J. Indian Inst. Sci.
A Multidisciplinary Reviews Journal
ISSN: 0970-4140 Coden-JIISAD

CrossMark

REVIEW ARTICLE

Linear and Non-linear Analysis of Breakup of Liquid Sheets: a Review

Debayan Dasgupta[1], Sujit Nath[1] and Achintya Mukhopadhyay[2]*

Abstract | Atomization primarily involves interaction between liquid and surrounding gaseous medium resulting in the growth of unstable waves on a distorted liquid surface. There is a huge volume of work surrounding various models and theories involved in investigation of interface dynamics during sheet breakup. The present work summarizes the linear and nonlinear studies for different geometrical shapes of liquid sheet such as planar, annular, and swirling. The techniques involved in deriving nonlinear governing equations such as perturbation technique, multiple scales approach, and long wave approximation have been outlined. Effects of various factors on sheet instability such as viscosity, compressibility, surface temperature difference, unequal velocity, lateral waves and gas swirl have been listed. Research lacunae and scope of future work are presented in the summary.

1 Introduction

Atomization can be encountered in a variety of engineering and industrial processes spanning medical, automotive, power generation, agriculture, food processing, etc. Desired spray characteristics usually depend upon the area of application. For instance, food processing industry requires food sprays to be produced such that it would not alter the complex material structure and essential product characteristics such as taste, nutrition and shelf life[1]. Fuel droplets having different sizes and velocities are required at different times in liquid-fueled combustion engines to achieve cold start-up, flame stability, etc. Bronchial sprays require very fine micro-scale droplets that can easily pass through narrow respiratory tracts. Hence, an accurate and detailed measurement of droplet properties like droplet size distribution or mean size (Sauter mean diameter or mass mean diameter), velocity, mass flow rate (flux), final splat dimension, among others is very essential to completely understand spray processes and validate spray models[2]. However, no intrusive technique is desired during the measurement of droplet properties that can disturb the spray pattern. The measurement technique ideally should have the capability to measure a large range of spatial and temporal distributions.

Literature reveals the existence of several comprehensive deterministic tools that can substitute the use of experimental methods in determining droplet size and velocity distributions. Such comprehensive methods usually combine two submodels namely, sheet breakup model and droplet distribution model. Sheet breakup is a very complex mechanism and is yet to be fully understood. In addition, hydrodynamic approximation becomes invalid at the time of rupture of fluid into discrete droplets when molecular dynamics comes into picture. However, to predict atomization features within low cost, efforts have been made to develop semi-empirical criteria of breakup that forms a link between the liquid destabilization model and resulting discrete droplet distribution[3]. Primary atomization involves aerodynamic interaction of liquid sheet with ambient gas in presence of four primarily important forces namely, gravity force ($\rho_l L^3 g$), inertia ($\rho_l L^2 V^2$), surface tension (σL) and viscous force ($\mu_l LV$) where ρ_l denotes liquid density, L is the characteristic length, V is the characteristic velocity, σ denotes surface tension, μ_l denotes kinematic viscosity of liquid and g

[1] Department of Mechanical Engineering, National Institute of Technology Silchar, Silchar, India.
[2] Department of Mechanical Engineering, Jadavpur University, Kolkata, India.
*achintya.mukhopadhyay@jadavpuruniversity.in

Springer IISc Press

denotes acceleration due to gravity[4]. In addition, there exist gravity, viscous and inertia forces of the ambient gases. Sheet breakup usually occurs when combined effect of the external disruptive forces exceeds the consolidating effect of surface tension force. A complex interaction between these forces generates waves on the gas–liquid surface which grow in amplitude as the sheet moves downstream. As the disturbance amplitude reaches a critical value, the liquid sheet breaks down into film bands under the influence of the most unstable disturbance. The consolidating effect of surface tension force contracts the bands into cylindrical ligaments which further interact with the surrounding medium and finally disintegrate into discrete droplets.

The sheet breakup model can be subdivided into linear and non-linear theories which use different flow parameters as input to predict breakup length/time and ligament diameter as a result of primary and secondary sheet breakup. While linear theories can predict the onset of instabilities and initial temporal/spatial growth rates when the perturbations are infinitesimal, they are not suitable to predict sheet behavior when the disturbance amplitude becomes too large[5]. However, inclusion of second- or third-order terms in non-linear theories has proved to be useful in predicting interface dynamics with sufficient accuracy at later stages of sheet development when the wave amplitude and sheet thickness are of the same order[6, 7]. The droplet distribution model uses information from the nonlinear model like breakup length/time and mean drop diameter to finally predict droplet size and velocity distribution. The two models can be represented as shown in Fig. 1.

Except for few recent literatures, which developed a comprehensive model combining both the modules, such as Kim et al.[8], Movahednejad[9], Nath et al.[10] and Dasgupta et al.[11], most of the existing works have focused either on the breakup module or the droplet distribution module. Studies on droplet distribution module usually use analytical approaches based upon statistical description and probabilistic consideration such as maximum entropy principle (MEP) and discrete probability function (DPF) and are not included in the present paper in the interest of space limitations. The current study mainly tries to review existing works on modelling of sheet breakup based upon linear and non-linear instability analysis and numerical simulations. The objective is to focus on the recent developments which can be included in a comprehensive model to elaborate the physics of primary sheet breakup.

2 Instability Analysis of Liquid Sheet

Instability theories have been found to provide a sufficiently reasonable description of sheet breakup and a framework of organizing breakup regimes. Such theories usually consider unstable growth of waves on a distorting liquid surface. There are good reviews on instability theories and numerical simulation in Lasheras and Hopfinger[12], Ibrahim and Jog[13], Le Moyne[3], Sirignano and Mehring[4] and Tharakan et al.[14].

2.1 Linear Stability Theories

The linear theory of hydrodynamic stability assumes that perturbations are infinitesimal. Such theories consider an initially known small disturbance η_0 as perturbation parameter and express perturbation as $\eta = \eta_0 \exp(\omega t + ikx)$ where ω is a complex variable representing angular frequency, t is non-dimensional time and k represents wave number. Since time scale of atomization is often much smaller than heat transfer phenomenon, energy equations are rarely employed. The governing equations usually comprise of continuity equation and momentum equation subjected to two boundary conditions defined as kinematic and dynamic boundary conditions. The kinematic boundary condition considers the gas–liquid interface as a material surface and assumes that the normal velocity component is continuous along the interface, i.e., $v = D\eta/Dt$ where v is the y component of velocity and η is perturbation amplitude. The dynamic

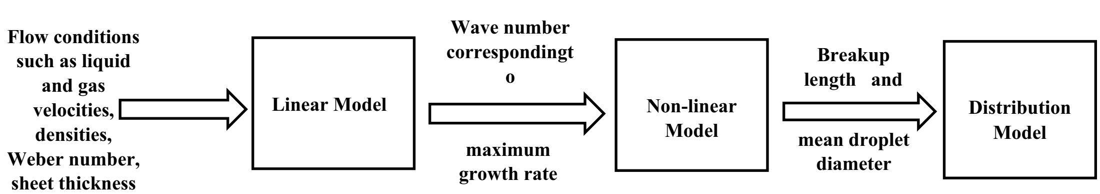

Figure 1: Model for sheet breakup and droplet distribution.

boundary condition says that the normal stress across the gas–liquid interface is continuous and is balanced by pressure induced by surface tension. There may be other boundary conditions such as no penetration condition or far boundary condition depending upon the presence or absence of confinement. Linear closed form solutions of velocity fields provide the dispersion equation which gives the relation between growth rate of disturbance and unstable wave number. Though linear theories are not suitable to accurately depict sheet behavior at later stages of sheet development, they have been extensively adopted to predict the most dominant wave number. The following section provides a review of literature on linear instability analysis of planar and annular liquid sheets.

2.1.1 Planar Liquid Sheet

Though the shape of liquid sheets emanating from nozzles is generally annular, the radius of curvature is often much greater than liquid sheet thickness at the breakup region. Hence, in an attempt to simplify the analysis, several literatures have made a planar or flat sheet assumption without losing main features of sheet breakup.

