J. Indian Inst. Sci.
A Multidisciplinary Reviews Journal
ISSN: 0970-4140 Coden-JIISAD

REVIEW ARTICLE

Understanding and Modelling Disorderly Traffic Streams

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Abstract | Disorderly traffic streams are those that, simply stated, do not have parallel lines (or lanes) of vehicles but have vehicles distributed more haphazardly in the road space. Vehicles in such streams, while moving longitudinally, change their lateral positions frequently. Their trajectories have a more pronounced wander along the width or the lateral dimension as opposed to those vehicles that primarily move in lanes. This property of disorderly streams dictates that its mathematical models must admit two spatial dimensions (the longitudinal and the lateral). Further, the observed impact of road geometry features like width, curvature, etc., on stream behavior, irrespective of whether the stream is disorderly, also suggests that realistic models of traffic streams must describe the streams using two spatial dimensions. Unfortunately, most of the theories of traffic dynamics are one-dimensional—they only consider the longitudinal dimension. This paper, while describing many of the existing approaches to modelling vehicular traffic behavior builds a case for strengthening two-dimensional modelling approaches that are all, still in their infancy. Given the (1) large increase in computation and data handling capabilities over the last decade and (2) significant strides made in developing tools for observing traffic dynamics at scales and accuracy levels that were previously unimaginable, the authors believe the time has come to develop, calibrate and validate reasonable two-dimensional models of traffic dynamics.

1 Introduction

Traffic flow in many developing economies, including India and a number of other south and south-east Asian countries, is different from that in most of Europe and the Americas. The traffic and the roads in most developing economies, where the extent and quality of transportation infrastructure has not yet reached desired levels, is characterized by (1) weak lane discipline, (2) widely varying geometry and, especially in urban areas, frequent lateral infringements from roadside activities and pedestrian movements, (3) heterogeneous vehicle mix with widely varying vehicle dimensions (especially in the lateral direction) and operating characteristics (4) stepped or multi-stage gap-acceptance where a vehicle may accept a series of smaller gaps to complete its manoeuver, and (5) weaker reliance on driver–driver communication, leading to heightened sense of threat to safety from unexpected behavior of the surrounding vehicles. All of these lead to a "disorderly" flow of traffic. The apparent disorderliness comes from the haphazard arrangement of vehicles on the roads as opposed to lines of vehicles one is used to observing in "orderly" or "laned" traffic.

Modelling such heterogeneous, disorderly traffic flow requires a paradigm shift from the extant one-dimensional models to somewhat nascent two-dimensional models of traffic flow. Typically, traffic flow has been modelled assuming that vehicles predominantly move in the

One-dimensional models: These are models of traffic dynamics that represent the behavior over time using only one spatial dimension—the longitudinal dimension (i.e., the dimension along the length of the road).

Two-dimensional models: These are models of traffic dynamics that represent the behavior over time using two spatial dimensions—the longitudinal dimension and the lateral dimension (i.e., the dimension along the width of the road)

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Springer IISc Press

longitudinal direction with episodic changes in lateral positions (though lane-change manoeuvers). However, when the characteristics mentioned in the previous paragraph are present, streams can no longer be viewed as one-dimensional systems where vehicles interact only in the longitudinal direction. One must develop models that admit that a vehicle's response depends on its surroundings and that vehicles interact with (1) the geometry in the lateral direction and (2) one another in both the lateral and longitudinal directions. Such two-dimensional frameworks also hold the promise of taking into account the widely varying dimensions of vehicles in traffic streams such as those in India.

Interestingly, but not surprisingly, a recent study reports the existence of lateral interactions among vehicles moving in the same direction but on adjacent lanes in traffic streams that follow lane discipline.[43] In any case, it is well-known and acknowledged that even in lane-disciplined traffic, geometry features such as lane-width, lateral clearance, curvature, etc. impact the way drivers drive and consequently how streams behave. Manuals on highway traffic like the Highway Capacity Manual[20] of USA and others incorporate empirically obtained graphs and formulae to capture the impact. Even after almost eight decades of work on various aspects of traffic stream behavior (for lane-disciplined traffic) engineers have to rely on purely empirical results to make predictions on capacity and level-of-service of roads under various geometric conditions. The prime reason for this is the lack of a well-tested model of traffic flow that includes both lateral and longitudinal interactions. The development of robust two-dimensional traffic flow models will obviate the need for undue reliance on empirically derived formulae and graphs for the design of roads. It is interesting to note that most of the parameters that engineers need to decide on while designing a road, be it an expressway or an arterial, comes from purely empirical relations. This needs to change.

The one-dimensional view of traffic streams also created the need for partitioning driving into various tasks, often somewhat artificially. For example, car-following and lane-changing are thought of as different tasks (see figure developed by Federal Highway Administration of USA under NGSIM and reproduced in Ni[40],[1]) even though they are ramifications of the same basic forces of need to minimize travel time (urgency) and concern for safety.[6] Analysts have to create these task divisions in a one-dimensional view of traffic streams since car-following theories that move vehicles in a lane cannot admit the lateral dimension and by extension cannot represent lanes and lane-changes. Two-dimensional models will eradicate the need for such artificial divisions. They will be able to develop vehicle trajectories as an expression of a single comprehensive model of traffic flow.

