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A personal computer-based analysis and evaluation system for switch-mode power converters

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Abstract

This paper describes a personal computer (PC)-based system for the analysis and automated testing of switched-mode power supplies (SMPS). Such a system will prove useful in the design and testing iterations for the development of SMPS.

Key words: Switched-mode power supply, automated testing, computer-aided analysis, state-space averaging method.

1. Introduction

The availability of fast-power switching devices has opened up the application of switched-mode power supplies (SMPS) for practically all power supply applications. SMPS feature high efficiency, small size and weight and low cost. The operation of the basic switching converter circuits has been extensively covered in the literature[1]. Many modelling techniques such as circuit averaging[2], describing functions[3], state-space averaging[4], current injected modelling[5], and variable structure system[6] have been successfully applied for the analysis of SMPS. The state-space averaging method has emerged as popular among these methods and is well suited for computer-aided analysis.

This paper reviews briefly the state-space averaging method and develops a convenient formulation of the system equations suitable for computer solution. A software is developed based on this formulation, which takes input data from circuit parameters and provides the performance functions of the converter. The performance functions are the control transfer function, noise susceptibility and input and output impedance, as a function of frequency either in pole-zero form or in a more convenient Bode plot. An SMPS evaluator has been built which provides the necessary input signals such as control input modulation, switching frequency modulation, and input voltage modulation to test any SMPS. An automated measurement system consisting of a PC, a frequency-response analyser and the SMPS evaluator have been integrated for the measurement and verification of the basic

SMPS performance functions. Analysis and measurement results on a few basic converters are presented.

2. State-space averaging

SMPS circuits incorporate switches for efficient handling of power. They are mathematically time variant and discontinuous systems. Linear circuit analysis methods may not be directly applied to SMPS. State-space averaging method overcomes this problem by obtaining an equivalent linear continuous system description of the switching converter. The method is well suited for the analysis of pulse width-modulated (PWM) SMPS. We briefly review the method through the example of a Cuk converter (Fig. 1a).

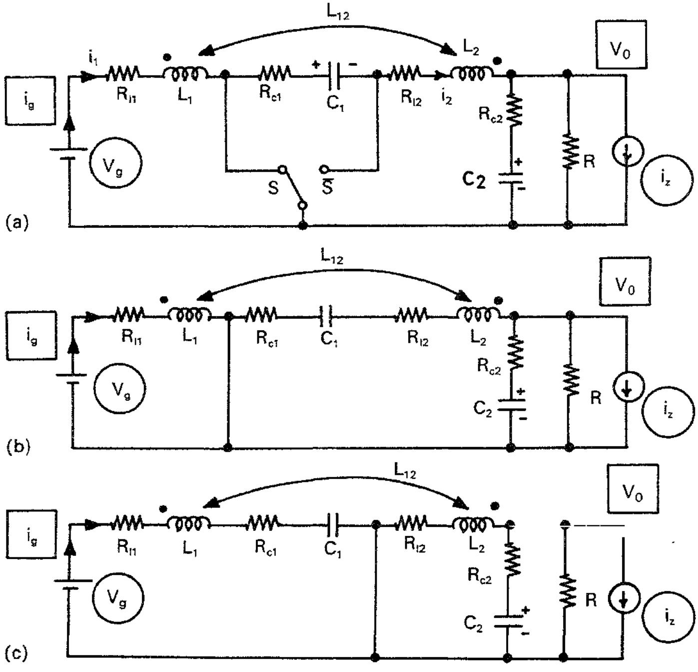


FIG. 1. Cuk converter (a), and the equivalent circuits in each of the subperiods (b and c) in a cycle.

2.1. *Circuit operation*

The above example is chosen so that it brings out the generalised form of the defining equations. The circuit of a Cuk converter is shown in Fig. 1a. Its operation is as follows. The switches S and $\bar{S}$ operate at a high-switching frequency. For a part of the switching period (dTs), S is ON. For the rest of the switching period $[(1-d)Ts]$, $\bar{S}$ is ON. When S is ON, L_1 is charged from v_g and C_1 discharges into the output circuit. When $\bar{S}$ is ON, C_1 is charging from v_g and output filter (L_2, C_2) is discharging to the output. Figures 1b and c show the circuit of the converter during the two intervals in a switching period. In operation, the converter alternates between these two topologies.

