Journal of the Indian Institute of Science
A Multidisciplinary Reviews Journal
ISSN: 0970-4140 Coden-JIISAD

REVIEWS

Spin Polarised Tunneling Probe for Two and Three Dimensional Dirac Metals

Sourin Das

Abstract | One of the most promising features of the time reversal symmetry protected two and three dimensional topological band insulators lies in the fact that their surface spectrum closely mimics the spectrum of massless Dirac fermions leading to a spin-momentum locked surface state. Hence, probing this property of topological insulators in a electrical transport set up is desirable. In this article we will review the possibility of probing the spin-momentum locked nature of the surface state spectrum via spin polarized tunneling probes. We will introduce a multi-terminal avatar of TMR measurement which could lead to a direct measurement of the spin momentum locking angle and the related spin texture of the surface state spectrum.

1 Introduction

Characterization of materials based on their electrical transport property are broadly classified into three types, namely metals, semi-conductors and insulators. Recently there has been a tremendous progress both on the theoretical and experimental front for discovery of new class of insulators (band insulators), which have metallic surface state owing to the nontrivial topological properties of their band structure.[1,2] These surface states are of particular interest both form theoretical and application point of view. The spectrum of the surface states closely mimics spectrum of massless Dirac fermions where the electron spin is locked to its momenta.

In this article we will review an idea pertaining to probing the spin momentum locked nature of the surface state spectrum in the ballistic limit by employing spin polarized tunneling probes. Motivated by the spin valve (SV) effect, spin polarized tunneling probes are something which naturally comes to our mind in the context of determining spin polarization state of the electron which lives on the surface of topological insulator (TI). We intend to employ the tunnel magneto resistance (TMR[3]) response between the TI surface state and a spin polarized tunneling probe to infer the spin momentum locking angle of the surface electrons. Though the dispersion of the surface state have nontrivial spin texture, they do not have a netspin polarization owing to time reversal symmetry, and hence a magneto resistance response is not expected, in general. Putting it differently, as the Fermi surface of the surface states of TI have no net spin polarization, the tunneling current injected form a magnetic electrode into the surface state will be independent of direction of magnetization of the tunneling probe. Hence, a simple minded spin polarized probe is not expected to fetch us any information regarding the spin momentum locking angle. The point to note here is the fact that a finite patch of the Fermi surface can have a net spin polarization due to spin texturing of the Fermi surface, but it is only the total Fermi surface whose net spin polarization must be zero. This in terns implies that the current injected form the magnetic probe into the momentum states belonging to finite patch of the Fermi surface will be sensitive to relative angle between the magnetization direction of the probe and the net spin polarization direction of the finite patch of the Fermi surface hence leading to a finite TMR response. Therefore, measuring the magnetoresistance response of the finite patch of the Fermi surface can lead, to possibilities of measurement of the spin orientation and hence the spin momentum locking angle of the surface states.

Now the final hurdle lies in finding a way to measure the current carried by electrons, which are injected form the tunneling probe into a

Department of Physics and Astrophysics, University of Delhi, Delhi 110 007, India.
sdas.du@gmail.com

finite patch of the Fermi surface of the surface state. This calls of a multi terminal current measurement. To understand this point let us consider the case of two dimensional TIs that host an one dimensional surface state (edge state). The edge state is described by a spectrum that mimics a one dimensional massless Dirac fermion with has a perfect spin momentum locking. The Fermi surface of this systems comprises two points, the left Fermi point (left movers) and the right Fermi points (right movers).

The spin polarization associate with the states at left and right Fermi point are exactly opposite of each other as demanded by time reversal symmetry. Individually both the state at the two Fermi points have finite spin polarization, but the net spin polarization of total Fermi sea is zero. When spin polarized current is injected locally into the edge state at small bias (linear response limit), these electron go and occupy states that are close to these two Fermi points. Hence, the total injected current can be thought of as sum of current carried by states belonging to left and right Fermi points.

