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Influence charts for rigid pavements for edge load

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Abstract

Influence charts are presented for obtaining theoretical deflections and moments in concrete airport runway slabs for edge loading. The subgrade material on which the concrete pavement rests is idealized by Filonenko-Borodich model. The charts are similar to those presented by Pickett and Ray in which the subgrade medium is idealized by Winkler model.

Key words : Airports, deflections, edge loading, influence charts, moments, rigid pavements, runways.

1. Introduction

Influence charts for obtaining deflection and moments in concrete slabs for interior loading were presented by Siva Reddy and Pranesh[1]. The subgrade material on which the concrete pavement rests is idealized by Filonenko–Borodich and Reissner models which possess continuity. Siva Reddy[2] presented similar charts for Hetenyi model for interior loading. In this paper influence charts are presented for obtaining theoretical deflections and moments in concrete airport runway slabs for edge loading. The subgrade material in this case is idealized by Filonenko–Borodich model. The charts are similar to those presented by Pickett and Ray[3] in which the subgrade medium is idealized by Winkler model.

2. Derivation of equations

Figure 1 shows a semi-infinite concrete slab resting on Filonenko–Borodich foundation. The interaction between the spring elements is achieved by connecting a stressed elastic membrane at the top ends of the springs. The stretched membrane is subjected to a constant tensile force field T.

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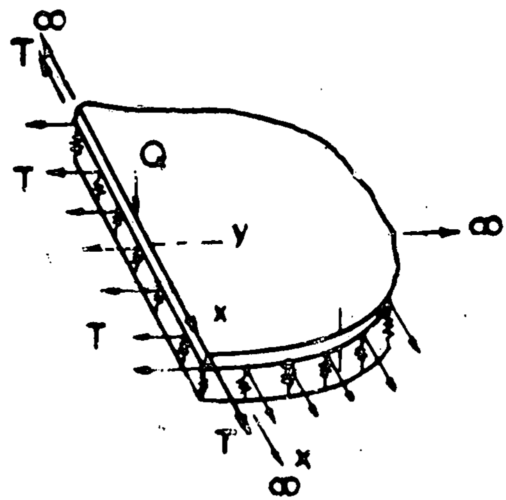

FIG. 1. Semi-infinite concrete slab on Filonenko–Borodich model.

The governing differential equation for the slab is

$$D\nabla^4 w = q - p \tag{1}$$

in which w = deflection of the concrete slab,

$D = \dfrac{Eh^3}{12(1-\mu^2)}$,

E = modulus of elasticity of concrete slab,

h = thickness of slab,

μ = Poisson's ratio of slab,

$\nabla^4 = \left(\dfrac{\delta^2}{\delta x^2} + \dfrac{\delta^2}{\delta y^2}\right)\nabla^2$,

$\nabla^2 = \dfrac{\delta^2}{\delta x^2} + \dfrac{\delta^2}{\delta y^2}$,

q = uniformly distributed load,

p = reaction at the bottom of the slab.

The reaction, p, on the slab is given by[4]

$$p = Kw - T\nabla^2 w \tag{2}$$

in which K = modulus of subgrade reaction and T = tensile force.

From (1) and (2) when only a concentrated load is acting on the slab

$$D\left(\frac{\delta^4 w}{\delta x^4} + 2\frac{\delta^4 w}{\delta x^2\,\delta y^2} + \frac{\delta^4 w}{\delta y^4}\right) - T\left(\frac{\delta^2 w}{\delta x^2} + \frac{\delta^2 w}{\delta y^2}\right) + Kw = 0. \tag{3}$$

Introducing the nondimensional variables

$$x' = x/l,\ y' = y/l,\ W = w/l \text{ where } l = 4\sqrt{D/K}$$

and assuming $W = \int_0^\infty \cos ax\ g(y')\,da$, we get from (3)

$$g^{\text{IV}} - (2a^2 + A')\,g'' + g\,(1 + A'a^2 + a^4) = 0 \tag{4}$$

in which

$$A' = \frac{Tl^2}{D},\ g'' = \frac{d^2 g}{dy^2} \text{ and } g^{\text{IV}} = \frac{d^4 g}{dy^4}.$$

The general solution for (3) is then given by

$$W = \left[E_1 e^{+\gamma y/l}\sin\frac{\beta y}{l} + E_2 e^{\gamma y/l}\sin\frac{\beta y}{l} + E_3 e^{-\gamma y/l}\cos\frac{\beta y}{l}\right.$$
$$\left. + E_4 e^{-\gamma y/l}\sin\frac{\beta y}{l}\cos\frac{ax}{l}\right] \tag{5}$$

in which

$$\gamma = \sqrt{\frac{\frac{2a^2 + A'}{2} + \sqrt{1 + a^2 A' + a^4}}{2}}$$

$$\beta = \sqrt{\frac{-\left(\frac{2a^2 + A'}{2}\right) + \sqrt{1 + a^2 A' + a^4}}{2}}.$$

Considering the case of a semi-infinite slab acted on by a concentrated load Q at the origin which is on the edge of the slab, the four constants of (5) can be evaluated from the following boundary conditions[4].

(i) $W(\infty) = 0$

(ii) $W'(\infty) = 0$

(iii) $M_y(0) = 0$

(iv) $V_y(0) = \underset{\epsilon \to 0}{Lt}\ \frac{Q}{\epsilon}$ (6)

Expressing the shear condition $V_y(0)$ in Fourier integral and applying the boundary conditions

$$W = \frac{2}{\sqrt{4 - A'}} \frac{2Q}{\pi l^2 K}$$

$$\times \int_0^\infty \frac{\gamma \cos \frac{\alpha x}{l} \left[\frac{\sqrt{4 - A'^2}}{2} \cos \frac{\beta y}{l} + \left\{ \frac{2\alpha^2 + A'}{2} - \mu \alpha^2 \right\} \sin \frac{\beta y}{l} \right] e^{(\gamma y/l)} d\alpha}{1 + 4(1 - \mu) \alpha^2 \gamma^2 - (1 - \mu^2) \alpha^4} \tag{7}$$

2.1. *Deflections from distributed load*

From Maxwell's reciprocal relations, (7) can also be considered to be the deflection at the origin caused by a concentrated load at variable coordinate point. Then by use of the principle of superposition and by integration, one may find the deflection at the origin due to a distributed load. The load Q is replaced by the load $q\,dx\,dy$ and then integrated with respect to x between a_1 and a_2 and with respect to y between b_1 and b_2 as shown in Fig. 2 to determine the number of blocks for given region.