2.1.1.1 Inviscid Liquid Sheet Theoretical investigation of planar liquid sheet was pioneered by Squire[15], who proposed two modes of disturbances namely antisymmetric or sinuous mode and symmetric or varicose mode. The phase difference at gas liquid interface for sinuous mode was observed to be zero, while it was ϖ for varicose mode. This observation was further validated by Hagerty and Shea[16] and Taylor[17]. Hagerty and Shea[16] reported that the antisymmetric mode dominated the sheet breakup under typical conditions of atomization. Taylor[17] also observed that the symmetric mode was dispersive and propagated much slower than the antisymmetric mode. He further experimentally identified a point disturbance where both the waves were produced simultaneously. Fraser et al.[18] and Dombrowski and Hooper[19] performed analytical and experimental investigation of the effects of ambient density on planar spray sheets generated by single hole fan spray nozzles. While Fraser et al.[18] observed that at sub-atmospheric ambient pressure, the wavy sheet turned into a perforated sheet followed by a sudden increase in droplet size, Dombrowski and Hooper[19] identified a critical ambient density below and above which drop size decrease and increase, respectively, with increase in density. Luca and Costa[20] studied the asymptotic behavior of local perturbation in their linear stability analysis of inviscid liquid sheet under the effect of gravity in still gas. In case of sinuous disturbances, a critical value of local Weber number equal to unity was identified below and above which the sheet was absolutely and convectively unstable, respectively. Rao and Ramamurthi[21] reported linear stability analysis of an inviscid liquid sheet having continuous increase/decrease of velocities across the thickness. Even when aerodynamic shear was not present, such velocity profiles were found to be inherently unstable due to differential convective acceleration in gas–liquid interfaces. Convective acceleration from velocity gradient had stabilizing effect on longer wavelengths, while the shorter wave growth rate was dampened by overall effect of surface tension and reduced difference in velocities between liquid and gas phase at the interface. Matas et al.[22] performed spatial linear stability analysis of a slow moving liquid stream exposed to parallel gas stream moving at high velocity with a splitter plate at the gas–liquid interface. The analytic study was further compared with the experimental results. The presence of splitter plate at the interface caused velocity deficit which increased the frequency significantly. While the predicted frequency showed good agreement with experimental results, the experimental growth rate was much higher than the predicted growth rate. They concluded that inclusion of finite gas stream thickness, liquid viscosity and considering a cylindrical liquid shape in the analytical study might have a significant effect on the predicted results. In recent years, Yang et al.[23] performed a linear stability analysis of thin power law liquid sheet moving between two gas streams under symmetric disturbances. The study identified a critical value of consistency coefficient K above which an increase in the coefficient destabilized the sheet and below which the effects were opposite. Among all the parameters studied, the gas liquid relative velocity was identified as the key factor for sheet instability.

2.1.1.2 Viscous Liquid Sheet Dombrowski and John[24] extended the studies of Squire[15] and Hagerty and Shea[16] to examine a more realistic case of sheet breakup by considering the effects of diminishing sheet thickness and liquid viscosity. The study showed that unlike the inviscid sheet, the disturbance amplitude and wave number of a viscous-attenuating sheet depended upon sheet thickness. Both growth rate and dominant wave number increased as the liquid sheet attenuated. Lin et al.[25] also investigated linear stabilities in viscous liquid sheet exposed to ambient gas. At low values of liquid Weber number and zero gas–liquid density ratio, the sinuous mode was found to be stable and otherwise, convectively unstable. However, the varicose mode was unstable for all Weber numbers at non-zero gas–liquid density ratio. Li and Tankin[26] reported temporal instability analysis of a planar viscous liquid moving in inviscid ambient gas. While surface tension always dampened growth rate of disturbances, relative gas–liquid velocity enhanced the onset and growth of instabilities. In addition to aerodynamically driven instability, a new instability mode enhanced by viscosity was identified. Viscosity had a dual effect on instabilities depending upon the liquid Weber number. While it destabilized the flow at a low Weber number, it had a stabilizing effect at a larger Weber number. However, at an intermediate range of Weber number, two local maxima of growth rate curve were reported, one belonging to aerodynamic instability and the other corresponding to viscosity-enhanced instability. Li[27] studied spatial instabilities in a planar viscous liquid sheet exposed to inviscid stationary ambient gas. In case of sinuous mode, the study found that the disturbances were convective in nature at high liquid sheet velocity. However, when the liquid sheet velocity became smaller than characteristic velocity $(\mathrm{We})^{1/2}$, Green's function was present at all spatial locations at all times but was bounded in nature. Such type of instability was referred to as pseudo-absolute instability. Whereas, in case of varicose mode, instabilities were found to be convective at all flow conditions. Unless the liquid was highly viscous, the spatial growth rate of sinuous waves was observed to be less than that of varicose waves at Weber number slightly higher than unity. Viscosity played a dual role based on Weber number in case of sinuous mode, whereas it always had a stabilizing effect on the varicose mode, a phenomenon also observed by Ibrahim[28]. The study showed that surface tension dampened instabilities at all flow conditions. Antisymmetric mode influenced the instabilities at a higher Weber number, while symmetric mode dominated the flow at a low Weber number.

Cousin and Dumouchel[29] performed linear stability analysis of flat viscous incompressible liquid sheet to find out the conditions under which viscosity had no or a negligible effect on sheet breakup. The study showed that effect of viscosity on the growth of sinusoidal waves was minor and it did not influence cut-off wave number at all. Moreover, a dimensionless parameter defined as $\mathrm{We}_g^2 / \mathrm{Re}_l$ where We_g represents gas Weber number and Re_l represents liquid Reynolds number, was identified whose value separated the viscous and the inviscid regimes. Ibrahim and Akpan[30, 31] performed linear stability analysis of both two-dimensional and three-dimensional disturbances in a viscous and inviscid liquid sheet, respectively, issued in an inviscid gas medium. Both the studies showed that 2D disturbances dominated symmetric and asymmetric instabilities at low Weber number. With increase of Weber number, the 3D disturbances became more unstable for symmetric disturbances. However, in case of asymmetric mode, 2D disturbances always dominated the instabilities irrespective of Weber number or gas–liquid density ratios. Growth rate of both symmetric and asymmetric waves was observed to be equal for short waves. In case of inviscid liquid sheet emanating in inviscid gaseous surrounding, the study identified a critical wave number beyond which 3D disturbances became unstable.

Barreras et al.[32] studied linear instabilities in a viscous liquid sheet disintegrated by moving viscous gases, a situation commonly encountered in air blast atomizers. Results showed that keeping gas–liquid velocity ratio unchanged, an increase in Reynolds number caused an increase in growth rate of disturbances. A comparison with experimental investigation showed a good agreement of the qualitative behavior of liquid sheet. However, inclusion of viscosity in basic velocity profile and assumptions regarding velocity of wave propagation led to severe quantitative discrepancies between predicted and experimental results. Heislbetz et al.[33] conducted a theoretical stability analysis of three types of plane Newtonian liquid sheet namely, low viscous fluid, Eulerian fluid and highly viscous fluid flowing in a gaseous medium. Results predicted by the theoretical study was compared with experimental data obtained for high and low viscous fluids disintegrated by impinging jet injectors. Among the three types of fluids considered in the study, critical wave length and breakup length for highly viscous fluid agreed well with experimental results. Altimira et al.[34] performed spatial stability analysis of two-dimensional viscous liquid sheet surrounded by two similar viscous gases using spectral methods. Among the two velocity profiles studied, velocity profile based on quadratic functions gave a higher growth rate and larger critical wave number than the one based on error functions. Also, viscosity played a contrasting role for each velocity profile. While it destabilized the liquid sheet when quadratic functions were adopted, it dampened the growth rate for velocity profile with error functions. The spatial analysis results were compared with results obtained from temporal analysis using Gaster transformation and a good agreement was obtained. The study showed that consideration of gas boundary layer in the analysis was very important to accurately predict disturbance amplification. This was also validated by Matas[35] in his spatio-temporal linear instability analysis of viscous gas and liquid mixing layer. The study showed that the inclusion of finite gas and liquid thickness resulted in better agreement with experimental result as compared to previous studies with infinite gas and liquid thickness assumption.

2.1.1.3 Liquid Sheet in Confined Configuration In several practical applications of atomization such as gas turbine and liquid rocket jets, injectors are provided with confinement to have better mixing and improved spray characteristics.

Subramaniam and Parthasarathy[36] performed linear absolute instability analysis of an inviscid incompressible gas jet subjected to two stationary viscous liquid streams within a tube. The study showed the effect of tube radius on growth rate of varicose disturbances at different Weber numbers and liquid viscosities. At low liquid viscosity, confinement showed no significant effect on the highest growth rate and dominant unstable wave number. However, at higher values of liquid viscosity, an increase in tube radius significantly enhanced growth rates and increased the range of unstable wave numbers. The study suggested that the stabilizing influence of liquid viscosity on disturbance growth rate can be countered by the presence of confinement. Juniper[37] performed a linear spatio-temporal study to predict the onset of global modes for two-dimensional wakes confined between two flat plates in transverse direction for a range of density ratios. The study implied that the presence of confinement created regions of absolute instability within regions of local convective instability which could enhance mixing in industrial shear injectors. Maximum instability was recorded when the wave number of the fundamental mode in the wake and the surrounding fluids were equal. Rees and Juniper[38] extended the study of Juniper[37] to include the effects of viscosity and shear thickness on jets and wakes with continuous velocity profile. The spatio-temporal analysis showed that while viscosity had a stabilizing effect for $\mathrm{Re} < 1000$, confinement destabilized both viscous and inviscid flows. Shear layers with small finite thickness were found to be more unstable than thick or infinitely thin shear layers. Recently, Mohanta et al.[39] presented temporal linear stability analysis of two coaxial jets flowing in a circular tube with conductive heat transfer between the wall and the interface and convective heat transfer from the interface to the liquid. Results showed that while heat and mass transfer stabilized the flow, confinement enhanced initial disturbance amplitude. Also, for inner and outer fluid velocity ratio less than unity, breakup took place at higher wave number, whereas equal fluid velocities reduced the wave number at which break up occurred.