The important point to note here is the fact that a fraction of the total that is injected around the left Fermi point and the right Fermi point will individually depend on the relative angle between the magnetization direction of the tunneling probe and the direction of polarization of the spin of electronic states at the respective Fermi points. If we can measure the current carried by injected electrons at the left and the right Fermi point separately, then by studying the variation of these current as a function of the magnetization direction of the tunneling probe, we can infer the direction of polarization of spin of electronic state at the left and the right Fermi points. This essentially implies that we can measure the spin momentum locking angle for the surface state. Note that the current injected at the left Fermi point can be collected at contact that is placed on the left side of the point of injection and the current injected at the right Fermi point can be collect at a contact placed at the right side of the point of electron injection. Hence, the minimal requirement of measuring the spin-momentum locking angle is a three terminal geometry, namely the magnetic tunneling contact form where current is to injected in to the surface state and the left contact and the right contact mentioned above. This idea can be straightforwardly extended to probe the surface state of three dimensional TI. In this article we first review the application of the idea discussed to edge states of two dimensional TIs in presence of electron-electron interaction.[4] Then we extend the analysis to the case of three dimensional TIs, but we restrict ourselves to the case with no electron-electron interaction.[5]

STM: Physics of scanning tunneling microscopy (STM) is based up on the simple phenomenon of tunneling which is a purely quantum mechanical effect. This technique was developed by *Gerd Binnig* and *Heinrich Rohrer* to image the surfaces of various solids at atomic scales for which they got the Nobel prize in 1986.

2 Probing Surface State of Two Dimensional TI

For a two-dimensional topological insulator, a pair of one-dimensional counter propagating modes appear on the edges,[1,6] which are transformed into helical Luttinger liquids (HLL) due to inter-mode Coulomb interactions.[7] The central point about the edge state lie in the fact that the spin orientation of the edge electrons is correlated with the direction of motion of the electron, *i.e.*, opposite spin modes counter propagate. The existence of such edge channels have already been detected experimentally in a multi-terminal Hall bar setup.[8] As discussed in the introduction, here we introduce a three terminal set up to probe the spin texture of these edge states.

2.1 *Proposed set up and the model*

We propose a three terminal junction as shown in Fig. 1. The spin of the electrons in the edge states are polarized in some direction depending on details of the spin-orbit interaction in the bulk. We use a coordinate system that has its $\hat{Z}$-axis along the direction of orientation of the spin of the edge electrons and the plane containing the polarization direction of the edge electron and the tip electron is assumed to be the $\hat{X}-\hat{Z}$ plane (see Fig. 1). Note that here we have assumed that the edge is smooth and is along a straight line, so that there is a well defined quantization direction for the electron spin living on the edge.

The Hamiltonian for the HLL is given by

$$H_0 = v\int_{-L/2}^{L/2} dx\left[K(\partial_x\Phi)^2 + K^{-1}(\partial_x\Theta)^2\right], \tag{1}$$

where $\Phi = (\phi_{R\uparrow} + \phi_{L\downarrow})/2, \Theta = (\phi_{R\uparrow} - \phi_{L\downarrow})/2$ and the $\phi_{R\uparrow/L\downarrow}$ are related to the up and down electron operators in the edge by the standard bosonization identity $\psi_{R\uparrow}(x) \sim \frac{1}{\sqrt{2\pi\zeta}}e^{ik_F x}e^{i\phi_{R\uparrow}(x)}$, $\psi_{L\downarrow}(x) \sim \frac{1}{\sqrt{2\pi\zeta}}e^{-ik_F x}e^{i\phi_{L\downarrow}(x)}$. ζ and K are the short distance cut-off and the Luttinger parameter respectively. Unlike the standard LL, here the spin orientation is correlated with the direction of motion. We drop Klein factors as they are irrelevant for our computations.

The Hamiltonian for the STM is assumed to be that of a free electron in 1–D. The tunneling Hamiltonian between the tip and the helical edge at a position $x = 0, x' = 0$ is given by

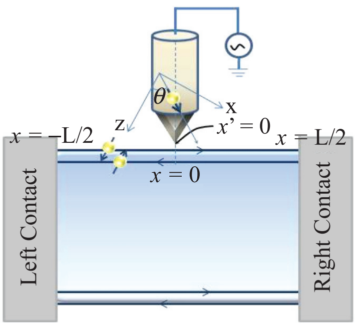

Figure 1: A schematic of the geometry of the proposed setup. The direction of orientation of the electron spin in the HLL is along the $\hat{Z}$ axis. The angle between direction of orientation of the spin of electrons in the edge and the majority spin in the STM tip is θ and they are assumed to lie in the $\hat{X}-\hat{Z}$ plane. The $\hat{Y}$-axis points out of the plane of the paper. Here x and x' represent the intrinsic one dimensional coordinates of the STM tip and the wire.

$$H_t = t\,[\,\psi^{\dagger}_{i\alpha}(x=0)\,\chi_{\alpha}(x'=0) + \text{h.c.}\,], \tag{2}$$

where $i = R, L$ denotes right and left movers and α denotes the spin index, $\psi_{i\alpha}$ and χ_{α} denote the electron destruction operator in the HLL and the STM respectively. Voltage bias in the tunneling operator can be introduced simply by replacing $\chi_{\alpha}(x) \to \chi_{\alpha}(x)e^{-iVt/\hbar}$. We will, henceforth, drop the index *i, j* denoting the direction of motion.

