

## Error rate performance of NEC receivers of Duoquaternary FSK

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### Abstract

Wireless communications have become indispensable and spectrum resources have already reached their limits. Special care needs to be taken for constant envelope modulations that can achieve higher bandwidth efficiency while keeping circuitry at simple levels. Partial response signalling and correlative encoding are techniques that maintain immunity to nonlinearities due to constant envelope property as well as high bandwidth efficiency. Duoquaternary frequency shift keying (FSK) makes use of correlative encoding and premodulation filtering to increase spectrum efficiency in addition to having better power efficiency than MSK. Therefore, it should be considered for applications in mobile, portable and/or fixed digital data transmission systems. This type of signaling, apart from reducing the out-of-band power levels, helps increase the symbol error rate. Nonredundant error correction (NEC) is utilised to improve the system performance under white gaussian noise conditions. The receiver consists of a band of differential detectors. It utilises their outputs to correct data patterns containing random errors in accordance with the concept of a convolutional error-correcting code without sacrificing any more bandwidth or power.

In this paper, a single- and a double-error-correcting receiver of duoquaternary FSK are proposed. For the first time correlative multilevel modulations of higher efficiency are theoretically evaluated with NEC in additive white gaussian noise environments. Computer-simulated results of SER performance are also given, indicating significant performance improvements over conventional differential detection.

### 1. Introduction

Wireless communications have evolved through the years to support satellite communications, fixed point-to-point microwave links and, during the last quarter of our century, mobile communications. However, the big challenge of cellular systems is that now frequencies allocated for the service are shared among the subscribers in a direct way rather than providing the means for the trunk networks of the carrier/service providers. The frequency spectrum allocated to mobile communications operators is limited, but the success of new services is dependent on the flexibility of the transmitted signalling format. The scientific community has therefore been working towards increasing the efficiency of the transmitted signals.

Efficiency in communications is measured in many dimensions such as frequency, power and space. In general, frequency efficiency changes in reverse to power efficiency, and thus, a trade-off should be considered when trying to improve the performance of a communications system. For example, multilevel modulations have better frequency efficiency (number of bits per second per Hertz) but worse power efficiency (bit error rate at some fixed signal-to-noise power ratio) as the signal set increases and subsequently the minimum distance in the signal

space constellation diagram decreases. Another advantage of multilevel signalling when deployed in mobile channels is that it exhibits flat fading rather than frequency selective fading, thus avoiding the employment of equalisers to remove intersymbol interference (ISI).<sup>1</sup> Apart from decreasing the number of transmitted symbols within a signalling period, it is also the compactness of the signal spectrum (frequency bandwidth that contains 99% of the power for a given symbol rate or pulse duration) that plays an important role in communication systems design. In frequency division multiplexing systems, this characteristic would be very desirable as an adjacent channel interference reduction mechanism.

Many techniques have been developed in an attempt to further limit the signal power spreading out of the main transmission frequency band. Premodulation filtering is a technique of this kind which, by pulse shaping, aims at reducing the spectral sidelobes of the modulation and, hence, increasing the percentage of power which is contained within a frequency interval equal to the symbol rate. In another technique, proper phase shifts are inserted between the I and Q components of quadrature modulations in order to decrease the maximum phase shift applied between successive symbol periods.<sup>2</sup>

Correlative encoding is another technique, which instead of using pulse shapes extending over several symbol periods correlates the data just at the output of the source before any filtering stage.<sup>3</sup> The application of correlative encoding to data transmission by FM was first introduced by Lender<sup>4</sup> and was systematically examined by D. Muilwijk in the class of phase shift keying signals.<sup>5</sup> Duobinary MSK, which belongs to the above class of signals, has been used in practical systems and has also been studied extensively.<sup>6,7</sup> There also exist other signals of this class in the literature such as tamed frequency modulation (TFM), where the correlation of the symbols takes place among three consecutive symbol periods. TFM is a binary waveform that is characterised by great bandwidth efficiency.<sup>8</sup> An area lacking in results is the nonbinary or multilevel data when correlated in order to achieve higher efficiencies.

One method of improving the performance of differentially detected signals employs the nonredundant error correction (NEC) technique.<sup>9</sup> As shown by Masamura *et al.* for an MSK signal by using modulo-2 arithmetic and by employing differential detectors of order up to two, the output sequence is a codeword sequence of a rate  $\frac{1}{2}$  convolutional code which has an error correction capability of one. This error correction capability, however, is achieved without using additional bandwidth, as conventional coding techniques would require. In<sup>10,11</sup> the performance of DMSK was investigated with single NEC in an AWGN channel and double NEC in a bandlimited (with InterSymbol Interference (ISI)) channel, respectively. Samejima *et al.* have employed the NEC technique with M-ary differential PSK (DPSK) and extended the theoretical evaluation by using modulo-M arithmetic and by using differential detectors of order up to L.<sup>12</sup> Finally, in<sup>13</sup> for the first time that the NEC technique is proposed for binary correlative encoded signalling and the results are more than encouraging.