The circled quantities v_g and i_z are the input variables. The squared quantities v_o and i_g are the output variables. In standard literature the input-output pair i_z and i_g are not usually considered. The addition of these quantities in the formulation is a mathematical artifice which will enable us to derive the impedance functions of the converter later. R_{11}, R_{12}, R_{c1} and R_{c2} are the parasitic resistances of L_1, L_2, C_1 and C_2, respectively.

2.2. *State-space formulation*

The converter alternates between two linear circuits in each of the switching period (Figs 1b and c). The state equations for each of these circuits are as follows. During the ON period

$$L_1 \mathrm{d}i_1/\mathrm{d}t - L_{12}\mathrm{d}i_2/\mathrm{d}t = v_g - i_1 R_{11} \tag{1}$$

$$L_2 \mathrm{d}i_2/\mathrm{d}t - L_{21}\mathrm{d}i_1/\mathrm{d}t = -v_{c1} - i_2 R_{c1} - v_0 - i_2 R_{12} \tag{2}$$

$$C_1 \mathrm{d}v_{c1}/\mathrm{d}t = i_2 \tag{3}$$

$$C_2 \mathrm{d}v_{c2}/\mathrm{d}t = i_2 - i_z - v_0/R \tag{4}$$

$$v_0 = v_{c2}R/(R + R_{c2}) + i_2(R \| R_{c2}) - i_z(R \| R_{c2}) \tag{5}$$

$$i_g = i_1 . \tag{6}$$

The above equations may be put in the following form

$$P\dot{x} = A_1 x + b_1 v_g + m_1 i_z \tag{7}$$

$$v_0 = q_1 x + k_1 i_z \tag{8}$$

$$i_g = p_1 x \tag{9}$$

$$x = [i_1 i_2 v_{c1} v_{c2}]^T$$

$$P = \begin{bmatrix} L_1 & -L_{12} & 0 & 0 \\ -L_{12} & L_2 & 0 & 0 \\ 0 & 0 & C_1 & 0 \\ 0 & 0 & 0 & C_2 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -R_{11} & 0 & 0 & 0 \\ 0 & -R_{c1} - R_{12} - (R \| R_{c2}) & -1 & -R/(R+R_{c2}) \\ 0 & 1 & 0 & 0 \\ 0 & R/(R+R_{c2}) & 0 & -1/(R+R_{c2}) \end{bmatrix}$$

$$b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}; m_1 = \begin{bmatrix} 0 \\ R \| R_{c2} \\ 0 \\ -R/(R+R_{c2}) \end{bmatrix}$$

$$q_1 = [0 \quad R \| R_{c2} \quad 0 \quad R/(R+R_{c2})]$$

$$k_1 = -(R \| R_{c2})$$

$$P_1 = [1 \quad 0 \quad 0 \quad 0].$$

In a similar way, during the OFF period the state equation for the circuit in Fig. 1c may be put in the following form

$$P\dot{x} = A_2 x + b_2 v_g + m_2 i_z \tag{10}$$

$$v_0 = q_2 x + k_2 i_z \tag{11}$$

$$i_g = p_2 x \tag{12}$$

$$A_2 = \begin{bmatrix} -R_{11} - R_{c1} & 0 & -1 & 0 \\ 0 & -R_{12} - R \| R_{c2} & 0 & -R/(R+R_{c2}) \\ 1 & 0 & 0 & 0 \\ 0 & R/R+R_{c2} & 0 & -1/(R+R_{c2}) \end{bmatrix}$$

$$b_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad m_2 = \begin{bmatrix} 0 \\ R \| R_{c2} \\ 0 \\ -R/(R+R_{c2}) \end{bmatrix}$$

$$q_2 = [0 \quad R \| R_{c2} \quad 0 \quad R/(R+R_{c2})]$$

$$k_2 = -(R \| R_{c2})$$

$$p_2 = [1 \quad 0 \quad 0 \quad 0].$$

It is to be noted that P and A are square matrices b, m, column vectors, and p, q, row vectors and k is a scalar.

2.3. *Averaging*

The converter switches between two linear circuits in each of the switching period. Under the assumption that the state velocity ($\dot{x}$) is constant in each of the subperiods, we may approximate the state velocity with an equivalent state velocity which is the average of the state velocities in the two different subperiods. The assumption that the state velocity is constant in each interval will be valid provided the circuit time constants[3] are much higher than the switching period. The averaged system description then is given by

$$P\dot{x} = [A_1 d + A_2(1-d)]x + [b_1 d + b_2(1-d)]v_g + [m_1 d + m_2(1-d)]i_z \tag{13}$$

$$v_0 = [q_1 d + q_2(1-d)]x + [k_1 d + k_2(1-d)]i_z \tag{14}$$

$$i_g = [p_1 d + p_2(1-d)]x \tag{15}$$

where d is duty ratio. Thus the averaged system equations (13) to (15) give an approximate equivalent continuous system which may be analysed following standard state-space methods. Notice that P matrix will be a diagonal matrix in converters where there are no coupled inductors.