Since the tunneling conserves spin, using a fully polarized STM with polarization direction tuned along the positive or negative direction of $\hat{Z}$-axis will naturally allow for chiral injection *i.e.*, injecting only right (↑) or left (↓) movers. In the absence of interactions in the HLL, the chirally injected electron will cause both charge current and spin current to flow only to the right or to the left lead, hence leading to a left-right asymmetry. In the presence of interactions in the HLL, due to Coulomb scattering between the right and left movers, the chirally injected charge and spin degrees of freedom of the electron get fractionalized and move in both directions; however, in general, the left-right asymmetry still survives.

Now, let us consider the fully polarized STM tip with the polarization direction making an arbitrary angle θ with respect to the spin of the HLL electron. In the quantization basis of the HLL spins, the tip spinor can be written as $\chi_{rot} = e^{-i\theta\sigma\cdot\hat{Y}/2}\chi_T$, where χ_T is the tip spinor in a basis where the spin quantization axis is along the STM polarization direction *i.e.*, $\chi_T = (\chi_{\uparrow}, 0)$. So $\chi_{rot\uparrow} = \cos(\theta/2)\,\chi_{\uparrow}$ and $\chi_{rot\downarrow} = \sin(\theta/2)\,\chi_{\uparrow}$. In other words, the electron in the tip has both ↑ and ↓ spins, but the effective tunnel amplitudes are asymmetric (except when $\theta = \pi/2$) and hence, the current asymmetry survives. As a function of the rotation angle θ, the chiral injection goes from being a pure right-mover at $\theta = 0$ to a pure left mover at $\theta = \pi$.

2.2 *Charge current*

The tunneling Hamiltonian can now be rewritten in terms of $\chi_{\uparrow}$ as

$$H_t = \left[t_{\uparrow}\,\psi^{\dagger}_{\uparrow}\,\chi_{\uparrow} + t_{\downarrow}\,\psi^{\dagger}_{\downarrow}\,\chi_{\uparrow} + \text{h.c.}\right], \tag{3}$$

where $t_{\uparrow} = t\cos(\theta/2)$ and $t_{\downarrow} = t\sin(\theta/2)$ can be tuned by tuning θ. The Boguliobov fields $\tilde{\phi}_{L/R}$, which move unhindered to right and left direction (henceforth we call them the right chiraland left chiral fields) are given by

$$\phi_{\uparrow/\downarrow} = \frac{1}{2\sqrt{K}}\left[(1\pm K)\,\tilde{\phi}_R + (1\mp K)\,\tilde{\phi}_L\right]. \tag{4}$$

Note that the total electron density on the HLL wire can be expressed in terms of the chiral fields as $\rho(x) = (\sqrt{K}/2\pi)\partial_x\tilde{\phi}_R - (\sqrt{K}/2\pi)\partial_x\tilde{\phi}_L$, thus defining the chiral right (left) densities and the corresponding number operators as

$$\tilde{N}_{R/L} = \int_{-L/2}^{L/2} dx\,\tilde{\rho}(x)_{R/L} = \pm\frac{\sqrt{K}}{2\pi}\int_{-L/2}^{L/2} dx\,\partial_x(\tilde{\phi}_{R/L}). \tag{5}$$

Next we define the operator corresponding to the chiral decomposition of the total charge current as $I_{t\alpha} = d\tilde{N}_{\alpha}/dt = -i[\tilde{N}_{\alpha}, H_t]$, where we have set $\hbar = 1$ and electron charge $e = 1$ and $\alpha = R/L$. Using the standard commutation relations of chiral fields, $[\tilde{\phi}_{\uparrow/\downarrow}(x), \tilde{\phi}_{\uparrow/\downarrow}(x')] = \pm i\pi\,\text{sgn}(x-x')$ the chiral currents were found to be

$$I_{tR/L}(\theta) = \frac{1}{2}[\,(1\pm K)\cos(\theta/2)\,I_t(\theta=0) + (1\mp K)\sin(\theta/2)\,I_t(\theta=\pi)\,]. \tag{6}$$

$I_t(\theta) = I_{tL}(\theta) + I_{tR}(\theta)$ is the total tunneling charge current operator for an arbitrary value of θ and $I_t(\theta = 0/\pi) = it(\chi^\dagger_\uparrow \psi_{\uparrow/\downarrow} - \psi^\dagger_{\uparrow/\downarrow}\chi_\uparrow)$.