In this paper, we attempt a theoretical evaluation of the behaviour of multilevel modulations through correlation in order to increase the bandwidth efficiency. During the same analysis, we employ single-error and double-error NEC receivers to improve the performance in case of noncoherent detection. Section II presents the differential detection of duoquaternary FSK modulation. Section III refers to NEC with single-error and double-error correction capability for duoquaternary FSK. Theoretical analysis of error rate performance in AWGN envi-

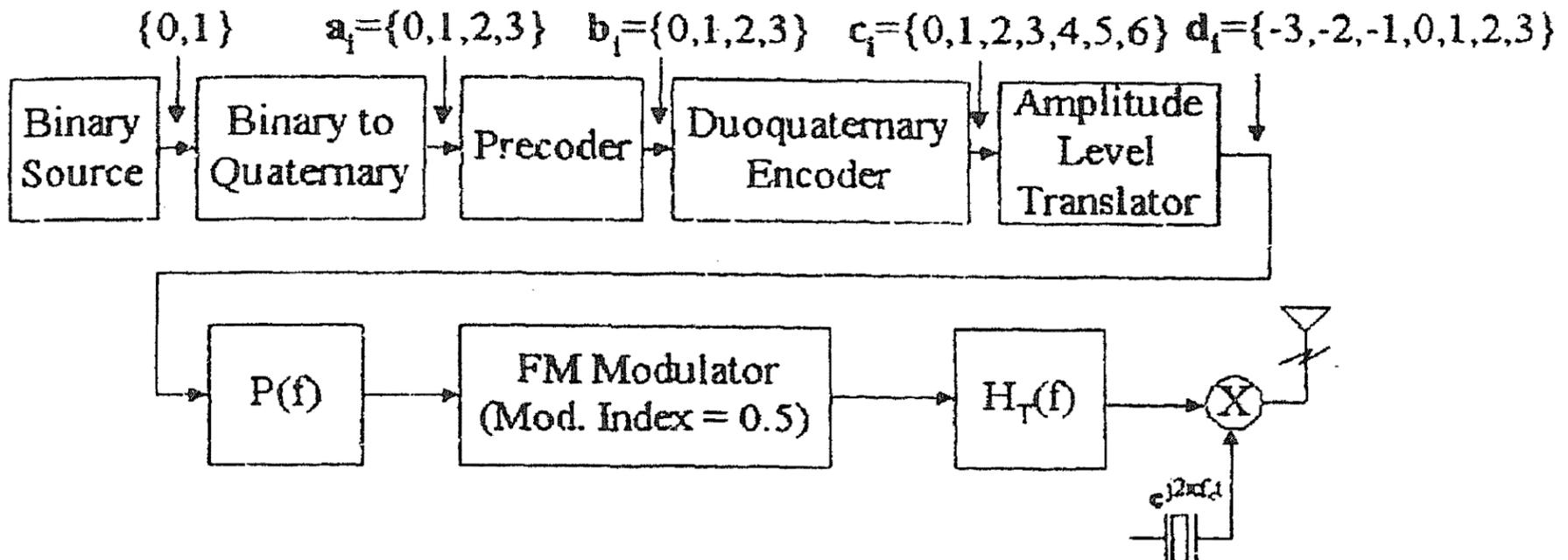


FIG. 1. Block diagram of duoquaternary FSK transmitter.

ronment follows in section IV. Simulation results are presented and useful conclusions are finally derived in section V.

### 2. Differential detection of duoquaternary FSK

The block diagram of the transmitter is shown in Fig. 1. It consists of the binary information source, the binary-to-quaternary transformation circuit, the precoder, the duoquaternary encoder, the amplitude level translator, the premodulation filter, the FM modulator and the band-pass transmit filter. The sequence  $\{a_i\}$  at the output of the information source represents the original quaternary data, and  $a_i$  is any one of the four symbols  $\{0, 1, 2, 3\}$  with equal probability. The precoding is performed just at the output of the quaternary source and is expressed by the equation

Table I

$a_i$	$\bar{b}_{i-1}$	$b_i$	$b_{i-1}$	$c_i$	$d_i$	$\Delta\phi_i = d_i * \pi/2$	$r_i$
0	0	0	0	0	-3	$-3\pi/2$	1
0	1	1	3	4	1	$\pi/2$	1
0	2	2	2	4	1	$\pi/2$	1
0	3	3	1	4	1	$\pi/2$	1
1	0	1	0	1	-2	$-\pi$	2
1	1	2	3	5	2	$\pi$	2
1	2	3	2	5	2	$\pi$	2
1	3	0	1	1	-2	$-\pi$	2
2	0	2	0	2	-1	$-\pi/2$	3
2	1	3	3	6	3	$3\pi/2$	3
2	2	0	2	2	-1	$-\pi/2$	3
2	3	1	1	2	-1	$-\pi/2$	3
3	0	3	0	3	0	0	0
3	1	0	3	3	0	0	0
3	2	1	2	3	0	0	0
3	3	2	1	3	0	0	0

Table II

$c_i$	$d_i$	$\Delta\phi_i = d_i * \pi/2$	$P(c_i)$
0	-3	$-3\pi/2$	1/16
1	-2	$-\pi$	2/16
2	-1	$-\pi/2$	3/16
3	0	0	4/16
4	1	$\pi/2$	3/16
5	2	$\pi$	2/16
6	3	$3\pi/2$	1/16

$$b_i = a_i + \bar{b}_{i-1} \text{ mod } 4 \quad (1)$$

The output of the precoder  $b_i$  consists also of four symbols  $\{0, 1, 2, 3\}$  with equal probability. The duoquaternary encoder is described by the polynomial  $F(D) = (1 + D)$ . According to the patterns of successive input data  $\{b_i, b_{i-1}\}$  that it correlates, it yields the following seven symbols  $\{0, 1, 2, 3, 4, 5, 6\}$  each with different occurrence probability as shown in Table II.