2.1.1.4 Liquid Sheet Subjected to Unequal Velocities Atomization at low injection pressure is often augmented by having forced aerodynamic instabilities using moving air or gas streams, a beneficial effect similar to presence of shroud air in fan spray and pressure swirl atomization. Such type of atomization aided by kinetic energy of co-flowing gases is commonly found in industrial gas turbines and different types of air craft engines[2]. In addition, certain applications like the preparation of fuel in jet engines use different inner and outer gas velocities to further enhance aerodynamic interaction. Linear stability analysis of viscous planar liquid sheets surrounded by gas streams with unequal velocities performed by Li[40] identified two independent modes of instabilities, namely, para sinuous and para varicose mode. While phase difference between gas and liquid interface for para sinuous mode was close to zero, it was close to ϖ for para varicose mode. The study suggested that at practical conditions encountered in liquid atomization, where Weber number is usually large, para sinuous mode always dominated the instabilities over the para varicose mode. Gas velocity asymmetry across the interfaces induced by Kelvin–Helmholtz instability, was identified as the main driving factor for instabilities. While viscosity could stabilize or destabilize the liquid sheet depending upon wavelength, surface tension always had a stabilizing effect on the liquid sheet. Tharakan and Ramamurthi[41] extended the study of Li[40] to include the effects of longitudinal and lateral waves. The study showed that lateral waves did not have any effect on growth rate at a Weber number higher than 10. However, the presence of lateral waves restricted the droplet size within smaller range of diameters as compared to wider distribution observed in the absence of lateral waves. Yang et al.[42] considered the case of a non-Newtonian liquid sheet to study the effects of non-zero unequal gas velocities on linear instabilities. The study identified that instabilities are primarily governed by the larger gas velocity for para sinuous mode and smaller gas velocity for para varicose mode. Though increase in gas velocity difference across the interfaces always destabilized the liquid flow, the extent of enhanced destabilizing effect depended upon other flow parameters as well. But it was generally suggested that a co-flowing air surrounding the liquid sheet could significantly improve atomization.

Nath et al.[43] presented temporal instability analysis of liquid sheet subjected to para sinuous and para varicose disturbances, surrounded by non-zero unequal gas velocities using a perturbation technique. Similar to[40], an increase in the gas velocity difference enhanced the breakup process. Droplet size distribution and velocity distribution were predicted using maximum entropy formulation. While an increase in gas velocity asymmetry significantly reduced the droplet size, it had a relatively slight effect on droplet velocity distribution. Dasgupta et al.[44] studied the combined simultaneous effect of confinement and unequal gas velocities on planar inviscid liquid sheet under the influence of both para sinuous and para varicose instabilities. Figure 2 shows the schematic of two planar sheets trapped between gas streams of unequal velocities bounded by solid walls, for (a) para sinuous mode (b) para varicose mode.

Results showed that para sinuous mode recorded a higher growth rate compared to para varicose mode at all flow conditions. While gas velocity asymmetry increased the growth rate of disturbance for both the modes, an increase in solid wall separation had a stabilizing effect on the liquid sheet instabilities.

Gas compressibility also plays an important role in case of effervescent and high-speed air-assisted atomizers where the gas velocities become close to velocity of sound. Ibrahim and Jackson[45] and Ibrahim[46] performed spatial instability analysis of a compressible liquid sheet exposed to compressible gas medium. Though a rise in gas Mach number increased highest growth rate, dominant wave number and unstable wave number range, liquid compressibility had a negligible effect on instabilities. Antisymmetric disturbances showed a higher growth rate and a lower dominant wave number than that of symmetric disturbances at a low Weber number. However, both the modes recorded almost similar highest growth rate and dominant wave number at a high Weber number. Cao and Li[47] extended Li's[40] study on effect of unequal gas velocities on instabilities in a planar liquid sheet by including gas compressibility and liquid viscosity. Similar to[40], para sinuous mode was predominant at a large Weber number, a practical condition found at atomizers. Inclusion of gas compressibility showed a destabilizing effect upon the liquid sheet and caused an increase in both growth rate and unstable wave number range. Tharakan and Ramamurthi[48] also extended their study on spatial instabilities in a liquid sheet in presence of longitudinal and lateral waves[41], by introducing liquid and gas compressibility. The study showed that inclusion of gas compressibility enhanced growth rate of para sinuous instabilities. As also observed by Ibrahim[46], liquid compressibility had a minor influence on growth of para varicose waves. Yang et al.[49] studied the influence of Mach number on interfacial instability in a liquid film surrounded by compressible gas, a situation encountered during supersonic combustion in hypersonic aircraft. The study showed that gas compressibility slightly dampened instabilities in case of asymmetric mode. Growth of distribution was found to be independent of Mach number for symmetric mode.

2.1.2 Annular Liquid Sheet

Annular jets are encountered in most practical atomizers. Study of annular jets provides an insight into the effect of lateral curvature on wave growth. Crapper and Dombrowski[50] were among the first few to perform first-order analysis of Kelvin–Helmholtz instabilities on cylindrical liquid sheet. The study showed that wave velocity increased with increase in radius of curvature and decreased sheet thickness for asymmetric mode. However, this effect became less significant at higher frequencies. Symmetric waves were invariant to changes in sheet curvature and thickness. This phenomenon could also be seen in the analytical instability investigation of Meyer and Weihs[51] for a viscous annular liquid jet exposed to an inviscid gas medium. The study identified a critical value of penetration thickness, above

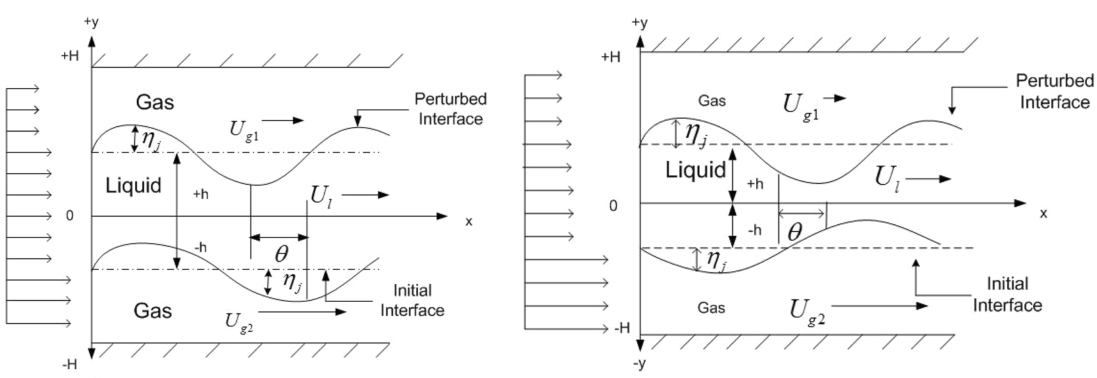

Figure 2: Schematic of a moving plane liquid sheet exposed to two gas streams of unequal velocity bounded by two solid walls for **a** para sinuous mode **b** para varicose mode.

which the annular jet behaved like that of a full liquid jet and the only observed unstable instabilities were axisymmetric and independent of sheet thickness. Below the critical thickness, the annular jet behaved like a 2D liquid sheet, where the antisymmetric perturbations were mostly unstable and their growth rate increased with increase in jet thickness. Biswas et al.[52] studied linear instabilities in a cylindrical liquid sheet with finite thickness exposed to quiescent gas medium. The study showed that axisymmetric disturbances become unstable when its wavelength became more than the outer circumference of the sheet. For a Weber number more than 2.5, the sheet was observed to be unstable at large wave lengths and the highest growth rate enhanced with rise in Weber number, keeping sheet thickness constant. Also, at constant Weber number, the wave length corresponding to highest growth rate showed a slow increase with increase in sheet thickness. Shen and Li[53, 54] considered a case of unequal inner and outer velocities of gas in their linear stability analysis of annular viscous liquid jet. Though both absolute velocities of gas and liquid and relative velocity between them had a destabilizing effect, the effect of relative velocity was much more predominant. Among the two gas streams, a high velocity of the inner gas enhanced the instabilities more than an equivalent velocity of the outer gas jet. Shijun et al.[55] performed linear spatial instability analysis to study the effect of three-dimensional disturbances on annular viscous liquid jet. A dimensionless parameter was identified above and below which axisymmetric and asymmetric mode of disturbance dominated the breakup process, respectively. While the axisymmetric instability was induced by surface tension for low-speed liquid jet, the asymmetric instabilities were induced by aerodynamic interaction of the liquid jet and surrounding gases for high-speed liquid. Yang and Fu[56] investigated temporal and spatio-temporal instabilities in recessed gas–liquid shear flow, a situation usually found in liquid propellant rocket engine. Temporal analysis showed that growth rate for a strong confinement was higher than that of a gas liquid shear flow with weak confinement. An increase in outer radius, liquid Weber number, velocity ratio and gas–liquid density ratio produced a stabilizing effect on liquid flow. Spatiotemporal analysis depicted that the flow was absolutely unstable in case of strong confinement at all flow conditions and convectively unstable in case of weak confinement.