2.3 *Results and conclusion*

The expectation values of the currents operator in linear response is given by

$$\langle I_t(\theta)\rangle = -\frac{i}{\hbar}\int_{-\infty}^{0} d\tau \langle [\, I_t(\theta, \tau = 0), H_t(\tau)\,]\rangle. \quad (7)$$

Since the HLL Hamiltonian is left-right symmetric in the absence of the tip and the tip is fully polarised, the value is equal for $\theta = 0$ and $\theta = \pi$ and given by $\langle I_t(\theta = 0)\rangle = \langle I_t(\theta = \pi)\rangle = I_0$. Using the well-known correlation function of LL liquid at finite temperature T, we find

$$I_0 = \frac{e^2}{h} |t^2| \frac{(T/\Lambda)^\nu}{(\hbar v_F)^2\, \Gamma(\nu + 1)} \times V, \quad (8)$$

where Λ is an ultra-violet cutoff and ν is the Luttinger tunneling exponent given by $\nu = -1 + (K + K^{-1})/2$. Here we have assumed that $T \gg T_L$, T_V, where T_L is the temperature equivalent of the length of the wire defined by $\nu/L = k_B T$ and $T_V = eV/k_B$, is the temperature equivalent of bias voltage.

Using these values, we now obtain the current heading to the right and left ends of the wire as

$$\langle I_{tR,L}(\theta)\rangle = \frac{(1 \pm K\cos\theta)}{2} I_0\,. \quad (9)$$

Note that even though the left and right chiral currents that are measured at the right and left contact depend on θ, the total tunneling current $I_t(\theta) = I_{tL}(\theta) + I_{tR}(\theta)$ is independent of θ. Thus, we show that unlike the two terminal tunnel current, the three terminal current is clearly not independent of θ. This fact tells us that indeed the current measured at the right and left contact shows a TMR type response as a function of θ. Further, note that percentage asymmetry in the left and right current given by $A = (\langle I_{tL}(\theta)\rangle - \langle I_{tR}(\theta)\rangle)/(\langle I_{tL}(\theta)\rangle + \langle I_{tR}(\theta)\rangle) = K\cos\theta$ is independent of I_0, which carry all the details of tunnel junction. Hence, measuring this quantity as a function of θ can provide a direct information regarding the exact orientation of the spin of the electron associated with the right and left moving states. Lastly, it is worth mentioning that our strategy of using a spin polarized tunneling probe for detecting the spin momentum locking nature of the edge spectrum works even in presence of electron-electron interactions as is evident form the above results. Next we consider surface state of three dimensional TIs where we only focus on the case of non-interacting electrons.

3 Probing Surface State of Three Dimensional TI

The two popular probes used to scan the surface states of[1,2] 3-D TIs are spin polarised ARPES or STM. Spin polarised ARPES seems to have an edge over the STM as it couples more directly to the spin texture of the Fermi surface. Here we propose a multi-terminal set up involving spin-polarised STM (SPSTM)[9] which directly couples to the spin texture of the Fermi surface leading its straightforward read readout. This read out is theoretically understood in terms of a new kind of TMR response between the magnetized STM and the non-magnetic TI surface which relies crucially on its multi-terminal character as described below.

3.1 *Proposed set up*

The proposed set up comprises two contact pads placed diametrically opposite to each other on the surface of the TI while electrons are injected from the SPSTM placed at the centre of the sample as shown in Fig. 2. The surface can be imagined to be divided into two halves by a line through the centre of the sample perpendicular to the direction joining the two contacts, (for future reference, we mention that the angle made by this partitioning line with the x-axis is denoted by γ, see Fig. 2). Each contact measures the current flowing in the surface in its own half. We show that the total current $I_0 = I_L + I_R$ is insensitive to current anisotropy discussed above owing to zero magnetisation of Fermi surface, but $\Delta I = I_L - I_R$ is very sensitive to the current anisotropy and leads to a finite TMR response with the spin polarised STM which oscillates as a function of γ. Note that γ can be changed simply by rotating the sample with respect to the tip about z-axis.

We show that ΔI measured as a function of γ leads to a direct reconstruction of in-plane spin texture in the momentum space, i.e., we can extract the angle of spin-momentum locking (θ_L) and the chirality; left chiral or right chiral from this study.