In summary, it can be said that extensive and focused research on developing two-dimensional models of traffic flow is required not only to understand heterogeneous, disorderly traffic but also to better understand orderly or lane-disciplined traffic. Improved understanding will help reduce reliance on empirically derived formulae in design and ultimately improve the quality of traffic engineering.

This short document is divided into six sections of which this is the first. The next section presents and describes some empirical observations that highlight the extent of (1) the effect of geometry on individual vehicles as well as on the entire stream and (2) two-dimensional motion during driving in Indian and other such disorderly traffic conditions. The third section presents a brief discussion on understanding the two dimensional nature of both disorderly and orderly traffic streams. Sections 4 and 5, respectively, describe some of the microscopic and macroscopic modelling efforts on disorderly traffic streams. The last section presents a commentary on how traffic engineering and design may change in the presence of established two-dimensional models of traffic flow.

2 Empirical Observations on Some Aspects of Driver and Stream Behavior

The purpose of presenting some empirical observations (from published sources) on driver and stream behavior is to highlight the two-dimensional nature of traffic streams. Figure 1 shows a screenshot of speed–flow (u–q) diagram from the 2010 U.S. Highway Capacity Manual.[20] The figure shows that for different free-flow speeds, which are, inter alia, dependent on the geometry of the roads, the stream behaviors as expressed through the u–q plots are different. Although this dependency of stream behavior on geometry has been known and acknowledged for many decades the modelling framework had, by and large, remained one-dimensional—thereby precluding the possibility of integrating the effect of geometry into stream behavior models. It may be pointed out that traffic streams in USA are lane-disciplined.

[1] Also available at http://home.iitk.ac.in/~partha/poster.jpg.

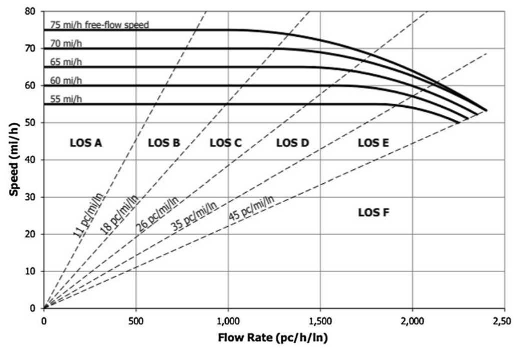

Figure 1: Speed-flow relations in 2010 US highway capacity manual (Source: http://onlinepubs.trb.org/onlinepubs/trnews/trnews273hcm2010.pdf).

The effect of road geometry (specifically, road width) on individual vehicles in weak lane-disciplined traffic can be seen from the following data collected on a two-lane, two-way, undivided road of varying width in Kanpur, India and shown in Fig. 2.[7] In the figure, LPR stands for the mean lateral position of vehicles (as measured from the left-edge of the road) as a fraction of the width of the road and speed refers to mean free speed of vehicles at a given width. As can be seen from Part (a) of the figure a vehicle moves from almost "middle" of the entire road (LPR $\approx$ 0.5) to almost "middle" of the left-half of the road (LPR $\approx$ 0.25) as the width of the road increases from about 6 m to about 8 m. Also (see Part (b)) the mean free speed of vehicles increase as road width increases. Finally Part (c) of the figure shows that at lower widths drivers frequently change their actions (in terms of acceleration) giving rise to higher acceleration noise at lower widths. In a way, drivers seem to be more jittery on lower width roads than on higher width ones.

Figure 3 shows some vehicle trajectories from a disorderly traffic stream in Chennai, India. The data were collected by Kanagaraj et al.[25] From the