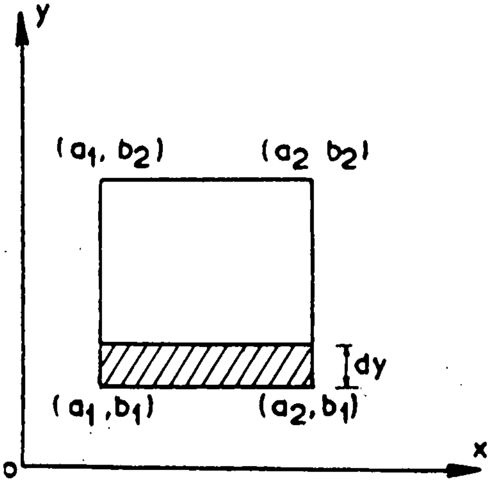

FIG. 2. Region of integration.

The result of integration with respect to x and y was expressed in the form

$$w = w(a_2, b_2) - w(a_1, b_2) - w(a_2, b_1) + w(a_1, b_1) \tag{8}$$

in which the general form $w(a, b)$ is as follows:

$$[(\sqrt{4 - A'^2})/2]\, w = \frac{2ql^4}{\pi D}$$

$$\times \int_0^\infty \frac{\gamma^2 \left[(A + 2B\beta^2) \left(1 - \cos \frac{\beta b}{l} e^{-\gamma b/l} \right) + (2A\beta^2 - B) \sin \frac{\beta b}{l} e^{-\gamma b/l} \right] \sin \frac{\alpha a}{l} \, d\alpha}{\alpha (\gamma^2 + \beta^2) [1 + 4(1 - \mu) \alpha^2 \gamma^2 - (1 - \mu)^2 \alpha^4]} \tag{9}$$

in which

$$\beta = \sqrt{\left\{-\frac{(2\alpha^2 + A')}{2} + \sqrt{1 + A'\alpha^2 + \alpha^4}\right\}\Big/2}$$

$$\gamma = \sqrt{\left\{\left(\frac{2\alpha^2 + A'}{2}\right) + \sqrt{1 + A'\alpha^2 + \alpha^4}\right\}\Big/2}$$

$$A = \frac{\sqrt{4 - A'^2}}{2}$$

$$B = \left(\frac{2\alpha^2 + A'}{2} - \mu\alpha^2\right)$$

$$l = \left(\frac{D}{K}\right)^{1/4}$$

E = Young's modulus of slab

μ = Poisson's ratio for slab, assumed to be 0·15 in all computations.

2.2. *Moments from distributed load*

The moment at point (x, y) in a direction of reference axis $O - n$ due to a load at the origin[3] is

$$M_x = -D\left(\frac{\delta^2 w}{\delta x^2} + \mu\frac{\delta^2 w}{\delta y^2}\right) \quad (10)$$

Due to reciprocal theorem, (10) is also the moment at the origin in the direction of the references axis O–n due to load at the point (x, y). In the equation for deflection the load Q is replaced by $qdxdy$ and the moment due to the load on area bounded by (a_1, b_1), (a_2, b_1), (a_1, b_2) and (a_2, b_2) is arrived at.

The result of integration with respect to x and y was expressed in the form

$$M_x = M(a_2, b_2) - M(a_1, b_2) - M(a_2, b_1) + M(a_1, b_1). \quad (11)$$

The general term $M(a, b)$ can be written as follows:

$$M(a, b) = \frac{2ql^2}{\pi A}\int_0^\infty \frac{\gamma^2 \sin\frac{\alpha a}{l}\left[S\left(1 - \cos\frac{\beta b}{l}e^{-\gamma b/l}\right) - T_1 \sin\frac{\beta b}{l}e^{-\gamma b/l}\right]d\alpha}{\alpha(\beta^2 + \gamma^2)[1 + 4(1-\mu)\alpha^2\gamma^2 - (1-\mu)^2\alpha^4]} \quad (12)$$

in which

$$S = (1 - \mu^2)\alpha^2[1 + 2(1 - \mu)\alpha^2\beta^2]$$

$$T_1 = (1 - \mu^2)\alpha^2[(1 - \mu)\alpha^2 - 2\beta^2].$$

The integrals for both deflection and moment are evaluated numerically using trapezoidal rule. The integral was evaluated for values of α at $0\cdot1$ intervals in the range from 0 to $2\cdot0$ and at $0\cdot5$ intervals in the range from $2\cdot0$ to $5\cdot0$. In all cases the integral for value α in excess of $5\cdot0$ is considered negligible.

3. Results and discussion

The parameters that constitute the foundation models, *viz.*, Tensile force (T) in the case of Filonenko–Borodien model can be determined by conducting a rigid stamp test on the subgrade and analysing the load deflection curve beyond the stamp[5]. It has been shown that the values of A' can range up to $0\cdot25$ and with $A'=0$, the Filonenko–Borodich model reduces to Winkler foundation[5].

Since T depends on soil type and other conditions like density, moisture content, etc., results are given up to $A'=0\cdot25$.

Influence charts for deflection and moments for values $A'=0\cdot01$, $0\cdot05$, $0\cdot1$ and $0\cdot25$ are presented in Figs. 3 to 10. The deflection w and moment M_n at the origin 0 are put in the form

$$w=\frac{0\cdot0005ql^4N}{AD} \tag{13}$$

$$M_n=\frac{ql^2N}{10000A} \tag{14}$$

in which

$N=$ number of blocks enclosed,

$M_n=$ moment at the origin in the n-direction.

$A=(\sqrt{4-A'^2})/2.$

In Figs. 7 to 10 a line divides the figures into two regions. The region to the left of the line consists of positive blocks and the region to the right of the line consists of negative blocks. In arriving at the values of the number of blocks N in the formula for bending moment, the number of blocks covered by the contact area is to be given positive sign when they happen to be to the left of the dividing line and the number of blocks covered by the contact area to the right of the line is to be given a negative sign. The sum of the blocks in the positive and negative regions is the value of N to be used in the formula for M_n.

The procedure for finding deflections and moments by using the influence charts is similar to that of Pickett and Ray[3]. A numerical example is worked out herein to show the use of charts.