To minimise the maximum phase shift required by the FM modulator, an amplitude level translator circuit converts the quaternary input signal levels  $\{0, 1, 2, 3, 4, 5, 6\}$  into a seven level symmetric output signal  $d_i = \{-3, -2, -1, 0, 1, 2, 3\}$ . The translated output symbols of the correlative encoder must correspond with the phase shifts  $\{-3\pi/2, -\pi, -\pi/2, 0, +\pi/2, +\pi, 3\pi/2\}$  at the output of the FM modulator. The low-pass premodulation filter  $P(f)$  is inserted to obtain a smoother phase path. It meets the Nyquist's third criterion so that the output phase of the frequency modulator passes through the fixed phase points at the end of each symbol time slot. The input to the FM modulator is derived from the equations (2) whereas the output phase shift is derived from the equation (3):

$$\begin{aligned} c_i &= (b_i + b_{i-1}) \\ d_i &= (c_i - 3) \end{aligned} \quad (2)$$

$$\Delta\phi_i = r_i * \pi/2 \quad (3)$$

At this point, an important observation should be made. The correlation process extends the input symbol set  $\{b_i\}$  into the set  $\{c_i\}$  that consists of seven non-equiprobable symbols. The amplitude level translator which follows, manages to yield a symmetric output symbol set  $\{d_i\}$  proper for feeding the FM modulator. Finally, the FM modulator with maximum frequency deviation of  $(3\pi/2)$  yields such a signal that, when captured by a 1-bit differential detector, is seen as a set of four equiprobable phase shifts  $\{0, \pi/2, \pi, 3\pi/2\}$  due to the inherent modulo- $2\pi$  property, Table II, Fig. 2. It is now possible to consider that due to the modulo- $2\pi$  property the phase shift  $(-3\pi/2)$  is equivalent to the phase shift  $(\pi/2)$ . In the same manner, the phase shift  $(-\pi/2)$  is equivalent to the phase shift  $(3\pi/2)$  and the phase shift  $(-\pi)$  is also equivalent to the phase shift  $(\pi)$ . The occurrence probability of each pair of equivalent phase shifts equals to 0.25, and the seventh phase shift of zero rads has also occurrence probability of 0.25, Table II. If we resort now to Table I, we will also observe that each pair of equivalent phase shifts and the remaining phase shift of zero rads are uniquely corresponded to the symbols of the original quaternary data stream  $a_i$ . It is thus concluded that by utilising a 1-bit differential detector at the receiver, it is possible to decide on  $a_i = \{0, 1, 2, 3\}$  symbols which, despite the cor-

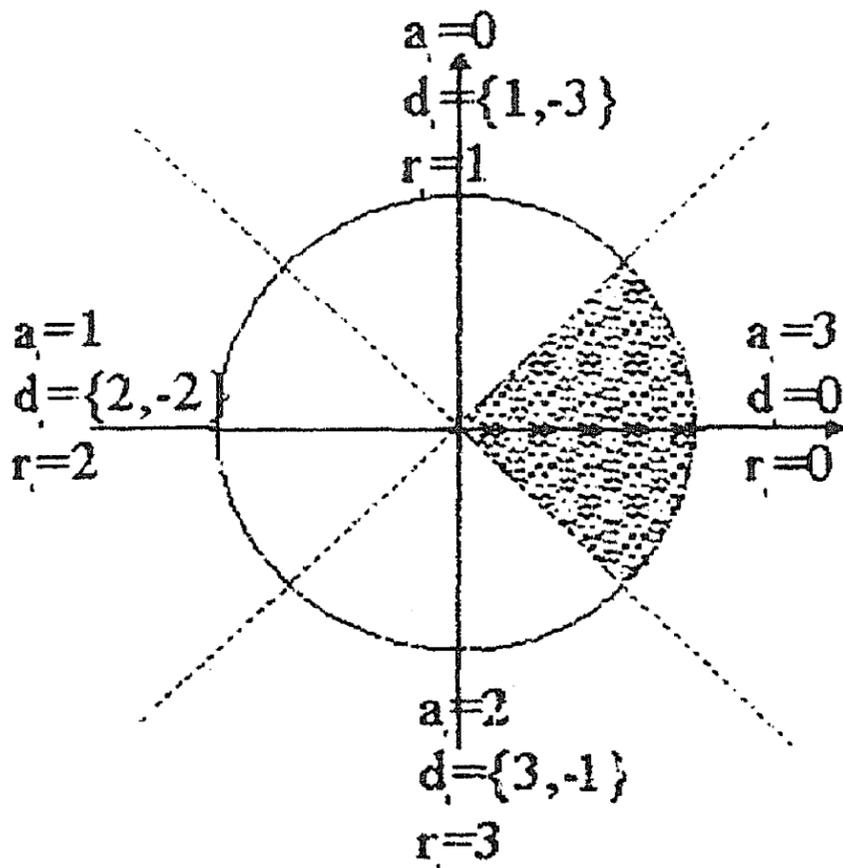


FIG. 2. Duoquaternary FSK signal constellation diagram.

relation process taking place at the modulator, are represented by equiprobable symbols processed at the receiver.

### 3. Nonredundant error correction for duoquaternary FSK systems

The state-space signal diagram of the duoquaternary FSK system in Fig. 2, similarly to PSK systems, shows that the transmitted signal points are selected from a signal group of seven points  $\{-3, -2, -1, 0, 1, 2, 3\}$ . However, due to the modulo- $2\pi$  property of signal phase, as explained in the previous section, the number of the possible phase shifts, within a time slot is limited to four:  $0, \pi/2, \pi,$  and  $3\pi/2$ . For example, when the phase shift due to a data symbol  $d_i = -3$  or  $d_i = +1$  is  $-3\pi/2$  or  $\pi/2$ , respectively, a 1-bit differential detector can only recognize a phase shift of  $\pi/2$  or equivalently a data symbol  $r_i = +1$ . Both transmitted data symbols  $\{-3, +1\}$  represent a source symbol  $a_i = 0$  and therefore  $r_i = +1$  helps recover the information data sequence  $\{a_i\}$ . The carrier phase angle of the proposed modulation at the  $i$ th time slot,  $\varphi_i$  is given by