2.1.3 Swirling Liquid Sheet

Plain orifice atomizers usually generate a narrow cone angle which is considered as a limitation. To have a wide cone angle, many practical applications such as chemical processing, and spray cooling use a conical swirl chamber having a small orifice at the vertex. The liquid jet is introduced into the swirl chamber via tangential ports and allowed to swirl. The swirling liquid then comes out of the outlet and spreads out of the orifice under the influence of both axial and radial forces. Moreover, in air blast atomizers, the swirling liquid flow is further exposed to swirling inner and outer air streams to further enhance the instabilities. Figure 3 shows the schematic diagram of an annular liquid sheet exposed to two swirling gas streams of unequal velocities.

There are several works in the literature that investigated the effect of swirl in atomization. Ponstein[57] was the pioneer in studying the instabilities in a rotating cylindrical jet under the influence of inertial effects of jet and surface tension. He considered three types of jets, namely, solid jet, hollow jet and infinitely thick jet. The inertial effects of ambient air were negligible when gas–liquid density ratio was low and velocity field was continuous at interface. In absence of liquid viscosity and inertial effects of ambient gas, the instabilities increased with the increase in dynamic surface tension ratio and liquid density. The study also showed that the non-axisymmetric and axisymmetric modes dominated instabilities in presence and absence of liquid swirl, respectively. Panchagnula et al.[58] employed a linear model for 3D disturbances in an annular, inviscid liquid sheet with swirl, in the presence of inner and outer gas streams moving at unequal velocity. They observed that swirl reduced the growth rate and most unstable wave number for axial disturbances. However, an increase in swirl Weber number above stabilizing region increased growth rate for axial mode and range of unstable wave number for both axial and circumferential mode. In absence of swirl, increasing the axial Weber number increased unstable axial disturbances range and the most unstable wave number. Liao et al.[59] developed a comprehensive model to predict the performance of a simplex atomizer using computational, experimental and theoretical methods. It was observed that liquid and gas phase relative velocity, density ratio and surface curvature had a destabilizing effect on the liquid sheet. Also, combined effect of axial and swirl velocity was more predominating in destabilization than only axial velocity.

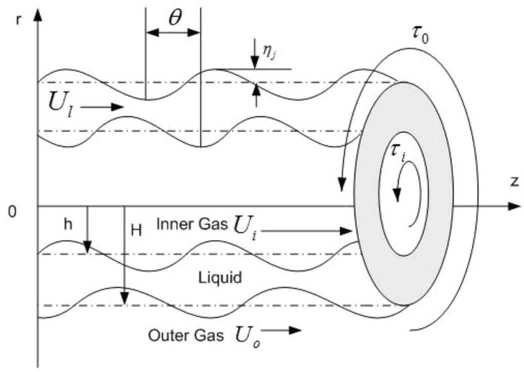

Figure 3: Schematic diagram of an annular liquid sheet subjected to two swirling gas streams of unequal velocities.

Mehring and Sirignano[60] employed reduced dimensional approach to study non-linear distortion caused by sinuous and dilational modulations in axisymmetric inviscid liquid jet with swirl within a void at no gravity. Non-linear effects generated fluid rings with thin connecting shells and sheet divergence increased breakup time/length for both the modes of instabilities. The study identified that presence of swirl velocity component enhanced instabilities and generated shorter breakup length and time. Liao et al.[61] developed a 3D model to predict instabilities in an annular liquid sheet exposed to two coaxial swirling air streams. The gas–liquid relative axial velocity increased growth rate and dominant unstable wave number. Combination of inner and outer air velocities was more destabilizing than only inner or outer airstream. A similar result could be observed in case of air swirl as well. Among the two, inner air stream was more effective than the outer one. Liao et al.[62] extended their previous analysis[61] on 3D instabilities of annular swirling liquid sheet by introducing the effect of liquid viscosity in their attempt to model air blast atomizers. The study showed that an increase in gas–liquid relative axial velocity had a significant effect in improving fuel atomization. Similar to their previous analysis[61], combined effect of inner and outer gas streams was found to be more destabilizing than only inner or outer gas stream. While air swirl made helical mode more dominant than axisymmetric mode, the axisymmetric mode was more dominant when the air swirl was absent. Viscosity caused a reduction in growth rate of unstable waves and shifted dispersion curve towards long waves. Ibrahim et al.[63] performed a temporal linear instability analysis to investigate the effects of liquid swirl velocity profile on annular liquid sheet with co-flowing inner and outer gas streams. Liquid swirl velocity was observed to stabilize and destabilize the flow at low and high liquid swirl Weber number, respectively. Liquid swirl also had a dual effect on the two modes of instabilities. While it increased growth rate for para sinuous instabilities, it had a damping effect on growth rate for para varicose instabilities. Ibrahim and Jog[13] extended their previous study[63] to include the effects of swirling inner and outer gas streams on linear temporal instabilities of a swirling annular liquid sheet. The study showed that in the absence of swirl component of liquid velocity, inner axial air stream was more predominant than outer air axial stream in destabilizing the liquid sheet. The influence of air stream orientation on sheet instability was studied by considering four different combinations of swirling air streams in reference to direction of liquid swirl. Results showed that while counter rotating inner and outer air streams produced highest unstable wave numbers at high liquid Weber number, coupled co-inner and counter-outer swirl resulted in highest unstable wave numbers at low liquid Weber number. Du and Li[64], and Du et al.[65] carried out linear temporal instability analysis of annular viscous liquid jet subjected to two gas jets swirling with unequal velocities. The study showed that while para sinuous mode was related to the inner gas liquid interface, para varicose mode was associated with the outer gas liquid interface. Since para sinuous mode was generally more dominant at practical liquid atomizing conditions, it was suggested that increasing inner gas stream rotation could significantly enhance the instabilities. Temporal linear instability analysis of annular viscous polymer sheet subjected to swirling air streams performed by Herrero et al.[66] showed that a decrease in polymer flow decreased breakup length and produced finer droplets due to formation of thinner films. However, in contrast, an increase in air flow rate enhanced aerodynamic interaction at the interfaces and the instabilities. Chatterjee et al.[67] carried out temporal linear instability analysis of swirling annular liquid jet subjected to co-flowing inner and outer gas flows within solid walls using a perturbation technique. The presence of confinement enhanced disturbance growth rate and corresponding dominant unstable wave number. Similar to the study conducted by Liao et al.[60], helical mode was observed to be predominated over axisymmetric mode when air swirl was present while reverse was true for no air swirl. Mahdavi et al.[68] considered four different orientations of swirling air in their linear stability analysis of inviscid annular liquid sheet subjected to two swirling air streams and non-swirl round liquid jet. While combined co-inner and counter-outer air stream led to shortest breakup length and highest wave number with maximum instability at low liquid swirl Weber number, combined inner and outer air stream rotating simultaneously with the liquid sheet provided the maximum growth rate. Yan et al.[69] studied linear spatial instabilities in an annular swirling viscous liquid sheet flowing in an inviscid gas medium under the influence of both para sinuous and para varicose instabilities. The study identified two separate regions—an initial region where an increase in liquid swirl strength decreased the highest growth rate and a region where increase in swirl strength monotonically increased the highest growth rate. At higher liquid swirl strength, antisymmetric mode was predominant, while axisymmetric mode became more dominant at low annular liquid curvature. Maximum growth rate and dominant wave number reduced with increase in viscosity. The effect of radius of curvature was limited to high swirl strength where it stabilized liquid sheet breakup. Surface tension had a complex dual (stabling and destabilizing) effect depending upon liquid swirl strength, gas–liquid density ratio and liquid Weber number. Vadivukkarasani and Panchagnula[70] studied temporal linear instabilities in an annular inviscid liquid sheet under the influence of combined Rayleigh–Taylor and Kelvin–Helmholtz mechanisms. The study identified four different modes of instabilities namely, Taylor, sinuous, flute and helical mode and inferred that the choice of instability depends upon Bond number and inner and outer Weber numbers. Rostami et al.[71] performed linear temporal stability analysis of swirling annular viscous liquid jet exposed to two gas streams with axial and swirl velocity components under the influence of axisymmetric disturbances. Solutions of the dispersion equation showed that presence of gas swirl enhanced instabilities and produced shorter breakup length and finer droplets.