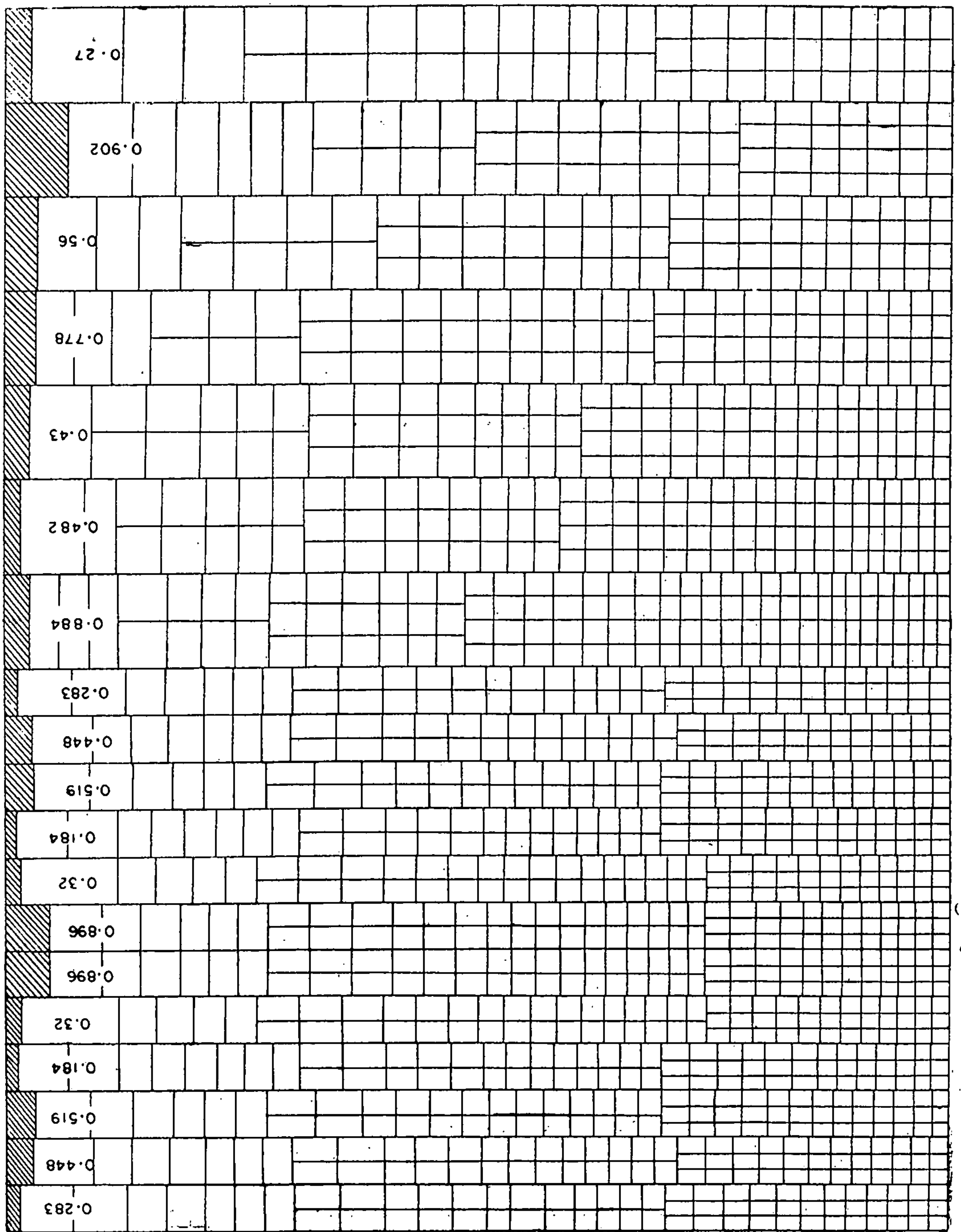

FIG. 3. Influence chart for deflection of slab for edge loading on Filonenko–Borodich model. $w = \frac{0 \cdot 0005\, q l^4 N}{DA}$ and $A' = 0 \cdot 01$.