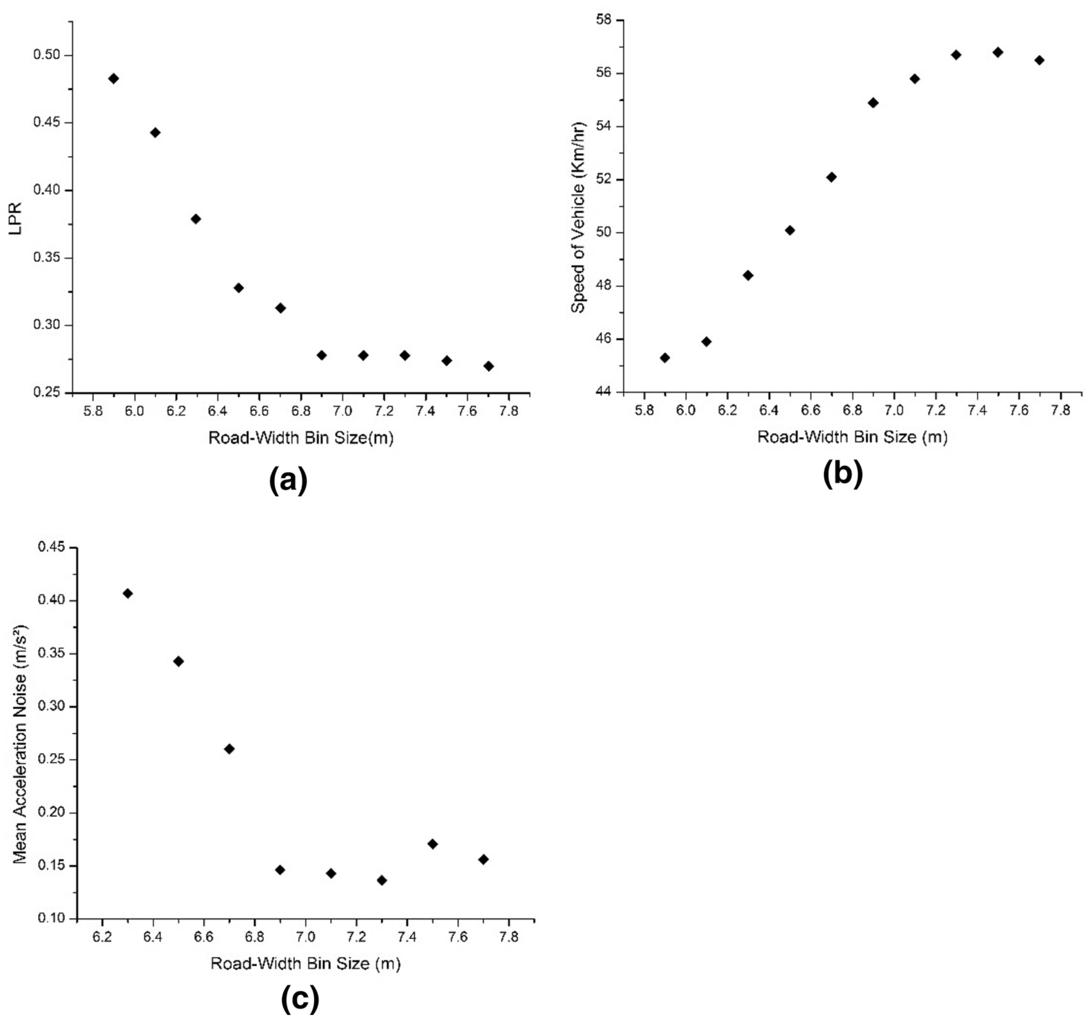

Figure 2: Impact of road width on **a** lateral location **b** speed and **c** acceleration noise of vehicles moving on a two-lane, two-way, undivided road with varying widths (Source: Chakroborty et al.[7])

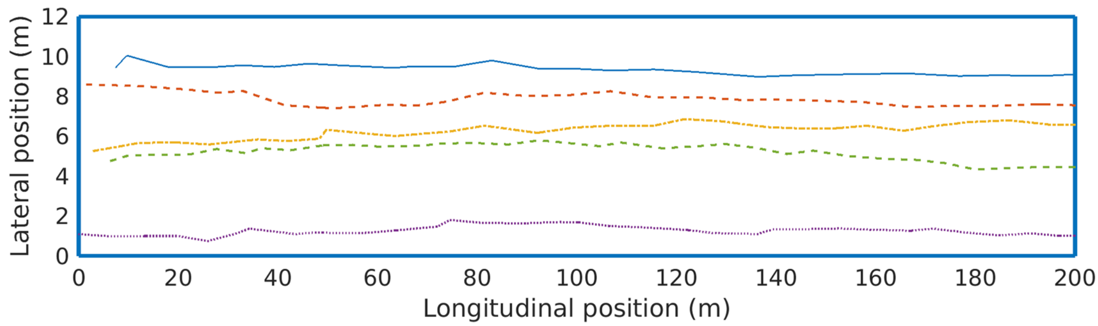

Figure 3: A sample of vehicle trajectories on a Chennai (India) road (Source: Kanagaraj et al.[25])

figure, it can be seen one can at best model only the vehicle with a solid-line trajectory in a one-dimensional framework. The trajectory for the rest clearly shows significant variations in the lateral position that cannot be ignored as one would do in a one-dimensional modelling framework. Nor are these variations episodic (i.e., happens over a short span)—the view taken by lane-change models. Given the impact of geometry and lateral interactions between vehicles that must exist when vehicles wander freely without much concern for lane-discipline the focus must shift towards developing two-dimensional models.

3 Modelling Traffic Streams

Traffic streams, at a macroscopic level, are viewed in terms of flow (q), speed (u) and density (k). In a one-dimensional view of the stream, densities are linear concentration of vehicles (say in units of vehicles per km) and speeds, as the name suggests, are treated as a scalar quantity because there is only one space dimension—the longitudinal direction. Microscopically, the predominant models of traffic streams are car-following models. These were variously referred to as "fundamental to the theory of traffic flow"[13] and one of the "foundations of the science of vehicular traffic".[21] These models are an outcome of the process where one set out to conceive "a theoretical basis for describing the dynamical behavior of vehicles on a single-lane highway, with no passing, in terms of human behavior".[21]

Car-following models: They are one-dimensional microscopic models that describe the actions of the following vehicle (in terms of acceleration/deceleration) only in response to the actions of the vehicle immediately ahead.

Microscopic models: These models describe, over time, individual driver behavior as a response to (1) actions of other vehicles in his/her vicinity, and (2) features of the road (like the geometry of the road, presence of parked vehicles, etc.).

