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REVIEW ARTICLE

A Review of Automatic Vehicle Following Systems

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Abstract | There has been considerable interest recently in the development of connected and autonomous vehicles (CAVs). Automatic vehicle following capability is central for CAVs; in this article, we provide a review of the critical issues in the longitudinal control design for automatic vehicle following systems (AVFS) employed by CAVs. This expository review differs from others in providing a review of underlying methodologies for design of AVFS and the impact of AVFS on traffic mobility and safety.

Keywords: *ACC, CACC, String stability, Traffic modeling, Traffic safety*

1 Introduction

Connected and autonomous vehicles (CAVs) are being developed by automobile companies, and are receiving considerable attention from researchers and policy makers.[1] An autonomous vehicle (AV) decides its control actions based on the available information from its on-board sensors; a CAV has an additional wireless communication capability, by means of which it can gather information about vehicles or the traffic, it may not readily sense with its on-board sensors. A vehicle equipped with advanced driver assist systems (ADAS) such as adaptive cruise control system (ACC) or lane keep assist may be viewed as an inceptive AV. One can expect to see CACC or advanced CACC systems (referred to as CACC+ systems) in the first generation of CAVs. ACC relies only on on-board sensory information, while vehicles equipped with a CACC System, currently in production only in Japan, utilize the preceding vehicle's acceleration communicated by a wireless system. Next generation of CAVs is expected to be equipped with CACC+ systems to exploit vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) connectivity to obtain information about multiple preceding vehicles or traffic downstream and respond appropriately.

The *anticipated* benefits of CAVs include enhanced traffic safety stemming from a reduced reaction time of machines, convenience and improved mobility and fuel economy, especially in the case of trucks. The *projected* path to the eventual deployment of CAVs from the current state passes through future states of technology where vehicles possess different levels of autonomy:[2, 3] Level 0 describes the (past) state of vehicles where drivers are completely in charge. Level 1 (current state) of autonomy still has the driver in control of the vehicle ,but includes Advanced Driver Assistance Systems (ADAS) that automate certain driving tasks, i.e., maintaining speed, vehicle following, or lane keeping is automated. In Level 2 of autonomy (referred to as partial automation), the driver monitors the environment at all times and is engaged in driving tasks, but the combined task of lane keeping and vehicle following is automated. In Level 3 of autonomy (referred to as conditional autonomy), the driver is not required to monitor the environment at all times, but must be prepared to take control of the vehicle at all times with a short notice. In Level 4 of autonomy (referred to as high automation), the vehicle will be capable of driving autonomously in some situations; the driver will have the option of controlling the vehicle. In Level 5 of autonomy (referred to as full automation), the vehicle will be capable of driving autonomously in all situations; yet, the driver will have the option of controlling the vehicle. It is safe to say that the current state of technology in commercially available cars has not provably surpassed Level 2 (partial automation).

An automatic vehicle following system (AVFS) is a central subsystem of AVs and CAVs. ACC and CACC systems are two types of AVFS. Design of an AVFS, a topic of active investigation for the

Adaptive cruise control: A driver assistance feature found in many modern cars that uses sensors like radars and cameras to measure distance to the vehicle in front and regulate speed accordingly.

Cooperative adaptive cruise control: Instead of relying only on passive sensors, a CACC system communicates with other cars to obtain better estimates of the state of surrounding vehicles while typically broadcasting its own state. This collectively improves vehicle following.

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past 50 years, has two important components: choice of a spacing policy and an associated control system. A spacing policy dictates the desired following distance of a CAV as a function of its speed, while the control system is concerned with regulating the following distance at its desired value with the information available to the CAV. AVFS couples the motion of CAVs (or simply vehicles) through feedback, and consequently, errors in maintaining a desired following distance and velocity can propagate in a collection of vehicles employing AVFS. Feedback requires information about other vehicles in traffic; an underlying information flow graph specifies the set of vehicles in the traffic whose information the controlled vehicle possesses. Depending on the underlying information flow graph, amplification of errors in spacing can occur and often result in chain collisions or pileups.

Time headway: Time headway between two vehicles can be interpreted as the difference between the time taken for the two vehicles to pass some common point in their path

Enhancements in traffic safety, in the context of AVFS for CAVs, can be characterized through a reduction in, if not elimination of, rear-end collisions. Enhancing traffic safety at the expense of traffic efficiency or vice versa is not desirable. For example, mobility is enhanced (via increased throughput and associated congestion relief) when smaller following distances are employed in automatic vehicle following or platooning applications;[4, 5]; however, from a safety viewpoint, tight following distances also leave little margins to account for parasitic actuation/sensing lags or imperfections in communication and control. The problem of safety can be compounded by heterogeneity of vehicles stemming from having different sensing, actuation, braking, and communication systems.

Rear-end collisions can occur with or without a following vehicle fully applying its inputs (brakes or throttle). As CAVs are coupled dynamically by feedback, the latter type of collisions can be examined via string stability, i.e., attenuation of spacing errors in maintaining desired safe following distances. Often, spacing error amplification in a string results in pileups and is a function of the information flow among vehicles, the desired following distance, parasitic sensing/actuation lags, communication imperfections such as packet drops, quantization of transmitted signals, etc. Maneuvers of the lead vehicle that do not result in any vehicle fully applying its throttle/brake will be referred to as nominal maneuvers. Rear-end collisions are more likely to occur in an emergency braking maneuver when a lead vehicle in a string applies its brake fully (resulting in its maximum deceleration) to avoid colliding with an obstacle in front.

The scope of AVFS literature is too vast and any review is bound to be restrictive and biased towards the taste of the authors. In this review article, we will briefly review various aspects of designing AVFS systems—(a) vehicle models, (b) string stability in nominal maneuvers, (c) information flow and its relation to string stability, (d) traffic flow modeling with CAVs, and (e) traffic safety in emergency braking maneuvers. This article is organized as follows: in Sect. 2, we discuss prominent models of vehicles that have been employed to design vehicle following control algorithms. In Sect. 3, we revisit the definition of string stability. In Sect. 4, we review two spacing policies—namely, constant spacing policy and constant time headway policy and review the literature on control algorithms for a vehicle employing these policies. In Sect. 5, we review the issues associated with modeling the flow of traffic with CAVs and provide a preliminary model. In Sect. 6, we review metrization of traffic safety for AVFS, especially during emergency braking maneuvers.

Notation The field of real numbers is denoted by $\Re$; the set of non-negative real numbers is denoted by $\Re_+$. The Euclidean norm on $\Re^n$ is denoted by $\|\cdot\|$. A function $\alpha : \Re_+ \to \Re_+$ belongs to class $\mathcal{K}$ if it is continuously, strictly increasing, and $\alpha(0) = 0$; if, in addition, $\lim_{l\to\infty} \alpha(l) = \infty$, the function α is said to belong to $\mathcal{K}_\infty$. A function $\beta : \Re_+ \times \Re_+ \to \Re+$ belongs to class $\mathcal{KL}$ if (a) for each fixed $s \in \Re_+$, $\beta(\cdot, s) \in \mathcal{K}$ and (b) for each fixed $l \in \Re_+$, $\beta(l, \cdot)$ is strictly decreasing and $\lim_{s\to\infty} \beta(l, s) = 0$. If $f : \Re_+ \to \Re$, then we will denote $F(s)$ to denote the Laplace transformation of $f(t)$ and use $f(t) \leftrightarrow F(s)$ to represent them as transform pairs. If $f : \Re_+ \to \Re^n$ is bounded, $\|f(t)\|_\infty = \sup_{t\in\Re_+} \|f(t)\|$; similarly, for every $p > 1, \|f(t)\|_p := [\int_0^\infty \|f(t)\|^p \mathrm{d}t]^{\frac{1}{p}}$.

2 Vehicle Models

To design a controller for an AVFS using available information, one requires a vehicle model. One often uses a model that captures the essence of longitudinal dynamics—to accurately predict the motion of a vehicle when subject to inputs that are varying sufficiently slowly. Since one does not expect velocity of a vehicle to decrease rapidly, and since inertia of a vehicle is sufficiently large, and the objective is to regulate the speed and position of a vehicle, the model must be reasonably accurate to predict the speed of a vehicle in response to low-frequency brake and throttle inputs. In Ref.[6], several models for a vehicle with Spark-Ignition (SI) engine were provided whose fidelity depends on the underlying assumptions.

They provide the following *lumped parameter* model involving five states—m_a (mass of air in the intake manifold), w_e (rotational speed of a Spark-Ignition (SI) engine), w_t (speed of the turbine of the torque converter), v (vehicle's longitudinal speed), and T_b (brake torque); the variable P_m (pressure in the intake manifold) is related to m_a through the ideal gas law:

$$P_m = m_a \frac{RT_m}{V_m},$$

where R is the gas constant for air, T_m is the intake manifold temperature (assumed to be the ambient temperature), and V_m is the volume of the intake manifold, a constant. Pressure in the intake manifold evolves according to the balance of mass of air, while the speed evolution of engine, transmission, and vehicle speed are governed by the balance of linear and angular momentum equations. Brake pressure (and hence brake torque, T_b) evolution depends on the bulk modulus of the brake fluid in the master cylinder and is described by the Navier–Stokes fluid model. Inputs to the vehicle are α (throttle angle) and T_{bc} (Brake torque commanded). The parameter r_t represents the tire radius. The form of the evolution equations is given below:

$$\dot{m}_a = \dot{m}_{a,in}(\alpha, P_m) - \dot{m}_{a,out}(P_m, w_e), \tag{1}$$

$$I_e \dot{w}_e = T_{net}(w_e, P_m) - T_{pump}(w_e, w_t), \tag{2}$$

$$I_t \dot{w}_t = T_{turb}(w_e, w_t) - R^*(T_b + r_t F_{tr}), \tag{3}$$

$$M\dot{v} = F_{tr} - c_a v^2 - F_f, \tag{4}$$

$$\tau \dot{T}_b + T_b = T_{bc}. \tag{5}$$

In (1), the right-hand side represents the net rate of accumulation of mass of air; the incoming rate, $\dot{m}_{a,in}$, is a function of the throttle angle and the pressure in the intake manifold. Higher intake manifold pressure or lower throttle angle lessens the entry rate of mass of air into the intake manifold. Similarly, the outgoing rate of mass of air, $\dot{m}_{a,out}$ into the cylinders is a function of the engine speed and intake manifold pressure. Note that $\dot{m}_{a,out}$ is an increasing function of its arguments. The term I_e in (2) represents the engine's lumped rotational inertia (including inertia of the flywheel), the net engine torque, and T_{net} is a function of w_e and P_m. For vehicles with automatic transmission, the engine is connected to the pump side of the torque converter, while the transmission is connected to the turbine side. The torques at the pump side and turbine side are T_{pump} and T_{turb}, respectively. A tacit assumption in the models given in Ref.[6] is that all the shafts are rigid. The term I_t in (3) represents the reflected inertia of all gears and the term F_{tr} is the traction force at the tire–road interface, and the gear reduction (due to differential and gear box) is represented by R^*. In (4), M denotes the mass of the vehicle, c_a is the aerodynamic drag coefficient, and F_f is the rolling resistance. The time constant for the brake subsystem is assumed to be a constant τ as given in (5).