2.1.4 Dual Mode Linear Analysis

Though most of the linear studies considered an individual case of antisymmetric and symmetric instability, in reality, both modes grow simultaneously. Mitra et al.[72] conducted linear stability analysis of planar viscous liquid sheet within two gas streams under the dual influence of sinuous and varicose disturbances by applying initial condition for both the modes simultaneously at the two gas–liquid interfaces. The study showed that while breakup occurred at full wave length for individual sinuous and varicose mode, it shifted between half and full wave lengths when both the initial conditions were applied simultaneously. An increase in proportion of varicose mode in the initial conditions also decreased the breakup time. Fu et al.[73] performed a linear stability analysis to study conical liquid sheet breakup exposed to dual effect of sinuous and varicose mode and validated the predicted results experimentally. Pressure drop had a destabilizing effect on the breakup of liquid sheet which was dominated by the sinuous mode. However, the conical sheet presented a varicose shape at the time of breakup. An increase in geometrical characteristics decreased breakup length and time, while the phase angle had minor effect on breakup.

2.2 Nonlinear Analysis

All the above literature focused on linear theories to predict disturbance growth rate and breakup of liquid sheet. However, Asare et al.[5] and Mitra et al.[72] showed significant variation between sheet behavior predicted by linear analysis and experimental results during sheet breakup. Also, the linear theory decouples antisymmetric waves from symmetric waves and as a result, sinuous mode cannot produce breakup due to constant distance between the two interfaces during the growth of unstable waves. Droplet distribution model also requires information about mean droplet size which can be obtained from volume of ligaments predicted by non-linear analysis at the breakup point.

2.2.1 Planar Liquid Sheet

Existing literature shows that there have been several works on investigation of non-linear instabilities in planar liquid sheets, which provided a good approximation of actual sheet breakup.

2.2.1.1 Inviscid Liquid Sheet Clarke and Dombrowski[6] pioneered non-linear instability analysis of inviscid liquid sheet under the influence of fundamental antisymmetric mode and its first harmonic. The long wave approximation showed that the first harmonic caused sheet thinning and subsequent breakup at 3/8 and 7/8 of the fundamental wave. Rangel and Sirignano[74] theoretically and computationally investigated linear and non-linear Kelvin–Helmholtz instabilities for a two-dimensional planar liquid sheet moving in ambient gaseous medium. A critical sheet thickness which was a function of density ratio was identified, below which all sinuous waves were observed to be stable. While sinuous waves resulted in formation of ligaments at half wave length, dilatational mode produced ligaments at full wave length. Jazayeri and Li[7] retained terms up to third order in their temporal instability study of a planar inviscid liquid sheet surrounded by stationary gas medium. The first harmonic of fundamental sinuous instabilities was observed to be varicose in nature, which caused sheet thinning and breakup at half wave length. An increase in initial amplitude, Weber number or gas density ratio enhanced instabilities and resulted in shorter breakup time. However, effect of Weber number and gas density became less significant at their high values. Breakup time was also found to be a weak function of wave number except when it approached cut-off wave number. Tharakan et al.[75] studied nonlinear growth of disturbances in a thin planar liquid sheet using control volume method. Liquid Weber number was found to have strong influence on the size and shape of ligaments formed after breakup. While antisymmetric waves produced thin ligaments at high Weber number, these disturbances transformed into asymmetric disturbances producing larger ligaments at low Weber number. Yoshinaga and Kan[76] derived unsteady non-linear equations for a falling inviscid liquid sheet in absence of surrounding fluid using membrane approximation and validated the results with numerical analysis. The study showed that only the antisymmetric mode was amplified as the wave propagated downstream, from which symmetric mode was induced subsequently. A comparison of weakly nonlinear studies with numerical results showed a good agreement for amplification of antisymmetric mode. However, magnitude of amplification of symmetric mode obtained from non-linear study was much less than that of symmetric mode. The previous non-linear studies[6, 7, 75] on breakup of liquid sheet considered gas medium to be quiescent. The influence of gas velocities (both higher and lower than liquid velocity) on disintegration of inviscid plane liquid sheet was studied by Nath et al.[10]. For gas–liquid velocity ratio higher than unity, antisymmetric waves showed a higher growth rate than symmetric waves. However, for gas–liquid velocity ratio less than unity, the study identified a critical Weber number at which the dominant mode changed from antisymmetric to symmetric mode.

2.2.1.2 Viscous Liquid Sheet Linear studies[26, 27, 29] have shown that viscosity may enhance or reduce disturbance growth depending upon the liquid Weber number. The effect of viscosity on non-linear instabilities was studied by Cheuch[77], who considered spatially traveling sinuous disturbances in a two-dimensional viscous liquid sheet. The study identified Weber number as the most dominating factor causing discrepancies between temporal and spatial instability results. An increase in Weber number or gas–liquid density ratio enhanced instabilities and shifted disturbance with highest growth rate towards shorter waves. It was suggested that higher flow rate and pressure would result in generation of finer droplets. An increase in liquid viscosity and surface tension stabilized the flow and produced larger droplets. The study also identified a critical Weber number and gas–liquid density ratio that separated the stable regime from the unstable regime. Cao et al. [78] studied sinuous and varicose instabilities for a planar viscous liquid film surrounded by gas medium and experimentally validated the model. The predicted wave shapes and breakup length produced by varicose disturbances showed good agreement with the experimental results. An increase in liquid velocity and sheet thickness had a stabilizing effect on the liquid sheet. As aspect ratio of nozzle exit was changed from larger to smaller value, a transition region from sinuous to varicose waves and decreasing to increasing breakup length was observed with increase in injected velocity. Yang et al.[79] performed temporal and spatial analysis of antisymmetric disturbances in a viscous planar liquid jet moving in a quiescent gas medium. Both temporal and spatial growth rates were observed to increase with rise in gas–liquid density ratio and Weber number, thereby resulting in more pronounced sheet distortion and shorter breakup time. The first harmonic of antisymmetric waves was observed to be dilational in shape and caused sheet breakup at half wavelength, a phenomenon also observed in some non-linear studies on inviscid liquid sheet[7, 75]. Viscosity stabilized first-order temporal and spatial growth rate. However, the effect of viscosity on second-order disturbance amplitude was minor and complicated. At practical conditions of atomization where Weber number is usually greater than 400 and gas–liquid density ratio is less than 0.001, viscosity had a weak stabilizing effect for Reynolds number less than 10 or higher than 150. However, at Reynolds number ranging between 10 and 150, viscosity weakly destabilized the flow. Wang et al.[80] studied non-linear instabilities produced by an initial sinuous wave in a two-dimensional viscoelastic liquid sheet surrounded by ambient gas medium. First harmonic of sinuous waves were observed to be dilational and subsequently caused sheet thinning and breakup. At dominant unstable wave number, second-order disturbance amplitude increased with increase in elasticity or decrease in deformation retardation time.

2.2.1.3 Liquid Sheet in Confined Configuration Kan and Yoshinaga[81] adopted the discrete vortex method to perform non-linear stability analysis of planar liquid sheet with surrounding fluids bounded by two solid walls. Linear study showed that a decrease in separation distance between solid walls resulted in higher growth rate and increase in unstable wave number range for both antisymmetric and dilational mode. This effect was more pronounced at low gas to liquid density ratio and liquid Weber number. Second-order results showed that the presence of confinement led to enhanced non-linearity, leading to more sheet deformation and distortion. However, non-linearity reduced linear growth rate except for long dilational wave. Monotonic or periodic energy exchange between the two modes made the dilational mode more strongly inducing than the antisymmetric mode. Nath et al.[82] developed a comprehensive model to investigate non-linear instabilities in an inviscid planar liquid sheet exposed to two moving gases bounded by two solid walls and analyzed the effects of confinement on droplet size and velocity distribution. Linear analysis showed that, for gas to liquid velocity ratio greater than unity, the growth rate for both sinuous and varicose mode increased with decrease in confinement height, a phenomenon also observed by Kan and Yoshinaga[81]. However, the effect of confinement was minor and became significant only at low Weber number and gas phase velocity. Non-linear analysis and droplet distribution model showed that, though presence of confinement resulted in relatively shorter breakup length, it increased the size of the droplets. The effect was more prominent for varicose mode with possible formation of satellite droplets. Yang et al.[83] included the effect of liquid viscosity in their spatial non-linear instability analysis of sinuous disturbances in a 2D planar liquid sheet exposed to co-flowing surrounded gases within two solid walls. However, the emphasis of the study was to investigate the effects of surrounding gas velocity rather than confinement. The first harmonic of sinuous mode was observed to be varicose in shape which caused sheet thinning and breakup. An enhanced growth rate with a higher second-order initial amplitude and a smaller breakup length was noticed when gas to liquid velocity ratio deviated from unity. However, increasing the gas and liquid velocity equally dampened growth rate, increased breakup length and had no significant effect on second-order initial amplitude and disturbance wave length. The growth rate predicted by non-linear analysis, based upon shortest breakup length, was shorter than the one predicted by linear analysis and agreed better with experimental results. Dasgupta et al.[11] extended the temporal instability analysis of Nath et al.[82] to include the effects of unequal gas velocity on breakup of liquid sheet flowing within two solid walls. Para sinuous mode dominated the disintegration process at all practical liquid atomization conditions at high liquid Weber number. Though presence of confinement enhanced instabilities, a further increase in separation distance between solid walls led to longer breakup length. Increase in gas velocity asymmetry across two interfaces produced higher aerodynamic interaction at the interfaces and resulted in finer droplet.