Even if, one argues that the one-dimensional framework of traffic streams outlined in the thoughts of the founding fathers[2] of the theory of traffic science was an initial simplification, the fact is, that is how they remained for decades to come. Even works like that of Gunay[19] that explicitly acknowledged and tried to incorporate the impact of lateral interactions in car-following still represented the car-following behavior through longitudinal positions of the following vehicle and its speed (not velocity as it should be in a two-dimensional framework). Similar reliance on one-dimensional notions can be seen in recent (macroscopic) continuum models (see Mohan and Ramadurai[38]) that introduce concepts like "area occupancy" but continue to work with one-dimensional representation of the continuity equation and the acceleration equation (more on these is presented later).

The overwhelming popularity of one-dimensional view of the traffic stream notwithstanding, some researchers are trying to understand driving and traffic streams in a two-dimensional or spatial (as opposed to linear) setting. As mentioned earlier, Chakroborty et al.[6] proposed that driving, in general, is an outcome of two competing forces in the human mind—that of concern for safety and need for time-savings (or urgency). They argued the need for, what they called, a "comprehensive" microscopic model of driver behavior—one of the first such demands. Chakroborty[5] reiterated the need for such microscopic models. It can be further argued that two-dimensional models of disorderly flow can and must subsume within it models of one-dimensional, orderly, lane-based flow.

Even though microscopically there have been attempts to develop two-dimensional models of driver behavior with Kanagaraj and Trieber[26] being one of the recent ones, macroscopically there have been little attempt in this direction. Vikram[52] is the only notable exception to that rule. In the next two sections, a more detailed description of attempts at developing

[2] Herman, Gazis, Rothery, Lighthill, Whitham, Richards, Prigogine along with a few others like Pipes can be considered as the founding fathers of traffic science for their seminal contributions in the 1950s and 1960s (see Pipes[42], Lighthill and Whitham[30], Richards[47], Chandler et al.[11], Gazis et al.[16], Prigogine and Herman[44]).

microscopic and macroscopic models of disorderly flow are presented. The macroscopic models discussed here are all continuum models of traffic flow (empirical models are not described).

Macroscopic models: These models describe, over time, the behavior of a vehicular stream under different traffic conditions and features of the road.

4 Microscopic Models

As described in the introduction and subsequent sections, traffic stream in many developing economies including India are disorderly due to weak lane discipline, frequently varying road geometry and other features, and heterogeneous vehicle mix. Road geometry and other road features generally include road width, lateral clearance, lane markings, curves, road surface condition and static obstacles like lane barricades, parked vehicles, etc. In such traffic streams, vehicles occupy any lateral position to minimize their travel time and maximize their safety. Drivers in such streams need to decide not only their speed (which can be varied through acceleration and deceleration) but also their direction of movement (which can be varied through steering angle) and that too simultaneously. Therefore, microscopic traffic flow models for such streams need to predict over time the vector, velocity (as opposed to the scalar, speed) of vehicles based on the existing traffic scenario.

As discussed earlier, majority of the microscopic traffic flow models developed in the past are one-dimensional models; i.e., they incorporate only longitudinal interaction between vehicles and predict only longitudinal speeds (or longitudinal accelerations) over time. Although these can be used for lane-disciplined traffic, they fail to incorporate and predict the effect of road geometry even for such streams. In these approaches, to address the episodic changes in lateral positions (i.e., lane changes manoeuvers), a layered structure of tasks are created. For example, car-following and lane changing are considered as two separate tasks rather than outcomes of the single task of driving. Later, however, some attempts have been made to develop comprehensive (or two dimensional) microscopic models of driving (for example, see Chakroborty et al.[6], Maurya[35], Kanagaraj and Treiber[26]).

4.1 One Dimensional Car-Following Models and Their Modifications

In microscopic traffic flow models, the behavior of each vehicle (driver) is modelled using mathematical formulations. Action of each vehicle is determined based on the action of the vehicle ahead, driver characteristics, and sometimes, surrounding vehicles' positions, etc.[12] Most of the well-established behavioral models are related to car-following (CF) theories in which a driver only interacts longitudinally with its immediate leader (referred to as the one dimensional CF model). The one-dimensional CF theory was first conceptualized by Reuschel[46] and Pipes[42] and taken forward by Herman and others (see Footnote #2). The popular CF models can be categorized as stimulus-response models, safe-distance models, optimal velocity models, psychophysical/action point models, rule-based models (including cellular automata models), etc. Many authors like Brackstone and McDonald[4], Toledo[50], Li and Sun[29] and Aghabayk et al.[1] have presented more detailed reviews of CF models and hence these are not repeated here.

Most of the initial models, however, fail to capture variations between drivers and vehicles. Later, several piecemeal attempts were made to fix the limitations of these models. Some notable efforts are discussed here. Siuhi and Kaseko[48] address vehicle heterogeneity in stimulus-response CF models. Gunay[19] modified Gipps safe-distance approach[17] CF models to incorporate effects of lateral separation—a prominent feature in weak lane disciplined traffic. Ravishankar and Mathew[45] further incorporated vehicle-type specific parameters in the modified Gipps model. Another effort to incorporate lateral clearance between interacting vehicles was made by Jin et al.[24] who proposed a modified full velocity difference CF model.