Torque converter: A hydraulic coupling typically used in automatic transmission vehicles in place of a mechanical clutch.

The functions $\dot{m}_{a,in}, \dot{m}_{a,out}, T_{net}$ are obtained experimentally and are usually in the form of a look-up table. The same is the case with the torque converter charts which specify T_{pump} and T_{turb} in a tabular manner using Kotwicki's[7] quasi-static model. A constitutive model is specified for F_{tr} as a function of longitudinal slip and the axle load. This model is not amenable for control design for a variety of reasons—for example, one may not have the measurement of speeds (especially vehicle speed of sufficient accuracy that can used for the computation of longitudinal slip). In this case, when all the state measurements are unavailable, the corresponding output feedback control design problem is still *open* for non-linear systems. Second, it is not clear if the performance improvement based on a detailed model justifies the need for additional sensory information that is required of the corresponding controllers.

Since the control inputs are α and T_{bc}, the disturbances (such as (i) uncertainty entering the system owing to inadequate knowledge of engine maps or (ii) torque converter charts or the constitutive relationships for the traction force) influence the outputs (velocity or position of the vehicle) sooner than the control inputs; in this case, matching conditions are not satisfied for the non-linear vehicle model listed above. This mismatched non-linear system together with inaccurate knowledge of engine maps or torque converter charts or the constitutive relationships makes the control problem even harder.

Engine map: The function that describes the torque output by an engine, given the current angular speed of the engine shaft and throttle input.

One, therefore, resorts to simplifying assumptions to get a simpler model for control; for example, the following assumptions lead to a simplification of the vehicle model to a second-order model[6]: (a) pressure evolution in the intake manifold is much faster compared to the evolution of engine speed, (b) torque converter is locked, (c) there is no slip between the wheel and the ground, and (d) braking is instantaneous, i.e., $\tau = 0$. Using these simplifying assumptions, the resulting equation of motion of a vehicle is given by:

$$\begin{aligned}(I_e + (R^*)^2 M r_t^2)\dot{v} = R^* r_t T_{net}(\frac{v}{R^* r_t}, \alpha) \\ - (R^*)^2 r_t (T_b + r_t F_r - c_a r_t v^2).\end{aligned} \tag{6}$$

This model is amenable for control design by treating $T_{\text{net}} - R^* T_{bc}$ as a control input; by choosing

$$T_{\text{net}} - R^* T_{bc} = R^* r_t F_r + R^* r_t c_a v^2 + (I_e + (R^*)^2 M r_t^2) u,$$

where u is the synthetic control input (commanded acceleration). A similar approach is adopted for trucks,[8] [9]] where u (commanded acceleration) is considered as a synthetic input; the same I/O linearization approach holds with trucks that are equipped with Compression-Ignition (CI) engines. In this case, T_{net} is a function of the velocity and throttle (fuel) command and the braking dynamics is different owing to pneumatic brakes being used in trucks in place of passenger brakes. This leads to the following input–output linearized vehicle model for control design:

$$\ddot{x}_i = u_i, \tag{7}$$

where x_i is the position of the ith vehicle and u_i is the control input (force/unit mass) to the ith vehicle.

The model should be notionally understood as follows: the acceleration of a vehicle can be controlled arbitrarily (through an appropriate choice of throttle/brake). In this sense, heterogeneity should not be a problem for nominal vehicle maneuvers as the values of acceleration/deceleration are well within the capabilities of vehicles, despite their being different structurally. It must be admitted that feedback linearization presumes exact knowledge of parameters, functional forms for resistance, and unlimited actuation ability; however, one may view this as a first step in the study of heterogeneous collections in this quantitative fashion. This model is extensively used in the development of vehicle following control laws for the following reasons: (1) lower level control design employs feedback linearization to render the model to be homogeneous and linear (I/O linearized); (2) braking or acceleration limits are not attained during most vehicle maneuvers; and (3) platooning experiments in the literature, and especially, those at California PATH, conducted under a number of scenarios with controllers developed with this approach have been satisfactory.

While this model assumes instantaneous actuation, it is never the case in practice. To test the robustness of controllers, one usually analyzes the designed controllers for robustness by considering the following singularly perturbed model for longitudinal vehicle dynamics:

$$\ddot{x}_i = f_i, \quad \tau \dot{f}_i + f_i = u_i, \tag{8}$$

where f_i is the force/unit mass applied to the vehicle and τ is an uncertain parameter that models the time constant of the inner loop control systems that regulate brake pressure in the wheels or the engine speed based on the commanded acceleration command. As mentioned before, it is difficult to model the inner loop dynamics of engines with the associated complexities of combustion and fluid flow in the engine, intake manifold and torque converter, compliance in the shafts, and, more importantly, the tire–road interactions. From the speed response of a vehicle to throttle and brake pedal input, one can empirically bound the maximum lag in the response; in reality, there is time delay in the braking and torque production processes.[10–13] This model may be seen as an approximation for the time-delay systems, where the delay is lumped along with the lag in the response. We use τ_0 to represent the upper bound of τ.

Other vehicle models have also been used in the literature; for electric vehicles such as Toyota Prius, Ploeg et al.[14] used a third-order linear model for experimentally demonstrating the reduction in time headway possible through Cooperative Adaptive Cruise Control (CACC):

$$\tau \dddot{x}_i + \ddot{x}_i = u_i.$$

Sheikholeslam and Desoer[15] used this model, where τ is a function of $\dot{x}_i$. As mentioned before, the functional characterization of τ is far from easy;[11] Bender et al.[16] and Takasaki et al.[17] have used the best linear fit of τ from the experimental data. This model is suitable for electric power trains as the model for the electrical part of the motor can be better approximated by a deterministic first-order linear system unlike vehicles with gasoline or diesel engines—as mentioned earlier, the uncertainty in the engine maps and the lag in their response is higher. The results in this paper also apply to this model. Swaroop and Hedrick[18–20] were the first to introduce singular perturbations for string stability analysis in CAVs; the introduction was carried through with this first-order lag model.

Traffic engineering researchers[21, 22] used a delayed acceleration model for their car-following theories and this is relevant here:

$$\ddot{x}_i = u_i(t - \Delta),$$

where Δ is the reaction time of the combined vehicle–driver system. While this model reflects the time delays in reaction time of the drivers, the controller incorporated velocity feedback with respect to the leader. In the case of Ref.[23], there were additional feedback terms reflecting the delay in obtaining measurements from preceding vehicles.

Since parameters such as mass of the vehicle can change from one trip to another depending on the number of passengers and their attendant baggage, Eq. (6) is adapted for this purpose:[24]

$$M_{\text{total}}\ddot{x} = F_i - C\dot{x}^2 - F_f,$$

where $M_{\text{total}} = I_e + (R^*)^2 M r_t^2, F_i = T_{\text{net}} - R^* T_b$, $C = (R^*)^2 c_a r_t^2$, and $F_f = (R^*)^2 r_t^2 F_r$; in this model, the uncertain parameters that are to be estimated in real time are M_{total}—the effective inertia of the vehicle, C—the effective aerodynamic drag coefficient, and F_f—the effective tire drag coefficient.

Autonomous ground vehicle platforms are being commercially supplied by companies such as AutonomouStuff Inc. In these platforms, the companies provide an inner loop controller, the command to which is the desired velocity of the vehicle. One would naturally be tempted to use a first-order integrator model in such a scenario:

$$\dot{x}_i = u_i,$$

where u_i is the commanded speed. Such a first-order integrator model is simply unsuited for the analysis of string stability in AVFS as documented in Ref.[25]

3 String Stability

Consider a string of vehicles traveling in a single straight lane with a leader indexed by 0 and following vehicles indexed in sequence by $\{1, 2, \ldots\}$. String stability concerns the following scenario: the leader makes an acceleration/deceleration maneuver and every following vehicle tries to maintain a desired following distance specified by the AVFS. When the leader makes a maneuver where there is a step change in its steady-state velocity, it is required that the spacing error remains bounded spatially and temporally. Moreover, the error must decay asymptotically in time while following its predecessor. If $e_i(t)$ represents the spacing error of the ith following vehicle, the following definitions of string stability, introduced by Chu[26] and generalized in Ref.[18], have been used extensively in the literature:

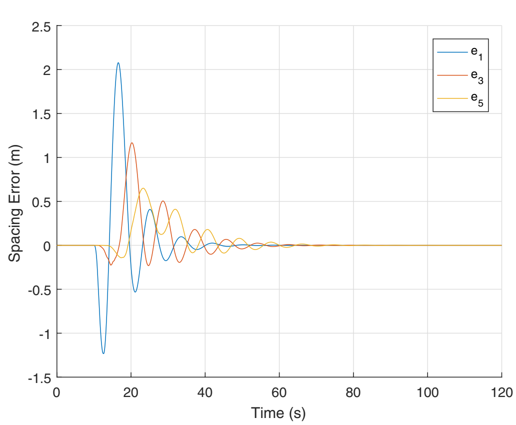

Figure 1: Typical spacing error plot for a string stable platoon.