3. Performance functions of the converter

The system equation given by eqn (13) indicates the state velocities of the equivalent system as a function of the input variables (duty ratio d, input voltage v_g, load change i_z) and the system state x. Equations (14) and (15) give the outputs of the converter as a function of the state x, and input variables d, v_g and i_z.

3.1. *Steady-state solution*

Under steady state the converter operates with input voltage V_g, duty ratio D and load R ($i_z = 0$). The system equations reduce to a set of algebraic equations ($\dot{x} = 0$). The steady-state solution is then

$$X = -A^{-1} b V_g \tag{16}$$

$$V_0 = qX \tag{17}$$

$$I_g = pX \tag{18}$$

where

$$A = A_1 D + A_2(1-D)$$

$$b = b_1 D + b_2(1-D)$$

$$q = q_1 D + q_2(1-D)$$

$$p = p_1 D + p_2(1-D)$$

D = steady-state duty ratio.

3.2. *Dynamic performance equations*

The control of the converter is through the duty ratio d. Usually the converter is operated in closed loop such that the output voltage is maintained through controlling d against variations in v_g or load (R). To obtain various dynamic performance functions of the converter, the system input variables may be perturbed ($d = D + \hat{d}$, $v_g = V_g + \hat{v}_g$, $i_z = \hat{\imath}_z$,

$x = X + \hat{x}$), and separated into steady state and small signal parts. The small signal model thus obtained may be linearised (dropping the nonlinear product terms) to obtain the following linear small signal model of the converter.

$$P\hat{x} = A\hat{x} + b\hat{v}_g + m\hat{\imath}_z + f\hat{d} \tag{19}$$

$$\hat{v}_0 = (q_1 - q_2)X\hat{d} + q\hat{x} + k\hat{\imath}_z \tag{20}$$

$$\hat{\imath}_g = (p_1 - p_2)X\hat{d} + p\hat{x} \tag{21}$$

$$k = k_1 D + k_2(1 - D)$$

$$f = [(A_1 - A_2)X + (b_1 - b_2)V_g]$$

where ^ indicates small signal ac variables. Equation (19) may be rearranged as

$$\dot{\hat{x}} = A^*\hat{x} + b^*\hat{v}_g + m^*\hat{\imath}_z + f^*\hat{d}. \tag{22}$$

From eqns (20), (21) and (22) we may obtain the following dynamic performance equations of the converter.

a) *Audio susceptibility*
The transfer function $\hat{v}_0/\hat{v}_g(\hat{d} = 0; \hat{\imath}_z = 0)$ gives the output on account of disturbances in input voltage $\hat{v}_g$ and is defined as audio susceptibility F.

$$\text{F} = q(\text{SI} - A^*)^{-1}b^*. \tag{23}$$

b) *Input admittance*
The transfer function $\hat{\imath}_g/\hat{v}_g$ $(\hat{d} = 0; \hat{\imath}_z = 0)$, defined as the input admittance Y of the converter, is useful in determining the compatibility of the converter with the source v_g.

$$Y = p(\text{SI} - A^*)^{-1}b^*. \tag{24}$$

c) *Output impedance*
The transfer function $\hat{v}_0/\hat{\imath}_z(\hat{d} = 0;\ \hat{v}_g = 0)$ is the output impedance Z of the converter and is useful in determining the effects of dynamic load on the converter

$$Z = q(\text{SI} - A^*)^{-1}m^* + k. \tag{25}$$

d) *Control gain*
The converter is controlled through the duty ratio d and hence the most important transfer function is the control gain $G = \hat{v}_0/\hat{d}(\hat{v}_g = 0;\ \hat{\imath}_z = 0)$

$$G = (q_1 - q_2)X + q(\text{SI} - A^*)^{-1}f^*. \tag{26}$$

A knowledge of G is essential to design feedback circuits for closed loop operation. The following sections utilize the above formulations for computer-aided evaluation of these performance functions.