The initial rule-based models that incorporated uncertainty arising out of human control in CF was conceived by Kikuchi and Chakroborty[27] with relative spacing, relative speed and acceleration of the leading vehicle (LV) as inputs (also see Chakroborty and Kikuchi[8]). Other rule-based CF models have also been developed (for example, Gao et al.[15]). Later Chakroborty and Kikuchi[9] created a self-learning version of their rule-based CF model—an early attempt at achieving automated vehicles. Another class of rule-based models are collectively referred to as cellular automata (CA) models (for a detailed review see Chakroborty and Maurya[10]). Luo et al.[32] modified the standard CA models to simulate car and bicycle mixed traffic streams on urban roads.

Before leaving this section it may be pertinent to provide a discussion on the plethora of microscopic simulation tools/models available commercially or otherwise that seem to simulate disorderly traffic (for example, VISSIM, AIMSUM, Mallikarjuna and Rao[34], Mathew et al.[36] and others). These tools continue to work with one-dimensional models of car-following (and consequently treat lane-changing as a separate task). They often adapt these one-dimensional models for "disorderely" streams by dividing lanes into narrow artificial longitudinal strips (or in CA-based approaches use artificially narrow cells) which tend to reduce the suddenness of lane-changing. Attempts are also made by Arasan and Koshy[2] and Venkatesan et al.[51] who use an overtaking maneuver strategy (where the maneuver is represented through instantaneous lateral shifts to "adequate" gaps to the left or right of the subject vehicle) with certain primitive car-following strategy (where speed of the subject/following vehicle is equated with that of the leading vehicle if overtaking is not possible) to represent the effect of disorderliness in traffic streams. Further, as an aside, it must be mentioned that most of these tools use a large number of user-specified parameters whose values are often difficult (if not impossible) to uniquely identify from real-world data. One may refer to Mahapatra et al.[33] for a more detailed discussion on many of the simulation tools.

4.2 Two-Dimensional Microscopic Traffic Flow Models

The discussion on one-dimensional car-following models and their modifications (presented in the previous section) reveal that these models, fundamentally, cannot represent disorderly traffic streams. In such traffic, drivers continuously look for possible opportunities to progress through available gaps by maintaining a safe distance with the surrounding vehicles. During this process, vehicles keep changing their lateral position to utilise opportunities for safe increases in velocities. Therefore, modelling such driver behavior requires a framework to find gaps or opportunities, in a two-dimensional search space, that allows maximal improvement in velocities.

Maiden attempt to develop a two dimensional comprehensive microscopic model for disorderly traffic was made by Chakroborty et al.[6] using a force field (potential field[3]) based approach. Two response models are used to control the steering angle and acceleration of vehicles for any driving scenario. The model assumes that every goal or destination emanates attractive (or negative) potentials while other road features (like parked vehicles, road edges, moving vehicles, etc.) are obstacles that emanate repulsive (or positive) potentials. The total potential at a point is assumed to represent a driver's perceived threat from that point. Since velocity exacerbates the threat perception it is assumed that higher the potential at a point lower is the velocity that can be maintained at that point. Therefore, a driver chooses a minimum potential path. The model performed satisfactorily for different driving scenarios. The most significant limitations of this model are its computation-intensive nature and lack of calibration schemes (from field data) for its parameters.

Later, Maurya[35] proposed a comprehensive traffic simulation model that employs two-dimensional vehicle interactions. The lateral control module identifies the best path for movement (i.e., direction) based on certain goodness criteria (like available distance headway, difficulty in maneuvering in terms of steering angle requirement, the propensity of conflict with neighbouring vehicles, presence of obstacles, etc.). The longitudinal control module predicts the safe speed based on characteristics of vehicles in the vicinity of the identified best path. The control modules together provide the safe speed along with the steering angle for each subject vehicle (i.e., these models predict the safe velocities over time). Although this model promises to replicate the behavior of disorderly, heterogeneous traffic streams it is still to be extensively calibrated and validated using real-world traffic data.

In a recent attempt, Kanagaraj and Trieber[26] have proposed a general multi-particle model for self-driven "high-speed particles (vehicle)" to represent disorderly traffic flow. This model is also a force-field based model. Similar to previous models, due to heavy parametrisation, this also requires extensive calibration and validation from field data.

4.3 Strengths and Shortcomings

Microscopic traffic flow models for disorderly streams need a unified approach to predict the two-dimensional vehicle-vehicle and vehicle-geometry interactions. The piecemeal and ad hoc arrangements to adapt one-dimensional traffic flow models for two-dimensional streams are not ideal solutions.

Simulation results from two-dimensional comprehensive traffic flow models presented in the earlier sections are encouraging. These models mostly consider the longitudinal and lateral control modules together to predict vehicular velocity (speed as well as steering angle). Some, like Maurya[35], however, apply these modules sequentially in their implementation. Such sequential application

[3] The potential field based approach was motivated by robot motion planning schemes (see Latombe[28] for more details).

raises the unanswered question as to which module should get primacy.

Proposed comprehensive models employ a large number of parameters to replicate the observed two-dimensional behavior of traffic streams. Calibration of many such parameters are quite challenging as they are sometimes confounded, sometimes "deeply" latent, etc. For example, calibration of potential (or force) field functions in Chakroborty et al.[6] for different types of obstacles is quite tricky. Further, such models are computationally expensive and this restricts its use for network-level simulation.