Definition (*String Stability*) A string is stable if given $\epsilon > 0$, there exists a $\delta(\epsilon) > 0$ (independent of the size of the string), such that

$$\sup_i \max\{|e_i(0)|, |\dot{e}_i(0)|\} < \delta \implies \sup_i \left\{ \sup_{t\geq 0} \max\{|e_i(t)|, |\dot{e}_i(t)|\} \right\} < \epsilon. \tag{9}$$

A string is asymptotically string stable if it is string stable and $\lim_{t\to\infty} \sup_i |e_i(t)| \to 0$.

A convenient way to visualize string stability is to use spacing error plots. Using simulated or empirical data, by observing errors $e_i(t)$ for each vehicle i, the stability of a platoon can be ascertained. Figures 1 and 2 provide examples of spacing error trajectories from a stable and an unstable platoon respectively, when the platoon leader performs an emergency braking maneuver. Only the first, third, and fifth errors are shown here for clarity. While all the errors in both figures eventually diminish with time in this case, we can observe that the $\sup_{t\geq 0} |e_5|$ is smaller than $\sup_{t\geq 0} |e_1|$ in the string stable case, indicating that errors are bounded and diminish down the platoon. On the other hand, in Fig. 2, the spacing errors appear to amplify as we move away from the platoon leader, indicating an unstable string.

One may question even the relevance of the $\epsilon - \delta$ definition in so far as its ability to capture the collision-avoidance capability of a AVFS system in CAVs. As long as ϵ is sufficiently smaller than the minimum desired following distance of CAVs in a string, collision avoidance can be guaranteed. However, this would require the initial spacing errors to be sufficiently small, as well; here is where the trouble could possibly lie. Another shortcoming of this definition is that the effect of disturbing forces (disturbances in control systems parlance) on the propagation of spacing errors is not captured. Recently, Ploeg et al.[27], and Besselink and Knorn[28] introduced the following definition of "scalable" input-state stability: consider a string of N vehicles with S_i

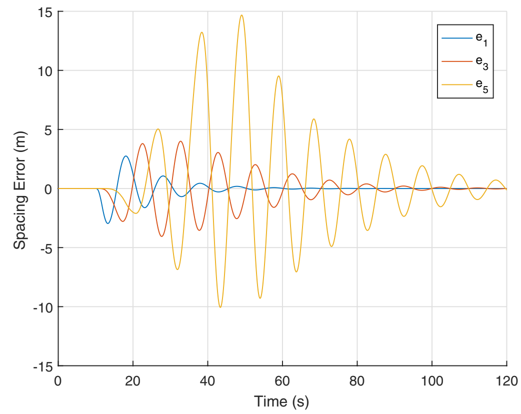

Figure 2: Typical spacing error plot for an asymptotically unstable platoon.

being the set of vehicles whose states influence the evolution of the state, ζ_i of ith vehicle. Let $d_i(t)$ be the disturbance acting on the ith vehicle. Let $\mathcal{I}_N := \{1, 2, \ldots, N\}$. The evolution of spacing errors may be described by an equation of the form for some appropriate functions f_{ij} and h_i:

$$\dot{\zeta}_i = \sum_{j \in S_i} f_{ij}(\zeta_i, \zeta_j, d_i), \quad e_i = h_i(\zeta_i), \quad i \in \mathcal{I}_N.$$

Note that when the disturbances are absent, $\zeta_i = 0,\ i \in \mathcal{I}_N$ is an equilibrium solution of the above set of coupled evolution equations.

Definition (*Scalable input-state stability*) The non-linear system is said to be scalable input–output stable if there exist functions $\beta \in \mathcal{KL}$ and $\sigma \in \mathcal{K}$ and a number $N_{\min}$, such that for any $N \geq N_{\min}$ and for any bounded disturbances $d_i(t),\ i \in \mathcal{I}_N$:

$$\max_{i \in \mathcal{I}_N} \|\zeta_i(t)\| \leq \beta(\max_{i \in \mathcal{I}_N} \|x_i(0)\|, t) + \sigma(\max_{i \in \mathcal{I}_N} \|d_i(t)\|_\infty).$$

This definition subsumes the definition by Chu[26] and Swaroop and Hedrick[20]; second, it provides a way to accommodate disturbances acting on the system. In some cases, it is easier to analyze (especially, when there are bidirectional interactions) the deleterious effects of amplification of disturbances.[29–31] For example, analysis appealing to this approach is heavily used by treating the sinusoidal acceleration of the lead vehicle as the only disturbance acting on the string of vehicles and, subsequently, analyzing how the spacing error amplitudes evolve spatially.

Before Chu's introduction of this $\epsilon - \delta$ definition of string stability (as an instance of Lyapunov stability for vehicle strings), asymptotic stability referred to spatial attenuation of spacing errors.[16, 32] The analysis of string stability relies on controllers designed based on feedback linearization of the vehicle model or Jacobi linearization of the vehicle model. This allows for analysis of propagation of spacing errors using Laplace transformations of the relevant signals. Let $E_i(s)$ denote the Laplace transformation of the spacing error $e_i(t)$; furthermore, let $a_0(t)$ be the acceleration of the lead vehicle with its Laplace transformation denoted by $A_0(s)$. Let $H(s)$, $G(s)$ denote proper rational transfer functions. For appropriate $H(s)$, $G(s)$ that are determined by the control laws employed by the AVFS, the error propagation equations associated with a single-vehicle look-ahead control system may be described using Laplace transformation as:

$$E_1(s) = G(s)A_0(s), \quad E_i(s) = H(s)E_{i-1}(s), \quad \forall i \geq 2. \tag{10}$$

It is usually the case that the poles of the transfer function $G(s)$ and the error propagation transfer function $H(s)$ are identical and they can be designed to ensure that every pole has a negative real part. For a single-vehicle look-ahead, the error propagation transfer function is constrained, so that $H(0) = 1$.

Fenton and Bender[16, 32] use the velocity $v_0(t)$, of the lead vehicle as the input to transfer function $G(s)$ in place of $a_0(t)$; they provide a frequency domain condition $\|G(jw)\|_\infty \leq 1$ for non-amplification of any velocity disturbances from the lead vehicle to the first following vehicle. In the subsequent literature, for instance[15, 18, 33–35] use $\|H(jw)\|_\infty \leq 1$ to specify the attenuation of spacing errors. Utilizing this condition as a definition for string stability still has some shortcomings:

- Error propagation with controllers utilizing feedback from multiple vehicles has different structure of error propagation equations than Eq. (10). Hence, this definition will not hold in this scenario both in theory and practice.
- Even for a single-vehicle look-ahead scenario, one must show that $\|H(jw)\|_\infty \leq 1$ leads to a vehicle platoon satisfying string stability according to its $\epsilon - \delta$ definition above. From Linear System Theory,[36] $\|H(jw)\|_\infty$ norm relates the two-norm of the output, $\|e_i(t)\|_2$, with the two-norm of the input, $\|e_{i-1}(t)\|_2$, using the following inequality:

$$\|e_i(t)\|_2 \leq \|H(jw)\|_\infty \|e_{i-1}(t)\|_2.$$

While $\|H(jw)\|_\infty \leq 1$ ensures that the two norms of spacing errors do not amplify, from a safety point of view, what is required is a relation that ensures that the maximum spacing errors do not amplify for collision avoidance, i.e.,

$$\|e_i(t)\|_\infty \leq \|e_{i-1}(t)\|_\infty.$$

It is known from Ref.[36] that

$$\|e_i(t)\|_\infty \leq \|h(t)\|_1 \|e_{i-1}(t)\|_\infty,$$

where $h(t)$ is the unit impulse response of the transfer function $H(s)$. For any natural number p, it is also known that if γ_p is the pth induced norm of the error propagation transfer function, $H(s)$, that is

$$\gamma_p := \sup_{\|e_{i-1}\|_p=1} \|e_i(t)\|_p,$$

then

$$\int_0^\infty h(\tau)d\tau = H(0) \leq \gamma_2 = \|H(jw)\|_\infty \leq \gamma_p \leq \|h(t)\|_1.$$

Either (a) one must design the error propagation transfer function, $H(s)$ to have its unit impulse response, $h(t) \geq 0$, so that $\|h(t)\|_1 = H(0) = 1$, thereby guaranteeing $\|e_i(t)\|_\infty \leq \|e_{i-1}(t)\|_\infty$ or (b) use mixed-norm bounds, specifically, those that relate $\|e_{i-1}(t)\|_2$ to $\|e_i(t)\|_\infty$. Sheikholeslam and Desoer[15] specify the desirability of $h(t) \geq 0$. For a general plant, it is difficult to ascertain if $h(t)$ can be made non-negative with the choice of controller parameter gains that affect the coefficients of the numerator and denominator polynomials of the error propagation transfer function $H(s)$; this problem is tied closely to the *open* problem of transient control in linear control systems. If $\|H(jw)\|_\infty < 1$ (and this can be the case if the lead vehicle's position information is available and appropriately fed back to every vehicle in the string), then one can make use of the inequalities that relate the $\|h(t)\|_1$ with $\|H(jw)\|_\infty$[37, 38] through the McMillan degree, n, of $H(s)$:

$$\|h(t)\|_1 \leq (2n+1)\|H(jw)\|_\infty.$$

If one were to consider the head-to-tail transfer function,[38] $\frac{E_i(s)}{E_1(s)} = H^{i-1}(s)$, its McMillan degree will increase linearly (as the number of states for its realization will increase linearly), while $\|H^{i-1}(jw)\|_\infty = \|H(jw)\|^{i-1}$ decreases geometrically. Combining, one can conclude that

$$\|e_i\|_\infty \leq (2ni+1)\|H(jw)\|_\infty^{i-1}\|e_1\|_\infty.$$

However, this method can only provide some sort of assurance for non-amplification of spacing errors if $\|H(jw)\|_\infty < 1$ and certainly, not when $H(0) = 1$.