4. Computer-aided analysis of converters

When the system parameters are known (P, A_1, A_2, b_1, b_2, m_1, m_2, p_1, p_2, q_1, q_2, k_1, k_2, v_g, D), eqns (16) to (18) and (23) to (26) may be used to evaluate the steady state and dynamic performance of the converter. The flow chart of the program is given in Fig. 2. The ON and OFF period system matrices are input to the program. The program evaluates the steady-state solution at first. Then the dynamic performance functions are evaluated as a function of frequency. The transfer functions thus evaluated are available either in normalized pole-zero form or in a more convenient Bode plot[7].

5. SMPS evaluator

To measure the performance functions of the converter, an SMPS evaluator was built (Fig. 3). The evaluator provides a voltage source v_g (to be used as input power for the converter under test) and a set of drive signals to be used to drive the switches in the converter under test. The steady-state switching frequency (F_s) and the duty ratio (D) may be set anywhere in the range of 20 to 100 kHz and 0·2 to 0·8, respectively. The steady-state source voltage (V_g) may be set in the range of 15 to 20 volts. Further, sinusoidal modulations may be superimposed using a signal generator on V_g, F_s, D at any frequency.

6. Computer-aided measurement of performance functions

The measurement of the dynamic performance function is done using a Schlumberger 1250 frequency response analyser (FRA, Fig. 4). FRA has one output channel, which is a signal generator whose amplitude and frequency are controllable (0–15 V; 0–64 kHz), and two input channels. To measure any dynamic performance function (say $\hat{v}_0/\hat{v}_g$), the generator output is connected to the appropriate input ($\hat{v}_g$) of the SMPS evaluator. The evaluator outputs v_g and drive signals are connected to the converter under test. Appropriate measurement signals from the converter under test (v_0 and v_g) are connected to the channels 2 and 1 of the FRA, respectively. On starting the test, FRA carries out the test and displays the measured function in magnitude and phase format. By repeating the test at different generator frequencies, the transfer function as a function of frequency may be generated. Since the testing involves a number of settings and repetition at different frequencies, it is well suited for automation[8] (Fig. 5). A resident program in the PC does the instrument settings and collects the test data. The results are displayed in the Bode plot format so that a fast comparison with theoretical prediction may be made.

7. Experimental results

To test the effectiveness of the system, a few simple converters were built (Figs 6a, b and c). The performance functions of control gain G, audio susceptibility F and output impedance Z of these converters were plotted following analysis and then by actual testing using the

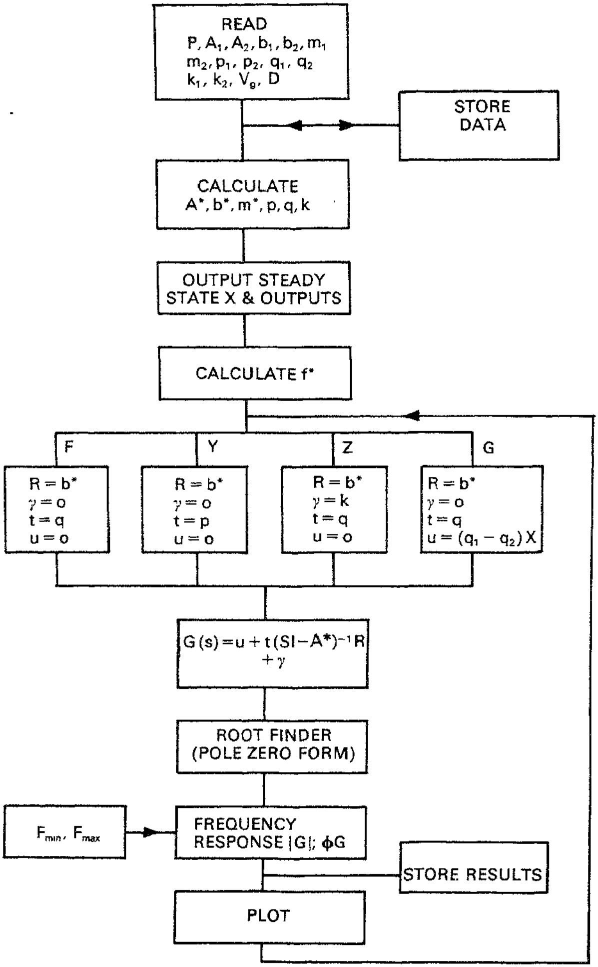


FIG. 2. Flow chart for computer-aided analysis of converters.

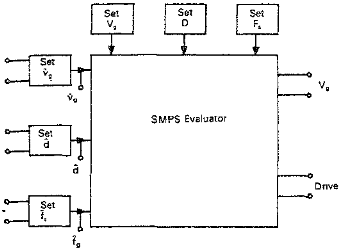


FIG. 3. Block diagram of SMPS evaluator.