4.4 Future

With computing powers increasing almost exponentially[4] the problems of two-dimensional microscopic models arising out of their computation requirements will progressively become less. Further, given (1) the initial success of two-dimensional microscopic models (especially the potential field or force field models), (2) their flexibility in handling driver, vehicle and geometric heterogeneity, and (3) their easy-to-understand nature, it is felt that their use will increase in future. However, for that to happen, the challenges associated with the use of large number of parameters, their calibration and availability of good quality microscopic data on driver behavior (to be used for calibration) must be resolved.

Today, a large number of parameters used in these models have been introduced to achieve quick fix solutions for more deep-rooted problems. With more focused research the need for such ad hoc solutions and consequently introduction of unnecessary parameters will go down. With more transportation engineers now working in the cutting edge of econometrics and estimation science one hopes that customized tools will be developed for calibrating parameters in these models. Finally with significant progress being made in data collection methods for microscopic driver behavior (through instrumented probe vehicles or stabilized quadcopters with high-resolution cameras) one hopes to collect large quantities of good quality data for purposes of calibration and validation.

[4] Although, how long Moore's law (see https://en.wikipedia.org/wiki/Moore%27s_law for more details) will remain valid is an open question and some estimates say that it will collapse in another two decades.

5 Continuum Models (Macroscopic Models)

The previous section briefly discussed microscopic modelling of traffic streams. The section ended with a discussion on the future of such microscopic strategies. This section focuses on concisely presenting the state of the art on macroscopic (continuum) models of traffic streams. The section is divided into five subsections.

The first subsection gives a general description of continuum models. The second concentrates on certain extensions of one-dimensional continuum models that try to incorporate the impact of geometry and heterogeneity. The third subsection briefly describes the attempt to develop a two-dimensional continuum model of disorderly traffic streams. The fourth and fifth subsections, respectively provide discussions on the strength and weaknesses of these models as well as on their future.

5.1 A General Description of Continuum Models

Continuum models of traffic flow can be divided into three categories: (1) LWR (Lighthill–Whitham–Richards) models, (2) one-dimensional behavioral models (for example, Greenberg[18], Payne[41], Aw and Rascle[3], Zhang[56]), and (3) two-dimensional behavioral model[52]. First two categories of continuum models define speed and flow of a traffic stream as one-dimensional quantities and density as a linear density whereas the third category defines speed (rather velocity) and flow as two-dimensional quantities and density as an area density. However, the following three characteristics are common to all continuum models of traffic flow: (1) conservation of number of vehicles plying on a road, (2) utilization of fundamental equation of traffic flow, and (3) description of stream or driver behavior.

One-dimensional models like LWR models describe traffic streams through (1) one-dimensional continuity equation $\left(\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0\right)$ representing the conservation of number of vehicles, (2) fundamental relation of traffic flow ($q = u \times k$), and (3) an algebraic equation representing equilibrium relation between u and k, for example $u = f(k)$. Different assumptions on $f(k)$ give rise to different LWR models.

One-dimensional behavioral models describe traffic streams through same first two equations. However, instead of requiring the stream to obey the equilibrium equation at all times, an acceleration/deceleration equation (capturing driver behavior) is employed to model the dynamics of a traffic stream. The structure of the acceleration/deceleration equation is as follows:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = g(k)\frac{\partial k}{\partial x},$$

where $g(k)$, the sensitivity term, is typically a function of density (k) while the spatial (longitudinal) gradient of density is the stimulus term. Different forms of acceleration/deceleration equations give rise to different one-dimensional behavioral models.

The two-dimensional behavioral model, on the other hand, uses a two-dimensional form of the continuity equation to represent conservation of number of vehicles. The fundamental equation of traffic flow now takes the form $\boldsymbol{q} = k \times \boldsymbol{u}$ ($\boldsymbol{q}$ and $\boldsymbol{u}$ are now vectors as opposed to scalars in one-dimensional models). Driver behavior, in this model, is described using two acceleration/deceleration equations; one each for the longitudinal and lateral directions.

From the nature of the partial differential equations (PDEs) that arise in continuum models of traffic streams, it can be inferred that the variation of each traffic flow parameter, under certain conditions, will have discontinuities that may move with time. In other words, shock waves will occur in traffic streams. Further, the PDEs that arise here need to be solved numerically. Any numerical scheme that is based on derivatives may not be a suitable solution technique because of the possibility of discontinuities. One also requires shock capturing terms to avoid Gibb's phenomena (for more details see Vikram et al.[53]). Additionally, it can be observed from the PDEs that speed (velocity) and density terms are intertwined. These properties make it difficult to develop a numerical scheme for solving such PDEs. However, Vikram et al.[53] were successful in devising a numerical method using FEM (an integration based scheme) that overcame these challenges.

5.2 Some Extensions on One-Dimensional Continuum Models

An attempt was made by Vikram et al.[54] to incorporate the impact of road geometry on traffic flow by extending the one-dimensional behavioral model. In this study, only the acceleration/deceleration equation is modified. The modified equation includes the spatial gradient of road width (and/or curvature) as additional stimulus/stimuli. Interestingly, the results from the study show that if one knows speed-density relation for a given road geometry then one can determine the speed density relations for all other road geometries. All the results from test cases reveal that the model is able to replicate phenomena that are observed empirically on a road with varying geometry.