$$\varphi_i = \varphi_{i-1} + \Delta\varphi_i \quad (4)$$

where  $\Delta\varphi_i$  is the differential phase which takes values from the alphabet  $\{0, \pi/2, \pi, 3\pi/2\}$  and is mathematically expressed as

$$\Delta\varphi_{i,i} = \varphi(iT) - \varphi(iT - T) = \frac{\pi}{2} r_i \quad (5)$$

where  $r_i$  is now the transmitted  $d_i$  but at the output of a mod 4 operator and thus it is characterized by an alphabet of 4 symbols  $\{0, 1, 2, 3\}$ . Generally, the carrier phase  $\varphi_i$  at any time slot can be expressed as the modulo- $2\pi$  sum of the carrier phase  $k$  time slots earlier,  $\varphi_{i-k}$ , and the phase shifts occurring during these time slots. Because of the *duoquaternary FSK* signal con-

stellation (see Fig. 2 and Table II), it appears that, for the *duoquaternary FSK* signal, a mod 4 addition is required so that  $\varphi_i$  can be expressed as

$$\varphi_i = \varphi_{i-k} + \left[ \sum_{j=0}^{k-1} \left( \frac{\pi}{2} \right) r_{i-j} \right] \text{mod } 2\pi, \quad (6)$$

where  $r_{i-j} \in \{0, +1, +2, +3\}$ . If it is assumed that no noise or interference is present, the  $k$ -bit differential detector output at the  $i$ th time slot is given from (6) by

$$r_{k,i} = \left[ \sum_{j=0}^{k-1} r_{i-j} \right] \text{mod } 4. \quad (7)$$

However, due to the various channel imperfections (noise, fading, ISI, etc.), the output of the  $k$ -bit differential detector at the  $i$ th time slot is<sup>12</sup>

$$r_{k,i} = \left[ \sum_{j=0}^{k-1} r_{i-j} + e_{k,i} \right] \text{mod } 4, \quad (8)$$

where  $e_{k,i}$  represents the possible error symbol of the  $k$ -bit differential detector at the  $i$ th time slot. The syndrome  $S_{k,i}$  can be obtained by the output symbol as follows:<sup>9</sup>

$$S_{k,i} = \left[ \sum_{j=0}^k r_{1,i-j} - r_{k+1,i} \right] \text{mod } 4 = \left[ \sum_{j=0}^k e_{1,i-j} - e_{k+1,i} \right] \text{mod } 4 \quad (9)$$

where  $k = 1, 2, \dots, (L-1)$  and  $L$  is the number of differential detectors. Decoding performed at the  $(i+L-1)$ th time slot determines the error symbol  $e_{1,i-L+1}$ . As long as the total number of errors is less than  $(L-1)$  which is equal to the error correction capability of the NEC system, the  $(L-1)L$  syndromes differ for every error symbol. Consequently, by calculating the syndromes and comparing them with predetermined detection patterns, the error symbol  $e_{1,i-L+1}$  can be determined. Incorrect error decision would result if the received signal crosses the decision region boundary. For the *duoquaternary FSK* under moderate noisy and/or interference conditions, the signal may go only into its neighboring decision regions. Due to the integer number signal assignment and the mod 4 arithmetic, an error symbol of 1 or 3 ( $= -1 \text{ mod } 4$ ) would result.

### 3.1. NEC with Single Error Correction Capability for Duoquaternary FSK

Similar to<sup>7</sup>, a NEC duoquaternary FSK system with single error correction capability can be implemented by using 1-bit and 2-bit differential detectors (Fig. 3). The equations of the two syndromes which are used to detect the error symbol are the following:

$$S_{1,i} = (e_{1,i} + e_{1,i-1} - e_{2,i}) \text{mod } 4 \quad (10)$$

$$S_{1,i-1} = (e_{1,i-1} - e_{2,i-1}) \text{mod } 4 \quad (11)$$

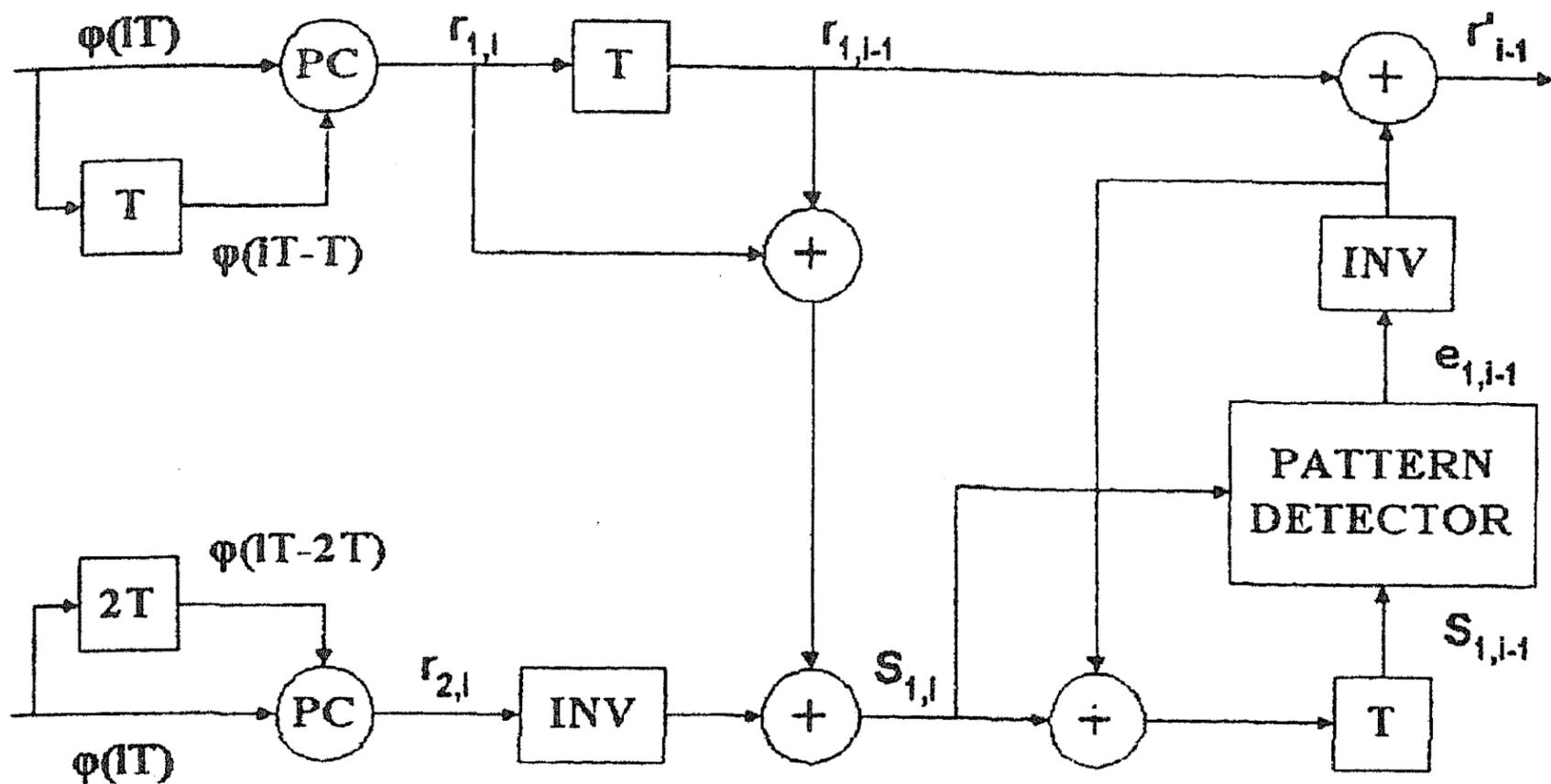


FIG. 3. Single error correction NEC receiver.

Detection performed at the  $i$ th time slot determines the error symbol  $e_{1,i-1}$ . As long as the total number of errors is less or equal to one, the two syndromes are both equal to the error symbol  $e_{1,i-1}$ . The correction circuit in Fig. 3 operates as follows: Compare the syndromes  $S_{1,i}$  and  $S_{1,i-1}$ . If they are nonzero and equal to each other then a decision is made that the error symbol  $e_{1,i-1}$  is equal to the syndrome value  $S_{1,i}$ . The error symbol  $e_{1,i-1}$  is subtracted from the received symbol  $r_{1,i-1}$ .

### 3.2. NEC with Double Error Correction Capability for Duoquaternary FSK

The block diagram of a NEC receiver which utilises up to 3-bit differential detectors is illustrated in Fig. 4. A total of six syndromes, which can be derived by substituting the appropriate values of  $k$  into (9), are

$$S_{1,i} = (e_{1,i} + e_{1,i-1} - e_{2,i}) \bmod 4 \quad (12)$$

$$S_{1,i-1} = (e_{1,i-1} + e_{1,i-2} - e_{2,i-1}) \bmod 4 \quad (13)$$

$$S_{1,i-2} = (e_{1,i-2} - e_{2,i-2}) \bmod 4 \quad (14)$$

$$S_{2,i} = (e_{1,i} + e_{1,i-1} + e_{1,i-2} - e_{3,i}) \bmod 4 \quad (15)$$

$$S_{2,i-1} = (e_{1,i-1} + e_{1,i-2} - e_{3,i-1}) \bmod 4 \quad (16)$$

$$S_{2,i-2} = (e_{1,i-2} - e_{3,i-2}) \bmod 4 \quad (17)$$

In these syndromes, the error symbols  $e_{1,i-3}$  and  $e_{1,i-4}$  are assumed to be corrected through the output of the pattern detector, which also acts as a feedback to update the syndromes that contain it. The error symbol  $e_{1,i-2}$  can only be corrected if the number of errors is less than or