Only recently, Vegamoor and Darbha[39] explored the option (b): under the reasonable assumptions that the acceleration of the lead vehicle is (i) bounded and (ii) square integrable, results from Ref.[36] assure that spacing error in any following vehicle is bounded temporally and is squared integrable, i.e., $\|e_i(t)\|_\infty$ and $\|e_i(t)\|_2$ are bounded; furthermore, that $\|e_i(t)\|_2 \leq \|e_{i-1}(t)\|_2$ for $i \geq 2$. However, from Corless, Zhu, and Skelton,[40] one can show that there is a $K, L > 0$, such that $\|e_i\|_\infty \leq \|e_{i-1}\|_2$ and $\|e_1(t)\|_\infty \leq L\|a_0(t)\|_2$. This would in turn imply that for all $i \geq 2$:

$$\|e_i(t)\|_\infty \leq K|e_{i-1}|_2 \leq K\|e_1\|_2 \leq KL\|a_0(t)\|_2,$$

indicating that the maximum spacing error does not amplify spatially.

In general, if $\mathcal{S}_i$ is the set of vehicles whose information is available and fed back by the ith vehicle in the platoon, the error propagation will be of the form:

$$E_i(s) = \sum_{j\in\mathcal{S}_i} H_j(s)E_j(s) + W_i(s)D_i(s),$$

where $W_i(s)$ relates how the disturbance, $D_i(s)$, influences the spacing error in the ith vehicle. The lead vehicle's acceleration can be treated as a disturbance and lumped with individual disturbances acting on the system in defining $D_i(s)$. One can associate an information flow graph, where vehicles in the string serve as nodes and the directed edge (i, j) is in the allowable set of edges in the graph if $j \in \mathcal{S}_i$. The propagation of spacing errors is not only influenced by the underlying information flow graph but also dependent on the spacing policy employed by CAVs.

Information flow graphs studied initially in the literature concerned with $\mathcal{S}_i = \{i-1, i\}$ (triangular or look-ahead structure[16, 26]). Peppard et al.[33] considered the case $\mathcal{S}_i = \{i-1, i, i+1\}$. Until the California PATH program, V2V communication capability was virtually absent and information was primarily sensed from neighboring vehicles ,and hence, the information flow graphs were naturally restricted. Swaroop and Hedrick[18, 41] were the first to generalize the structure to $\mathcal{S}_i = \{i-r \leq j \leq i\}$. Fax and Murray[42] considered general architectures, including the symmetric architectures. For symmetric architectures, Graph Laplacians were first introduced for analysis of rigid formations in Ref.[42].

3.1 Methods of Analysis of String Stability

The problem of string stability has a temporal variable and a spatial variable. Consequently, string stability can be analyzed by transforming the errors with respect to the spatial variable first and considering the resulting differential equation (this approach is taken in Refs.[26,43]. One can also transform the temporal variable along with the spatial variable and adopt a 2-D polynomial approach to study string stability, as in Refs.[44,45] based on the results of Kamen's work in the collection.[46] Essentially, this approach implies the following polynomial, $\Delta(s,z)$, to be Hurwitz for every $z : |z| = 1$ for the system to be Bounded Input Bounded Output (BIBO) stable:

$$\Delta(s,z) := z^r - \sum_{j=1}^{r} H_j(s) z^{r-j}.$$

Another approach is to use Laplace transformation to transform the temporal signals and analyze the sequence of signals—the approach that has already been introduced tacitly in this paper. This approach has the advantage that it can be generalized to analyzing the string stability of interconnected non-linear systems[18] through the converse stability theorems; one often is left with difference inequalities as opposed to difference equations with this approach, but comparison principle[47] can be employed to make the desired inference.

4 Design of AVFS

The design of an AVFS requires a spacing policy, i.e., the specification of desired following distance as a function of vehicle's speed. Spacing policies are of two types: (a) the Constant Spacing (CS) policy, where the desired following distance does not vary with vehicle's speed, and (b) a variable spacing policy, where the desired following distance changes with vehicle's speed. Of the variable spacing policies, the Constant-Time Headway (CTH) policy is the most widely considered in the literature; in this policy, the desired following distance varies linearly with vehicle's speed and the proportionality constant is referred to the time headway or time gap.

The choice of a spacing policy can have wide ramifications on traffic safety and mobility; not only will the spacing policy dictate the information that should be available to a controlled vehicle to guarantee string stability, but also will influence the dynamics and stability of the movement of traffic, thereby traffic mobility. When studying traffic dynamics, there are two approaches that have traditionally been used—the microscopic or car-following approach, much like the design of AVFS, and the macroscopic or traffic flow modeling approach. In the former approach, one models the car-following behavior of traffic, and in the later, one looks at traffic at a larger spatial scale and is interested in the aggregate dynamics of vehicles in a section of a highway. Steady-state behavior of traffic at the macroscopic level is often characterized by Fundamental Traffic Characteristic (FTC) in civil engineering literature; essentially, this curve relates how the following distance between vehicles (reciprocally related to "traffic density") affects the speed and, hence, the traffic throughput in a section.

The connection between these two approaches for traffic consisting of manually driven vehicles was first reported by Gazis, Herman, and Potts;[48] essentially, they were interested in reconciling the empirical traffic data of Greenberg[49] with possible "follow-the-leader" microscopic models for manual driving. Clearly, in the case of designing AVFS, the specification of spacing policy is equivalent to specifying the FTC and directly affects traffic mobility and operations; this connection was *first* explicitly made by Darbha and Rajagopal[50] and poses an inverse problem of designing a variable spacing policy for AVFS based on a traffic specification;[50,51] subsequently, the problem of spacing policy design has been revisited by other researchers.[52,53] We will briefly review the control algorithms for AVFS in the following subsections.

4.1 Control Architectures for a Constant Spacing Policy

Let L_0 be sum of the car length and the constant desired following distance. Let $x_i(t), x_0(t)$ denote the respective positions of the ith following vehicle and the lead vehicle in a string of vehicles traveling in a straight lane. This yields the following definition of spacing error:

$$e_i(t) := x_i(t) - x_0(t) - L_0.$$

4.1.1 PD Algorithm

The simplest control law for an AVFS[33,54] for maintaining a constant spacing is given by:

$$u_i(t) = -k_p e_i(t) - k_v \dot{e}_i(t).$$

Note that the measurements of $e_i(t), \dot{e}_i(t)$ can be obtained from on-board sensors. Since vehicles are coupled dynamically by feedback, spacing errors propagate; the errors propagate according to the following evolution equations for $i \geq 2$:

$$\ddot{e}_i + k_v\dot{e}_i + k_p e_i = k_v e_{i-1} + k_p e_{i-1},$$

and using Laplace transformation, one can show that

$$H(s) := \frac{E_i(s)}{E_{i-1}(s)} = \frac{k_v s + k_p}{s^2 + k_v s + k_p},$$

and

$$|H(jw)|^2 = \frac{k_p^2 + k_v^2 w^2}{(k_p - w^2)^2 + k_v^2 w^2}.$$

If the lead vehicle was to perform a sinusoidal acceleration at a sufficiently low frequency (which is usually the case in highway driving conditions), the error in the first following vehicle will also vary with the same low frequency. From the error propagation transfer function, it is clear that for a sufficiently small w, the denominator is smaller than the numerator, implying that the amplitudes of oscillation of the spacing error will amplify geometrically from one vehicle to another, leading to a pileup. Caudill and Garrard[54] were the first to provide a proof of string instability with this scheme.

First-order model for a vehicle is inappropriate for studying spacing error propagation:

$$\dot{x}_i = u_i.$$

Since spacing error can be fed back, it is clear that

$$u_i = -k_p e_i,$$

one gets an error propagation transfer function with just the information of the preceding vehicle to be:

$$H(s) = \frac{k_p}{s + k_p}.$$

Clearly, this model would imply that string stability can be guaranteed for constant spacing policy with just on-board sensing information.

4.1.2 Dynamic Feedback of Spacing and Velocity Errors

Seiler, Pant, and Hedrick[29] and later, Khatir and Davison[55] considered dynamic feedback of the form:

$$U_i(s) = -K(s)E_i(s),$$

where the control input for the ith following vehicle is the output of a controller transfer function with $E_i(s)$ as its input and $K(s)$ is a proper, rational controller transfer function. While Seiler et al.[29] use Bode-integral to show that this scheme cannot lead to string stability; the approach by Khatir and Davison[55] is simpler. The error propagation transfer function in this case is:

$$H(s) = \frac{E_i(s)}{E_{i-1}(s)} = \frac{K(s)}{s^2 + K(s)},$$

and

$$|H(jw)|^2 = \frac{K_r(w)^2 + w^2 K_i(w)^2}{(K_r(w^2) - w^2)^2 + w^2 K_i^2 w_i^2},$$

where $K(jw) = K_r(w) + jwK_i(w)$. It is not difficult to see that $K_r(0) > 0$ and $K_i(0) > 0$ for stability. Consequently, for a sufficiently small w, $|H(jw)|^2 > 1$ leading to string instability.