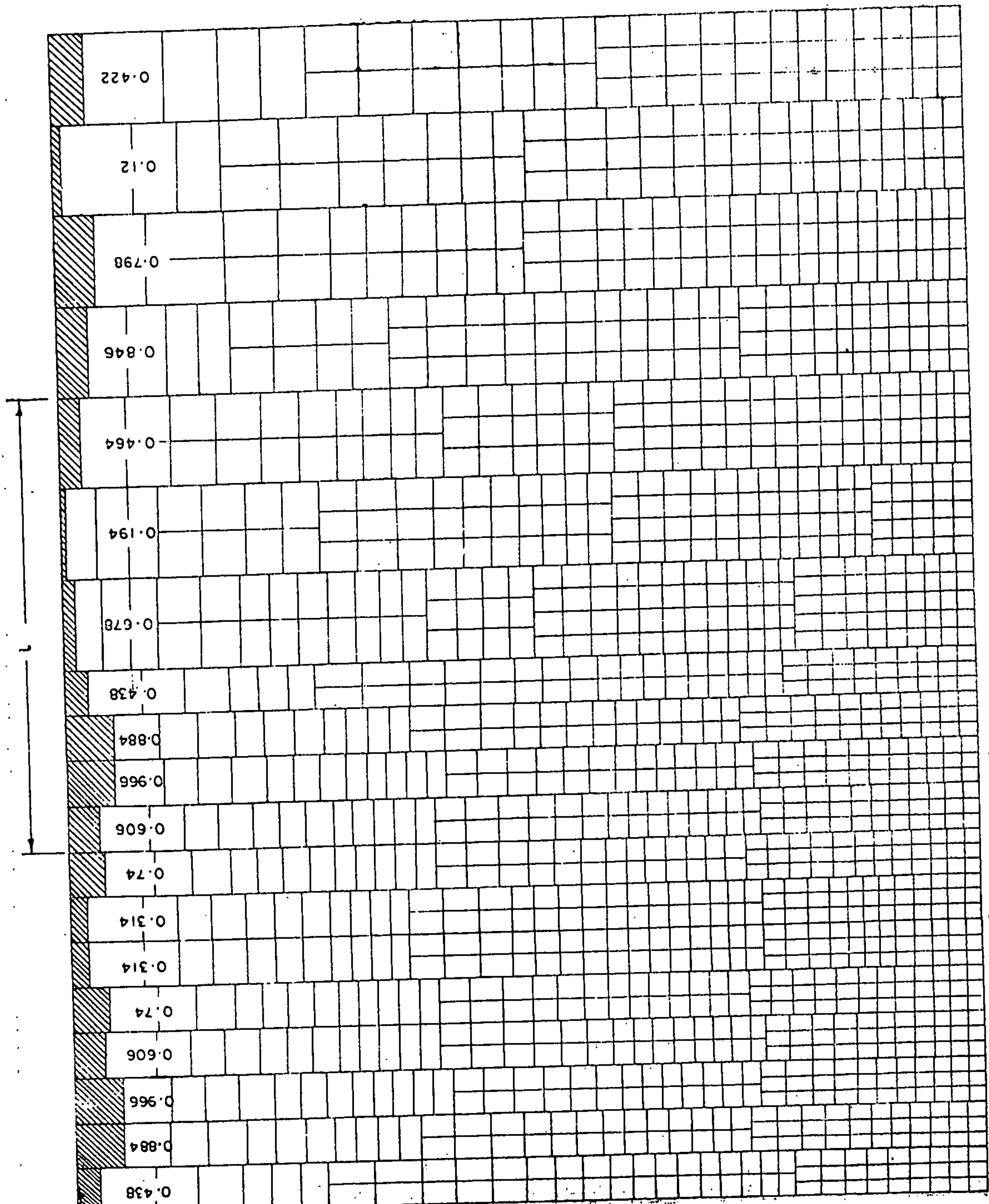

FIG. 4. Influence chart for deflection of slab for edge loading on Filonenko-Borodich model. $w = \frac{0\cdot 0005\, ql^4 N}{DA}$ and $A' = 0\cdot 05$.

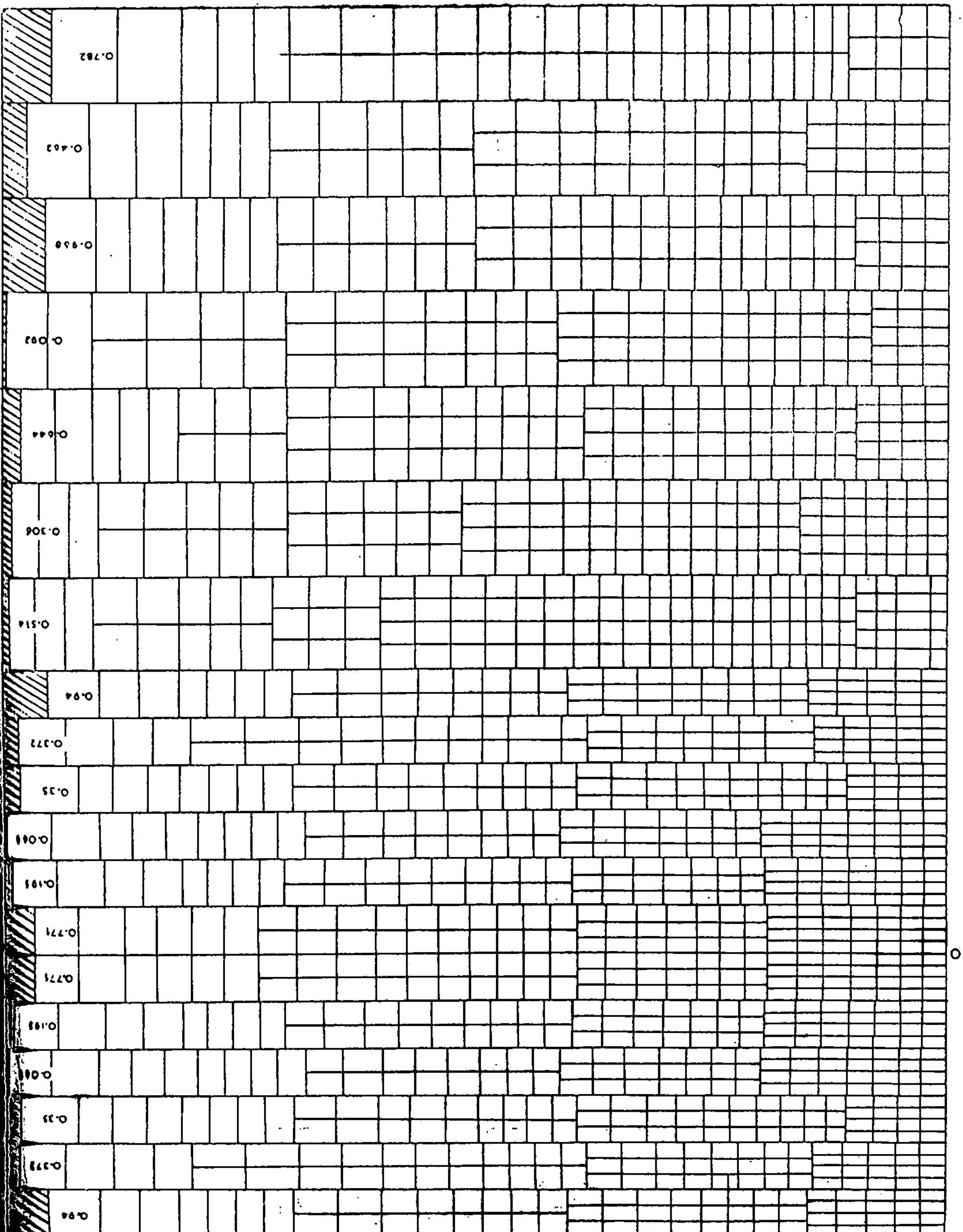

FIG. 5. Influence chart for deflection of slab for edge loading on Filonenko–Borodich model. $w = \frac{0.0005\, q l^4 N}{D.A}$ and $A' = 0.1$.