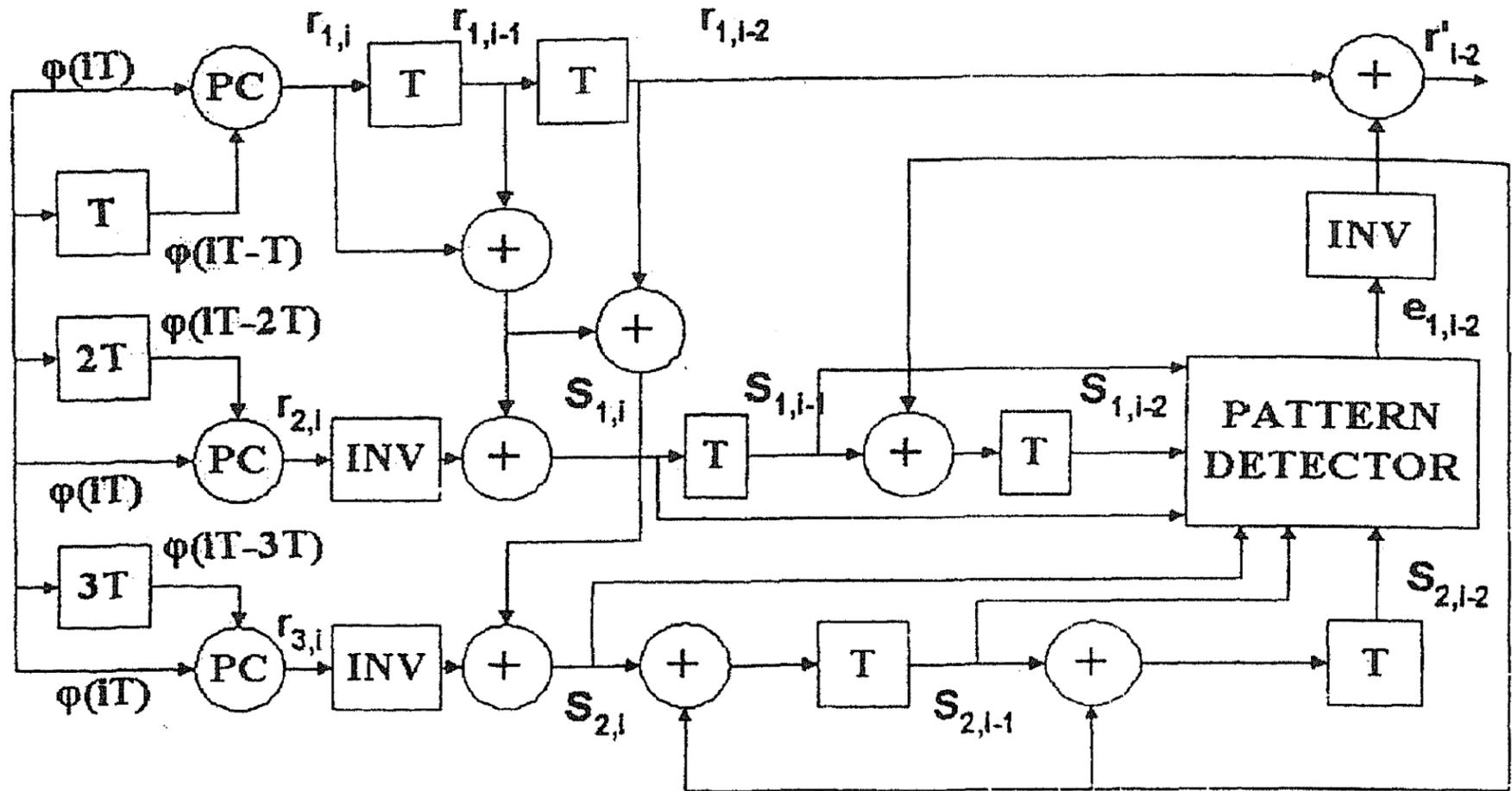


FIG. 4. Double error correction NEC receiver.

equal to two. The pattern detector consists of the patterns containing only one or two errors. In the first case, the error symbol of interest  $e_{1,i-2}$  is assumed to be the only error, which takes values in the subset  $\{1, 3\}$ . In the second case an extra error among the  $e_{1,i}, e_{1,i-1}, e_{2,i}, e_{2,i-1}, e_{2,i-2}, e_{3,i}, e_{3,i-1}$  and  $e_{3,i-2}$  is assumed to take values in the subset  $\{1, 3\}$ . If any one of these patterns is detected, the error estimate of  $e_{1,i-2}$  would be assigned a value of 1 or 3 respectively, otherwise it will be assigned the value of zero. If none of the above patterns occurs, it is meant that either the number of errors is more than two or there is no error. The output symbol  $r_{1,i-2} = r_{i-2} + e_{1,i-2}$  is then corrected by subtracting the estimate of error  $e_{1,i-2}$  at the  $(i+2)$ th time slot. The pointed error symbol  $e_{1,i-2}$  will be also subtracted from the syndromes  $S_{1,i-1}, S_{2,i}$  and  $S_{2,i-1}$  so that it would not appear as  $e_{1,i-3}$  at  $S_{1,i-2}, S_{2,i-1}$  and  $S_{2,i-2}$  in the  $(i+3)$ th time slot and as  $e_{1,i-4}$  at  $S_{2,i-2}$  in the  $(i+4)$ th time slot.

#### 4. Theoretical analysis of the NEC in AWGN environment

A block diagram of the digital communication system under consideration, illustrated in Fig. 5, consists of a duoquaternary FSK transmitter with an output  $x(t)$ , an additive white gaussian noise (AWGN) source  $n(t)$ , a bandpass filter  $H_R(f)$  and a NEC receiver with either single or double error correction capability. The received signal at the input of the NEC receiver,  $s(t)$ , can be expressed as

$$s(t) = x_r(t) + n_r(t), \quad (18)$$

where  $x_r(t), n_r(t)$  are the filtered signals of the transmitter and the noise source respectively at the output of the receiver bandpass filter  $H_R(f)$ . Assuming that both the transmitter and the receiver bandpass filters are square-root-raised cosine filters having a transfer function

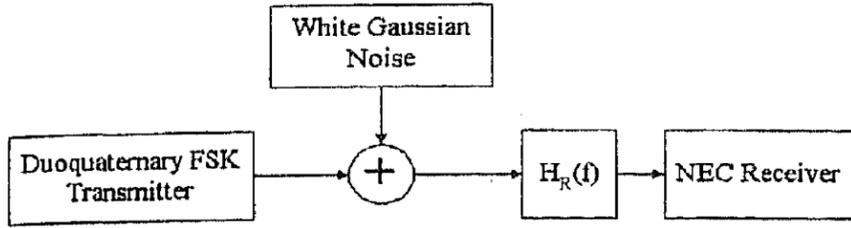


FIG. 5. Block diagram of digital transmission system.