Long wave approximation as employed to investigate instabilities in falling liquid sheet[76] was also applied to study instabilities in free viscous liquid sheet triggered by difference in temperature or van der Waal's force. Yoshinaga and Uchiyama[84] and Yoshinaga and Yoshida[85] found that, for viscous liquid sheet with two different surface temperatures, there existed a critical temperature difference, beyond which the sheet became linearly unstable. At a large temperature difference, the varicose mode was induced upon the antisymmetric mode which subsequently caused sheet thinning and breakup. Viscosity was observed to have a duel effect such that it enhanced instabilities when sheet was unstable and stabilized the disturbances when sheet was stable. Prevost and Gallez[86], Erneux and Davis[87] and Sharma et al.[88] studied the effect of Van der Waal's force on rupture of free viscous liquid film. The studies showed that while Van der Wall's force enhanced instabilities and caused rupture of liquid sheet, non-linearity further accelerated the rupture process. Hwang et al.[89] and Lin et al.[90] performed nonlinear investigation of dynamic rupture of thin liquid sheet with insoluble and soluble surfactant, respectively. The studies showed that presence of surfactant dampened the instabilities and an increase in surfactant solubility enhanced the instabilities of the film system.

2.2.2 Dual Mode Non-linear Analysis

Wang et al.[91] extended linear stability analysis of Mitra to explore the effect of non-linearity on temporal instabilities of a planar inviscid liquid sheet in surrounding gas medium exposed to combined effect of sinuous and varicose instabilities. The results predicted by non-linear analysis were further validated with numerical analysis. The comparison showed good agreement in terms of sheet behavior and qualitatively explained instability mechanism at the time of sheet breakup. First harmonic of both antisymmetric and symmetric modes was symmetric in shape. However, first harmonic produced by coupling of antisymmetric and symmetric mode was antisymmetric. Linear analysis showed that except for initial varicose disturbance, sinuous mode dominated sheet shape at large disturbance amplitudes. At regions of relatively large wavenumber, a further increase in wave number reduced the wave amplitude. A decrease in initial disturbance amplitude caused the first harmonic of initial sinuous mode to be predominant. However, except for the initial sinuous mode, all other modes namely, initial varicose, first harmonic varicose and sinuous mode were able to cause sheet breakup.

2.2.3 Annular Sheet

Non-linear stability analysis of annular sheet, which is the practical shape of sheet in most common atomizers, provides a relatively realistic approach to predict actual breakup. Mehring and Sirignano[92] performed a non-linear temporal instability analysis of a thin inviscid annular liquid sheet inside a surrounding void with non-zero pressure gas core in the absence of gravity. While only dilatational mode was dispersive in case of planar sheet, both sinuous and dilational modes were found to be dispersive for the case of annular liquid sheet. Dilatational oscillation was noticed at large annular radius while otherwise, coupled sinuous and dilatational modulations was found even in linear representation. Non-linear effects caused an increase in breakup time for dilatational mode, while it produced shorter breakup time for sinuous mode. Yoshinaga[93] and Yoshinaga and Yamamoto[94] performed analytical investigation of non-linear instabilities in a viscous compound liquid jet consisting of a core and a surrounding annular phase. They adopted a long wave approximation and derived set of non-linear equations for large deformations. Breakup phenomenon for the semi-infinite jet was numerically investigated in the presence of antisymmetric disturbances. At low Reynolds number, choking of core phase at bottleneck could be observed in the presence of jet pinching, followed by ballooning of annular phase along the upstream direction. The study identified a critical wave number below and above which liquid jet transitioned from being convectively unstable to being absolutely unstable. Yang et al.[95] extended their linear study of power law planar liquid[79] to include the effects of non-linearity in an annular power law liquid subjected to inner and outer gas streams. The temporal analysis showed that liquid instability could be increased with increase of either inner or outer gas stream but instabilities were more enhanced in the presence of both gas streams. However, outer interface gas stream was more influencing in enhancing instabilities than inner gas stream. Surface tension and thickness of liquid sheet stabilized liquid sheet and dampened the highest growth rate, most unstable wave number and range of unstable wave numbers. Yan et al.[96] performed non-linear spatial instability analysis of an annular viscous liquid sheet surrounded by inner and outer gas streams with motion in axial direction using a perturbation technique and validated the model with available experimental data. An increase in curvature of annular sheet or difference in velocity of gas and liquid flow increased the highest growth rates of both first and second order. Increasing relative velocity between gas and liquid streams also caused large deformation in the sheet resulting shorter breakup length. The study identified the shear effect generated due to velocity difference in outer and inner gas streams as the main source of instability.

2.2.4 Swirling Liquid Sheet

Ibrahim and Jog[97] performed non-linear temporal analysis of annular liquid sheet with co-flowing inner and outer gas streams using a perturbation expansion method. The study showed that while linear theory could only predict sheet behavior at onset of instabilities, a non-linear study was required to accurately predict wave propagation during sheet breakup. The presence of gas swirl enhanced instabilities in the liquid jet and made the helical mode more dominant as compared to axisymmetric mode. Effect of axially moving inner gas stream was more than that of outer gas stream in disintegrating the liquid sheet. Combined destabilizing effect of axially moving outer and inner gas streams was more than that of any one gas stream. An increase in swirl strength of outer gas increased highest growth rate and corresponding unstable circumferential wave number leading to shorter breakup length and thinner ligaments. Ibrahim and Jog[98] performed both computational analysis using volume of fluid method for modeling the hydrodynamics inside the atomizer and non-linear stability analysis to study instabilities in a liquid sheet coming out of a pressure swirl atomizer. An increase in ambient pressure led to reduced air core diameter due to higher air liquid momentum transport and subsequently resulted in shorter breakup length. The coupled computational and sheet instability analysis showed good agreement with experimental results and provided a comprehensive way to model atomization in pressure swirl atomizers. Yan et al.[99] employed the perturbation technique to perform non-linear analysis of instabilities in an annular viscous liquid sheet with co-flowing inner and outer gas jets. The study showed that liquid swirl had a stabilizing effect on liquid sheet at low values of swirl strength as it enhanced instabilities at its high values. Axisymmetric mode dominated the instabilities in absence of liquid swirl whereas the helical mode became more dominant with increase in swirl strength, a phenomenon also observed in[62]. In the presence of purely axial motion of liquid sheet, inner gas stream was more destabilizing than the outer gas streams, whereas the outer gas stream was predominantly responsible for causing disruption when liquid swirl was present. An increase in gas–liquid density ratio or radius of curvature produced shorter breakup length, while increase in liquid viscosity dampened disturbance growth and led to longer breakup length. Zakaria et al.[100] investigated non-linear temporal instabilities in a liquid jet flowing with alternating streamwise circulation in stationary gaseous medium using perturbation technique and multiple scales method. They derived non-secular conditions corresponding to second and third order and obtained non-linear evolution equations at different cases of resonance. The study showed that Weber number and uniform streamwise circulation stabilized the liquid jet. In different resonance cases, amplitude of periodic part of streamwise circulation destabilized the flow and led to sheet breakup. In case of non-resonance, the non-trivial solutions were damped with an increase in phenomenological number and vibrated as periodic amplitude number of streamwise circulation increased.

3 Summary

There is plethora of studies that has employed linear stability analyses to investigate dynamics of liquid sheet development, largely due to the simplicity of the linear model. Majority of the studies have considered planar or annular configuration in the presence and absence of gas swirl, while the actual conical shape of liquid sheet in injectors have mostly been ignored with one exception[73]. These studies have primarily focused on the onset of sheet instability and the conditions under which the sheet is convectively or absolutely unstable[20, 21, 25, 27]. However, the actual breakup process is highly non-linear and several studies have shown significant deviation in surface deformation predicted by linear studies and experimental investigation[5, 72].

While perturbation technique remains one of the most popular methods for studying non-linear instabilities in planar and annular sheet, other techniques such as multiple scale expansion method[100] and long wave asymptotic assumption have also been employed[76, 84, 85].

The amplitude of oscillations developed on the liquid surface usually grows spatially and remain invariant with time. But, majority of the literature considered time-dependent instabilities and employed Gaster transformation to convert breakup time into breakup length. Non-linear analysis by Yang et al.[79] have shown that, as both temporal and spatial growth rate is relatively smaller than oscillation frequency, the interface deformation for both the instabilities are observed to be similar. However, some discrepancies in disturbance growth rate for low Weber number have been reported by linear investigation of Altimira et al.[34]. Similar non-linear studies presenting comparison between spatial and temporal instabilities for slow moving flows has not been reported yet.

Recent developments in non-linear studies have investigated complex flow characteristics such as viscosity, presence of lateral waves, progressive change in velocity across thickness, liquid and gas swirl. However, investigation of boundary layer formation, turbulence, eddy interaction and cavitation need more attention. Though effect of pressure on gas liquid density ratios has been studied, cases of severe decompression caused by very high pressure have been ignored. Instabilities induced by temperature difference with emphasis on variation of surface tension gradient with temperature have been investigated[84, 85]. But temperature gradient large enough to cause a phase change as experienced in combustion chamber would need a revision of the definition of surface tension. Finally, inclusion of energy conservation equations which are mostly excluded on the basis of isothermal assumption may throw some light on many unknown facts of atomization process.