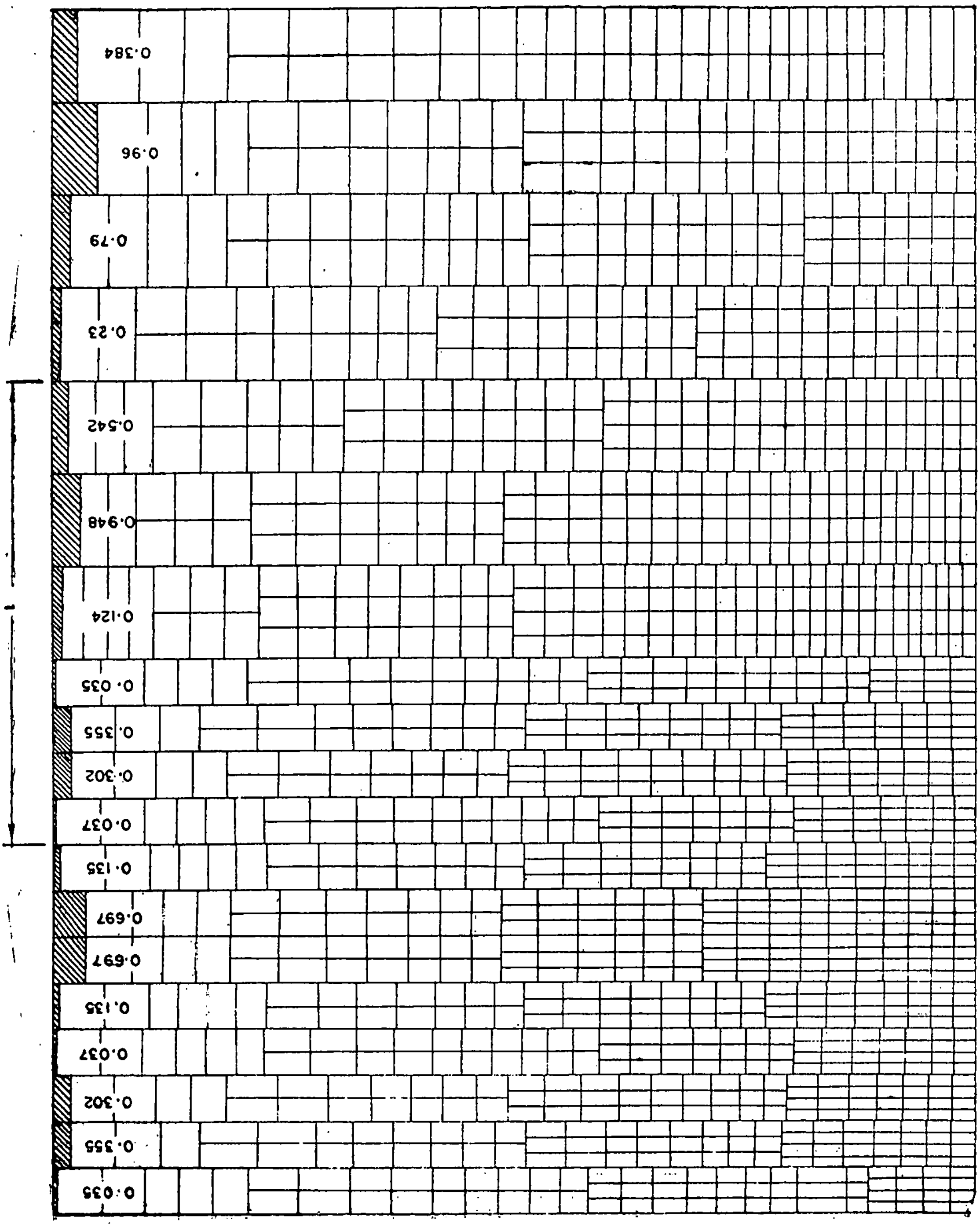

FIG. 6. Influence chart for deflection of slab for edge loading on Filonenko-Borodich model. $w = \frac{0 \cdot 0005\, q l^4 N}{DA}$ and $A' = 0 \cdot 25$.