symmetric around  $f_c$ , the modulated signal  $x_r(t)$  will be free of intersymbol interference (ISI) at the ideal sampling instant.<sup>2</sup> The noise is considered zero mean white gaussian noise with double-sided power spectral density  $N_0/2$ . The noise,  $n_r(t)$  after passing through the bandpass filter  $H_R(f)$  can be written as

$$n_r(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \tag{19}$$

where  $n_I(t)$  and  $n_Q(t)$  are the in-phase and quadrature phase low-pass equivalents of  $n(t)$ . Both  $n_I(t)$  and  $n_Q(t)$  are low-pass independent zero mean Gaussian random processes and have power equal to  $\sigma^2 = N_0/2$ .<sup>14</sup> With the assumption that  $n(t)$  is independent from symbol to symbol, the probability density function of the phase error  $\phi$ ,  $f(\phi)$  is derived in<sup>15</sup> as

$$f(\phi) = \left( \frac{e^{-R}}{2\pi} \right) + \frac{1}{2} \sqrt{\frac{R}{\pi}} e^{-R \sin^2 \phi} \cos \phi \left[ 1 + \operatorname{erf}(\sqrt{R} \cos \phi) \right] \tag{20}$$

The error rate performance of the NEC receiver is estimated similarly to convolutional encoders. Error symbol patterns, containing more errors than the error-correcting capability of the NEC receiver are used as input. Each pattern contains error symbols  $e_{k,i}$  which can be either +1 and  $-1 \bmod 4 = +3$  or +2 depending on the severity of the additive white gaussian noise that corrupts the relevant signal phases  $\phi(iT)$  and  $\phi(iT - kT)$ . A good approximation of the bit error rate can be obtained by using only those patterns having relatively high occurrence probability. We refer to patterns, whose errors occur only in neighbouring regions and, hence, the error symbols are only either +1 or -1. Tables III and IV show the number of error patterns, the number of remaining errors after the NEC operation and their occurrence probability  $P'_{L+j}$  of having  $(L + j)$  specific errors among the  $2L$  received signals, for the single-error ( $L = 2$ ) and double-error correcting ( $L = 3$ ) NEC receivers.

**Table III**  
**Single Error NEC Receiver**

Number of input errors	Number of NEC remaining errors	Number of patterns	Occurrence probability
1	0	8	$P'_1$
	0	10	
2	1	8	$P'_2$
	2	6	
3	1	8	$P'_3$
	2	24	
4	2	16	$P'_4$

**Table IV**  
**Double Error NEC Receiver**

Number of input errors	Number of NEC remaining errors	Number of patterns	Occurrence probability
1	0	12	$P'_1$
2	0	60	$P'_2$
3	0	96	$P'_3$
	1	24	
	2	40	
4	0	38	$P'_4$
	1	66	
	2	136	
5	1	52	$P'_5$
	2	140	
6	2	64	$P'_6$

In the following, we adopt a methodology which was first introduced by J. Oberst<sup>16</sup> in the calculation of the double-error probability in differential PSK to calculate the occurrence probability of the input error patterns. In a duoquaternary FSK system, like any differentially detected continuous-phase modulation system, the received signal phase  $\varphi(iT)$  of the  $i$ th time slot contributes to two error symbols for each differential detector, since it acts both as a "SIGNAL" as well as a "REFERENCE".<sup>12</sup> Assuming that a NEC receiver utilises up to  $L$ -bit differential detectors, simultaneous errors among the  $2L$  received symbols (10–11, 12–17) which involve the signal phase at the  $(i - L + 1)$ th time slot as "reference" occur with relatively high probability. For example, in the case of a single-error-correcting NEC receiver ( $L = 2$ ) an incorrect signal phase  $\varphi(iT - T)$  could result in four potential errors  $e_{1,i}$ ,  $e_{1,i-1}$ ,  $e_{2,i-1}$  and  $e_{2,i+1}$ , where  $e_{k,i} = \varphi(iT) - \varphi(iT - kT)$ .

The occurrence probability of each pattern containing  $L$  or *more* errors in the aforementioned  $2L$  received symbols can be readily calculated. The error probability at the detector output, under the condition that the phase error of the reference signal is  $\phi_1$ , can be expressed as

$$P(\text{error} / \phi_1) = 1 - \int_{\phi_1 - (\pi/4)}^{\phi_1 + (\pi/4)} f(\phi_2) d\phi_2 \quad (21)$$

and the occurrence probability of multifold errors is

$$P_m = \int_{-\pi}^{\pi} [P(\text{error} / \phi_1)]^m f(\phi_1) d\phi_1 \quad (22)$$

assuming that noise samples one time slot apart are independent. The occurrence probability of a pattern with  $(L + j)$  specific errors among the  $2L$  received signals is derived by using the probability of multifold errors

$$P'_{L+j} = \frac{1}{2^{L+j}} \sum_{k=j}^L (-1)^{k-j} \binom{L-j}{k-j} P_{L+k} \quad (23)$$

Table V

Time Phase	$(i-5)T$	$(i-4)T$	$(i-3)T$	$(i-2)T$	$(i-1)T$	$iT$	$(i+1)T$
Error	$\varphi(iT-5T)$	$\varphi(iT-4T)$	$\varphi(iT-3T)$	$\varphi(iT-2T)$	$\varphi(iT-T)$	$\varphi(iT)$	$\varphi(iT+T)$
$L=2$	$e_{1,i}$				REFERENCE	SIGNAL	
	$e_{1,i-1}$			SIGNAL	REFERENCE		
	$e_{2,i-1}$		SIGNAL		REFERENCE		
	$e_{2,i+1}$				REFERENCE		SIGNAL
	$e_{1,i-1}$			REFERENCE	SIGNAL		
$L=3$	$e_{1,i-2}$		SIGNAL	REFERENCE			
	$e_{2,i}$			REFERENCE		SIGNAL	
	$e_{2,i-2}$	SIGNAL		REFERENCE			
	$e_{3,i+1}$			REFERENCE			
	$e_{3,i-2}$	SIGNAL		REFERENCE			SIGNAL