4.1.3 Semi-autonomous Control

With communication devices or with other means of estimating the acceleration of the preceding vehicle, Swaroop and Hedrick[38] consider the following control law for AVFS:

$$u_i = k_a\ddot{x}_{i-1} - k_v\dot{e}_i - k_p e_i.$$

A schematic of such an AVFS is shown in Fig. 3, where the dotted lines indicate flow of information via wireless channels. In this case, $\ddot{x}_{i-1}$ is received by each vehicle i. With this control law, spacing errors propagate as

$$H(s) = \frac{E_i(s)}{E_{i-1}(s)} = \frac{k_a s^2 + k_v s + k_p}{s^2 + k_v s + k_p}.$$

It is clear that $k_a \in (0, 1]$ for $\|H(jw)\| \leq 1$; moreover, if $k_a \neq 1$, at low frequencies, the numerator of $\|H(jw)\|^2$ will exceed that of the denominator as $(k_p - k_a w^2) > k_p - w^2$. When $k_a = 1$, $H(s) \equiv 1$; however, if one considers a singularly perturbed model of a vehicle, it is clear that

$$H_p(s) = \frac{E_i(s)}{E_{i-1}(s)} = \frac{s^2 + k_v s + k_p}{\tau s^3 + s^2 + k_v s + k_p}.$$

Even in this case, for any $\tau > 0$ (needed for stability and typical of any parasitic "stable" actuation), $|H_p(jw)|^2 > 1$ for a sufficiently low frequency w as $k_v > k_v - \tau w^2$. This case was the primary motivation for the introduction of singular perturbation theory in the analysis of robustness in string stability by Swaroop.[19,56]

4.1.4 Control with Information from Multiple Vehicles Ahead

If every vehicle has information of r preceding vehicles in traffic, then consider the following control law:

$$u_i = \sum_{j=1}^{r}(k_{aj}\ddot{x}_{i-j} - k_{vj}(\dot{x}_i - \dot{x}_{i-j}) - k_{pj}(x_i - x_{i-j} + L_{0j}),$$

where the control gains k_{aj}, k_{vj}, k_{pj} are to be designed. In this case, if the initial spacing errors are zero and the lead vehicle performs a maneuver, the spacing errors for $i > r$ propagate according to:

$$E_i(s) = \sum_{j=1}^{r} \frac{k_{aj}s^2 + k_{vj}s + k_{pj}}{s^2 + \left(\sum_{j=1}^{r} k_{vj}\right)s + \sum_{j=1}^{r} k_{pj}} E_{i-j}(s).$$

An information flow diagram for a two-vehicle lookup ($r = 2$) platoon is shown in Fig. 4 as an example.

Numerical simulations in Swaroop and Hedrick[38] seem to indicate that errors will amplify in large strings geometrically; only recently has this scheme been shown to be string unstable;[57] a generalization of the same result where the filtered versions of the spacing errors are fed back also leads to the same conclusions.[58]

4.1.5 Control with Lead and Preceding Vehicle Information

A general form of the control law with such a scheme is as follows[18]:

$$u_i = k_a\ddot{x}_{i-1} + k_l\ddot{x}_0 - k_v\dot{e}_i - k_p e_i$$
$$- c_v(\dot{x}_i - \dot{x}_0) - c_p(x_i - x_0 + iL_0),$$

where the gains k_a, k_l, k_v, k_p, c_p, c_v must be chosen for the specific AVFS. This is illustrated in Fig. 5. With this scheme, the error propagation obeys:

$$H(s) = \frac{E_i(s)}{E_{i-1}(s)} = \frac{k_a s^2 + k_v s + k_p}{s^2 + (k_v + c_v)s + (k_p + c_p)}.$$

To avoid the problems of string instability, Shladover took inspiration from "skyhook" damping in automotive suspensions[59, 60] and introduced lead vehicle's velocity feedback ($c_v > 0$). Shladover then demonstrated the attenuation of spacing errors using numerical simulations. Sheikholeslam and Desoer[15] demonstrated that it is possible to choose control gains k_a, k_v, c_v, k_l, k_p that render $\|H(jw)\|_\infty = 1$, so that spacing errors can be guaranteed not to amplify; in this work, the lead vehicle position information is not fed back, i.e., $c_p = 0$. In Ref.[18], Swaroop considered the case $c_p > 0$ and demonstrated geometric attenuation of spacing errors. Specifically, with lead vehicle velocity information, it is possible to design the impulse response of the transfer function to be non-negative,[56] and hence, $H(0) = \|h(t)\|_1 = 11$; with lead vehicle's position information, it can be further improved to $H(0) = \|h(t)\|_1 = \frac{k_p}{k_p + c_p} < 1$.

This control law was implemented in the NAHSC mandated Automated Highway Systems demonstration of California PATH held at San Diego in 1997.[61]

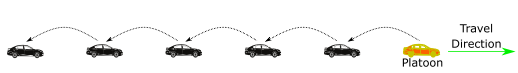

Figure 3: AVFS using information from one vehicle ahead.

4.1.6 Control Laws Incorporating Information of Following Vehicles

Peppard[33] considered a feedback of information of the preceding vehicle and following vehicle; however, it is not clear from this work which spacing policy was being adopted. Seiler, Pant, and Hedrick[29] revisited the problem of maintaining a constant following distance with information from the preceding and immediately following vehicles (as illustrated in Fig. 6):

$$u_i = -k_{vf}(\dot{x}_i - \dot{x}_{i-1}) - k_{pf}(x_i - x_{i-1} + L_0)$$
$$- k_{vb}(\dot{x}_i - \dot{x}_{i+1}) - c_{vb}(\dot{x}_i - \dot{x}_{i+1})$$
$$- c_{pb}(x_i - x_{i+1} - L_0)).$$

Using BIBO stability arguments, Seiler et al.[29] showed that a unit disturbance on every vehicle leads to a maximum spacing error that increases

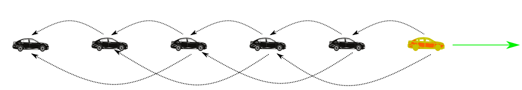

Figure 4: AVFS using information from two vehicles ahead.

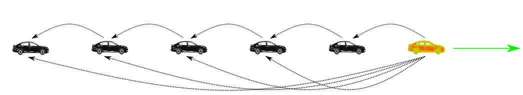

Figure 5: AVFS using information from platoon leader as well as from one vehicle ahead.

Figure 6: AVFS using information from immediately preceding and following vehicles.

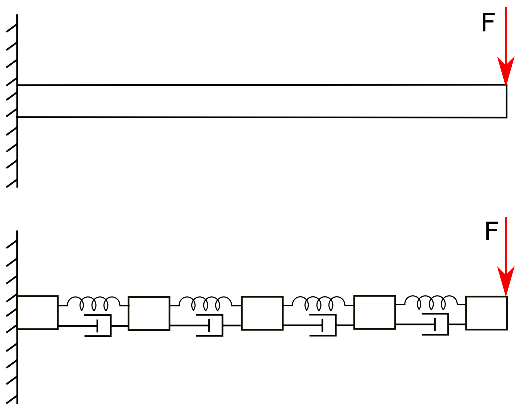

Figure 7: A discretized cantilever beam as an analogy for AVFS using information from following and preceding vehicles.

with the size of the string. The basic intuition is provided in Refs.[30, 31, 62]: if one were to take a spatially discretized cantilever beam (by taking a unit mass corresponding to a specified unit length of the beam), as shown in Fig. 7, then the resulting system resembles a vehicle follower system; the spring constants of the springs connecting masses are analogous to the position feedback gains, and the damper coefficients correspond to velocity feedback gains. Clearly, as the length of the beam increases, a unit force at the tip results in an increasing deflection at the tip; similarly, as the number of masses (vehicles) increases, the maximum spacing error also increases. The mechanism of this instability is that the lowest non-zero natural frequency of the discretized structure goes to zero as the size increases.

For general symmetric information flow graphs, where $j \in \mathcal{S}_i \iff i \in \mathcal{S}_j$, Fax and Murray[42] introduced Graph Laplacians for analyzing the stability of vehicle formations. A Graph Laplacian is analogous to a stiffness matrix in structural engineering literature and the algebraic connectivity of a graph is analogous to the lowest non-zero natural frequency of the structure. Using the basic intuition about the mechanism for string instability, Yadlapalli et al. were the *first* to demonstrate the limitations of symmetric information flow graphs for maintaining rigid formations.[30] The results in this paper imply that unless sufficient number of following vehicles (as a non-trivial percentage of vehicles in the string) have the lead or reference vehicle's information, spacing errors in maintaining the desired constant following distance will always increase with the size of the collection. Consider a bridge—unless there are sufficient props from the ground, as the length of the bridge increases, the maximum deflection will increase. The results in Yadlapalli et al.[30] provide insights into the structure of the controllers: if a controller takes aggregate error (computed from communicated/sensed information) as an input, that is

$$U_i(s) = -C(s) \sum_{j \in \mathcal{S}_i} E_j(s),$$

then $C(s)$ cannot be strictly proper, or must not incorporate integral action or must not have any non-minimum phase zero. In each of these cases, if one chooses $C(s)$, there is a limit on the size of string beyond which the motion of the vehicles in the string becomes unstable! Darbha and Pagilla[63] rule out symmetric information flow graphs for use in maintaining rigid formations for unmanned vehicles (or CAVs obeying a constant spacing policy) from practical considerations.

Results similar to Refs.[30, 31, 62] can also be found in subsequent papers.[64–67]

4.2 Control Architectures for a Constant-Time Headway Policy

Time headway was first considered in the operational analysis of traffic on highways by Pipes.[68] The advantage of employing this policy is that one can guarantee string stability with just on-board information.[35, 54, 56] Let h be the desired time headway, so that the error in spacing can be written as:

$$e_i = x_i - x_{i-1} + L_0 + h\dot{x}_i.$$

4.2.1 Adaptive Cruise Control (ACC)/Autonomous Intelligent Cruise Control (AICC)

The associated feedback control law may be given by:

$$u_i = -k_p e_i - k_v(\dot{x}_i - \dot{x}_{i-1}).$$

Note that e_i and $\dot{x}_i - \dot{x}_{i-1}$ can be computed/obtained from measurements using on-board sensors. It was hence referred to as autonomous intelligent cruise control (AICC) in Ref.[35]; nowadays, this control law is implemented in ACC systems. The spacing errors in this case propagate as:

$$H(s) := \frac{E_i(s)}{E_{i-1}(s)} = \frac{k_v s + k_p}{s^2 + (k_v + hk_p)s + k_p}.$$

It is shown in Ref.[56] that the impulse response of this transfer function can be made non-negative, rendering $1 = H(0) = \|h(t)\|_1$, and the control law is effective in ensuring string stability.