There are other attempts where researchers have tried to incorporate heterogeneity of a traffic stream. The models proposed in these studies are extensions of either LWR models or one-dimensional behavioral models. In these studies, the continuity equation and fundamental equation of traffic flow are considered to be valid for each class of vehicles, separately. Of course, the proposed models vary in their description of driver behavior. In the models that are extensions of LWR models (like, Wong and Wong[55] and Logghe and Immers[31]), the speed and flow rate of a class of vehicles are assumed to be directly dependent on a linear combination of densities of all classes of vehicles. In the models that are an extension of one-dimensional behavioral models (such as Tang et al.[49]), acceleration/deceleration of a class of vehicles is assumed to be dependent not only on the density of the same class of vehicles but also on the densities of other classes of vehicles.

In an early study, Herman and Prigogine[22] tried to incorporate the speed heterogeneity in traffic streams using a two-fluid model. Some later studies like Nair et al.[39] and Fan and Work[14] incorporated the effect of passing of speeding small-sized vehicles among slowly moving large-sized vehicles (although being one-dimensional models they could not model passing or overtaking per se). These one-dimensional continuum models can, at best, be described as stepping stones towards modeling disorderly traffic streams.

5.3 A Two-Dimensional Continuum Model

So far, the classes of models discussed through some exemplar papers (including those that tried to incorporate the effect of geometry or heterogeneity) all assume the movement of vehicles only in the longitudinal direction and thereby continue to work with one-dimensional representation of traffic flow. As mentioned earlier, the impact of road geometry or disorderliness of a traffic stream indicates the presence of lateral interactions in addition to longitudinal interactions. Lateral interactions cannot be modeled in a one-dimensional framework. Therefore, a two-dimensional framework is required.

Vikram[52] proposed a two-dimensional continuum model of traffic flow, possibly for the first time. In this study, a traffic stream is considered to be a two-dimensional entity where stream movement can take place in the plane formed by the longitudinal and lateral directions. Naturally, density can no longer be the linear density used in one-dimensional models but has to be an area density represented as number of vehicles per unit area of the road. Further, velocity, as opposed to the scalar quantity speed used in one-dimensional models, and flow rate of a traffic stream are each resolved into two components: (1) one along the longitudinal direction, and (2) the other along lateral direction of the road. For conservation of number of vehicles plying on a road, a two-dimensional continuity equation is employed in this model. The fundamental equation of traffic flow is assumed to be valid for both the longitudinal and lateral directions, separately. Driver behavior description is through two acceleration/deceleration equations—one each for the longitudinal and lateral components of stream movement. As a detailed discussion of this model is beyond the scope of the paper the following few paragraphs briefly present two of the various novel characteristics of the proposed model. A more complete discussion of this model and its results are the subject matter of a forthcoming publication.

Numerical experiments carried out using the proposed two-dimensional continuum model show the creation of various features like oblique shocks. These features are akin to those observed in high-speed compressible flow of fluids that are governed by Navier–Stokes equation. Presence of oblique shocks in a traffic stream represents discontinuity in traffic parameters at that location. Further, oblique shocks are always stationary and they are associated with sudden change in the velocity of vehicles at that location. The various features, created in the simulated traffic stream using the proposed model, indicate the possibility of the model to be able to capture many phenomena observed in real world traffic.

Among the various experiments conducted with the proposed two-dimensional model one was to observe the model's ability to capture the impact of road geometry on stream behavior. Numerical experiments were conducted on a road with the geometry shown in Fig. 4.

The inlet (Location A) and exit (Location D) sections of the road (see Fig. 4) are 10.5 m wide and in between the road smoothly narrows to a 7.0 m wide section (like in Location C). Location B represents an arbitrary location just upstream of the narrow section. Suitable initial and boundary conditions are provided to simulate traffic stream on such a road section. Numerical experiments are carried out for various inlet flow conditions. The inlet flow, in the test cases, increases from lean flow to high flow.

The results from these test cases reveal that at lean inlet flow the impact of road width on stream behavior at macroscopic (aggregate) level is negligible. At somewhat higher inlet flow the speed turns out to be lesser (and density higher) in the narrow section as compared to those in the wider sections. Further, flows in the narrow section and downstream of it remain same, over different inlet flows, when these inlet flows are high. That is, the narrow section puts a cap on the flow that pass through it. This indicates that the narrow section (as all road sections must) has a capacity and it does not allow flow higher than this. Interestingly, the flow just upstream of the narrowing section (i.e., Section B) also operates at the flow which is same as the capacity of the narrow section whenever the inlet flows are higher than the capacity of the narrow section. The traffic stream at the upstream location (Section B), as expected, operates in the congested regime and the congestion moves backward with time. It may be pointed out here that all these observations are consistent with the expectations associated with the flow on such roads.[5] The two-dimensional model naturally incorporates the impact of this varying road geometry on traffic stream behavior without considering variations in geometry as a separate stimulus in the behavioral equation (as was done in Vikram et al.[53]).

5.4 Strengths and Shortcomings

For lane-disciplined traffic in a reasonably non-varying geometry, the one-dimensional continuum models provide engineers simple tools such as shockwave analysis for studying the creation and dissipation of queues and congestion. It allows analysis of how conditions in streams evolve from one to another. With some tweaking to its behavioral equation one can replicate some of the effects of changes in geometry and heterogeneity on stream behavior. These benefits of one-dimensional models, notwithstanding, they essentially work in a modelling framework that grossly simplifies the physical system – in this case the traffic stream. This over-simplification causes many problems and these have been discussed earlier when describing one-dimensional models.