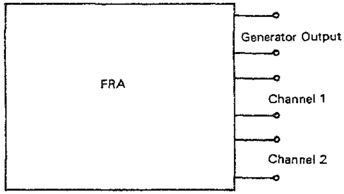
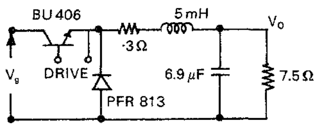
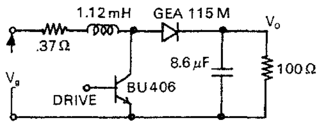


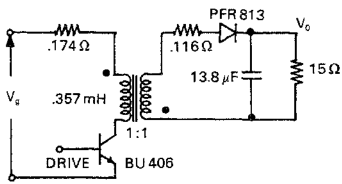
FIG. 4. Block diagram of frequency response analyser.



(a) Buck converter.



(b) Boost converter.



(c) Isolated flyback converter.

FIG. 6. Examples of switching converters tested with the analysis and measurement system.

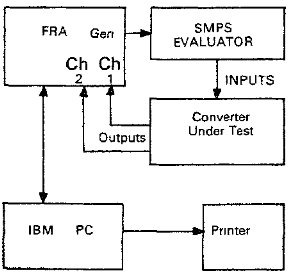


FIG. 5. Test set up for computer-aided measurement of performance functions of switching converters.

above system. The results are shown in Figs 7–9. It may be seen that the matching between analysis and measurement is very good. The discrepancy found in the output impedance functions at low frequency end requires study. It has been reported that the dynamic

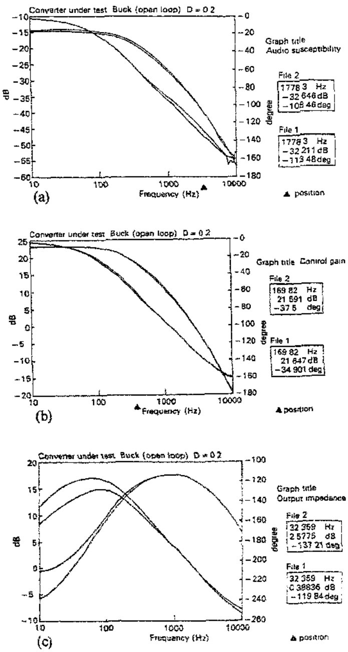


FIG. 7. Performance functions of buck converter.

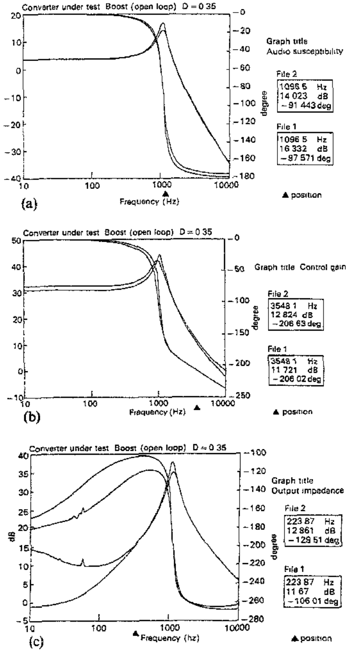
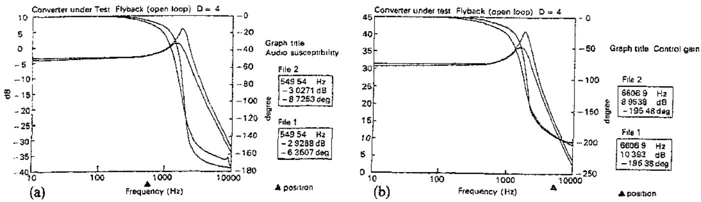


FIG. 8. Performance functions of the boost converter.

nonideality (storage delay time) of the transistor switch can explain this behaviour[9]. We intend to report on this in the future.

8. Conclusion

A PC-based analysis and evaluation system for switching converters has been developed and tested on simple converters for its effectiveness. Such a facility will be of help to analyse the effect of different nonidealities in switching converters, design and study of suitable closed-loop compensation and to study scaled-down models of larger power converters.



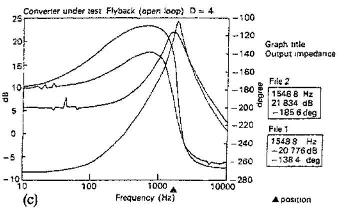


FIG. 9. Performance functions of the flyback converter.

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