Next, the number of errors at the NEC receiver output is obtained for each of the patterns with  $L$  or more errors in the aforementioned  $2L$  symbols. Let the total number of output errors for all patterns with  $(L+j)$  errors be designated as  $N_{L+j}$ . Using  $N_{L+j}$  and  $P'_{L+j}$ , the error probability  $P_{\text{out}}$  of the NEC receiver output can be obtained as follows:

$$P_{\text{out}} = \sum_{j=0}^L N_{L+j} * P'_{L+j}. \quad (24)$$

From Table III, for each set of patterns with the same number  $n = j + 2$  of input errors (1st column of Table), we compute the total number of remaining output errors  $N_{2+j}$  by multiplying the number of remaining errors with the number of relative input error patterns. Then, we make use of the formula (24) to calculate the error probability of the single error NEC receiver

$$P_{\text{single}} = 20P'_2 + 56P'_3 + 32P'_4 = 5P_2 - 3P_3. \quad (25)$$

Similarly, the symbol error probability of the double error NEC receiver can be obtained from Table IV as

$$P_{\text{double}} = 104P'_3 + 338P'_4 + 332P'_5 + 128P'_6 = 13P_3 - 17.875P_4 + 7.125P_5 - 0.25P_6 \quad (26)$$

## 5. Conclusions

For the first time the NEC hardware error correction technique has been used with a correlative encoded multilevel modulation such as duoquaternary FSK. A theoretical analysis of the proposed combination takes place in AWGN. Fig. 6a shows remarkable performance improvements for duoquaternary FSK with single- and double-error-correcting NEC receivers as compared to classical differential detection receivers.

Simulations performed lately indicate a real coincidence with the theoretical results for both single- and double-error-correcting NEC receivers, Fig. 6b. Further performance improvements may result by employing NEC receivers of higher correction capabilities but with reduced gains due to the error randomness. Since correlative encoded continuous-phase modulation is more and more adopted in communications applications worldwide, thanks to the bet-

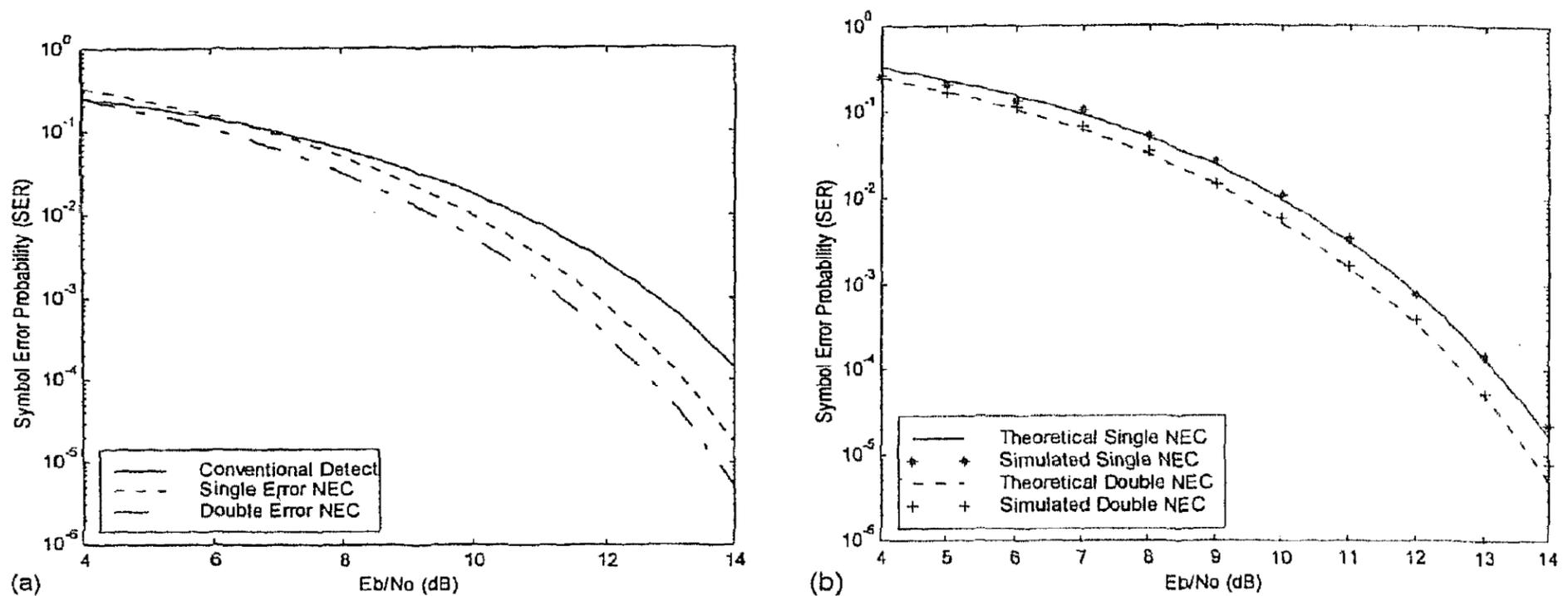


FIG. 6a. Duoquaternary FSK with single and double error NEC vs. conventional differential detection b) Comparison of theoretical and simulation performance results.

ter frequency spectrum manipulation it achieves, analysis should take into consideration more realistic transmission conditions. Duoquaternary FSK as a correlative encoded multilevel modulation succeeds not only to exhibit a compact spectrum although operated at higher bit rates than binary modulations but also to maintain a good power efficiency.

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