Received: 17 August 2018 Accepted: 13 September 2018
Published online: 1 October 2018

References

1. Tanner FX, Feigl K, Kaario O, Windhab EJ (2016) Modeling and simulation of air-assist atomizers with applications to food sprays. Appl Math Model 40:6121–6133
2. Liu H (1999) Science and engineering of droplets. William Andrew, New York
3. Le Moyne L (2010) Trends in atomization theory. Int J Spray Combust Dyn 2:49–84

4. Sirignano W, Mehring C (2000) Review of theory of distortion and disintegration of liquid streams. Prog Energy Combust Sci 26:609–655
5. Asare HR, Takahashi RK, Hoffman MA (1981) Liquid sheet jet experiments: comparison with linear theory. J Fluids Eng 103:595
6. Clark CJ, Dombrowski N (1972) Aerodynamic instability and disintegration of inviscid liquid sheets. Proc R Soc 329:467–478
7. Jazayeri SA, Li X (2000) Nonlinear instability of plane liquid sheets. J Fluid Mech 406:281–308
8. Kim WT, Mitra SK, Li X, Prociw LA, Hu TCJ (2003) A predictive model for the initial droplet size and velocity distributions in sprays and comparison with experiments. Part Part Syst Charact 20:135–149
9. Movahednejad E, Ommi F, Hosseinalipur S, Chen C, Mahdavi S (2001) Application of maximum entropy method for droplet size distribution prediction using instability analysis of liquid sheet. Heat Mass Transf 47:1591–1600
10. Nath S, Mukhopadhyay A, Sen S, Tharakan TJ (2010) Influence of gas velocity on break up of planar liquid sheets sandwiched between two gas streams. At Sprays 20:983–1003
11. Dasgupta D, Nath S, Bhanja D (2018) Dual-mode nonlinear instability analysis of a confined planar liquid sheet sandwiched between two gas streams of unequal velocities and prediction of droplet size and velocity distribution using maximum entropy formulation. Phys Fluid 30:044104
12. Lasheras JC, Hopfinger EJ (2000) Liquid jet instability and atomization in a coaxial gas stream. Annu Rev Fluid Mech 32:275–308
13. Ibrahim AA, Jog MA (2006) Effect of liquid and air swirl strength and relative rotational direction on the instability of an annular liquid sheet. Acta Mech 133:113–133
14. Tharakan TJ, Mukhopadhyay A, Datta A, Jog MA (2013) Trends in comprehensive modeling of spray formation. Int J Spray Combust Dyn 5:123–180
15. Squire HB (1953) Investigation of the instability of a moving liquid film. Br J Appl Phys 4:167–169
16. Hagerty WW, Shea JF (1955) A study of the stability of plane fluid sheets. J Appl Mech 22:509–514
17. Taylor G (1959) The dynamics of thin sheets of fluid. II. Waves on fluid sheets. Proc R Soc 253:296–312
18. Fraser RP, Eisenklam P, Dombrowski N, Hasson D (1962) Drop formation from rapidly moving liquid sheet. AIChE 8:672–680
19. Dombrowski N, Hooper PC (1962) The effect of ambient density on drop formation in sprays. Chem Eng Sci 17:291–305
20. Luca BLDE, Costa M (1997) Instability of a spatially developing liquid sheet. J Fluid Mech 331:127–144
21. Rao VN, Ramamurthi K (2009) Internal instability of thin liquid sheets. Phys Fluids 21:092106
22. Matas JP, Marty S, Cartellier A (2011) Experimental and analytical study of the shear instability of a gas-liquid mixing layer. Phys Fluids 23:094112
23. Yang L, Du M, Fu Q, Tong M, Wang C (2015) Temporal instability of a power-law planar liquid sheet. J Propuls Power 31:286–293
24. Dombrowski N, Johns WR (1963) The aerodynamic instability and disintegration of viscous liquid sheets. Chem Eng Sci 18:203–214
25. Lin SP, Lian ZW, Creighton BJ (1990) Absolute and convective instability of a viscous liquid sheet. J Fluid Mech 220:673–689
26. Li X, Tankin RS (1991) On the temporal instability of a two-dimensional viscous liquid sheet. J Fluid Mech 226:425–443
27. Li X (1993) Spatial instability of plane liquid sheet. Chem Eng Sci 48:2973–2981
28. Ibrahim EA (1995) Spatial instability of a viscous liquid sheet. J Propuls Power 11:146
29. Cousin J, Dumouchel C (1996) Effect of viscosity on the linear instability of a flat liquid sheet. At Sprays 6:563–576
30. Ibrahim EA, Akpan ET (1996) Three-dimensional instability of viscous liquid sheet. At Sprays 6:649–665
31. Ibrahim EA, Akpan E (1998) Liquid sheet instability. Acta Mech 167:153–167
32. Barreras F, Lozano A, Dopazo C (2001) Viscous linear instability analysis of the viscous longitudinal perturbation on an air-blasted liquid sheet. At Sprays 11:139–154
33. Heislbetz B, Madlener K, Ciezki HK (2007) Breakup characteristics of a Newtonian liquid sheet formed by a doublet impinging jet injector. In: 43rd AIAA Joint Propulsion Conference
34. Altimira M, Rivas A, Ramos JC, Anton R (2010) Linear spatial instability of viscous flow of a liquid sheet through gas. Phys Fluids 22:74103
35. Matas J (2015) Inviscid versus viscous instability mechanism of an air—water mixing layer. J Fluid Mech 768:375–387
36. Subramaniam K, Parthasarathy RN (1999) Effects of confinement on the temporal instability of gas jets injected in viscous liquids. Phys Fluids 89:1998–2001
37. Juniper MM (2006) The effect of confinement on the stability of two-dimensional shear flows. J Fluid Mech 565:171–195
38. Rees SJ, Juniper MM (2010) The effect of confinement on the stability of viscous planar jets and wakes. J Fluid Mech 656:309–336
39. Mohanta L, Cheung F, Bajorek SM (2016) Stability of coaxial jets confined in a tube with heat and mass transfer. Physica A 443:333–346
40. Li X (1994) On the instability of plane liquid sheets in two gas streams of unequal velocities. Acta Mech 106:137–156