Since the vehicle model was based on instantaneous actuation and the feedback is based on instantaneous sensing and the processing of sensing data is also assumed instantaneous, it will be prudent to check the robustness in string stability with a simple parasitic model for actuation which combines all the lags.[56] In such a case, the error propagation evolves according to:

$$H_p(s) = \frac{E_i(s)}{E_{i-1}(s)} = \frac{k_v s + k_p}{\tau s^3 + s^2 + (k_v + hk_p)s + k_p},$$

where τ is the unmodeled lag and one may assume that $\tau \le \tau_0$, and τ_0 is a known upper bound on the lag. In Refs.[18,69], it is shown that it is possible to pick k_p, k_v, so that $\|H_p(jw)\|_\infty = 1$ if $h \ge 2\tau_0$.

For example, Figs. 8 and 9 show the effect of selecting a time headway on either side of the minimum bound. For these simulations, the maximum parasitic lag in the platoon was taken as 0.5 s (i.e., $\tau_0 = 0.5$ s and the minimum bound, $2\tau_0 = 1$ s). The simulation entails the situation where the lead vehicle is initially moving at a constant speed, and at $t = 10$ s, performs a hard braking maneuver for 1 s and resumes constant velocity for the remaining part of the simulation. Gains (k_v, k_p) were set to $(0.8, 1)$. We can see that when the platoon leader performs the braking maneuver, spacing errors e_i diminish across the platoon in Fig. 9, since the time headway selected meets the minimum bound for string stability.

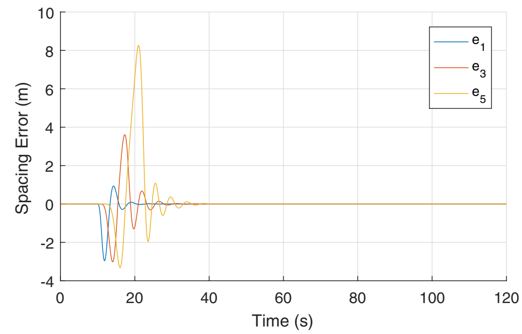

Figure 8: Spacing error plot for an unstable ACC platoon with time headway 0.7 s ($h < 2\tau_0$).

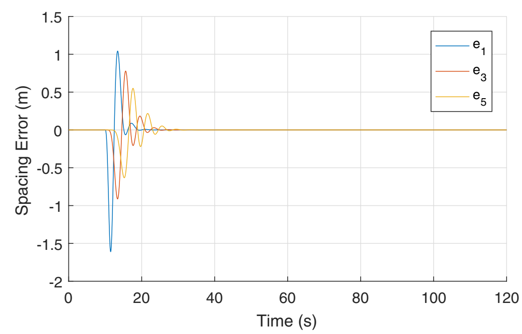

Figure 9: Spacing error plot for a stable ACC platoon with time headway 1.2 s ($h > 2\tau_0$).

4.2.2 Cooperative Adaptive Cruise Control (CACC)/Semi-autonomous Adaptive Cruise Control

The switch in focus of automotive companies from constant spacing to constant-time headway policy in their AVFS was necessitated by lack of adequate communication/connectivity capability in the late 1990s. However, the lower bound on h by unmodeled lags in the system limits how close vehicles can follow each other, thereby limiting mobility; for example, if the unmodeled lags total 500 ms, a minimum time headway of 1 second must be employed to ensure string stability. However, this would translate to roughly 30 m at 66 mph, a typical highway speed. Trucks need to operate at a time headway smaller than 1 s ($\approx$ 0.6 –0.7 s) to take advantage of aerodynamic drafting to reduce fuel consumption. Shladover[70–72] showed both in simulations and experiments that available preceding vehicle's acceleration can aid in string stability at lower time headways. The analytical bound on how small the time headway can be was supplied by Darbha, Konduri, and Pagilla.[73,74]

Associated with this control scheme, the control law is given by:

$$u_i = k_a \ddot{x}_{i-1} - k_p e_i - k_v(\dot{x}_i - \dot{x}_{i-1}).$$

The error propagation transfer function with perturbed vehicle dynamics is given by:

$$H_p(s) = \frac{E_i(s)}{E_{i-1}(s)} = \frac{k_a s^2 + k_v s + k_p}{\tau s^3 + s^2 + (k_v + hk_p)s + k_p}.$$

In Ref.[75], it is shown that for any given $k_a \in [0, 1)$, there exist control gains k_p, k_v, such that $\|H(jw)\|_\infty = 1$ if $h \ge \frac{2\tau_0}{1+k_a}$. In other words, one can guarantee string stability with a lower time headway for a string of vehicles employing CACC systems than with a string of vehicles employing ACC systems in their AVFS. Roughly, one can cut the time headway by a factor of half with CACC

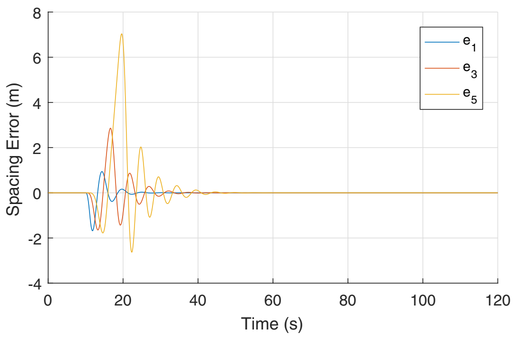

Figure 11: Spacing error plot for an unstable CACC platoon with time headway 0.4 s ($h < \frac{2\tau_0}{1+k_a}$).

and this is a quantification of the benefit of additional connectivity.

For example, using the earlier simulation setup with $\tau_0 = 0.5$ s, we can achieve string stability even with a time headway of 0.7 s, as shown in Fig. 10. However, if the time headway does not meet the minimum requirements of $\frac{2\tau_0}{1+ka} = 0.67$ s, stability may not be achieved (as shown in Fig. 11, with time headway of 0.4 s). Here, k_a was chosen to be 0.5.

The usage of preceding vehicle's acceleration in ACC systems was initially christened Semi-autonomous Adaptive Cruise Control and was first considered in Ref.[76] However, the result here seemed to suggest that the lower bound for time headway can be made arbitrarily small by the use of acceleration feedback of the controlled vehicle, that is

$$u_i = -k_1\ddot{x}_i + k_a\ddot{x}_{i-1} - k_v(\dot{x}_i - \dot{x}_{i-1}) - k_p(x_i - x_{i-1} + L_0 + h\dot{x}_i).$$

If we define $\bar{\tau} = \frac{\tau}{1+k_1}$, $\bar{k}_a - \frac{k_a}{1+k_1}$, $\bar{k}_v = \frac{k_v}{1+k_1}$, $\bar{k}_p = \frac{k_p}{1+k_1}$, the closed loop error propagation with this scheme is given by:

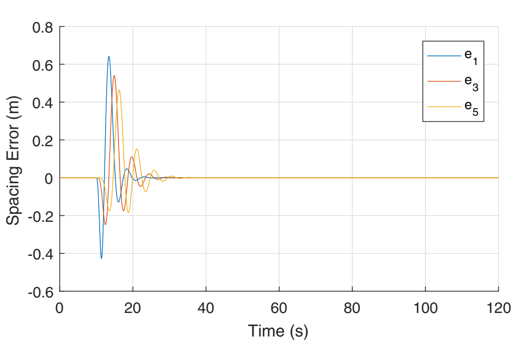

Figure 10: Spacing error plot for a stable CACC platoon with time headway 0.7 s ($h > \frac{2\tau_0}{1+k_a}$).

$$H_p(s) = \frac{\bar{k}_a s^2 + \bar{k}_v s + \bar{k}_p}{\bar{\tau}s^3 + s^2 + (\bar{k}_v + h\bar{k}_p)s + \bar{k}_p}.$$

Using the earlier result from Konduri et al.[75], it follows that:

$$h \geq \max_{\tau\in[0,\tau_0]} \frac{2\bar{\tau}}{1+\bar{k}_a}.$$

or equivalently:

$$h \geq \frac{2\tau_0}{1+k_1+k_a}.$$

Since there is no restriction on k_1 other than the restrictions for stability, $k_1 \geq 0$ could be chosen to arbitrarily reduce the bound. However, this seems to contradict what was established in Refs.[18, 29, 55] Clearly, when $h = 0$, the desired following distance does not vary with vehicle's speed; in this case, it is a constant spacing policy. In this case, semi-autonomous control does not lead to string stability. As $h \to 0$, this case should lead to the same conclusion. This can be inferred if a higher order perturbation was to be used or a delay (or even possibly, a lag) included in the measurement and communication of preceding vehicle's acceleration. Such a perturbed model will limit the values of k_1 (and consequently, k_a) and subsequently, limit h. A plausible argument against the use of controlled vehicle's acceleration feedback is as follows: For a non-zero but small delay, $\Delta > 0$, in the acceleration measurement, one obtains a quasi-polynomial (in the denominator of $H_p(s)$) as:

$$\tau s^3 + (1 + k_1 e^{-s\Delta})s^2 + (k_v + hk_p)s + k_p.$$

Stability when $\Delta = 0$ requires all the coefficients of the above quasi-polynomial to be of the same sign and

$$\tau k_p > (1 + k_1)(k_v + hk_p),$$

thereby limiting k_1. Moreover, from pages 104 to 106 of Ref.[77], instability sets in if $k_1 \notin [-1, 1]$ and can be even smaller for string instability. This can be a potential drawback in the scheme adopted by Bian et al.[78] as acceleration of the controlled vehicle is fed back.