3.2 *The model*

We start with the generic Hamiltonian for the 3D TI surface state given by

$$\mathcal{H}_{TI} = \hbar v_F \sum_{\vec{k}} \Psi^\dagger_{TI}(\vec{k})(\vec{\sigma} \times \vec{k})_z \Psi_{TI}(\vec{k}), \quad (10)$$

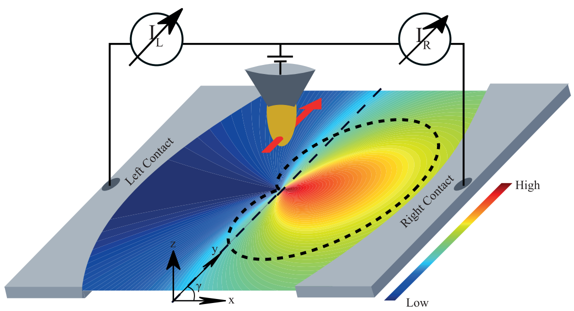

Figure 2: A schematic of the setup. The colour density shows the current density profile on the surface of the TI consistent with the polarisation direction of the tip shown by red arrow pointing along *y*-axis and the spin-momentum locking angle, $\theta_L = \pi/2$. Dotted curve is a polar plot of the tunnelling current amplitude at the point of injection showing the profile of the current anisotropy. $I_{L/R}$ are the current (in arbitrary units) carried by the left and right contact.

where $\Psi_{TI}(\vec{k}) = 1/\sqrt{2}(1, e^{i(\phi_{\vec{k}}+\theta_L)})^T \hat{c}_{\vec{k}}$ and $\hat{c}_{\vec{k}}$ is the annihilation operator with momentum $\vec{k}$. $\phi_{\vec{k}} = \tan^{-1}(k_y/k_x)$ is the polar angle of the momentum vector. For Eq. (10), the spin-momentum locking angle θ_L is $\pi/2$, consistent with popularly known TI materials. Assuming a flat density of states, a model Hamiltonian for a fully spin-polarised STM tip is written as $\mathcal{H}_{\mathrm{STM}} = \sum_{\vec{k}} \varepsilon_{\vec{k}} \hat{d}^{\dagger}_{\vec{k},\uparrow} \hat{d}_{\vec{k},\uparrow}$, where the STM electron annihilation operator in real space is given by $\Psi_{\mathrm{STM}}(\vec{r}) = \int d\vec{k} e^{i\vec{k}\cdot\vec{r}} (\cos(\theta_{\mathrm{STM}}/2), \sin(\theta_{STM}/2) e^{i\phi_{STM}})^T \hat{d}_{\vec{k},\uparrow}$ and $\varepsilon_{\vec{k}} = v_{\mathrm{STM}} |\vec{k}|$, where v_{STM} is the Fermi velocity of the STM and $\theta_{\mathrm{STM}}(\phi)$ is the polar (azimuthal) angle of the STM spin. The STM tip is assumed to be weakly coupled to the TI surface by a tunnelling Hamiltonian, $\mathcal{H}_{\mathrm{tunn}} = J(\Psi^{\dagger}_{\mathrm{TI}}(\vec{r}=0)\Psi_{\mathrm{STM}}(\vec{r}=0) + \mathrm{h.c.})$, which upon Fourier transforming gives $H_{tunn} = J\sum_{\vec{k},\vec{k}'} z_{\vec{k}} \hat{c}^{\dagger}_{\vec{k}} \hat{d}_{\vec{k}',\uparrow} + \mathrm{h.c.}$, where

$$z_{\vec{k}} = \frac{1}{\sqrt{2}}\left(\cos\frac{\theta_{STM}}{2} - i\sin\frac{\theta_{STM}}{2} e^{i(\phi_{STM}-\phi_{\vec{k}})}\right), \quad (11)$$

has the information about the overlap of the STM spinor and the TI spinor for each momentum mode $\vec{k}$ which ultimately decides how much injection happens in each $\vec{k}$, and hence the current anisotropy discussed above. The current operator is defined as $\hat{I} = d\hat{N}_{STM}/dt = \frac{i}{\hbar}[H, \hat{N}_{STM}]$, where $\hat{N}_{\mathrm{STM}} = \int d\vec{k} \hat{d}^{\dagger}_{\vec{k},\uparrow} \hat{d}_{\vec{k},\uparrow}$ and $\mathcal{H} = \mathcal{H}_{\mathrm{TI}} + \mathcal{H}_{\mathrm{STM}} + \mathcal{H}_{\mathrm{tunn}}$. The expectation value of the current at some time t is given by $\langle I(t)\rangle = \langle G|e^{i\mathcal{H}t}\,\hat{I}\,e^{-i\mathcal{H}t}|G\rangle$, where $|G\rangle$ is the ground state of $\mathcal{H}$ in presence of a chemical potential bias ($\mu_{\mathrm{STM}} \neq \mu_{\mathrm{TI}}$) between STM and TI. Treating $\mathcal{H}_{\mathrm{tunn}}$ perturbatively, we get

