid is set accurately at a point P_A' antipodal to P'. At this position tan $\psi/2$ gives the ratio of the axes of the ellipse. The figure also shows why when hs_1 is at its sensitive position, hs_2 is also at its sensitive position.

The analyser with birefringent half shades (Fig. 2 *b*).—Here the incident light represented by P, in its passage through hs_1 (Fig. 1 *b*) is divided into P_1 and P_2 by rotations about FS (and not C_L C_R). The quarter wave plate brings P_1 and P_2 to P_1' and P_2'. When hs_1 shows equality the axes of the elliptic vibration and the quarter wave plate would be coincident. A similar argument holds when the hs_1 is behind the quarter wave plate. In the lower field, however, as the half shade axes are at 45° to those of the quarter wave plate, the axis of rotation is GR which is at 90° to FS. Hence P is split into P_3 and P_4. The quarter wave plate brings them to P_3' and P_4'. As in the previous case, setting hs_2 to equality requires that the polaroid be placed exactly at a point P_A' antipodal to P'. tan FP'/2 gives the ratio of the axes.

It may be mentioned that if the quarter wave plate is not very accurately made, the setting at which hs_1 shows equality is independent of the setting of the polaroid in the second arrangement, while this is not the case with the bi-quartz arrangement. Hence with the analyser with birefringent half shades the constants of the elliptic vibration can be calculated from the knowledge of the retardation of the quarter wave plate.

Concluding Remarks

We have in this instrument an extremely easy and yet accurate method for analysing elliptically polarised light. The principal merit of the instrument is that it can be employed with equal facility for the analysis of light of any state of polarisation from plane to circular. Such an instrument can be put to a variety of uses in optical research.

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