4.2.3 Next-Generation Cooperative Adaptive Cruise Control (CACC+) Systems

Advances in connectivity can be exploited to enhance mobility and safety by incorporating information of multiple vehicles ahead of a controlled vehicle. For example, consider the following control scheme for a CACC+ system:[73, 74]

$$u_i = \sum_{l=1}^{r} \{ k_{al}\ddot{x}_{i-l} - k_{vl}(\dot{x}_i - \dot{x}_{i-l}) \\ -k_{pl}(x_i - x_{i-l} + lL_0 + lh\dot{x}_{i-l}) \}.$$

With this control law, spacing errors propagate as:[73, 74]

$$E_i(s) = \sum_{l=1}^{r} H_l(s) E_{i-l}(s),$$

where

$$H_l(s) = \frac{k_{al}s^2 + k_{vl}s + k_{pl}}{\tau s^3 + s^2 + \sum_{l=1}^{r}(k_{vl} + lhk_{pl})s + \sum_{l=1}^{r} k_{pl}}.$$

In particular, if $k_{aj} = k_a, k_{vj} = k_v, k_{pj} = k_p$, then

$$H(s) = H_l(s) = \frac{k_a s^2 + k_v s + k_p}{\tau s^3 + s^2 + \left(rk_v + \frac{r+1}{2}hk_p\right)s + rk_p},$$

and

$$E_i(s) = H(s) \sum_{l=1}^{r} E_{i-l}(s).$$

In this case, it is sufficient that $\|rH(jw)\|_\infty = 1$ for guaranteeing string stability; by letting $\bar{h} = \frac{r+1}{2}$, $K_a = rk_a, K_v = rk_v, K_p = rk_p$, one can exploit the headway bound for CACC to derive a bound for this CACC+ case: $rk_a \in [0, 1]$ and

$$\bar{h} \geq \frac{2\tau_0}{1 + rk_a} \Rightarrow h \geq \frac{4\tau_0}{(1+r)(1+rk_a)}.$$

In this case, it is clear that the bound for h decreases with r in the presence of ideal communication of information of preceding vehicles in the string. This benefit of connectivity was first reported by Darbha, Konduri, and Pagilla,[73, 74] and later, extensions of this result appeared in Ref.[78] An additional problem with Ref.[78] is that the reduction in headway is intimately tied to the value of $k_a \neq 0$; if controlled vehicle's acceleration may not be fed back, this guarantee fades away.

4.2.4 CACC with Imperfect Communication

A natural question that arises concerns how small the time headway can be when the communicated information is received in a distorted manner or not at all. If one were to model packet reception as a Bernoulli process with a packet reception probability of p, then in the extreme limits of $p = 1$, we have a lower bound for time headway of $\frac{2\tau_0}{1+k_a}$, and when $p = 0$, the corresponding lower bound is $2\tau_0$. It is shown in Ref.[79] that $h \geq \frac{2\tau_0}{1+pk_a}$ in this case, using a deterministic equivalent of the stochastic system.

For example, even if a platoon with $\tau_0 = 0.5$ s and $k_a = 0.5$ were to implement a time headway of 0.7 s $> (\frac{2\tau_0}{1+k_a})$, the platoon is observed to be unstable if packets are only received with a probability of 0.5 (see Fig. 12). However, if the time headway is readjusted to account for the expected packet reception rate (in this case, $p = 0.5$ and the minimum $\frac{2\tau_0}{1+pk_a} = 0.8$), then string stability can be regained, as shown in Fig. 13, with a time headway of 0.9 s.

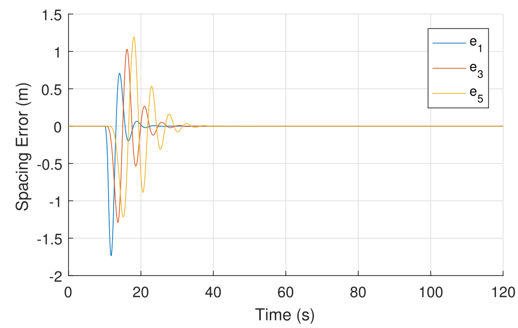

Figure 12: Spacing error plot for an unstable CACC platoon with time headway 0.7 s ($h > \frac{2\tau_0}{1+k_a}$, but $h < \frac{2\tau_0}{1+pk_a}$), operating under 50% packet reception.

4.3 Traffic Heterogeneity

Heterogeneity in vehicle strings has recently been gaining a significant attention.[80–85] Vehicle model represented by (7) or (8) makes any collection of structurally heterogeneous vehicles to obey the identical dynamics for analyzing vehicle motion under nominal maneuvers via feedback linearization. Consumers are usually given a choice of time headway from a small but discrete set (typically ranging from 0.5 to 2 s). If the vehicles are made by the same manufacturer and are of the same make, a basic problem to be understood is: how does heterogeneity in the choice of headway settings affects propagation of errors?

The distinguishing feature of the work in Ref.[86] is in its consideration of heterogeneity of traffic arising from drivers adopting different time headway even if they drive identical vehicles. The main result of this paper is that if there is a way to organize vehicles in a string, so that the time headway employed by vehicles in the string is monotonically decreasing, then string stability can be guaranteed. Simulation of results is also presented for cases where the ordering is different from this case.

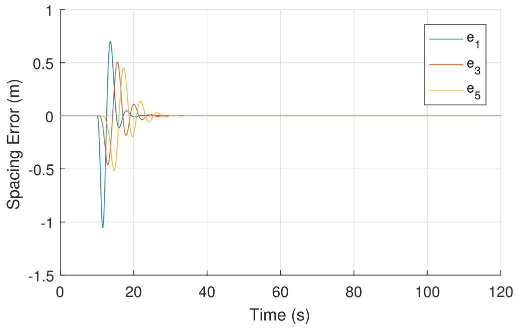

Figure 13: Spacing error plot for a stable CACC platoon with time headway 0.9 s ($h > \frac{2\tau_0}{1+pk_a}$), operating under 50% packet reception.

Preliminary results from Ankem[86] (a sample of it is shown in Fig. 14) on a heterogeneous collection of CAVs (consisting of CAVs equipped with ACC and others with CACC and employing different values of time headway) seem to suggest that attenuation of spacing errors is possible if the time headways form a *monotonically decreasing sequence* in the string and the worst-case amplification occurs if the time headway employed by vehicles forms a *monotonically increasing sequence*. Since the time headway values are chosen from a finite and bounded set, maximum spacing errors geometrically attenuate spatially at the tail of the string if the size of the string is sufficiently large. Hence, it seems sufficient to consider strings of small size (of the same order as the number of headway settings) to find if the order of vehicles in the strings contributes to a large spacing error.

At every stage of deployment of CAVs, one must contend with heterogeneity arising from vehicles of the same make but of different functionality. For example, vehicles equipped with ACC systems versus those equipped with CACC+ systems. This is a first step towards understanding the more complicated mixed traffic consisting of human driven vehicles and CAVs.

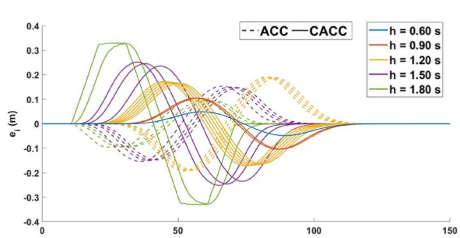

Figure 14: String stability with heterogeneous CAVs.

5 Traffic Modeling with CACC+ Systems

Traffic dynamics, when specified at the granularity of each vehicle, constitutes microscopic traffic models (or car-following models). As can be seen in a traffic consisting of CAVs, the microscopic traffic dynamics can be engineered and it will influence the macroscopic behavior of traffic. A natural question arises as to how the microscopic dynamics of traffic affects its macroscopic description? This question is important to understand the shift/change in traffic dynamics due to the deployment of CAVs and take appropriate traffic management actions. The question of relationship between macroscopic and microscopic descriptions of traffic was first broached for manually driven traffic in Gazis, Hermann, and Potts;[48] essentially, they fit a non-linear spacing policy for the microscopic models that matches the macroscopic "steady-state" observations in the form of a Fundamental Traffic Characteristic (FTC); since spacing policy is a design variable, Darbha and Rajagopal[50] related the spacing policy design with achievable FTC and posed an inverse problem of specifying the spacing policy to suit traffic flow specifications in the steady state. However, understanding the dynamics of traffic transients requires one to focus on the philosophy of modeling the flow of traffic.

The current traffic flow models even for CAVs are based on treating traffic as a continuum; usually, some modifications of the Navier–Stokes equation[87, 88] or Maxwell–Boltzmann equations[89, 90] are used to describe the macroscopic behavior of traffic. The principal objections to using these models are as follows:[91]

(i) Traffic does not consist of sufficiently large number of vehicles to be treated as a continuum.
(ii) Treating the traffic as a continuum seems to cause a problem with propagation of disturbances in the wrong direction, i.e., if vehicles in a section of a highway stop due to an accident or any other event, the velocities of both upstream and downstream sections get affected; this is counter to our observation with regard to the flow of traffic.
(iii) It is not clear why one should model CAVs or manually driven vehicles as a Navier–Stokes fluid or, for that matter, models that generalize Lighthill–Whitham–Richards (LWR) models[87, 88] or the Maxwell–Boltzmann's model based on kinetic theory of gases.

(iv) It is not clear how changes in the underlying information flow graph and associated control laws for CAVs get reflected in the macroscopic description of traffic consisting of CAVs.