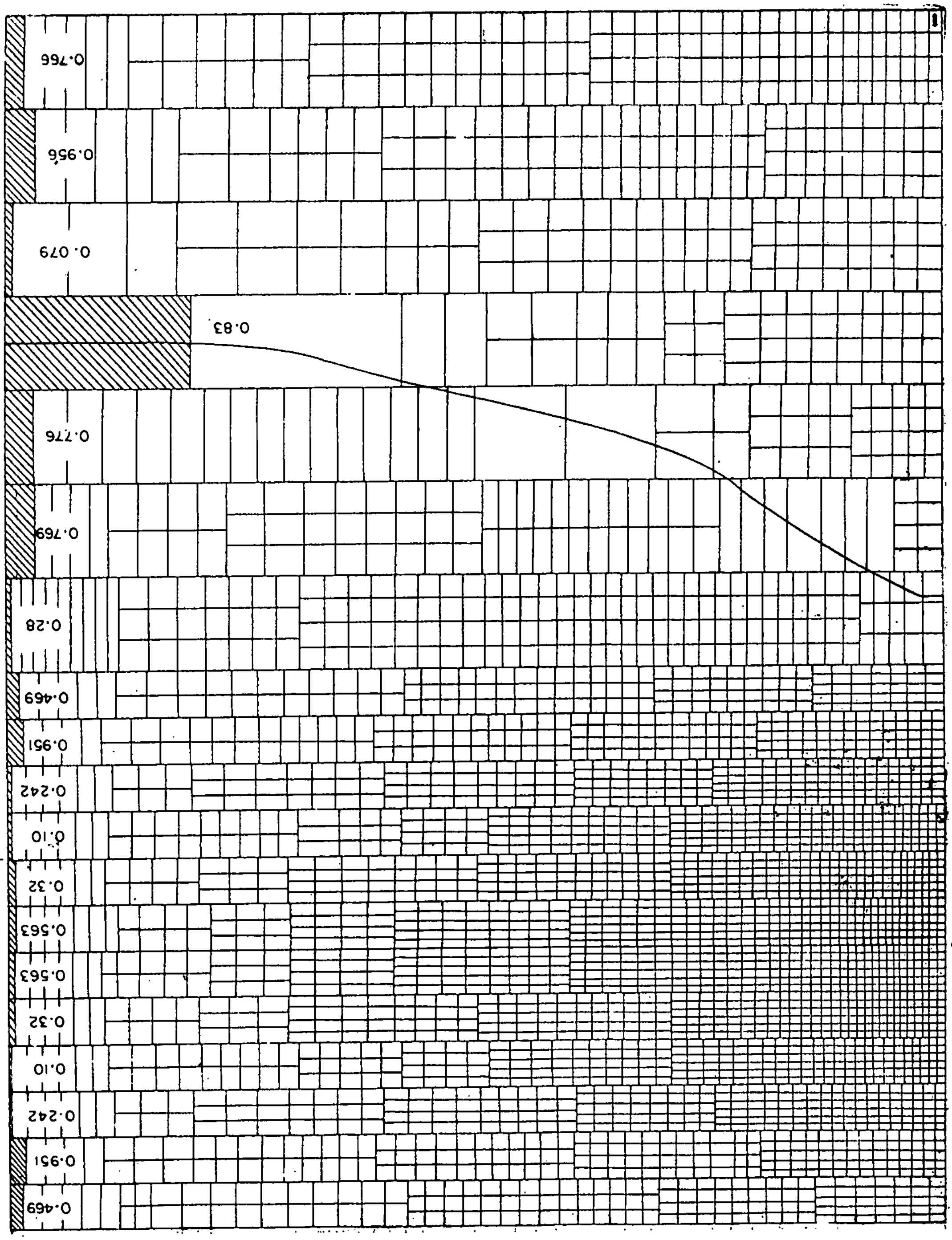

FIG. 7. Influence chart for moment $M_n = \frac{ql^2N}{10000\,A}$ for edge loading (N = number of positive blocks minus number of negative blocks, and Poisson's ratio for concrete = 0·15) of slab on Filonenko-Borodich model. $A' = 0·01$.

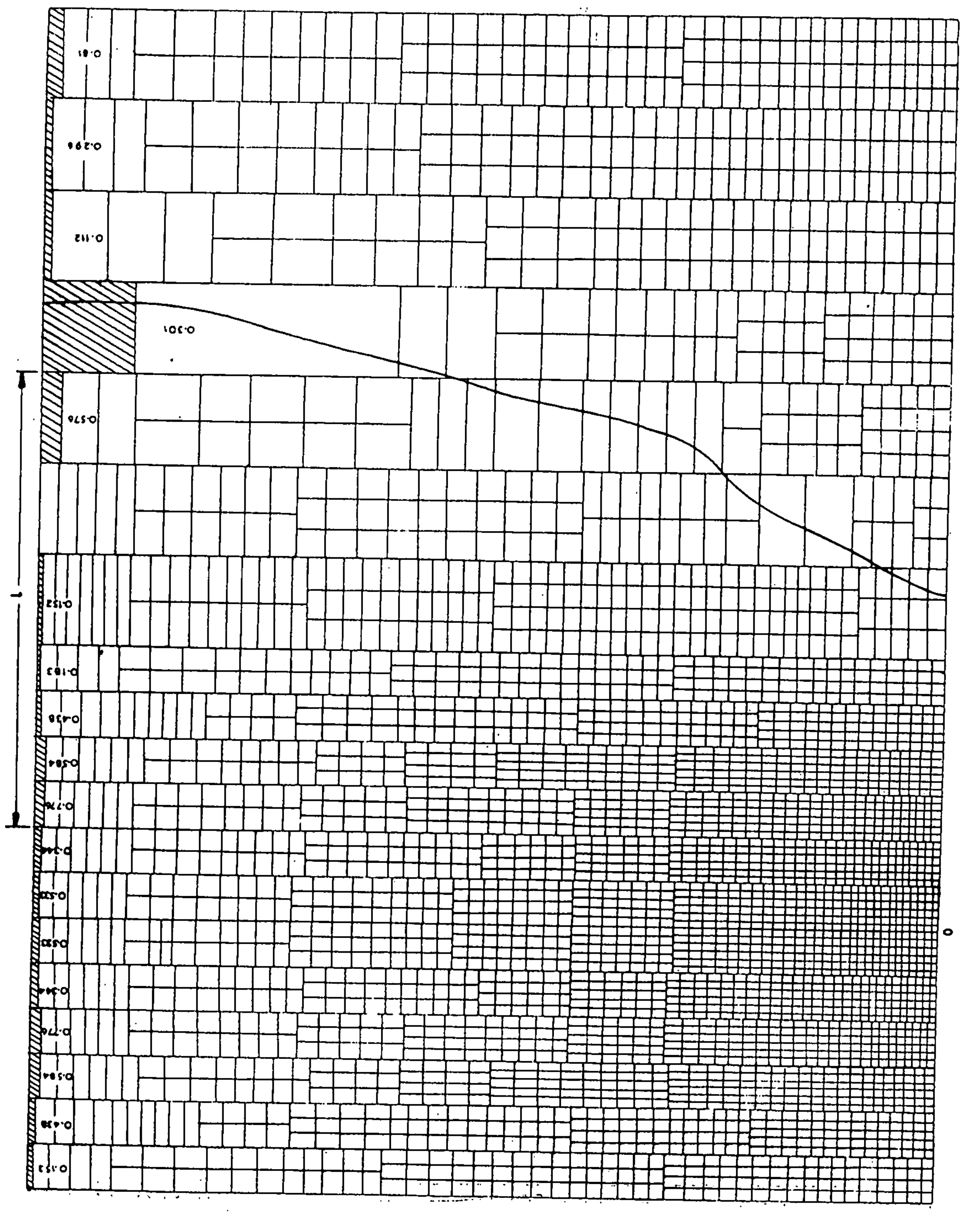

FIG. 8. Influence chart for moment $M_n = \frac{ql^2 N}{10000\,A}$ for edge loading (N = number of positive blocks minus number of negative blocks and Poisson's ratio for concrete = 0·15) of slab on Filonenko-Borodich model. $A' = 0{\cdot}05$.