[5] Expected flow through roads of geometry similar to that in Fig. 4 is discussed at length in May[37]. The stream behavior predicted by the Vikram model[52] is similar.

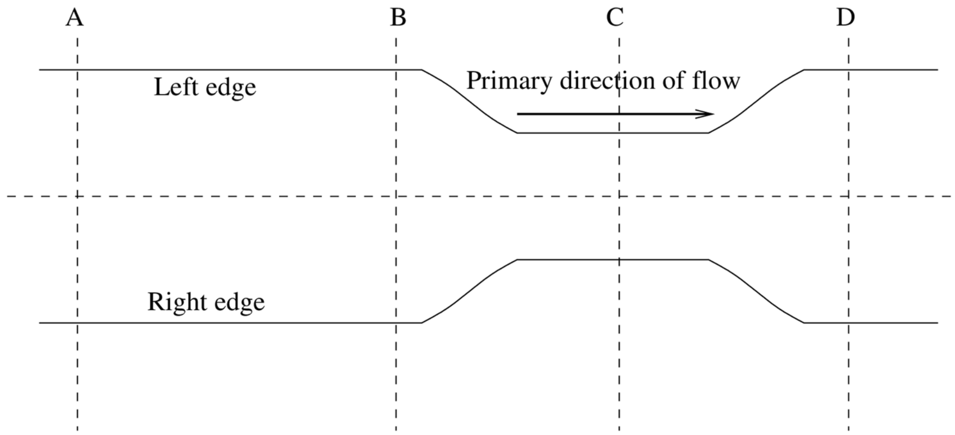

Figure 4: Schematic of a road section on which Vikram's[52] two-dimensional continuum model was tested.

The inability of one-dimensional continuum models to describe disorderly flow formed the motivation for developing the two-dimensional continuum model. Among the many strengths of the model (some of which are described earlier) one that immediately stands out is its ability to predict traffic relations like the speed-flow relations for various geometries given one such relation for a particular geometry as input (boundary condition). However, these models are computationally intensive and require high-performance computing. Their use in simulating large traffic streams (such as network-wide traffic), for the moment, is severely constrained.

Another shortcoming of such models arises from the fact that they, like most other models of traffic streams, ignore vehicle dimensions. Even though one can justify ignoring the length of the vehicle in one-dimensional models it is not wise to ignore the lateral dimension in two-dimensional models. The reason for this is that the width of a vehicle is not negligible when compared to the width of the channel (i.e., the road) and only a few can fit along the width of a road. Since these models ignore vehicle dimensions, their ability to incorporate all aspects of vehicle mix in a stream is also debatable.

5.5 Future

As mentioned earlier problems of two-dimensional models arising out of their computation requirements will progressively become less. Their usefulness in simulating large streams will consequently increase. However, in the future, two-dimensional macroscopic models of traffic dynamics must incorporate the heterogeneity in the lateral dimensions of a mixed stream. Later, heterogeneity in vehicle maneuverability and performance must also be integrated into the models. Ideas from the physics of granular flow[6] need to be also explored in the pursuit of a comprehensive model of traffic streams.

6 Impact on Traffic Engineering and Design

Today, most of the traffic design is done using empirically derived formulae and graphs. What-if questions are typically answered using the same empirical sources or through simulation programmes created from one-dimensional models of lane-based orderly traffic (see NGSIM figure in Ni[40] or the Footnote #1). This has various ramifications and some are listed here:

1. today, even in lane-based traffic systems, one cannot answer, with any degree of confidence, questions on how a stream would behave on roads with innovative or out-of-the-ordinary geometry (as there are no precedence from where data can be collected to develop empirical formulae),
2. today one cannot study the impact of varying road geometry on Indian roads since there are no simulation programmes built using two-dimensional models (some empirical studies have been recently codi-

[6] This, of course, is not a new idea as illustrated by the fact that a conference titled "Traffic and Granular Flow" is held once every 2 years from 1995. Nonetheless, integration of ideas between these two fields still needs a more focused effort.

fied in the Indo-HCM[23] but the complexities are large and even basic factors like passenger car equivalents are difficult to ascertain empirically),
3. today, for the same reasons, one cannot study the impact on safety if certain road features change on Indian roads,
4. today one cannot devise solutions to many traffic issues as one only knows under what conditions those occur (from empirical observations) and not why they occur—a key requirement for developing effective solutions.

The list can go on. The point is without two-dimensional models one cannot develop simulation programmes (an indispensable tool as traffic engineers often cannot experiment on real-world road because of safety concerns) that can answer questions related to effect of various traffic features (such as geometry, control strategies and vehicle mix) on stream behavior. This, in addition to affecting operational engineering, stifles innovative solutions. Sustainable development suffers as long-term effects of solutions proposed for implementation cannot be studied or predicted.

Although the beginning of traffic sciences in the 1950s was through one-dimensional models its future lies in the development of two-dimensional models.

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 16 October 2019 Accepted: 21 October 2019
Published online: 27 November 2019

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