$$\langle I(0)\rangle = \frac{i}{\hbar}\int_{-\infty}^{0} dt' \left\langle g\left|[H_{\mathrm{tunn,I}}(t'), I_{\mathrm{I}}(0)]\right|\right\rangle g, \quad (12)$$

where the I in the subscript stands for the interaction picture and $|g\rangle = |g\rangle_{\mathrm{TI}} \otimes |g\rangle_{\mathrm{STM}}$ is the non-interacting ground state with $\mathcal{H}_{\mathrm{tunn}}$ treated as interaction. Decomposing the current in momentum space we obtain, $\langle I\rangle = \frac{e}{\hbar}\int d\vec{k}\int d\vec{k}' |z_{\vec{k}}|^2 \chi_{\vec{k},\vec{k}'} \delta(\varepsilon_{\mathrm{STM}}(\vec{k}') - \varepsilon_{\mathrm{TI}}(\vec{k}))$, where $|z_{\vec{k}}|^2$ (Eq. (11)) has the information of the spinor overlaps, $\chi_{\vec{k},\vec{k}'} = \int_{-\infty}^{\infty} d\tau\, \mathrm{Im}[G_{\mathrm{TI}}(\vec{k}, 0; \vec{k}, \tau)\, G_{\mathrm{STM}}(\vec{k}', \tau; \vec{k}', 0)$ has the Green's functions, where G denotes the standard time ordered Fermionic Green's functions and the delta function ensures energy conservation. Hence, we obtain a momentum resolved current given by

$$\left\langle I(\vec{k})\right\rangle = \frac{e}{\hbar}|z_{\vec{k}}|^2 \chi_{\vec{k}}, \quad (13)$$

where $\chi_{\vec{k}} = \int_{-\infty}^{+\infty} d\vec{k}' \chi_{\vec{k},\vec{k}'} \delta(\varepsilon_{\mathrm{STM}}(\vec{k}') - \varepsilon_{\mathrm{TI}}(\vec{k}))$. The total tunnelling current is just a sum over $\left\langle I(\vec{k})\right\rangle$ for all possible $\vec{k}$ living in the bias window. The angular distribution of the injected current in real space is same as the angular distribution of momentum resolved current about the tunnelling point. As a consistency check for this, we evaluate the expectation value of the current vector operator for a pristine TI surface $\hat{\vec{j}}(\vec{r}) = (\sigma^y(\vec{r}), -\sigma^x(\vec{r}))$ at $\vec{r} = 0$ perturbatively to second order in tunnel Hamiltonian and obtain $\left\langle \hat{\vec{j}}(\vec{r}=0)\right\rangle = \sum_{\vec{k}} \left\langle I(\vec{k})\right\rangle \hat{n}_{\vec{k}}$, which reconfirms our interpretations of $\left\langle I(\vec{k})\right\rangle$ being the real space angular distribution of current about the injection point. Here $\hat{n}_{\vec{k}}$ is a unit vector pointing along $\vec{k}$. Owing to the azimuthally symmetric (in k – space) Fermi surface, $\left\langle I(\vec{k})\right\rangle$, turns out to be separable in its dependence on $|\vec{k}|$ and $\phi_{\vec{k}}$ as $\left\langle I(\vec{k})\right\rangle = I_{|\vec{k}|} I_{\phi_{\vec{k}}}$ with

$$I_{\phi_{\vec{k}}} = 1 + \sin\theta_{STM} \sin(\phi_{STM} - \phi_{\vec{k}}). \quad (14)$$

It is clear from the above result that the total injected current that is obtained by summing over all possible momenta in the bias window leads to a current, which is independent of the direction of STM tip magnetization. This fact is consistent with our observation for the case of edge state of two dimensional TI studied in the previous section. But the current asymmetry defined as $\Delta I = I_L - I_R = \left(\int_{\gamma}^{\gamma+\pi} - \int_{\gamma+\pi}^{\gamma+2\pi}\right) d\phi_{\vec{k}} I_{\phi_{\vec{k}}} \int |\vec{k}| \, d|\vec{k}| \, I_{|\vec{k}|}$, which could be measured directly in the set up depicted in Fig. 2 shows a finite TMR as function of γ given by

$$\Delta I = \frac{4eJ^2}{\hbar^4 v_f^2 v_{\mathrm{STM}}} \mathcal{F} \cos(\phi_{\mathrm{STM}} - \gamma) \sin \theta_{\mathrm{STM}}, \qquad (15)$$

where $\mathcal{F} = \int_{-\infty}^{+\infty} dE \, E\left(n_{\mathrm{F}}^{\mathrm{TI}} - n_{\mathrm{F}}^{\mathrm{STM}}\right)$ is obtained from the $|\vec{k}|$ integral by appropriately putting in the density of states, where $\mathrm{nF}(E, \mu, T)$ denotes the Fermi function.