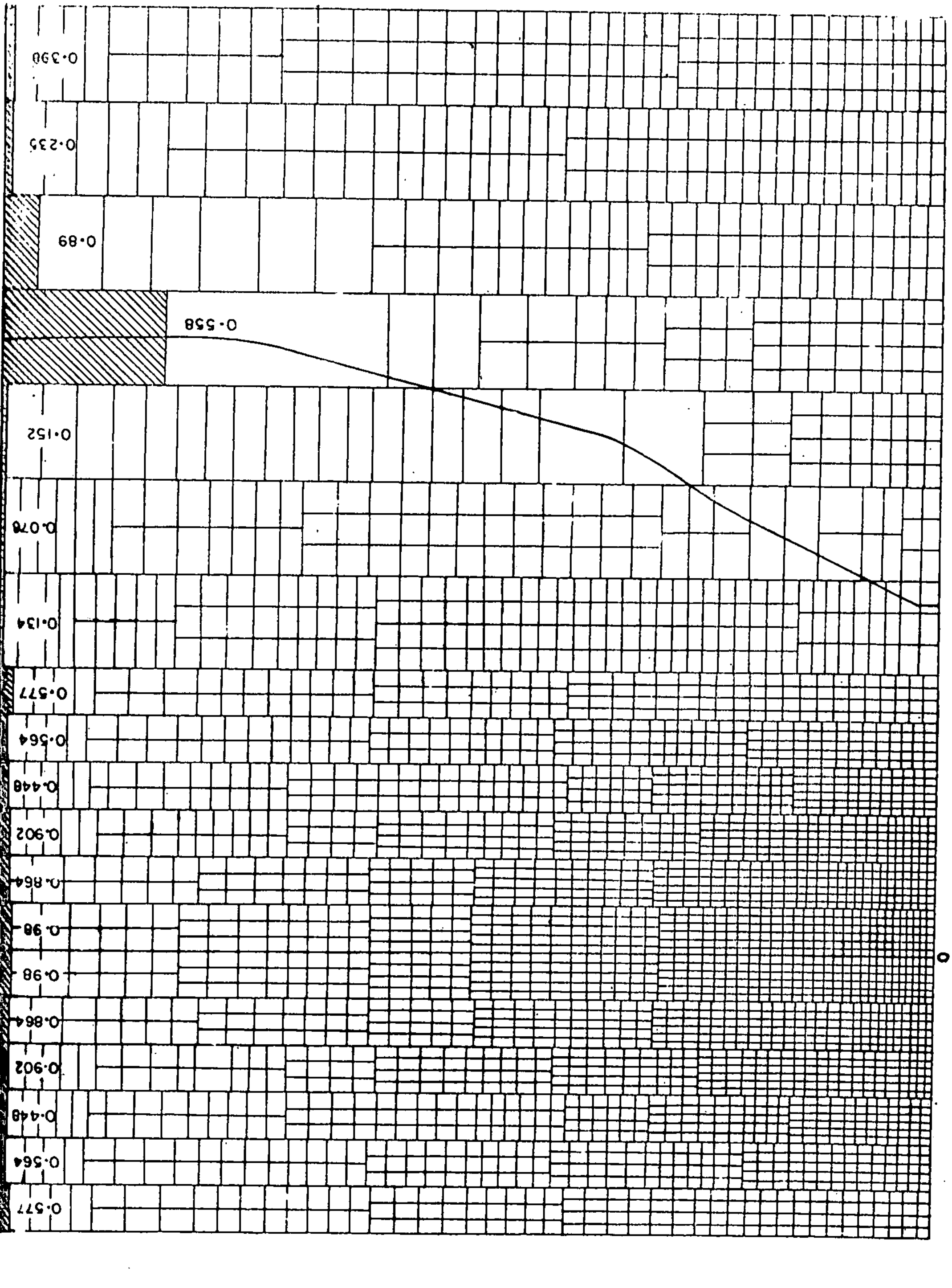

FIG. 9. Influence chart for moment $M_n = \frac{ql^2 N}{10000\,A}$ for edge loading (N = number of positive blocks minus number of negative blocks, and Poisson's ratio for concrete=0·15) of slab on Filonenko–Borodich model. $A' = 0{\cdot}1$.