In Ref.[91], Darbha and Rajagopal proposed a traffic model to overcome these limitations; the idea of this model is to select representative vehicles in a section and use their vehicle following laws as a representative for the evolution equations of traffic speed in the section; this serves as an analog of the balance of linear momentum in fluid-type models while directly reflecting the underlying automatic vehicle following behavior. As far as the accumulation of vehicles in a section is concerned, it is just the balance law for mass and is based on the number of vehicles of a representative type entering and leaving the section. Clearly, if the number of vehicles in a section increases, the corresponding average following decreases and it is reflected in an equation for average following distance on which the vehicle following behavior is based. This model seems to overcome the above-mentioned limitations and seems to be able to accommodate heterogeneity in traffic unlike the continuum-based models.

6 Traffic Safety

Metrizing traffic safety is important for assessing the safety benefits of CAVs; however, the task of metrizing traffic safety is difficult and can depend on the sequence of events under consideration. An emergency braking situation is relevant for AVFS in assessing safety metrics such as the probability, expected number, and severity of rear-end collisions in a string of vehicles equipped with ACC/CACC+ systems. These metrics can help one to compare the safety benefits of connectivity and different control laws.

It is intuitive that if all vehicles travel initially at the same velocity and brake at the same deceleration at the same time, there will not be any collision. This corresponds to the case when vehicles coordinate their braking either through communication and coordination or via vehicle following control laws. In the former case, every vehicle must be cognizant of its maximum deceleration and communicate this information to the lead vehicle, so that it brakes at the least value of the maximum deceleration of all vehicles in the string. In the latter case, the vehicle following control laws dictates how a vehicle must brake based on the available information. In both cases, if the commanded deceleration is within the maximum deceleration of all vehicles, then no collisions can occur if string stability is guaranteed and the stand-still separation is adequate. In the event that commanded deceleration exceeds the maximum deceleration of some vehicle in the string, string stability guarantees do not hold; heterogeneity in their decelerating capability will have a significant bearing on safety in an emergency braking situation.[86, 92]

In the Emergency Braking (EB) situation, all vehicles in a string have the same initial velocity but different decelerating capability. The maximum achievable deceleration can be different owing to a variety of factors such as the wear and tear of tires, condition of the road, and the make of vehicles. It is reasonable to assume that the maximum deceleration of every vehicle in the string is an independent random variable drawn from a known probability distribution. When a vehicle is commanded a deceleration in accordance with the control law, the commanded deceleration may not be realizable, because it may exceed the maximum deceleration of the vehicle. In such a case, the "effective" deceleration is the minimum of the commanded and maximum values of deceleration, and is a random variable. With a perfect coordination scheme alluded to earlier, the "effective" deceleration of all vehicles is the same, and hence, there is no variance in effective deceleration of the vehicles in a string. In this case, the benefit of coordination in this scenario is clear. Qualitatively, it is clear that if the variance of the probability distribution of "effective" braking is small, the corresponding safety quantities such as the probability of a collision, the expected number of collisions, and the expected relative velocity at impact will be small. AVFS reduces this variance in braking through the vehicle following control laws as evidenced in Ref.[92]

Another issue arises in the computation of metrics—that of the mechanics of collision. Consider a pileup where collisions between some vehicles cause collisions between other pairs of vehicles in a platoon, collisions that would not have otherwise happened. One can, therefore, think of primary and secondary collisions, with primary collisions being the antecedents and the secondary collisions being the consequents. If one is to deal with collisions, one must take into account both the primary and the secondary collisions; the occurrence of secondary collisions depends on the mechanics of the primary collisions. Since secondary collisions can be sensitive to the model of a collision, Darbha and Choi[92, 93] introduced the concept of a violation in a string to simplify the analysis of collisions as follows: a violation occurs in a platoon if one can find a successive pair of vehicles where the effective deceleration of the following vehicle is smaller than that of its predecessor. Clearly, the probability of a violation and the expected number of violations in a platoon will have a bearing on the probability (primary or secondary) and the expected number of collisions, respectively. Darbha and Choi[92] corroborated the computational benefits of using violation as a proxy for collision; they first computed the safety metrics computed via Monte Carlo simulations when vehicles employ constant spacing policy and the control law in Ref.[18]; they employed a simple model of collision involving coefficient of restitution. Darbha and Choi[92] then used a Markov Chain approach to analytically compute the probability of a violation, expected number of violations, and expected relative velocity when a violation occurs. The analytical computational results seem to correlate well with the Monte Carlo simulation results for different values of coefficients of restitution.

From the traffic safety viewpoint of CACC+ systems, the probability distribution of effective deceleration of each of the vehicles, and their relative spacing and velocity are critical.

For the purposes of illustration, consider the following discretized model of the *simplified* dynamics of the ith vehicle in a string of N vehicles employing a CACC+ system and assuming a control sample time of T seconds for every vehicle:

$$\begin{bmatrix} x_i(k+1) \\ v_i(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_i(k) \\ v_i(k) \end{bmatrix} + \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} u_i(k), \tag{11}$$

where $x_i(k), v_i(k)$, and $u_i(k)$ are, respectively, the position, velocity, and control input of the vehicle at time $t = kT$. For the Emergency Braking (EB) Scenario corresponding to a string of CACC+ vehicles, the lead vehicle brakes at its maximum deceleration, while the following vehicles obey CACC+ that employs a constant-time headway of h_w s. As long as the commanded deceleration is within the braking limit, the effective deceleration is the same as the commanded deceleration; otherwise, the vehicle brakes at its maximum deceleration (braking limit, D_i). This is reflected in the following equations where the EB is on the first vehicle:

$$\begin{aligned} u_1(k) &= -D_1, \\ u_{com,i}(k) &= -k_p[x_i(k) - x_{i-1}(k) + h_w v_i(k) + L_0] \\ &\quad - k_v[v_i(k) - v_{i-1}(k)] + k_a u_{i-1}(k), \end{aligned} \tag{12}$$

$$u_i = \max\{u_{com,i}, -D_i\}, \tag{13}$$

for all $k \geq 0$, such that $v_i(k) > 0$.

While the formulation provided in Ref.[93] allows for initial conditions to be stochastic, for the purposes of simplicity, initial spacing and velocity errors may be assumed to be zero: $v_i(0) = v_0$, $x_i(0) - x_{i-1}(0) = h_w v_0 + L_0$, where v_0, L_0 are deterministic. The fundamental problem here is to determine the statistics (mean and variance, if not the probability distribution at every sample time k) for the spacing errors, velocities, and effective deceleration of vehicles, respectively, given by $\epsilon_i(k)(:= x_i(k) - x_{i-1}(k) + L_0)$, $v_i(k)$, $u_i(k)$ for $i = 1, 2, \ldots, N$. Clearly, mean and variance indicate the spread of the spacing; the associated probability distributions can, indeed, be computed either analytically or computationally via sampling procedures. An example of the probability distribution for maximum deceleration is provided in Fig. 15. One can also subsequently calculate the probability of a violation, i.e., probability that $\epsilon_i(k) < 0$ for every k and the corresponding values of the relative velocity. This problem formulation is generic and can easily be modified to accommodate parasitic dynamics or delays in communication through changes in model given by Eq. (11) or changes in the vehicle following law given in Eqs. (12) and (13). This problem is challenging to solve due to the presence of the limits on the maximum acceleration values (Eq. 13).

An important feature of the proposed formulation is that it allows for quantifying the benefits of additional information while accounting for heterogeneity in traffic. Specifically, one can aim to understand the enhancement in safety due to the use of preceding vehicle's acceleration in a CACC (as against an ACC) system. To illustrate further, the model was implemented for two following vehicles in a three-vehicle string using a brute force approach. The variance in the distance of the two following vehicles is shown in Fig. 16. The curves with labels 'ACC1' and 'ACC2' correspond to the first and second following vehicles, each of them obeying an ACC control law; the corresponding curves for following vehicles obeying CACC are shown with labels 'CACC1' and 'CACC2'. Clearly, the plots show the value of communication in terms of reducing the variance of following distance for this simple example. A similar trend was observed for spacing errors and control inputs in larger vehicle platoons, implying that accidents, if ever they happen, will be restricted to a few vehicles at the head of the platoon and within a certain time window after the initiation of emergency braking. This plot further corroborates the safety benefit of connectivity associated with CACC system vis-a-vis ACC system.

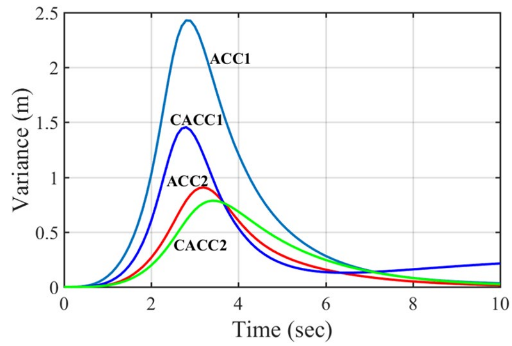

Figure 16: Variance in spacing in a two-follower string: ACC vs CACC.

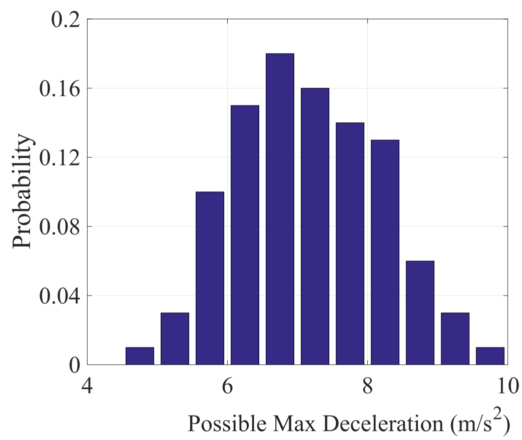

Figure 15: Probability distribution of maximum deceleration.

7 Conclusion

In this review article, we focused on central issues concerning the design of automatic vehicle following—vehicle models for AVFS, string stable control design, and its robustness to singular perturbations; we further reviewed the issues for traffic modeling and safety with AVFS. While connectivity has the potential to bring about enhancements in traffic safety and mobility, its exploitation requires a serious consideration. As can be seen from the article, the models for assessing the safety and mobility benefits of connectivity are in their infancy.

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