3.3 *Reconstructing spin texture*

We will now demonstrate that the above expression can be directly exploited to uniquely identify the spin-momentum locking angle and the chirality. At this point it is important to note that all the in-plane angles like γ and ϕ_{STM} are measured with respect to the positive direction of x-axis along the anti-clock wise direction. For the case of $\phi_{STM} = \pi/2$ observing a zero in ΔI at $\gamma = 0$ (see Fig. (3)), implies that the momentum modes pointing towards the left and right contact starting from the tip position have a locked-spin which is pointing perpendicular to the STM polarisation pointing along y-axis (see Fig. 2) so that the injected current gets symmetrically distributed between left and right contact. Assuming a planar spin texture, it directly tells us that the spin momentum locking angle $|\theta_{\mathrm{L}}| = \pi/2$. Of course this conclusion relies on the assumption that the Fermi-surface is circular in shape so that spin on each half can be added up symmetrically. Now we are left with two possibilities; the two oppositely directed momentum modes discussed above pointing towards left and right contact have a locked-spin either pointing parallel and anti-parallel to the x–axis or the other way round respectively. And this information is nothing but the spin chirality of the Fermi surface. To settle the chirality, we observe that ΔI is maximally negative for $\gamma = \pi/2$. This implies that maximal share of the injected current is flowing to the right contact (as depicted in Fig. 2) implying that the momentum mode pointing towards right starting from the tip position has a locked-spin which is parallel to the tip magnetisation direction. Hence, the study of ΔI also implies that the momentum mode pointing along x-axis has a locked-spin pointing along y-direction; therefore reading out the spin chirality of the Fermi surface in hand. Consequently, its leads to the conclusion that spin-momentum locked spin is uniquely given by $<\vec{\sigma}(\vec{k})> = (-\sin\phi_{\vec{k}}, \cos\phi_{\vec{k}})$ Hence, our claim of reconstructing the Fermi-surface spin texture using the proposed three-terminal TMR data is clearly demonstrated. For an arbitrary spin-momentum locking angle θ_L, $\Delta I \sim \sin(\gamma - \phi_{\mathrm{STM}} + \theta_{\mathrm{L}})$; the maxima in its magnitude occurs at $\gamma = \phi_{\mathrm{STM}} - \theta_{\mathrm{L}} + \pi/2$, and hence the spin-momentum locking angle can be extracted. The sign of the first maxima of ΔI as we increase γ from zero gives the chirality.

3.4 *Result and discussion*

Now we recast $I_{L/R}$ or equivalently ΔI in an explicit TMR form,[3] which puts our idea on a firm footing and adds further transparency to the discussion above. Note that a net spin polarisation vector can be obtained by performing a vector sum of spin polarisations of each momentum mode

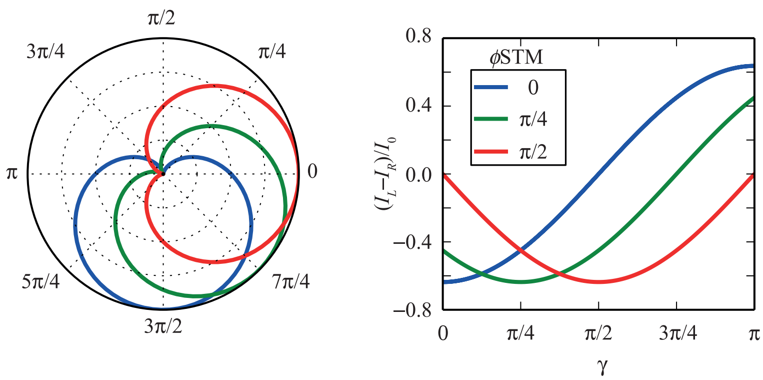

Figure 3: Left: The angular profile of the current as obtained from Eq. (14) for $\theta_{STM} = \pi/2$ and different values of ϕ_{STM} as mentioned in the legend. Right: The resulting current asymmetries as function of γ as obtained from Eq. (15).