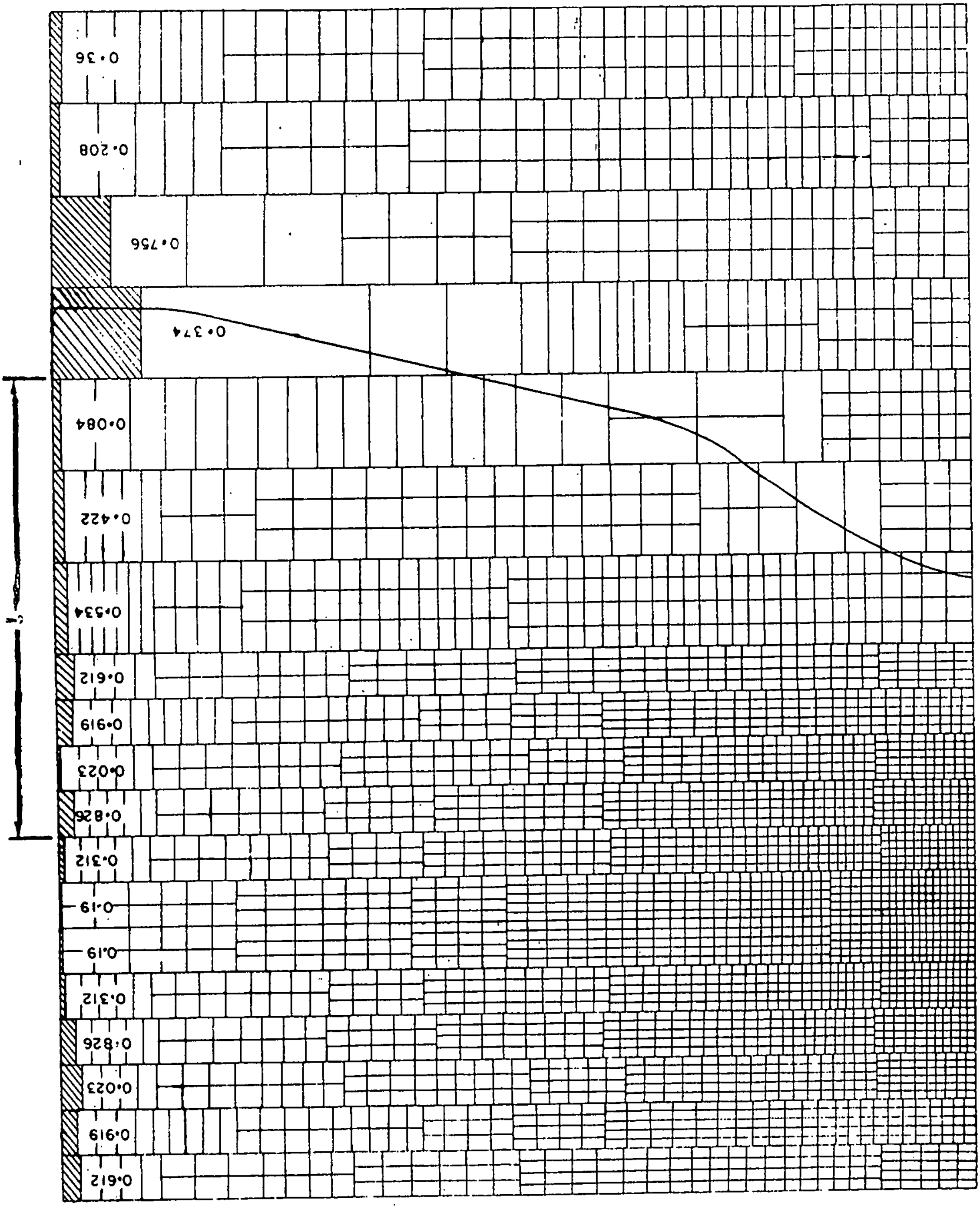

FIG. 10. Influence chart for moment $M_n = \frac{ql^2 N}{10000 A}$ for edge loading (N = number of positive blocks minus number of negative

4. Illustrative example

A dual tandem landing gear of a plane has a total load of 150,000 lb with fore and aft and dual spacing equal to 62″ and 31·5″ respectively. The other essential data are tyre pressure = 158 lb. per sq. inch, the thickness of concrete slab = 16″, $E = 4{,}000{,}000$ lb per sq. in. $\mu = 0\cdot 15$, $k = 100$ lb per cu. in. $A' = 0\cdot 25$ for Filonenko–Borodich foundation. The orientation of the gear with the axis O–n is as shown in Fig. 11.

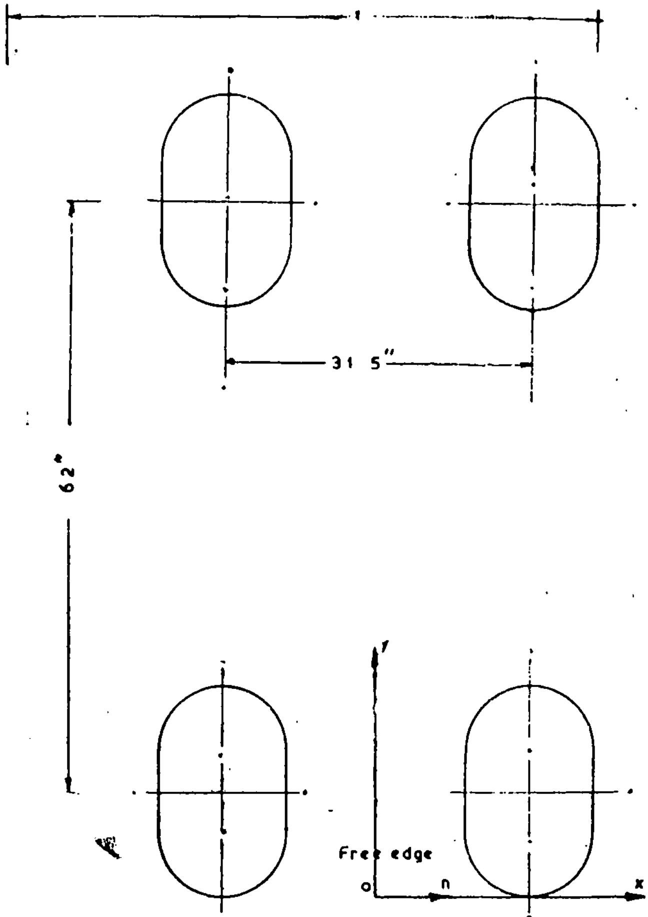

Fig. 11. Tyre imprints and their orientation for the numerical example.

Find the stress in the slab at point O.

Taking the shape of the contact area as a rectangle with semi-circular ends with width W_0 equal to six tenth of the length L and the contact pressure equal to tyre pressure, contact area $= 0\cdot 5227\, L^2$.

$$0\cdot 5227\, L^2 = \frac{37{,}500}{158} = 237\cdot 3 \text{ sq. in.}$$

$$L = \sqrt{\frac{237\cdot 3}{0\cdot 5227}} = 21\cdot 31 \text{ in.}$$

$$W_0 = 0\cdot 6 \times 21\cdot 31 = 12\cdot 79 \text{ in.}$$

$$l = \sqrt[4]{\frac{Eh^3}{12\,(1-\mu^2)\,K}} = \sqrt[4]{\frac{4{,}000{,}000 \times 16^3}{12\,(1-0\cdot 15^2)\,100}}$$

$$= 61\cdot 13 \text{ in.}$$

For drawing the tyre imprint the following relationship is used.

$$\frac{\text{Tracing dimension } (L \text{ or } W_0)}{\text{Actual contact dimension}} = \frac{\text{Length } l \text{ on chart}}{l \text{ of pavement}}$$

$$L = \frac{21\cdot 31}{61\cdot 13} \times 5\cdot 9 = 2\cdot 06 \text{ in.}$$

$$W_0 = 0\cdot 6\, L = 0\cdot 6 \times 2\cdot 06 = 1\cdot 236 \text{ in.}$$

Similarly, lateral tyre spacing on tracing

$$\frac{31\cdot 5}{61\cdot 13} \times 5\cdot 9 = 3\cdot 04 \text{ in.}$$

Fore and aft spacing on tracing

$$\frac{62\cdot 00}{61\cdot 13} \times 5\cdot 9 = 5\cdot 98 \text{ in.}$$

Using the dimensions obtained, the outline of the tyre imprints is drawn. For the orientation of the tyre imprint shown in Fig. 11, N, the number of blocks enclosed = 425.

$$M_n = \frac{158\,(61.13)^2}{A\ 10{,}000} \times 425 = 25344 \text{ in.lb.}$$

Stress in slab at point 'O' $= \frac{6M_n}{h^2} = \frac{6}{16^2} \times 25344 = 549$ psi.

5. Conclusion

Influence charts for edge loading for slabs on Filonenko-Borodich model are presented The charts can be used for the design of rigid airport pavement, resting on subgrade soil, which can be idealized as Filonenko-Borodich model. The charts are similar to those presented by Pickett and Ray[3], in which the subgrade is idealised as Winkler foundation.

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