41. Tharakan TJ, Ramamurthi K (2005) Growth of longitudinal waves in plane liquid sheets having lateral wave modes when exposed to two gas streams of unequal velocities. At Sprays 15:181–200
42. Yang L, Xu B, Fu Q (2012) Linear instability analysis of planar non-newtonian liquid sheets in two gas streams of unequal velocities. J Non Newton Fluid Mech 167:50–58
43. Nath S, Mukhopadhyay A, Datta A, Sen S (2012) Analysis of disintegration of planar liquid sheet sandwiched between gas streams with unequal velocities and resulting spray formation. Triennial international conference on liquid atomization and spray systems vol 12 pp 50–8
44. Dasgupta D, Nath S, Bhanja D (2017) Linear stability analysis of planar liquid sheet with unequal gas velocities confined between two solid walls. International conference on functional materials, characterization, solid state physics, power, thermal and combustion energy vol 20113
45. Ibrahim EA, Jackson SL (1996) Spatial instability of a liquid sheet in a compressible gas. J Colloid Interface Sci 180:629–631
46. Ibrahim EA (2010) Effects of compressibility on the instability of liquid sheets. Chem Eng Commun 161:37–41
47. Cao J, Li X (2000) Stability of plane liquid sheets in compressible gas streams. J Propuls Power 16:623–627
48. Tharakan TJ, Ramamurthi K (2010) Influence of liquid and gas compressibility on growth of thin liquid sheets. Fluid Dyn Res 42:035508
49. Yang HQ (1992) Interfacial instability between a liquid film and the surrounding compressible gas. 30th aerospace sciences meeting and exhibit
50. Crapper GD, Dombrowski N (1975) Kelvin–Helmholtz wave growth on cylindrical sheets. J Fluid Mech 68:497–502
51. Meyer J, Weihs D (1987) Capillary instability of an annular liquid jet. J Fluid Mech 179:531
52. Biswas G, Som SK, Gupta AS (1985) Instability of a moving cylindrical liquid sheet. J Fluid Eng 107:451–454
53. Shen J, Li X (1996) Instability of an annular viscous liquid jet. Acta Mech 114:167–183
54. Shen J, Li X (1996) Breakup of annular viscous liquid jets in two gas streams. J Propuls Power 12:752
55. Shijun Y, Maozhao X, Baixin C (1996) The breakup and atomization of a viscous liquid jet. Acta Mech Sin 12:124
56. Yang L, Fu Q (2012) Stability of confined gas—liquid shear flows in recessed shear coaxial injectors. J Propuls Power 28:1413
57. Ponstein J (1959) Instability of rotating cylindrical jets. Appl Sci Res A 8:425
58. Panchagnula MV, Sojka PE, Santangelo PJ (1996) On the three dimensional instability of a swirling, annular, inviscid liquid sheet subject to unequal gas velocities. Phys Fluids 8:3300
59. Liao Y, Sakman AT, Jeng SM, Jog MA, Benjamin MA (1999) A comprehensive model to predict simplex atomizer performance. J Eng Gas Turbine Power 121:285
60. Mehring C, Sirignano WA (2001) Nonlinear capillary waves on swirling, axisymmetric free liquid. Int J Multiph Flow 27:1707–1734
61. Liao Y, Jeng SM, Jog MA (2000) Instability of an annular liquid sheet surrounded by swirling airstreams. AIAA J 38:453
62. Liao Y, Jeng SM, Jog MA (2001) Advanced sub-model for airblast atomizers. J Prop 17:411
63. Ibrahim AA, Jog MA, Jeng SM (2006) Effect of liquid swirl velocity profile on the instability of a swirling annular liquid sheet. At Sprays 16:237
64. Du Q, Li X (2005) Effect of gas stream swirls on the instability of viscous annular liquid jets. Acta Mech 176:61–81
65. Du Q, Ning LIU, Jun YIN (2008) The study on the relationship between breakup modes and gas-liquid interfaces. Chin Sci Bull 53:2898–2906
66. Herrero EP, Martín EM, Valle D, Galán MA (2007) Instability study of a swirling annular liquid sheet of polymer produced by air-blast atomization. Chem Eng J 133:69–77
67. Chatterjee S, Samanta S, Mukhopadhyay A, Ghosh K, Sen S (2012) Effect of a confined outer air stream on instability of an annular liquid sheet exposed to gas flow. Proceedings of ASME 2012 gas turbine India conference pp 1–10
68. Mahdavi S, Movahednejad E, Ommi F, Hosseinalipur S (2013) Estimation of the breakup length for the annular and the round liquid jet using linear stability analysis. IJST Trans Mech Eng 37:217–232
69. Yan K, Ning Z, Lü M, Sun C, Fu J, Li Y, Yan K, Ning Z, Lü M, Sun C, Fu J, Li Y (2016) Spatial instability in annular swirling viscous liquid sheet. Phys Fluids 27:24101
70. Vadivukkarasan M, Panchagnula MV (2017) Combined Rayleigh–Taylor and Kelvin–Helmholtz instabilities on an annular liquid sheet. J Fluid Mech 812:152–177
71. Rostami E, Ommi F, Mirmohammadi A, Khodayari H (2016) Effect of gas-to-liquid density ratios at different liquid viscosity on the atomization of the air-blast atomizer. Aust J Mech Eng 15:1–12
72. Mitra SK, Li X, Renksizbulut M (2001) On the breakup of viscous liquid sheets by dual-mode linear analysis. J Propuls Power 17:728–735
73. Fu Q, Yang L, Qu Y, Gu B (2010) Linear stability analysis of a conical liquid sheet. J Propuls Power 26:955
74. Rangel RH, Sirignano WA (1991) The linear and nonlinear shear instability of a fluid sheet. Phys Fluids 10:2392–2400

75. Tharakan TJ, Ramamurthi K, Balakrishnan M (2002) Nonlinear breakup of thin liquid sheets. Acta Mech 156:29–46
76. Yoshinaga T, Kan K (2006) Breakup of a falling liquid sheet caused by amplified modulational waves. Fluid Dyn Res 38:313–338
77. Chuech SG (2006) spatial instability of a viscous liquid sheet. Int J Numer Methods Fluids 50:1461–1474
78. Cao JM, Feng ZY, Jian XP, Li GB, Chen ZW, Zhang QX, Wang L (2010) Nonlinear stability analysis of a liquid sheet injected from a plane slot nozzle, The 6th international symposium on multiphase flow, heat mass transfer and energy conversion
79. Yang L, Wang C, Fu Q, Du M, Tong M (2013) Weakly nonlinear instability of planar viscous sheets. J Fluid Mech 735:249–287
80. Wang C, Yang L, Xie L, Chen P (2015) Weakly nonlinear instability of planar viscoelastic sheets. Phys Fluids 27:13103
81. Kan K, Yoshinaga T (2007) Instability of a planar liquid sheet with surrounding fluids between two parallel walls. Fluid Dyn Res 39:389–412
82. Nath S, Mukhopadhyay A, Datta A (1873) Effect of confinement on breakup of planar liquid sheets sandwiched between two gas streams and resulting spray characteristics. Fluid Dyn Res 46:15511
83. Yang L, Chen P, Wang C (2014) Effect of gas velocity on the weakly nonlinear instability of a planar viscous sheet. Phys Fluids 26:74106
84. Yoshinaga T, Uchiyama T (2001) Behavior of a free liquid sheet subject to a temperature difference between both surfaces. J Phys Soc Jpn 70:103–110
85. Yoshinaga T, Yoshida T (2003) Nonlinear wave behavior of the sinuous mode on a free liquid sheet with variable surface tension and viscosity. J Phys Soc Jpn 72:3113–3122
86. Prévost M, Gallez D (1986) Nonlinear rupture of thin free liquid films. J Chem Phys 84:4043
87. Erneux T, Davis SH (1993) Nonlinear rupture of free films. Phys Fluids A Fluid Dyn 5:1117
88. Sharma A, Kishore CS, Salaniwal S, Ruckenstein E (1995) Nonlinear stability and rupture of ultrathin free films. Phys Fluids 7:1832–1840
89. Hwang CC, Lin C, Hou DC, Uen WY, Lin J (1998) Nonlinear rupture theory of a thin liquid film with insoluble surfactant. Trans ASME 120:598–604
90. Lin C, Hwang C, Uen W (2000) A nonlinear rupture theory of thin liquid films with soluble surfactant. J Colloid Interface Sci 231:379–393
91. Wang C, Yang L, Ye H (2015) Nonlinear dual-mode instability of planar liquid sheets. J Fluid Mech 778:621–652
92. Mehring C, Sirignano WA (2000) Axisymmetric capillary waves on thin annular liquid sheets. I. Temporal stability. Phys Fluid 12:1417
93. Yoshinaga T (2012) Instability and breakup of a gas-cored viscous annular jet. Nonlinear Accoustics 122:119–122
94. Yoshinaga T, Yamamoto K (2011) Nonlinear instability and breakup of a viscous compound liquid jet. J Fluid Sci Technol 6:477–486
95. Yang L, Du M, Fu Q (2014) Stability of an annular power-law liquid sheet. J Mech Eng Sci 229:2750–2759
96. Yan K, Ning Z, Lü M, Sun C, Fu J, Li Y (2015) Interface instability mechanism of an annular viscous liquid sheet exposed to axially moving inner and outer gas. Eur J Mech B Fluids 52:185–190
97. Ibrahim AA, Jog MA (2008) Nonlinear instability of an annular liquid sheet exposed to gas flow. Int J Multiph Flow 34:647–664
98. Ibrahim AA, Jog MA (2013) Nonlinear breakup model for a liquid sheet emanating from a pressure-swirl atomizer. J Eng Gas Turbine Power 129:945–953
99. Yan K, Jog MA, Ning Z (2013) Nonlinear spatial instability of an annular swirling viscous liquid sheet. Acta Mech 224:3071
100. Zakaria K, Sirwah MA, Adham M (2013) Nonlinear evolution of a column swirling jet. Zeitschrift für Angew Math und Phys 64:811–830

Mr. Debayan Dasgupta is pursuing his PhD in Mechanical Engineering Dept. with specialization in thermal engineering at NIT Silchar, completed his M. Tech from NERIST, Arunachal Pradesh with specialization in thermal engineering and B.E in Mechanical Engineering from Hindustan College of Engineering, Chennai, Tamil Nadu (affiliated to Anna University). His area of interest includes computational fluid dynamics, thermodynamics and heat transfer and he has also published many technical papers in referred International journals.

Dr. Sujit Nath received his Ph.D. Degree from Jadavpur University, Kolkata, India. Currently, he is an Assistant Professor in Department of Mechanical Engineering, National Institute of Technology Silchar, India. His research areas are CFD, heat transfer, atomization, solar energy, optimization, etc. He is a reviewer in many International journals of repute and also published many technical papers in referred International journals.

Dr. Achintya Mukhopadhyay is a Professor of Mechanical Engineering at Jadavpur University, Kolkata (Calcutta), India. He also served as Professor of Mechanical Engineering at Indian Institute of Technology Madras. Dr. Mukhopadhyay also held visiting positions at Technical University of Munich where he was an Alexander von Humboldt Fellow and University of Illinois at Chicago. Dr. Mukhopadhyay's teaching and research interests include thermodynamics, heat transfer, combustion, multiphase flows and design and analysis of thermal systems. Dr. Mukhopadhyay has over 270 research publications including over 100 international journal publica. Mukhopadhyay is a Fellow of the West Bengal Academy of Science and Technology and life member of Indian Society of Heat and Mass Transfer and Indian section of the Combustion Institute.