living on half of the Fermi surface of TI surface states, where the Fermi surface bipartition is done along the line defining γ. This quantity for each half of the Fermi Surface is given by $\vec{S}_{half,L/R}(\gamma,\mu) = \mp(\rho_\mu^{\mathrm{TI}}/\pi)(\cos\gamma\hat{x} + \sin\gamma\hat{y})$, which sums to zero due to TRS. Here, $\rho_\mu^{\mathrm{TI}} = 2\pi\mu/(\hbar v_F)^2$ is the density of states (DOS) of TI at chemical potential μ where μ is measured from the neutrality point. Then, by extending Eq. (15) to include finite polarisation of the STM tip, in linear response limit we obtain,

$$I_{L/R} = \frac{\pi J^2 e^2 \rho_\mu^{TI} \rho^{STM}}{h}\left[1 \mp \frac{2p}{\pi}\hat{S}_{half}(\gamma,\mu)\cdot\hat{S}_{STM}\right]V \qquad (16)$$

where ρ^{STM} is the spin averaged DOS of the tip, p is the polarisation of the tip given by $(\rho_\uparrow^{STM} - \rho_\downarrow^{STM})/(\rho_\uparrow^{STM} + \rho_\downarrow^{STM})$ and V is the applied voltage bias between tip and TI. $\hat{S}_{half}$ and $\hat{S}_{STM}$ are unit vectors along $\vec{S}_{half}$ and magnetisation direction of the tip. We see that indeed the left and right contacts shows a standard TMR response[3] having opposite signs (due to TRS) with the magnetised STM and the pure magnetic response can be extracted from it simply by taking an anti-symmetric combination of the two, which is nothing but ΔI. Hence, spin-momentum locking together with multi-terminal set up leads to this exotic situation where large TMR response is extracted out of a non-magnetic material that is shown to be useful for characterizing the material itself. This results actually provide a solid platform to apply the idea of multi-terminal TMR to more complicated surface state, which can exist on the surface of 3-D TI surfaces. For example TI materials like Bi_2Se_3, Bi_2Te_3 and Sb_2Te_3, the planar surface states are distinct from one another with regard to their spectrum and the associated spin texture for each angle θ, which the normal to the surface makes with the crystal growth axis. A detailed study partaining to these surface state can be found in Ref. [10] and application of our idea of multi-terminal magneto resistance for probing these surface state can be found in Ref. [11]. In general it is expected that the surface states may also host a variety of surface potentials that can distort the spin texture of the pristine TI surface state. Out idea of multi-terminal magnetoresistance probing can also be applied to identify and characterize these surface potentials.[12]

Sourin Das is an Assistant Professor at the Department of Physics and Astrophysics, University of Delhi. His research interest primarily spans over the field of Mesoscopic physics. More specifically his work is in the field of low dimensional quantum filed theory, physics of quantum dots, quantum wires, topological state of matter in low dimensional systems, quantum hall effect. He also works in various aspects of geometric phase and its detection in context of Mesoscopic physics. He has published over 30 journal and conference papers in his research areas.

Acknowledgements

SD would like to thank Sumathi Rao, Sthitadhi Roy, Kush Saha, Diptiman Sen, Abhiram Soori, Krishnendu Sengupta and Disha Wadhawan for many stimulating discussions.

Received 15 March 2016.

References

1. M.Z. Hasan and C.L. Kane, Reviews of Modern Physics **82**, 3045 (2010), 1002.3895.
2. X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. **83**, 1057 (2011).
3. J.C. Slonczewski, Phys. Rev. B **39**, 6995 (1989).
4. S. Das and S. Rao, Physical Review Letters **106**, 236403 (2011), 1006.2239.
5. S. Roy, A. Soori, and S. Das, Phys. Rev. B **91**, 041109 (2015), 1407.4407.
6. M. Büttiker, Science **325**, 278 (2009), ISSN 0036-8075.
7. C. Xu and J.E. Moore, Phys. Rev. B **73**, 045322 (2006).
8. A. Roth, C. Brüune, H. Buhmann, L.W. Molenkamp, J. Maciejko, X.-L. Qi, and S.-C. Zhang, Science **325**, 294 (2009).
9. K. Saha, S. Das, K. Sengupta, and D. Sen, Phys. Rev. B **84**, 165439 (2011), 1105.0132.
10. F. Zhang, C.L. Kane, and E.J. Mele, Phys. Rev. B **86**, 081303 (2012), 1203.6382.
11. S. Roy, K. Saha, and S. Das, Phys. Rev. B **91**, 195415 (2015).
12. S. Roy and S. Das, Phys. Rev. B **93**, 085422 (2016).